Solutions to JEE(Main) -2021

Test Date: 20^{th} July 2021 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from $0°C$ to $50°C$ when no work is done is ___________. (R is the universal gas constant)

(A) 175 R
(B) 750 R
(C) 250 R
(D) 500 R

Q2. A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic field. Let r_d and r_d be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is equal to :

(A) $\sqrt{2}$
(B) 2
(C) $\frac{1}{\sqrt{2}}$
(D) 1

Q3. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}\cdot\vec{B} = \left|\vec{A}\times\vec{B}\right|$. Then the value of $\left|\vec{A}-\vec{B}\right|$ will be :

(A) $\sqrt{A^2+B^2-\sqrt{2}AB}$
(B) $\sqrt{A^2+B^2}$
(C) $\sqrt{A^2+B^2+2AB}$
(D) $\sqrt{A^2+B^2+\sqrt{2}AB}$

Q4. Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

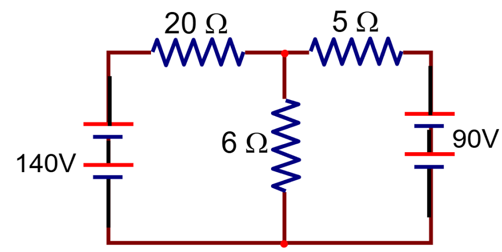
(A) $\frac{1}{\upsilon_A} > \frac{1}{\upsilon_B} > \frac{1}{\upsilon_C}$
(B) $\upsilon_A = \upsilon_B = \upsilon_C = 0$
(C) $\frac{1}{\upsilon_A} < \frac{1}{\upsilon_B} < \frac{1}{\upsilon_C}$
(D) $\upsilon_A = \upsilon_B \neq \upsilon_C$

Q5. A nucleus of mass M emits $\gamma-$ray photon of frequency 'v'. The loss of internal energy by the nucleus is : [Take 'c' as the speed of electromagnetic wave]

(A) $hv\left[1+\frac{hv}{2Mc^2}\right]$
(B) 0
(C) $hv\left[1-\frac{hv}{2Mc^2}\right]$
(D) hv

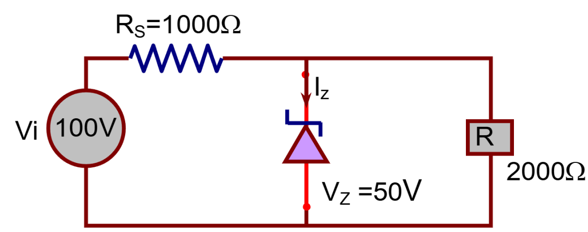
Q6. A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains? (Take ln 3 = 1.1)
(A) 340 years
(B) 1110 years
(C) 700 years
(D) 740 years

Q7. The value of current in the $6\,\Omega$ resistance is :
(A) 6 A
(B) 10 A
(C) 4 A
(D) 8 A

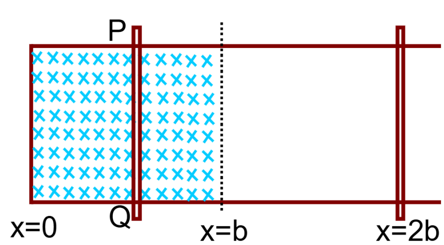


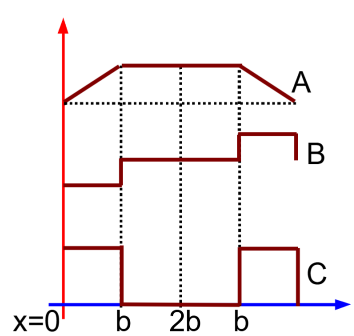
Q8. The radiation corresponding to $3 \rightarrow 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular circular path followed by these electrons is 7mm, the work function of the metal is : (Mass of electron = 9.1×10^{-31} kg)
(A) 0.82 eV
(B) 1.88 eV
(C) 1.36 eV
(D) 0.16 eV

Q9. For the circuit shown below, calculate the value of I_z :
(A) 0.15 A
(B) 0.1 A
(C) 25 mA
(D) 0.05 A



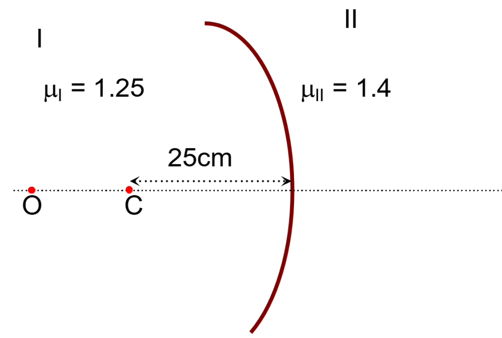
Q10. The arm PQ of a rectangular conductor is moving from $x = 0$ to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph the variation of different quantities with distance.





(A) A – Flux, B – EMF, C – Power dissipated
(B) A – EMF, B – Power dissipated, C – Flux
(C) A – Power dissipated, B – Flux, C – EMF
(D) A – Flux, B – Power dissipated, C – EMF

Q11. Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is :



(A) 9.52 cm
(B) 18.23 cm
(C) 37.58 cm
(D) 55.44 cm

Q12. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is __________ $\times 10^3\ kg\ m/s^2$.
[Given $\cos 30^\circ = 0.87, \mu_5 = 0.2$]

(A) 6.96
(B) 10.2
(C) 12.4
(D) 7.2

Q13. The entropy of any system is given by $S = \alpha^2 \beta\, \ell n\left[\frac{\mu k R}{J\beta^2} + 3\right]$ Where α and β are the constants. μ, J, k and R no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively . [Take $S = \frac{dQ}{T}$]
Choose the **incorrect** option from the following :

(A) S, β, k and μR have the same dimensions.
(B) α and k have the same dimensions.
(C) S and α have different dimensions.
(D) α and J have the same dimensions.

Q14. A current of 5 A is passing through a non-linear magnesium wire of cross – section $0.04\,m^2$. At every point the direction of current density is at an angle of 60° with unit vector of area of cross -section. The magnitude of electric field at every point of the conductor is : (Resistivity of magnesium $\rho = 44 \times 10^{-8}\,\Omega m$)

(A) $11 \times 10^{-7}\ V/m$
(B) $11 \times 10^{-2}\ V/m$
(C) $11 \times 10^{-5}\ V/m$
(D) $11 \times 10^{-3}\ V/m$

Q15. The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are ℓ_1 and ℓ_2 respectively. The actual length of the metal wire is :

(A) $\frac{T_1 \ell_2 - T_2 \ell_1}{T_1 - T_2}$
(B) $\sqrt{T_1 T_2 \ell_1 \ell_2}$
(C) $\frac{\ell_1 + \ell_2}{2}$
(D) $\frac{T_1 \ell_1 - T_2 \ell_2}{T_1 - T_2}$

Q16. AC voltage $V(t) = 20\sin\omega t$ of frequency $50\,Hz$ is applied to a parallel plate capacitor. The separation between the plates is 2mm and the area is $1\,m^2$. The amplitude of the oscillating displacement current for the applied Ac voltage is __________.
[Take $\varepsilon_0 = 8.85 \times 10^{-12}\,F/m$]

(A) $83.37\,\mu A$ (B) $55.58\,\mu A$
(C) $21.14\,\mu A$ (D) $27.79\,\mu A$

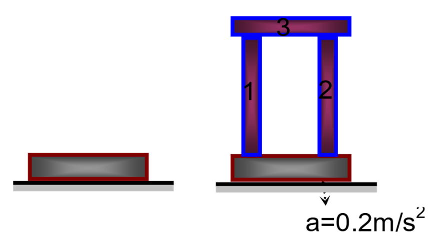
Q17. A butterfly is flying with a velocity $4\sqrt{2}\,m/s$ in North – East direction. Wind is slowly blowing at $1\,m/s$ from North to South. The resultant displacement of the butterfly in 3 seconds is :

(A) $15\,m$ (B) 20m
(C) 3 m (D) $12\sqrt{2}\,m$

Q18. A certain charge Q is divided into parts q and $(Q-q)$. How should the charges Q and q be divided so that q and (Q – q) placed at a certain distance apart experience maximum electrostatic repulsion ?

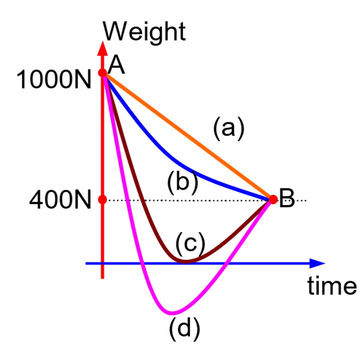
(A) $Q = \frac{q}{2}$ (B) $Q = 4q$
(C) $Q = 2q$ (D) $Q = 3q$

Q19. A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. The normal reaction R' by the floor if mass of the iron cylinders are equal and of $20\,kg$ each, is _________N.
[Take $g = 10\,m/s^2$ and $\mu_s = 0.2$]



(A) 714 (B) 684
(C) 716 (D) 686

Q20. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other object in sky and take acceleration due to gravity on the surface of the Earth and mars as $10\,m/s^2$ and $4\,m/s^2$ respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.



(A) (c) (B) (a)
(C) (b) (D) (d)

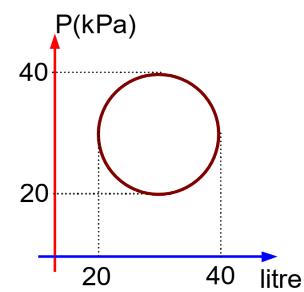
SECTION - B

(Numerical Answer Type)

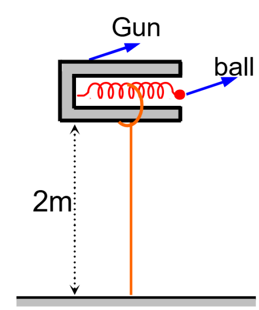
This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The amplitude of wave disturbance propagating in the positive x-direction is given by $y=\frac{1}{1+(x)^2}$ at time $t=0$ and $y=\frac{1}{1+(x-2)^2}$ at $t=1\,s$, where x and y are in metres. The shape of wave does not change during the propagation. The velocity of the wave will be ________ m /s.

Q2. In the reported figure, heat energy absorbed by a system in going through a cyclic process is __________ πJ.

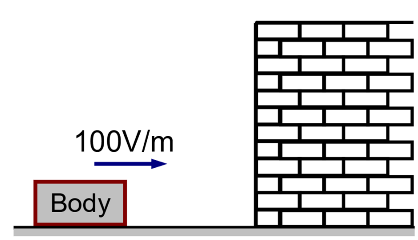


Q3. In a spring gun having spring constant $100\,N/m$ a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground . The value of d is ____________m.



Q4. The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m /s. Then the speed of car is __________km /h.

Q5. A body having specific charge $8\,\mu C/g$ is resting on a frictionless plane at a distance 10cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be ___________s.



Q6. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be ________.

Q7. An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image, A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to ________cm.

Q8. In an LCR series circuit, an inductor 30 mH and a resistor $1\,\Omega$ are connected to an Ac source of angular frequency 300 rad /s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x}\times10^{-3}\,F$. Then the value of x is ____________.

Q9. A circular disc reaches from top to bottom of an inclined plane of length 'L' when it slips down the plane, it takes time $'t_1'$. When it rolls down the plane, it takes time t_2. The value of $\frac{t_1}{t_2}$ is $\sqrt{\frac{3}{x}}$ The value of x will be ___________.

Q10. A carrier wave $V_C(t) = 160\sin(2\pi\times10^6 t)$ volts is made to very between $V_{max} = 200\,V$ and $V_{min} = 120\,V$ by a message signal $V_m(t) = A_m\sin(2\pi\times10^3 t)$ volts. The peak voltage A_m of the modulating single is ________V.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

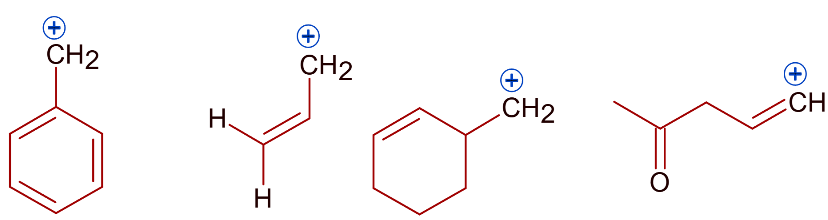
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and P_4O_{10} (in 4:1 ratio) is:
(A) basic
(B) neutral
(C) amphoteric
(D) acidic

Q2. Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are:
(A) Primary amine, nitrile compound, conc. HCl
(B) Primary amine, isonitrile compound , conc. HCl
(C) Secondary amine, isonitrile compound, conc NaOH
(D) Secondary amine, nitrile compound, conc. NaOH

Q3. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: The dihedral angles in H_2O_2 in gaseous phase is 90.2^o and in solid phase is 111.5^o.
Reason R: The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.
Choose the most appropriate answer from the options given below for **A** and **R**.
(A) **A** is correct but **R** is not correct.
(B) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.
(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(D) **A** is not correct but **R** is correct.

Q4.



(A) (B) (C) (D)

Among the given species the Resonance stabilized carbocations are:
(A) (A), (B) and (C) only
(B) (C) and (D) only
(C) (A), (B) and (D) only
(D) (A) and (B) only

Q5. A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2. The oxide is pale yellow in colour and paramagnetic. The element (M) is:
(A) Na
(B) K
(C) Mg
(D) Ca

Q6. Green chemistry in day- to day life is in the use of:
(A) Chlorine for bleaching of paper
(B) Large amount of water alone for washing clothes
(C) Liquified CO_2 for dry cleaning of clothes
(D) Tetrachloroethene for laundry

Q7. Identify the incorrect statement from the following:
(A) β-Glycosidic linkage makes cellulose polymer
(B) Amylose is a branched chain polymer of glucose
(C) Starch is a polymer of α- D glucose
(D) Glycogen is called as animal starch

Q8. The metal that can be purified economically by fractional distillation method is:
(A) Ni (B) Cu
(C) Fe (D) Zn

Q9. The set in which compounds have different nature is:
(A) $B(OH)_3$ and $Al(OH)_3$ (B) $B(OH)_3$ and H_3PO_3
(C) NaOH and $Ca(OH)_2$ (D) $Be(OH)_2$ and $Al(OH)_3$

Q10. The conditions given below are in the context of observing Tyndall effect in colloidal solutions:
(a) The diameter of the colloidal particles is comparable to the wavelength of light used.
(b) The diameter of the colloidal particles in much smaller than the wavelength of light used.
(c) The diameter of the colloidal particles is much larger than the wavelength of light used.
(d) The refractive indices of the dispersed phase and the dispersion medium are comparable.
(e) The dispersed phase has a very different refractive index from the dispersion medium.
Choose the most appropriate conditions from the options given below.
(A) (a) and (d) only (B) (a) and (e) only
(C) (c) and (d) only (D) (b) and (e) only

Q11. According to the valence bond theory the hybridization of central metal atom is dsp^2 for which one of the following compounds?
(A) $K_2[Ni(CN)_4]$ (B) $NiCl_2.6H_2O$
(C) $Na_2[NiCl_4]$ (D) $[Ni(CO)_4]$

Q12. In the given reaction 3-Bromo-2,2-dimethyl butane $\xrightarrow{C_2H_5OH}$ $\underset{\text{(Major Product)}}{\text{'A'}}$

Product A is:
(A) 2-Ethoxy-2,3-dimethyl butane (B) 2-Hydroxy-3,3-dimethyl butane.
(C) 2-Ethoxy-3,3-dimethyl butane (D) 1-Ethoxy-3,3-dimethyl butane.

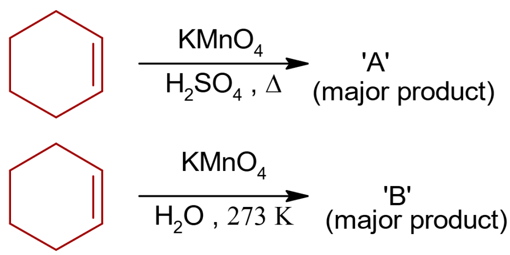
Q13. Orlon fibres are made up of:
(A) Polyamide (B) Polyacrylonitrile
(C) Cellulose (D) Polyesters

Q14. The species given below that does NOT show disproportionation reaction is:

(A) BrO^- (B) BrO_3^-

(C) BrO_2^- (D) BrO_4^-

Q15.



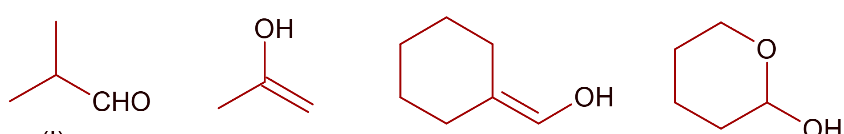
For above chemical reactions, identify the correct statement from the following

(A) Both compound ‘A’ and compound ‘B’ are dicarboxylic acids.
(B) Compound ‘A’ is dicarboxylic acid and compound ‘B’ is diol.
(C) Compound ‘A’ is diol and compound ‘B’ is dicarboxylic acid.
(D) Both compound ‘A’ and compound ‘B’ are diols.

Q16. An inorganic compound ‘X’ on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated H_2SO_4. Also compound ‘X’ gives precipitate ‘Y’, when its solution in dilute HCl is treated with H_2S gas. The precipitate ‘Y’ on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound ‘X’ is:

(A) $Cu(NO_3)_2$ (B) $Pb(NO_3)_2$
(C) $Co(NO_3)_2$ (D) $Pb(NO_2)_2$

Q17.



(I) (II) (III) (IV)

Which among the above compound/s does / do not form silver mirror when treated with Tollen’s reagent?

(A) Only (IV) (B) (III) and (IV) only
(C) Only (II) (D) (I), (III) and (IV) only

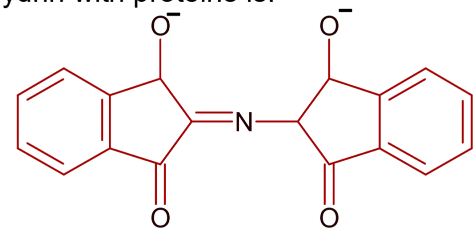
Q18. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

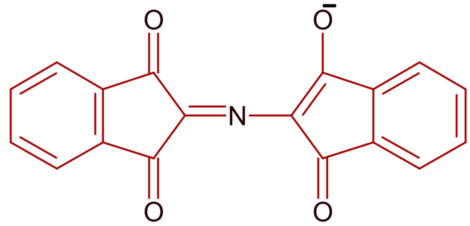
Assertion A: Sharp glass edge becomes smooth on heating it upto its melting point.
Reason R: The viscosity of glass decreases on melting.

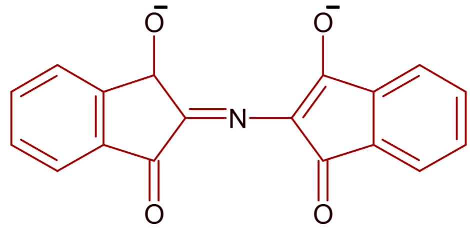
Choose the most appropriate answer from the options given below.

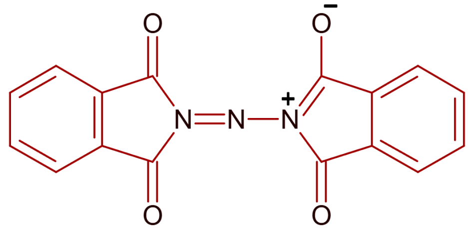
(A) **A** is false but **R** is true.
(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.
(C) **A** is true but **R** is false
(D) Both **A** and **R** are true and **R** is the correct explanation of **A**.

Q19. The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is:

(A) 

(B) 

(C) 

(D) 

Q20. The correct order of intensity of colors of the compounds is:

(A) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

(B) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(C) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

(D) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The number of lone pairs of electrons on the central I atom in I_3^- is___________.

Q2. An average person needs abut 10000 KJ energy per day.The amount of glucose (molar mass = 180.0 g mol^{-1}) needed to meet this energy requirement is_________ g, (Nearest integer)
(Use : $\Delta_c H$ (glucose) = – 2700kJ mol^{-1})

Q3. To synthesise 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate _________ equivalents of CH_3MgBr reagent will be required. (Integer value)

Q4. The spin only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is______ BM.
[At. No. of Co= 27]

Q5. At 20°C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20°C above an equimolar mixture of benzene and methyl benzene is ____________ $\times 10^{-2}$. (Nearest integer)

Q6. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$
In an equilibrium mixture, the partial pressures are
$P_{SO_3} = 43 kPa$; $P_{O_2} = 530$ Pa and
$P_{SO_2} = 45 kPa$. The equilibrium constant K_p = _____________ $\times 10^{-2}$ (Nearest integer)

Q7. 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is__________ $\times 10^{21}$ (Nearest integer)
$(N_A = 6.022 \times 10^{23})$

Q8. The number of nitrogen atoms in a semicarbazone molecule of acetone is__________.

Q9. The inactivation rate of a viral preparation in proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is ___________$\times 10^{-3} min^{-1}$. (Nearest integer)
[Use: $\ell n10$ = 2.303; $\log_{10} 3$=0.477; property of logarithm: $\log x^y = y \log x$]

Q10. The Azimuthal quantum number for the valence electrons of Ga^+ ion is____________.
(Atomic number of Ga= 31)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15, x \in R$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15, x \in R$ has a :

(A) local minimum at $x = \frac{3}{4}$
(B) local maximum at $x = \frac{3}{4}$
(C) local minimum at $x = -\frac{3}{4}$
(D) local maximum at $x = -\frac{3}{4}$

Q2. Let $A = [a_{ij}]$ be a 3 x 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i - j| = 1 \\ 2x + 1 & , \text{ otherwise} \end{cases}$$

Let a function $f : R \to R$ be defined as $f(x) = \det(A)$. Then the sum of maximum and minimum values of f on R is equal to :

(A) $-\frac{88}{27}$
(B) $\frac{20}{27}$
(C) $\frac{88}{27}$
(D) $-\frac{20}{27}$

Q3. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(w) = \frac{3\pi}{2}$, then $\arg\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

(A) $\frac{\pi}{4}$
(B) $-\frac{3\pi}{4}$
(C) $-\frac{\pi}{4}$
(D) $\frac{3\pi}{4}$

Q4. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

(A) $6 + 8\sqrt{3}$
(B) $8 + 2\sqrt{2}$
(C) $10 + 6\sqrt{2}$
(D) $4 + 2\sqrt{3}$

Q5. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

(A) $\frac{1}{9}$ (B) $\frac{1}{11}$

(C) $\frac{2}{11}$ (D) $\frac{1}{66}$

Q6. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is :

(A) $-{}^{100}C_{15}$ (B) ${}^{100}C_{16}$

(C) $-{}^{100}C_{16}$ (D) ${}^{100}C_{15}$

Q7. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$ is :

(A) $\frac{2}{9}$ (B) $\frac{1}{6}$

(C) $\frac{7}{36}$ (D) $\frac{1}{4}$

Q8. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(A) $q \Rightarrow p$ (B) $\sim q \Rightarrow p$

(C) $p \Rightarrow q$ (D) $p \Rightarrow \sim q$

Q9. Let a be a positive real numbers such that

$\int_0^a e^{x-[x]}dx = 10e - 9$

where [x] is the greatest integer less than or equal to x. Then a is equal to :

(A) $10 + \log_e 2$ (B) $10 - \log_e(1+e)$

(C) $10 + \log_e(1+e)$ (D) $10 + \log_e 3$

Q10. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If $\det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to :

(A) 18 (B) 36

(C) 24 (D) 45

Q11. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to :

(A) 56×3^{25} (B) 28×3^{25}

(C) 52×3^{24} (D) 56×3^{24}

Q12. The number of real roots of the equation

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is :

(A) 4
(B) 1
(C) 0
(D) 2

Q13. The value of the integral $\int_{-1}^{1}\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is equal to :

(A) $2\log_e 2+\frac{\pi}{4}-1$
(B) $\log_e 2+\frac{\pi}{2}-1$
(C) $\frac{1}{2}\log_e 2+\frac{\pi}{4}-\frac{3}{2}$
(D) $2\log_e 2+\frac{\pi}{2}-\frac{1}{2}$

Q14. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are :

(A) 10, 11
(B) 3, 18
(C) 1, 20
(D) 8, 13

Q15. Let the tangent to the parabola $S: y^2=2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to :

(A) $\frac{35}{2}$
(B) $\frac{15}{2}$
(C) 25
(D) $\frac{25}{2}$

Q16. Let a function $f: R \rightarrow R$ be defined as

$$f(x)=\begin{cases}\sin x-e^x & \text{if} \quad x\le 0\\ a+[-x] & \text{if} \quad 0<x<1\\ 2x-b & \text{if} \quad x\ge 1\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous on R, then $(a+b)$ is equal to :

(A) 2
(B) 5
(C) 3
(D) 4

Q17. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is :

(A) 3
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) 4

Q18. Let $y = y(x)$ be the solution of the differential equation

$$e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0,\ y(1) = -1$$

Then the value of $\left(y(3)\right)^2$ is equal to :

(A) $1+4e^6$ (B) $1-4e^3$
(C) $1+4e^3$ (D) $1-4e^6$

Q19. Let [x] denote the greatest integer $\le x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a)\cup[b, c)\cup[4, \infty)$, $a < b < c$, then the value of $a+b+c$ is:

(A) -3 (B) 8
(C) -2 (D) 1

Q20. Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,\ -1 \le x \le 1,\ y\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is :

(A) $\frac{1}{6}(\pi-1)$ (B) $\frac{1}{8}(\pi-1)$
(C) $\frac{1}{4}(\pi-2)$ (D) $\frac{1}{12}(\pi-3)$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^{7} + 2I$, where I is an identity matrix of order 3 x 3.

If $B = [b_{ij}]$, then b_{13} is equal to.........

Q2. Let $y = mx + c, m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x+10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to ....

Q3. Let a, b, c, d be in arithmetic progression with common difference λ.

If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$, then the value of λ^2 is equal to.........

Q4. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to.......

Q5. Let T be the tangent to the ellipse $E: x^2 + 4y^2 = 5$ at the point $P(1,1)$. If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to.....

Q6. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is ....

Q7. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is.....

Q8. Let P be a plane passing through the points $(1,0,1)$ $(1,-2,1)$ and $(0,1,-2)$. Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}.(\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals.......

Q9. If the value of $\lim_{x \to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to.......

Q10. If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in R$, $\alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in R$ is 9, then α is equal to.......

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**D**	2.	**A**	3.	**A**	4.	**C**
5.	**A**	6.	**D**	7.	**B**	8.	**A**
9.	**C**	10.	**A**	11.	**C**	12.	**B**
13.	**B**	14.	**C**	15.	**A**	16.	**D**
17.	**A**	18.	**C**	19.	**D**	20.	**A**

SECTION - B

1.	**2**	2.	**100**	3.	**1**	4.	**132**
5.	**1**	6.	**4**	7.	**25**	8.	**3**
9.	**2**	10.	**40**				

PART – B (CHEMISTRY)

SECTION - A

1.	**D**	2.	**B**	3.	**D**	4.	**D**
5.	**B**	6.	**C**	7.	**B**	8.	**D**
9.	**A**	10.	**B**	11.	**A**	12.	**A**
13.	**B**	14.	**D**	15.	**B**	16.	**A**
17.	**C**	18.	**B**	19.	**B**	20.	**A**

SECTION - B

1.	**3**	2.	**667**	3.	**2**	4.	**2**
5.	**78**	6.	**172**	7.	**226**	8.	**3**
9.	**106**	10.	**0**				

PART – C (MATHEMATICS)

SECTION - A

1.	**D**	2.	**A**	3.	**B**	4.	**A**
5.	**B**	6.	**D**	7.	**A**	8.	**C**
9.	**A**	10.	**B**	11.	**C**	12.	**C**
13.	**B**	14.	**A**	15.	**D**	16.	**C**
17.	**C**	18.	**D**	19.	**C**	20.	**B**

SECTION – B

1.	**910**	2.	**34**	3.	**1**	4.	**4**
5.	**1.25 (As per NTA 1)**			6.	**777**	7.	**21**
8.	**81**	9.	**3**	10.	**6**		

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. Since process is isochoric, so

$$Q = nC_V\Delta T = n\left(\frac{f}{2}R\right)\Delta T = 4\left(\frac{5}{2}\times R\right)(50) = 500R$$

Sol2. As we know that radius of circular path in magnetic field is given as

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}, \text{ so}$$

Charged particle	Charge	Mass
Deuteron	e	2m
Alpha particle	2e	4m

$$\frac{r_d}{r_\alpha} = \sqrt{\frac{m_d}{m_\alpha}}\times\frac{q_\alpha}{q_d} = \frac{1}{\sqrt{2}}\times 2 = \sqrt{2}$$

Sol3. $\vec{A}\cdot\vec{B} = \left|\vec{A}\times\vec{B}\right| \Rightarrow AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45^0$

$$\left|\vec{A}-\vec{B}\right| = \sqrt{A^2+B^2-2AB\cos\theta} = \sqrt{A^2+B^2-2AB\times\frac{1}{\sqrt{2}}} = \sqrt{A^2+B^2-\sqrt{2}AB}$$

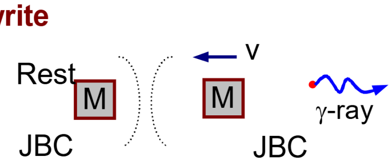
Sol4. As we know that root mean square speed is given as

$$v = \sqrt{\frac{3RT}{M}}, \text{ so}$$

$$v_A > v_B > v_C \Rightarrow \frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C} \quad \text{as} \quad m_A < m_B < m_C.$$

Sol5. Using conservation linear momentum, we can write

$$Mv = \frac{h}{\lambda} = \frac{h\nu}{c} \Rightarrow v = \frac{h\nu}{Mc}$$



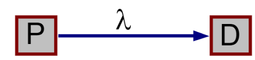
Loss of internal energy= Gain in kinetic energy+ energy of gamma ray

$$= \frac{1}{2}Mv^2 + h\nu = \frac{1}{2}M\left(\frac{h\nu}{Mc}\right)^2 + h\nu = h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$$

Difficulty Level: Difficult and Very good Question

Sol6. As we know that equivalent half life of radioactive material decays by simultaneous emissions, is given as

$$\frac{1}{T}=\frac{1}{T_1}+\frac{1}{T_2}=\frac{1}{700}+\frac{1}{1400}=\frac{3}{1400}\Rightarrow T=\frac{1400}{3}\text{ years}$$



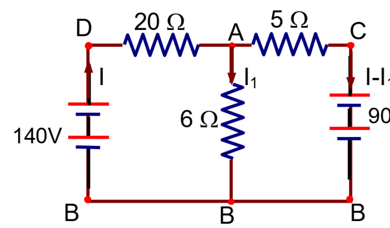
t = 0 N_0 0

t = t N

$$N=N_0e^{-\lambda t}\Rightarrow\frac{N_0}{3}=N_0e^{-\lambda t}\Rightarrow\frac{1}{3}=e^{-\lambda t}\Rightarrow\lambda t=\ell n(3)$$

$$\Rightarrow t=\frac{\ell n(3)}{\lambda}=\frac{\ell n(3)}{\ell n(2)}T=\frac{1.1}{0.693}\times\frac{1400}{3}\approx 740.74\text{ years}$$

Sol7. Method-I: Using Kirchhoff's Law for loop $\overrightarrow{BDABB}$, we can write



$-140+20I+6I_1=0\Rightarrow 10I+3I_1=70$ ....(i)

Using Kirchhoff's Law for loop $\overrightarrow{ACBBA}$, we can write

$5(I-I_1)+90-6I_1=0$

$\Rightarrow -5I+11I_1=90$ ....(ii)

Adding equation (1) with twice the equation (2), we have

$$25I_1=250\Rightarrow I_1=\frac{250}{25}=10\,A$$

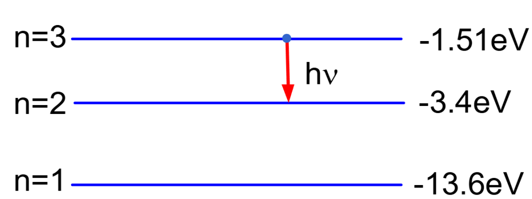
Method-II: Using concept of equivalent cell, we can write

$$V_{AB}=\frac{\frac{140}{20}+\frac{0}{6}+\frac{90}{5}}{\frac{1}{20}+\frac{1}{6}+\frac{1}{5}}=\frac{2100+0+5400}{15+50+60}$$

$$=\frac{7500}{125}=60\text{ Volt}$$

$$\Rightarrow I_1=\frac{60}{6}=10\,A$$

Sol8. $h\nu=3.4-1.51=1.89$ eV



As we know that radius of circular path in magnetic field is given as

$$r=\frac{\sqrt{2mK}}{qB}\Rightarrow K=\frac{r^2q^2B^2}{2m}$$

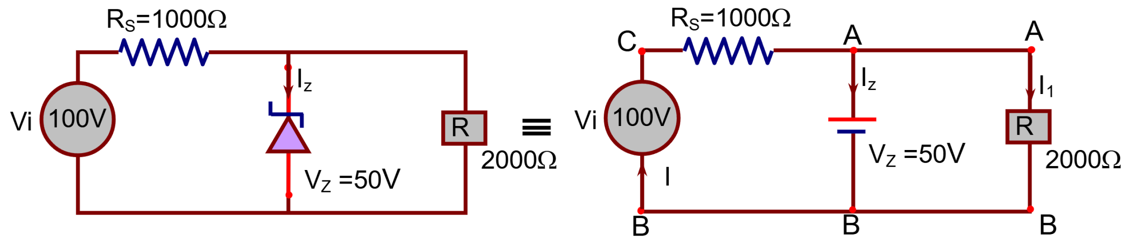
$$\Rightarrow K=\frac{(7\times10^{-3})^2\times(1.6\times10^{-19})\times(5\times10^{-4})^2}{2\times9.1\times10^{-31}}\text{eV}=107.7\times10^{-2}\text{eV}=1.08\text{ eV}$$

$$\Rightarrow\phi=h\nu-K=1.89-1.08=0.81\text{eV}$$

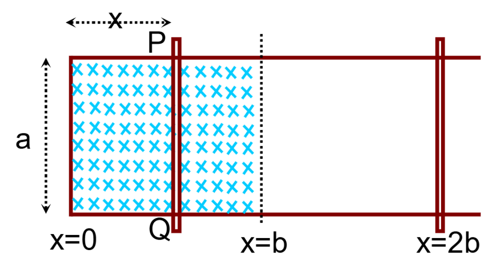
Difficulty Level: Difficult and Very good Question

Sol9. $I_1 = \dfrac{50}{2000} = 25\,\text{mA}$, and $I = \dfrac{V_i - V_z}{1000} = \dfrac{100-50}{1000} = 50\,\text{mA}$

$\Rightarrow I_z = I - I_1 = 25\,\text{mA}$



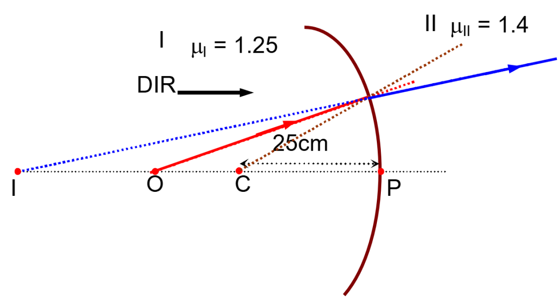
Sol10. When arm PQ of a rectangular conductor is moving from $x = 0$ to x = b, the flux(ϕ =ax) linked with loop increases and while moving from $x = b$ to x = 2b and from $x = 2b$ to x = b, flux (ϕ =ab) remains constant and then flux (ϕ =ax) deceases to zero as it moves from $x = b$ to x = 0 .



$e_{in} = -\dfrac{d\phi}{dt} = -a\dfrac{dx}{dt} = -av \Rightarrow$ Induced emf

$P = \dfrac{(e_{in})^2}{R} \Rightarrow$ Power dissipated

Sol11. **Since direction of incident ray (DIR) is from left to write , so considering refraction at point M , we can write**



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4 - 1.25}{-25}$$

$$\Rightarrow -\frac{1.4}{v} = \frac{0.15}{25} + \frac{1.25}{40} = \frac{1.2 + 6.25}{200} = \frac{7.45}{200}$$

$$\Rightarrow v = -\frac{200 \times 1.4}{7.45} = -37.58\,\text{cm}$$

Sol12. In the frame of vehicle, vehicle is in equilibrium under the influence of pseudo force F_P

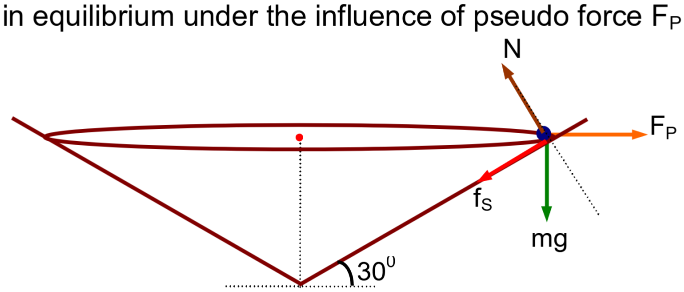
$$F_P = \frac{mv^2}{R}$$

$$N = mg\cos 30^0 + F_P \sin 30^0$$

$$\Rightarrow N = mg\cos 30^0 + \frac{mv^2}{R}\sin 30^0 \;.(i)$$

, and

$$f_S = \frac{mv^2}{R}\cos 30^0 - mg\sin 30^0$$



FBD in the Frame of vehicle

$\Rightarrow \mu N = \frac{mv^2}{R}\cos 30^0 - mg\sin 30^0 \ldots .(ii)$

By doing $(1)\times\cos 30^0 - (2)\times\sin 30^0$, we have

$N\left(\cos 30^0 - \mu\sin 30^0\right) = mg \Rightarrow N = \frac{mg}{\cos 30^0 - \mu\sin 30^0}$

$\Rightarrow N = \frac{800\times 10}{0.87 - 0.2\times 0.5} = \frac{800\times 10}{0.77} = 10.2\times 10^3\ kgm/s^2$

Difficulty Level: Difficult and Very good Question

Sol13.

Physical Quantity	Formula	Dimension
Entropy (S)	$S = \frac{dQ}{T}$	$\left[ML^2T^{-2}K^{-1}\right]$
Number of mole (μ)		**mol**
Mechanical equivalent of heat (J)	$J = \frac{W}{Q}$	**Dimensionless**
Boltzmann constant (k)	$k = \frac{Energy}{Temperature}$	$\left[ML^2T^{-2}K^{-1}\right]$
Gas constant (R)	$R = kN_a$	$\left[ML^2T^{-2}K^{-1}\right]$

$N_a \Rightarrow$ **Avogadro's Number**

$J\beta^2 \equiv [\mu kR] \Rightarrow \beta^2 \equiv [kR] \Rightarrow \beta \equiv [k] \equiv \left[ML^2T^{-2}K^{-1}\right]$, and

$S \equiv \alpha^2\beta \Rightarrow \left[ML^2T^{-2}K^{-1}\right] \equiv \alpha^2\left[ML^2T^{-2}K^{-1}\right] \Rightarrow \alpha \equiv \left[M^0L^2T^0K^0\right] \Rightarrow$ **Dimensionless**

Sol14. $I = JA\cos\theta \Rightarrow 5 = J\times 0.04\times\cos 60^0 \Rightarrow J = \frac{5}{0.02} = 250\ A/m^2$

$J = \frac{E}{\rho} \Rightarrow E = \rho J = 44\times 10^{-8}\times 250 = 11\times 10^{-5} V/m$

Sol15. Assuming Tension in metal wire suspended from roof varies linearly and 0 and T_0 (developed due its own weight) are tensions are ends of of wire of length ℓ. So

$T_1\ \alpha\ (\ell_1 - \ell)$, and $T_2\ \alpha\ (\ell_2 - \ell)$

$\frac{T_1}{T_2} = \frac{\ell_1 - \ell}{\ell_2 - \ell} \Rightarrow \ell = \frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$

Sol16. $C = \frac{\varepsilon_0 A}{d} \Rightarrow X_C = \frac{1}{C\omega} = \frac{d}{\varepsilon_0 A\omega} \Rightarrow I_0 = \frac{V_0}{X_C} = \frac{V_0\varepsilon_0 A\omega}{d} = \frac{2\pi f V_0\varepsilon_0 A}{d}$

$\Rightarrow I_0 = \frac{2\times 3.14\times 50\times 20\times 8.85\times 10^{-12}\times 1}{2\times 10^{-3}} = 27.79\times 10^{-6} A = 27.79\mu A$

Sol17. $\vec{v}_{BW} = 4\sqrt{2}\left(\cos 45^0\,\hat{i} + \sin 45^0\,\hat{j}\right) = 4\hat{i} + 4\hat{j}$, and $\vec{v}_{Wg} = 0\hat{i} - \hat{j}$

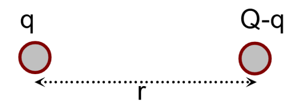
$\vec{v}_{Bg} = \vec{v}_{BW} + \vec{v}_{Wg} = \left(4\hat{i} + 4\hat{j}\right) + \left(0\hat{i} - \hat{j}\right) = 4\hat{i} + 3\hat{j}$

$\left|\vec{v}_{Bg}\right| = 5\ m/s \Rightarrow$ Speed of Butterfly

Magnitude of displacement of Butterfly= $\left|\vec{v}_{Bg}\right|\times t = 5\times 3 = 15\ m$

Sol18. Here r is fixed

$$F=\frac{q(Q-q)}{4\pi\varepsilon_0 r^2}\Rightarrow\frac{dF}{dq}=\frac{1}{4\pi\varepsilon_0 r^2}(Q-2q)\Rightarrow\frac{dF}{dq}=-\frac{1}{2\pi\varepsilon_0 r^2}$$



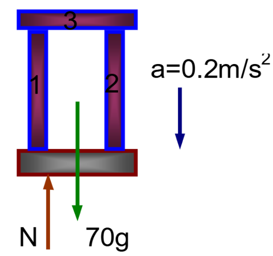
For maxima or minima of force , its first derivative should be zero.

$$\frac{dF}{dq}=\frac{1}{4\pi\varepsilon_0 r^2}(Q-2q)=0\Rightarrow q=\frac{Q}{2}$$

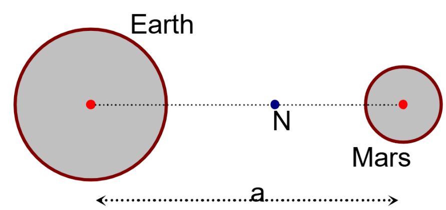
Since Second Derivative is always negative so maxima will occur at this value of q.

Sol19. $70g-N=70a=70\times0.2=14$

$\Rightarrow N=700-14=686N$



Sol20. $W_1=1000N\Rightarrow$ Weight of person at surface of earth $W_2=400N\Rightarrow$ Weight of person at surface of Mars. There will be a neutral point –N where weight will be zero because at this point net gravitational field due to earth and mars is zero.



SECTION - B

Sol1. The equation of wave at any time t will be $y=\frac{1}{1+(x-vt)^2}$, so $v\times1=2\Rightarrow v=2m/s$

Sol2. Heat absorbed in cyclic process = Work done = 100 π Joule

Sol3. Let ball starts its motion with horizontal velocity v_0 , so with the help of conservation of mechanical energy , we can write

$$\frac{1}{2}mv_0^2=\frac{1}{2}mx^2\Rightarrow v_0=x\sqrt{\frac{k}{m}}=0.05\times\sqrt{\frac{100}{0.1}}=0.5\times\sqrt{10}\,m/s$$

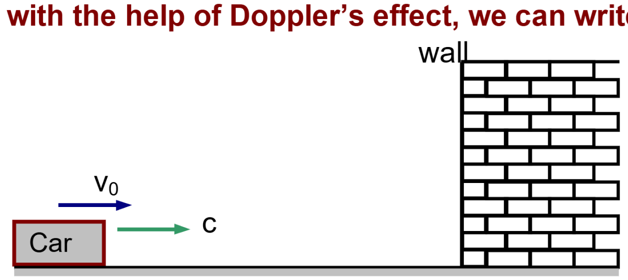
t = Time required to fall the ball $=\sqrt{\frac{2h}{g}}=2\times\sqrt{\frac{1}{10}}\,s$

$$\Rightarrow d=v_0\times t=0.5\times\sqrt{10}\times2\times\sqrt{\frac{1}{10}}=1\,m$$

Sol4. Initially wall will act as observer, so **with the help of Doppler's effect, we can write**

$$f'=\frac{c}{c-v_0}f_0$$

Now wall will act as source of sound of frequency f', so **With the help of Doppler's effect, we can write**



$$f'' = \frac{c+v_0}{c} f' = \frac{c+v_0}{c-v_0} f_0$$

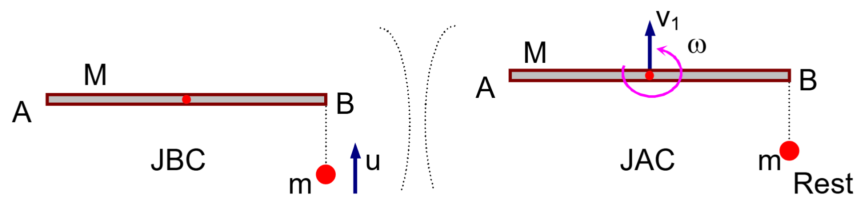
$$\Rightarrow 500 = \left(\frac{c+v_0}{c-v_0}\right) 400 \Rightarrow 5c - 5v_0 = 4c + 4v_0 \Rightarrow v_0 = \frac{c}{9} = \frac{330}{9} m/s = \frac{330}{9} \times \frac{18}{5} km/h = 132\ km/h$$

Sol5. $F = qE \Rightarrow$ Electric force acting on the body

$$a = \frac{F}{m} = \frac{q}{m}E = \frac{8\times 10^{-6}}{10^{-3}} \times 100 = 0.80\ m/s^2 \Rightarrow \text{Acceleration of body}$$

$$T = 2\sqrt{\frac{2\ell}{a}} = 2\sqrt{\frac{2\times 0.1}{0.8}} = 1\,s \Rightarrow \text{Time period of Motion}$$

Sol6. **Using conservation of linear momentum, we can write**



$$mu = Mv_1 \Rightarrow v_1 = \frac{mu}{M} \quad \ldots\ldots\ldots\ldots(1)$$

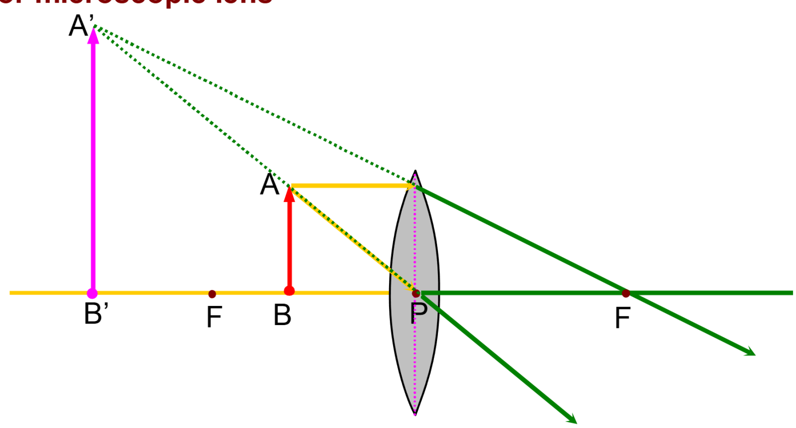
Using conservation of angular momentum about centre of mass of Rod, we can write

$$\frac{mu\ell}{2} = \frac{M\ell^2\omega}{12} + \frac{mv_2\ell}{2} \Rightarrow 6mu = M\ell\omega \Rightarrow \ell\omega = \frac{6mu}{M} \quad \ldots\ldots\ldots\ldots(2)$$

Using definition of e, we can write

$$v_1 + \frac{\ell\omega}{2} = u \Rightarrow \frac{mu}{M} + \frac{3mu}{M} = u \Rightarrow \frac{m}{M} = \frac{1}{4}$$

Sol7. **For microscopic lens**

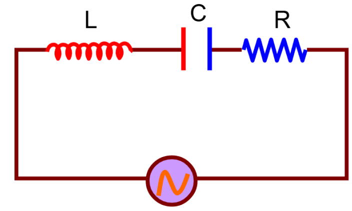


$$m = 1 + \frac{D}{f} \Rightarrow 6 = 1 + \frac{25}{f} \Rightarrow f = 5\,cm$$

Now in second case, this lens acts as objective of compound microscope, so according to question

$$m = m_O \times m_e = \frac{LD}{f_o f_e} \Rightarrow f_e = \frac{LD}{mf_o} = \frac{60\times 25}{12\times 5} = 25\ cm$$

Sol8. Here $\phi = 45^0$ and current leads voltage , so $X_C > X_L$



$$\tan\phi = \frac{X}{R} = \tan 45^0 = 1 \Rightarrow X = X_C - X_L = R$$

$$\Rightarrow X_C = R + L\omega \Rightarrow X_C = 1 + 30\times10^{-3}\times300 = 10$$

$$\Rightarrow \frac{1}{C\omega} = 10 \Rightarrow C = \frac{1}{10\times\omega} = \frac{1}{10\times300} = \frac{1}{3}\times10^{-3}\text{ F}$$

Sol9. **Case-I:** when disk slides down

$$t_1 = \sqrt{\frac{2L}{g\sin\theta}} \text{ ....(i)}$$

Case-II: when disk rolls down

$$t_2 = \sqrt{\frac{\frac{2L}{g\sin\theta}}{1+\beta^2}} = \sqrt{\frac{\frac{2L}{g\sin\theta}}{1+\left(\frac{k}{R}\right)^2}} = \sqrt{\frac{\frac{2L}{g\sin\theta}}{1+\left(\frac{R/\sqrt{2}}{R}\right)^2}} = \sqrt{\frac{\frac{2L}{g\sin\theta}}{1+\frac{1}{2}}} = \sqrt{\frac{4L}{3g\sin\theta}} \text{ .... (ii)}$$

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{2L}{g\sin\theta}}\times\sqrt{\frac{3g\sin\theta}{4L}} = \sqrt{\frac{3}{2}}$$

Sol10. $\mu = \frac{A_m}{A_c} = \frac{V_1 - V_2}{V_1 + V_2} = \frac{200-120}{200+120} = \frac{1}{4} \Rightarrow$ Modulation index

Here $V_C(t) = 160\sin\left(2\pi\times10^6 t\right)$ V and $V_m(t) = A_m\sin\left(2\pi\times10^3 t\right)$ V , so

$$A_m = \frac{A_c}{4} = \frac{160}{4} = 40\text{ V}$$

PART – B (CHEMISTRY)

SECTION - A

Sol1. Anhydride of HNO_3 produced by the treatment of P_4O_{10} (anhydring reagent) in ratio of 4:1 is N_2O_5 which is acidic in nature. $4HNO_3 + P_4O_{10} \longrightarrow \underbrace{2N_2O_5}_{\text{Acidic nature}} + (HPO_3)_4$

Sol2. 1° – Amine produced isocyanide (iso nitrile) compound with $CHCl_3$ and KOH

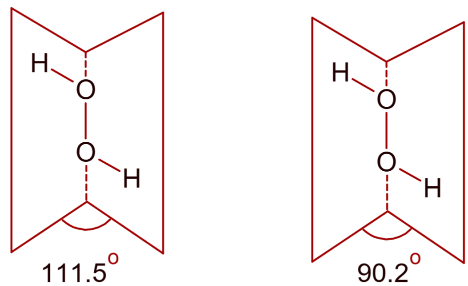
$$\underset{(A)}{R-NH_2} + CHCl_3 + 3KOH \longrightarrow \underset{(B)}{RNC} + 3KCl + 3H_2O$$

Isonitrile (toxic nature)

$\downarrow$ H_3O^+ / HCl (C)

$\underset{(A)}{R-NH_2}$ + HCOOH

Sol3. Following are the dihedral angles of H_2O_2 in gas phase and solid (Reverse with respect to question).



111.5° gas phase   90.2° Solid phase

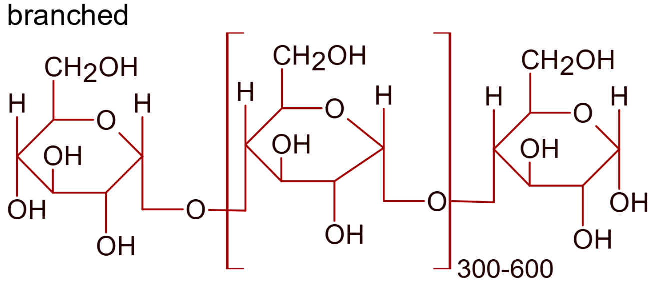
Change of angle is the changing of intermolecular interaction in different phase

Sol4. Only (A) and (B) structures have resonance effect. In (A) benzyllic resonance and in (B) allylic resonance of carbocation.

Sol5. An alkali metal oxide of formula MO_2 is super oxide which is formed by K and Rb. KO_2 has pale yellow colour and paramagnetic nature, while alkaline earth metal oxides formula MO_2 are peroxide and diamagnetic nature.

Sol6. Nowdays liquid CO_2 used as washing clothes, because Cl_2 and $C_2H_2Cl_4$ are toxic

Sol7. Amylose is polysaccharide made of α-D- glucose unit of linear chain polymer not branched



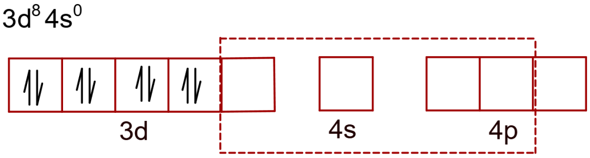
Sol8. Due to volatile nature , Zn – metal can be economically purified by fractional distillation refining process.

Sol9. $B(OH)_3$ is orthoboric acid having acidic nature, while $Al(OH)_3$ has amphoteric nature that is acidic and as well as basic

Sol10. Tyndall effect is the optical characteristics of light in which comparable particle size of colloids and bombarded light wavelength can produce this type of effect. For scattering effect of light in Tyndall effect refractive index of dispersed phase and dispersion medium should be greatly different.

Sol11. $K_2\left[Ni(CN)_4\right]$

$Ni^{2+} \rightarrow 3d^8 4s^0$

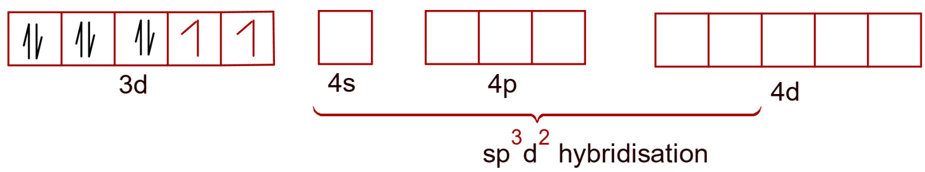


dsp^2 hybridisation

Pairing developed due to strong ligand effect.

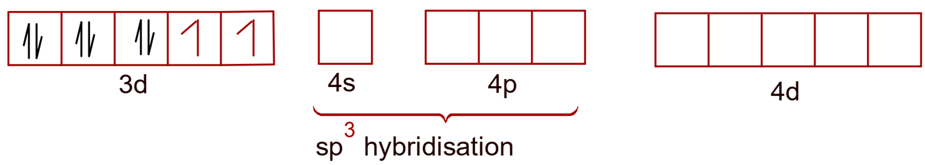
$NiCl_2.6H_2O$

$Ni^{2+} \rightarrow 3d^8 4s^0$



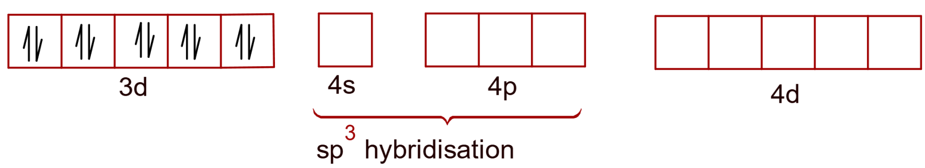
$Na_2[NiCl_4]$

$Ni^{2+} \rightarrow 3d^8 4s^0$



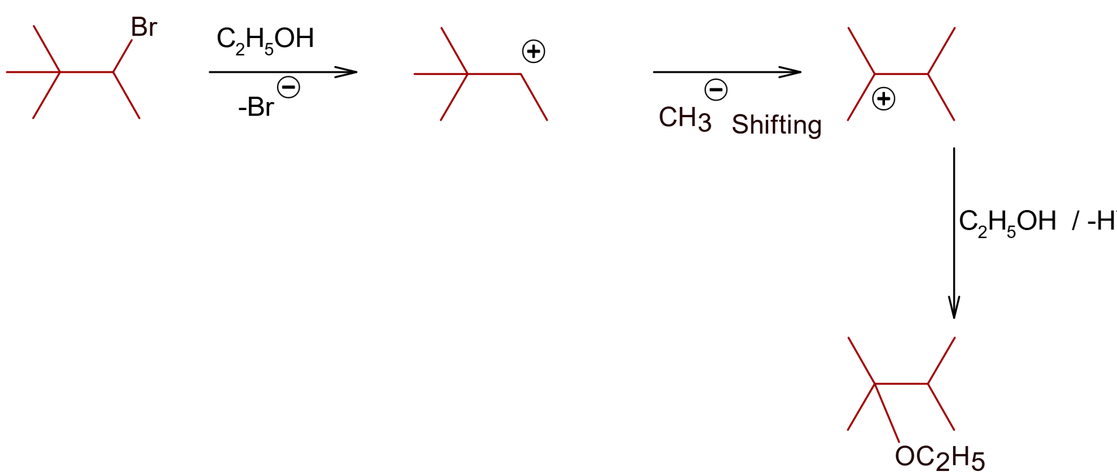
$[Ni(CO)_4]$

$Ni \rightarrow 3d^8 4s^2$



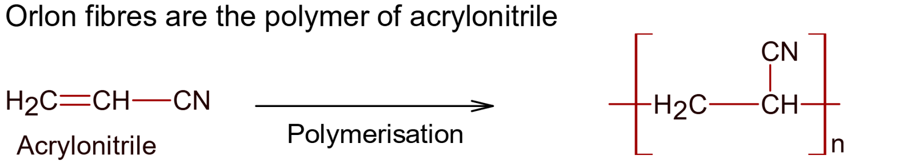
Pairing developed due to strong ligand (CO).

Sol12.



(2- Ethoxy-2,3- dimethyl butane)

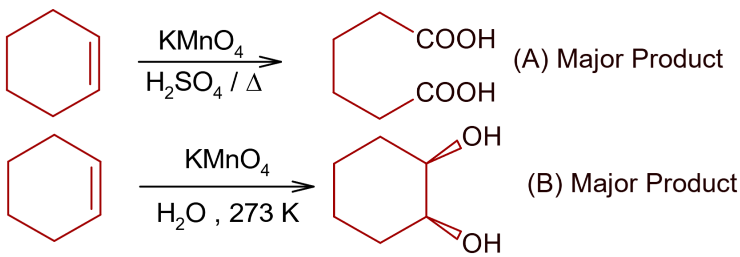
Sol13. Orlon fibres are the polymer of acrylonitrile



Polyacrylonitrile (PAN)

Sol14. In BrO_4^-, Br has +7 O.S, therefore it only acts as a oxidizing reagent , does not undergo disproportionation reaction

Sol15.

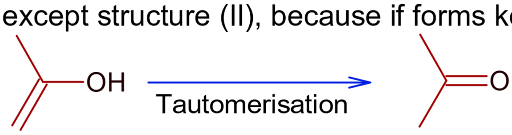


Sol16. $X \xrightarrow[H_2SO_4]{Conc}$ Brown fumes + Brown ring test with $FeSO_4$ / H_2SO_4

$X \xrightarrow[HCl]{H_2S} Y(ppt) \xrightarrow[HNO_3]{Conc}$ dissolved $\xrightarrow[NH_4OH]{}$ Deep blue colour solution

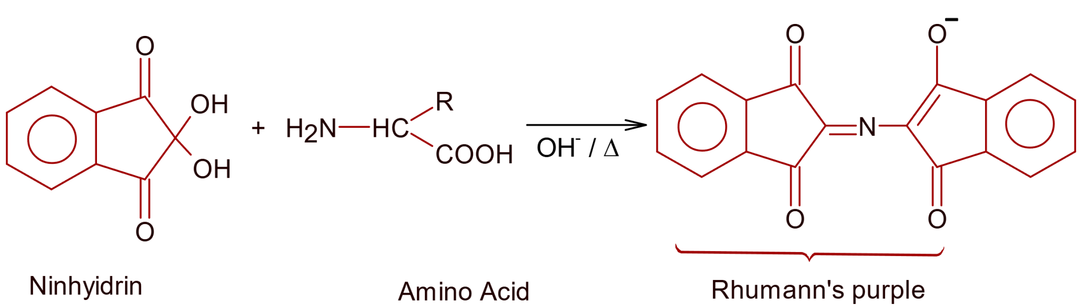
In cation analysis of Cu^{++} ions, precipitate formed is CuS on treating with H_2S and HCl which dissolved in HNO_3 and produced blue colour complex solution $\left[Cu(NH_3)_4\right]^{++}$ in NH_4OH. Brown fumes and brown ring performance given by nitrate sample. Hence salt is $Cu(NO_3)_2$.

Sol17. All structure can produce –CHO functional group which gives +ve test of Tollen's reagent except structure (II), because if forms ketonic group after tautomerisation



Sol18. Both statement are correct for the glass body heating , but reason is not correct explanation during heating process of glass, constituents unit rupture of glass body and gives the edge smoothness.

Sol19.



Sol20. For Ni^{2+}, crystal field CFSE magnitude shows the magnitude of absorbed light.

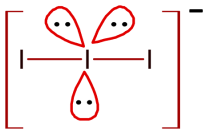
Following are the energy absorbed order $[NiCl_4]^{2-} < \left[Ni(H_2O)\right]^{2+} < \left[Ni(CN)_4\right]^{2-}$

Hence order of colour of compounds are.

$[NiCl_4]^{2-} > \left[Ni(H_2O)_6\right]^{2+} > \left[Ni(CN)_4\right]^{2-}$

SECTION - B

Sol1. I_3^- anion has three lone pair



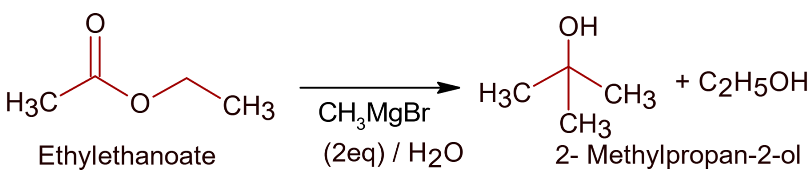
Sol2. 2700 kJ energy released from 180gm (1 mole) of glucose

$\therefore$ 1kJ energy released from $\frac{180}{2700}$ gm of glucose

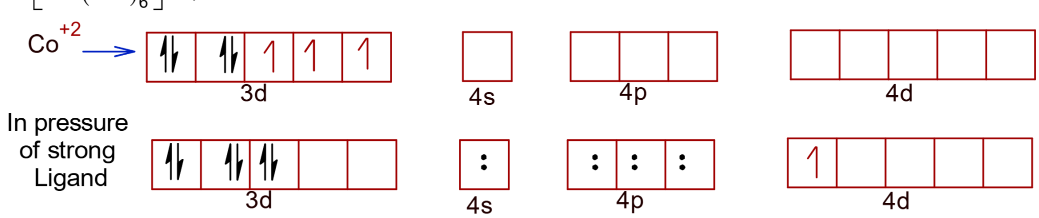
$\therefore$ 10000 kJ energy released from $\left(\frac{180 \times 10000}{2700}\right)$ gm of glucose

Amount of glucose = $\frac{18000}{27} = 666.66 \approx 667 gm$

Sol3.



Sol4. In $\left[Co(CN)_6\right]^{4-}$, Co has +2 oxidation state



Hence, electronic configuration of Co^{2+} is $[Ar]3d^7 4s^0$. In complex $\left[Co(CN)_6\right]^{4-}$, given ligand CN^- is strong hence, after pairing in d- subshell, total number of unpaired electron = 1

$\therefore$ spin magnetic moment $= \sqrt{1(1+2)} = \sqrt{3}$ = 1.73 BM

= 1.73BM $\approx$ 2.0 BM

Sol5. P_B^o = vapour pressure of benzene at $20^\circ C = 70 torr$

P_M^o = vapour pressure of toluene at $20^\circ C = 20 torr$

Mixture is equimolar, $X_B = 0.5$ and $X_M = 0.5$

Total vapour pressure $(P_T) = 70 \times 0.5 + 20 \times 0.5 = 45 torr$

Mole fraction of benzene in vapour phase $(X_B') = \frac{P_B^o X_B}{P_T} = \left(\frac{70 \times 0.5}{45}\right) torr$

$= 0.777 = 77.7 \times 10^{-2} torr$

Ans = 78 (nearest integer)

Sol6. $K_p = \dfrac{\left(P_{SO_3}\right)^2}{\left(P_{SO_2}\right)^2 \times \left(P_{O_2}\right)} = \dfrac{(43)^2}{(45)^2 \times (0.53)} = 1.72(kPa)^{-1}$

$k_P = 172 \times 10^{-2}(kPa)^{-1}$

Sol7.

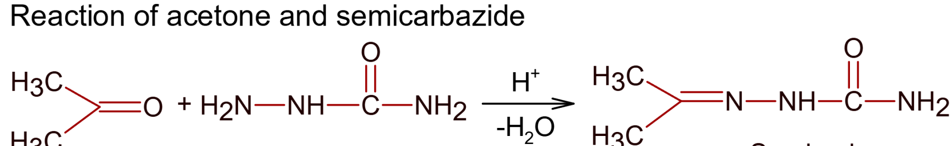
		NaOH +	HCl	$\longrightarrow$ NaCl +	H_2O
(Milimole)	t = 0	250×0.5	500×1.0	0	0
	$t = \infty$	–	375	125	125 millimole

unreacted HCl mole = $\dfrac{375}{1000}$

number of HCl molecule un reacted $= \dfrac{375}{1000} \times 6.023 \times 10^{23}$

$= 225.75 \times 10^{21} \approx 226 \times 10^{21}$

Sol8. Reaction of acetone and semicarbazide



Semicarbazide Semicarbazone

Sol9. Kt = 2.303 log $\dfrac{[A_0]}{[A_t]}$

Here, $[A_0] = 100$, $[A_t] = 90$ and t = 1 min

$\therefore K \times 1 = 2.303 \log_{10} \dfrac{100}{90}$

$\therefore K = 2.303 \times [\log 10 - \log 9] = 2.303[1 - 2\log 3]$

$\therefore K = 2.303[1 - 2 \times 0.477] = 0.105938 = 105.938 \times 10^{-3}\ \text{min}^{-1}$

Rounded = 106

Sol10. Electronic configuration of Ga^+ ion = [Ar] $3d^{10}\ 4s^2\ 4p^o$

Last electron goes into S- orbital, hence

Azimuthal quantum number (ℓ) for last electron = 0

PART – C (MATHEMATICS)

SECTION - A

Sol1. $f(x) = ax^2 + 6x - 15$

$f'(x) = 2ax + 6 = 0 \Rightarrow x = \dfrac{-3}{a} = \dfrac{3}{4} \Rightarrow a = -4$

$g(x) = -4x^2 - 6x + 15$

$g'(x) = 0 \Rightarrow -8x - 6 = 0 \Rightarrow x = -\dfrac{3}{4}$

$g''(x) = -8 < 0 \Rightarrow x = \dfrac{-3}{4}$ is a point of local maxima for g(x).

Sol2. $\det(A)=\begin{vmatrix}1 & -x & 2x+1\\ -x & 1 & -x\\ 2x+1 & -x & 1\end{vmatrix}$

$=1(1-x^2)-x(-2x^2-x+x)+(2x+1)(x^2-2x-1)$

$=1-x^2+2x^3+2x^3+x^2-4x^2-2x-2x-1$

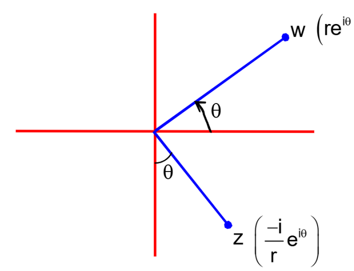
$f(x)=4x^3-4x^2-4x$

$f'(x)=4(3x^2-2x-1)=0\Rightarrow x=1,\frac{-1}{3}$

$f(1)+f\left(-\frac{1}{3}\right)=-4+\left(\frac{-4}{27}-\frac{4}{9}+\frac{4}{3}\right)$

$=-4+\frac{-4-12+36}{27}=-4+\frac{20}{27}=\frac{-88}{27}$

Sol3. Let $w=r.e^{i\theta}$



$\arg(w)=\theta$

$\therefore z=\frac{1}{r}(\sin\theta-i\cos\theta)$

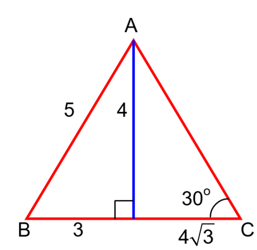
$\bar{z}w=\frac{i}{r}e^{-i\theta}.re^{i\theta}=i$

$\arg\left(\frac{1-2\bar{z}w}{1+3\bar{z}w}\right)=\arg\left(\frac{1-2i}{1+3i}\right)$

$\frac{1-2i}{1+3i}=\frac{(1-2i)(1-3i)}{1+9}=\frac{-5-5i}{10}=-\frac{1}{2}(1+i)$

$\arg\left(-\frac{1}{2}(1+i)\right)=\frac{-3\pi}{4}$

Sol4. $\cos B=\frac{3}{5}, R=5$



$\frac{5}{\sin C}=10\Rightarrow C=30^\circ$

$\Delta=\frac{1}{2}(4)(3+4\sqrt{3})=6+8\sqrt{3}$

Sol5. EXAMINATION, required probability

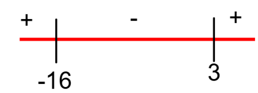
$=\frac{10!/2!\,2!\,2!}{11!/2!\,2!\,2!}=\frac{1}{11}$

Sol6. Coefficient of x^{256} in $\left\{(1-x^3)^{100}-x(1-x^3)^{100}\right\}$

= -Coefficient of x^{255} in $(1-x^3)^{100}$

$=-(-1)^{85}.\,{}^{100}C_{85}={}^{100}C_{15}$

Sol7. D<0



$$\Rightarrow 4\left[(a+4)^2-(64-5a)\right]<0$$

$$\Rightarrow a^2+13a-48<0$$

$$\Rightarrow (a+16)(a-3)<0$$

$$\Rightarrow -16<a<3$$

but $-5 \le a \le 30$

$$\Rightarrow -5 \le a < 3$$

$$\Rightarrow a=-5,-4,.....,1,2 \text{ (integers)}$$

Probability $=\frac{8}{36}=\frac{2}{9}$

Sol8. $(P\wedge\sim q)\rightarrow(q\vee\sim p)$

$$\equiv\sim(P\wedge\sim q)\vee(P\vee\sim p)$$

$$\equiv\sim p\vee q$$

$$\equiv p\rightarrow q$$

Sol9. $\int_0^a e^{\{x\}}dx=10(e-9)$

$$=\int_0^{[a]} e^{\{x\}}dx+\int_{[a]}^{a} e^{x}dx$$

$$=[a]\int_0^1 e^{x}dx+\int_0^{\{a\}} e^{[x]}dx$$

$$=[a](e-1)+e^{\{a\}}-1$$

$$=[a]e+e^{\{a\}}-([a]+1)$$

For $[a]=10$ & $e^{\{a\}}-11=-9$

i.e. $e^{\{a\}}=2$

$$a=10+\ln 2$$

Sol10. $P^T=P, Q^T=-Q$

$$A=\frac{(A+A^T)+(A-A^T)}{2}$$

$$P=\frac{A+A^T}{2}$$

$$=\frac{1}{2}\left(\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}+\begin{bmatrix}2 & a\\ 3 & 0\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}4 & 3+a\\ a+3 & 0\end{bmatrix}=\begin{bmatrix}2 & \frac{a+3}{2}\\ \frac{a+3}{2} & 0\end{bmatrix}$$

$$Q=\frac{A-A^T}{2}=\frac{1}{2}\begin{bmatrix}0 & 3-a\\ a-3 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{3-a}{2}\\ \frac{a-3}{2} & 0\end{bmatrix}$$

$$\det(Q) = 9 \Rightarrow \frac{(a-3)^2}{4} = 9 \Rightarrow a-3 = \pm 6 \Rightarrow a = 9, -3$$

For a = 9

$$\det(P) = \begin{vmatrix} 2 & 6 \\ 6 & 0 \end{vmatrix} = -36$$

For a = -3

$$\det(P) = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

Sol11. $x^2 + 3^{\frac{1}{4}}x + 3^{\frac{1}{2}} = 0$

Put $3^{\frac{1}{4}} = a$

Equation : $x^2 + ax + a^2 = 0$

$$\Rightarrow \left(\frac{x}{a}\right)^2 + \frac{x}{a} + 1 = \Rightarrow x = aw, aw^2$$

$\alpha = aw, \beta = aw^2$

$\alpha^{96} = a^{96}\ w^{96} = 3^{24}$

$\alpha^{12} - 1 = a^{12}\ w^{12} - 1 = 3^3 - 1 = 26$

$\alpha^{96}\left(\alpha^{12}-1\right) + \beta^{96}\left(\beta^{12}-1\right) = 52 \times 3^{24}$

Sol12. Domain of definition

$= \{-1, 0\}$

For $x = 0$ LHS = $\frac{\pi}{2}$

$x = -1$ LHS = $\frac{\pi}{2}$

No solution.

Sol13. $I = 2\int_0^1 \ln\left(\sqrt{1-x} + \sqrt{1+x}\right)dx$

Put $x = \cos\theta$

then $I = 2\int_{\frac{\pi}{2}}^{0} \ln\left(\sqrt{2}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)\right).\left(-\sin\theta\right)d\theta$

$$= 2\ln\sqrt{2} + 2\left[\left[\ln\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right).\left(-\cos\theta\right)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\sin\frac{\theta}{2} + \cos\frac{\theta}{2}}.\frac{1}{2}\cos\theta\ d\theta\right]$$

$$I = 2\ln\sqrt{2} + \int_0^{\frac{\pi}{2}} \left(1 - \sin\theta\right)d\theta$$

$$= \ln 2 + \left[\theta + \cos\theta\right]_0^{\frac{\pi}{2}}$$

$$= \ln 2 + \frac{\pi}{2} - (1)$$

$$= \ln 2 + \frac{\pi}{2} - 1$$

Sol14. $\frac{2+4+5+7+x+y}{6}=6.5$ ....(i)

$\frac{2^2+4^2+5^2+7^2+x^2+y^2}{6}-(6.5)^2=10.25$ ....(ii)

$(i)\Rightarrow x+y=21$

$(ii)\Rightarrow x^2+y^2=221$

$(i)\,\&\,(ii)\Rightarrow (x,y)=(10,11)$

Sol15. Tangent at $P(2,2)$ to $y^2=2x$ is $y(2)=(x+2)$

$\Rightarrow y=\frac{x}{2}+1$

Normal at P is $y=-2x+6$

$R\left(\frac{9}{2},-3\right)$

$Q(-2,0)$

$\Delta PQR=\frac{1}{2}\left|2(3)-2(-3-2)+\frac{9}{2}(2-0)\right|=\frac{1}{2}|6+10+9|=\frac{25}{2}$

Sol16. $f(0^-)=f(0^+)=f(0)$

$\Rightarrow -1=a-1=-1\Rightarrow a=0$

$f(1^-)=f(1^+)=f(1)$

$\Rightarrow a-1=2-b=2-b\Rightarrow a+b=3$

Sol17. $\vec{a}.\vec{c}=|\vec{c}|,\ |\vec{c}|^2+|\vec{a}|^2-2\vec{c}.\vec{a}=8,\ |\vec{a}|=3,\ |\vec{a}\times\vec{b}|=3$

$\Rightarrow |\vec{c}|^2+9-2|\vec{c}|=8\Rightarrow |\vec{c}|=1$

$\left|(\vec{a}\times\vec{b})\times\vec{c}\right|=|\vec{a}\times\vec{b}||\vec{c}|\sin\frac{\pi}{6}=\frac{3}{2}$

Sol18. $xe^x dx+\frac{ydy}{\sqrt{1-y^2}}=0$

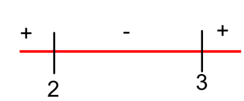
$\Rightarrow (x-1)e^x-\sqrt{1-y^2}=c$

Point $(1,-1)\Rightarrow c=0$

$x=3\Rightarrow 2e^3=\sqrt{1-y^2}$

$y^2=1-4e^6$

Sol19. $\frac{|[x]|-2}{|[x]|-3}\geq 0$



$\Rightarrow |[x]|\leq 2$ or $|[x]|>3$

$\Rightarrow -2\leq [x]\leq 2,\qquad [x]<-3$ or $[x]>3$

$\Rightarrow -2\leq x<3\,,\qquad x<-3$ or $x\geq 4$

$x\in(-\infty,-3)\cup[-2,3)\cup[4,\infty),\ a+b+c=-3-2+3=-2$

Sol20. $y = vx$

$$\Rightarrow \text{D.E. } \tan v.\left(v + x\frac{dv}{dx}\right) = \left(v\tan V - 1\right)$$

$$\Rightarrow \tan vx\frac{dv}{dx} + 1 = 0$$

$$\Rightarrow \tan v dv + \frac{1}{x}dx = 0 \Rightarrow x = c\cos\frac{y}{x}$$

Point $\left(\frac{1}{2}, \frac{\pi}{6}\right)$

$$\Rightarrow C = 1$$

$$x = \cos\frac{y}{x}$$

$$\Rightarrow y = x\cos^{-1}x$$

$$A = \int_0^{\frac{1}{\sqrt{2}}} x\cos^{-1}x dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \cos\theta.\theta.\left(-\sin\theta\right)d\theta$$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} -\frac{\theta}{2}\sin 2\theta d\theta = \frac{\pi - 1}{8}$$

SECTION - B

Sol1. $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$$B = 7A^{20} - 20A^7 + 2I$$

$$A^2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

a_{13} of $A, A^2, A^3, \ldots$

are 0, 1, 3, 6,……….

For 0, 1, 3, 6, ……..

$$S_n = 0 + 1 + 3 + 6 + \ldots + t_n$$

$$S_n = \quad 0 + 1 + 3 + \ldots + t_{n-1} + t_n$$

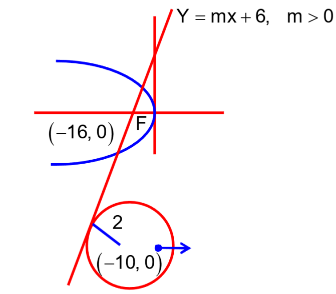
$$t_n = 1 + 2 + 3 + \ldots + \left(t_n - t_{n-1}\right)$$

$$= \frac{\left(n-1\right).n}{2}$$

$$b_{13} = 7\left(190\right) - 20\left(21\right) + 0$$

$$= 1330 - 420 = 910$$

Sol2. $0 = -16m + c$ ....(i)



$$\frac{|m(-10)+C|}{\sqrt{m^2+1}} = 2 \text{ ....(ii)}$$

(i) & (ii) $\Rightarrow 36m^2 = 4(m^2+1)$

$$m = \frac{1}{2\sqrt{2}}, C = \frac{8}{\sqrt{2}}$$

$$4\sqrt{2}(m+c) = 34$$

Sol3. $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a-c+1 & b-c & a-b \\ b-d-1 & c-d & b-c \\ x-b+d & x+d & x+c \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} a+1-b & 2b-c-a & a-b \\ b-1-c & 2c-b-d & b-c \\ -b & d-c & x+c \end{vmatrix} = \begin{vmatrix} -\lambda+1 & 0 & -\lambda \\ -\lambda-1 & 0 & -\lambda \\ -b & \lambda & x+c \end{vmatrix}, R_1 \to R_1 - R_2$$

$$= \begin{vmatrix} 2 & 0 & 0 \\ -\lambda-1 & 0 & -\lambda \\ -b & \lambda & x+c \end{vmatrix} = 2\lambda^2 = 2 \Rightarrow \lambda^2 = 1$$

Sol4. $|\vec{a}| = |\vec{b}| = |\vec{c}| = \ell$

$\vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{c} = \vec{c}\,.\,\vec{a} = 0$

$|\vec{a}+\vec{b}+\vec{c}|^2 = 3\ell^2$

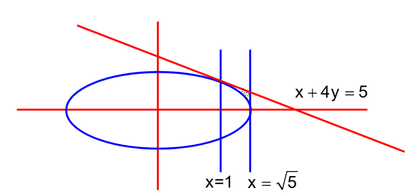
$\vec{a}\,.\,(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\theta$

$\Rightarrow |\vec{a}|^2 + 0 + 0 = |\vec{a}|\,|\vec{a}+\vec{b}+\vec{c}|\cos\theta$

$$\Rightarrow \ell^2 = \sqrt{3}\ell^2\cos\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$36\cos^2 2\theta = 36\left(\frac{2}{3}-1\right)^2 = 4$$

Sol5. Area = $\frac{1}{2}(\sqrt{5}-1)\frac{9-\sqrt{5}}{4} - \int_1^{\sqrt{5}} \frac{1}{2}\sqrt{5-x^2}\,dx$



$$= \frac{1}{8}(-14+10\sqrt{5}) - \frac{1}{2}\int_{\cos^{-1}\frac{1}{\sqrt{5}}}^{0} (-\sin^2\theta)\,d\theta$$

$$A = \frac{-14}{8} + \frac{5}{4}\sqrt{5} - \left(\frac{5}{4}\cos^{-1}\frac{1}{\sqrt{5}} - \frac{1}{2}\right)$$

$$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\frac{1}{\sqrt{5}}$$

$\alpha = \frac{5}{4}, \beta = \frac{-5}{4}, \gamma = \frac{-5}{4}$

$|\alpha + \beta + \gamma| = \frac{5}{4}$

Sol6.

Bowlers	Batsmen	Wicket Keepers
(6)	(7)	(2)
4	5	1
5	6	2
6	7	

Required number of ways

$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 + {}^6C_5 \times {}^7C_5 \times {}^2C_1 + {}^6C_4 \times {}^7C_5 \times {}^2C_2$

$= 15 \times 7 \times 2 + 6 \times 21 \times 2 + 15 \times 21 \times 1$

$= 210 + 252 + 315 = 777$

Sol7. $T_{r+1} = {}^{120}C_r\ 4^{\frac{120-r}{4}}\ .\ 5^{r/6}$

For T_{r+1} to be rational

r should be multiple of 6

Number of such terms = 21

Sol8. For the plane P,

$\vec{n} = \frac{1}{2}\left(-2\hat{j}\right) \times \left(-\hat{i} + \hat{j} - 3\hat{k}\right) = \hat{j} \times \hat{i} + 3\hat{j} \times \hat{k} = -\hat{k} + 3\hat{i}$

$\vec{a}\ .\ \vec{n} = 0 \Rightarrow 3\alpha - \gamma = 0$ ....(i)

$\vec{a}\ .\ (1,2,3) = 0 \Rightarrow \alpha + 2\beta + 3\gamma = 0$ ....(ii)

$\vec{a}\ .\ (1,1,2) = 2 \Rightarrow \alpha + \beta + 2\gamma = 2$ ....(iii)

(i), (ii) & (iii) $\Rightarrow \alpha = 1, \beta = -5, \gamma = 3$

$(\alpha - \beta + \gamma)^2 = 81$

Sol9. $\lim_{x \to 0}\left(2 - \cos x.\sqrt{\cos 2x}\right)^{\frac{x+2}{x^2}}$ $\quad (1^\infty)$

$= \lim_{x \to 0} e^{\left(1 - \cos x.\sqrt{\cos 2x}\right).\frac{x+2}{x^2}}$

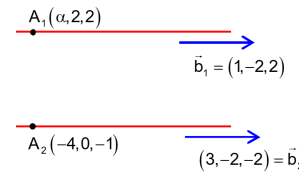
$= \lim_{x \to 0} e^{(x+2)\frac{\left(1 - \cos x\ .\ \sqrt{\cos 2x}\right)}{x^2}}$

$= \lim_{x \to 0} e^{2.\left(\frac{\sin x\sqrt{\cos 2x} - \cos x.\frac{1(-2\sin 2x)}{2\sqrt{\cos 2x}}}{2x}\right)}$

$= \lim_{x \to 0} e^{2.\left(\frac{1}{2}+1\right)} = e^3 = e^a$

$\Rightarrow a = 3$

Sol10. dist. $= \dfrac{\vec{A_1A_2}\cdot\left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$



$$\frac{\vec{b}_1 \times \vec{b}_2}{\left|\vec{b}_1 \times \vec{b}_2\right|} = \frac{(2,2,1)}{3}$$

$$= \frac{(-4-\alpha, -2, -3).(2,2,1)}{3}$$

$$= \frac{-8-2\alpha-4-3}{3} = -5 - \frac{2\alpha}{3}$$

$$\left|-5-\frac{2\alpha}{3}\right| = 9 \Rightarrow -5-\frac{2\alpha}{3} = \pm 9$$

$$\Rightarrow \frac{2\alpha}{3} = 4, -14 \text{, but } \alpha > 0$$

$$\alpha = 6$$