Solutions to JEE(Main) -2021

Test Date: 22nd July 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

<u>Important Instructions</u>:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to earth reach height h is_________s.

(A) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$ (B) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

(C) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$ (D) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

Q2. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is :

(A) $\frac{m_p}{m_e}$ (B) 1

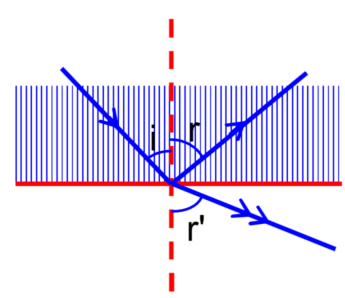
(C) $\frac{m_e}{m_p}$ (D) $\sqrt{\frac{m_p}{m_e}}$

Q3. What will be the projection of vector $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ on vector $\vec{B}=\hat{i}+\hat{j}$?

(A) $2(\hat{i}+\hat{j}+\hat{k})$ (B) $\sqrt{2}(\hat{i}+\hat{j})$

(C) $(\hat{i}+\hat{j})$ (D) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$

Q4. A ray of light passes from a denser medium to a rarer medium at an angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by :



(A) $\sin^{-1}(\cot r)$

(B) $\tan^{-1}(\sin i)$

(C) $\sin^{-1}(\tan r')$

(D) $\sin^{-1}(\tan r)$

Q5. A nucleus with number 184 initially at rest emits an $\alpha-$particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the $\alpha-$particle.

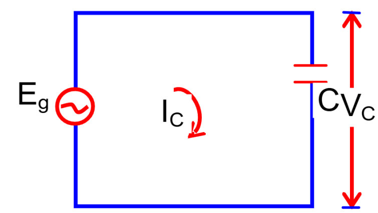
(A) 5.5 MeV (B) 5.38 MeV

(C) 5.0 MeV (D) 0.12 MeV

Q6. Intensity of sunlight is observed as $0.092\ \text{Wm}^{-2}$ at a point in free space. What will be the peak value of magnetic field at that point? $\left(\varepsilon_0 = 8.85\times10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}\right)$

(A) $5.88\ \text{T}$
(B) $1.96\times10^{-8}\ \text{T}$
(C) $8.31\ \text{T}$
(D) $2.77\times10^{-8}\ \text{T}$

Q7. In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0}\sin\omega t$, V_C and I_C are the voltage and current. Correct phase diagram for such circuit is :



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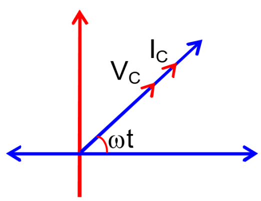

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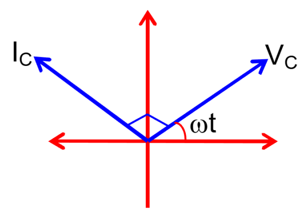

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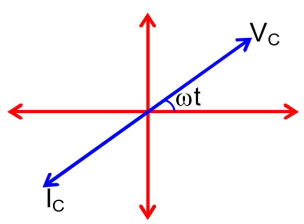

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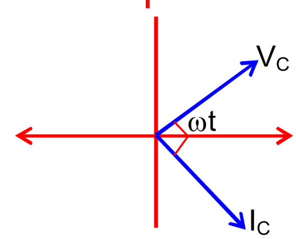


Q8. Consider a situation in which reverse biased current of a particular P – N junction increase when it is exposed to a light of wavelength ≤ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole – electron pairs. The value of band gap is nearly.

(A) 1 eV
(B) 4 eV
(C) 0.5 eV
(D) 2 eV

Q9. A porter lifts a heavy suitcase of mass 80kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase.
$\left(\text{take } g = 9.8\ \text{ms}^{-2}\right)$

(A) $-627.2\,\text{J}$
(B) -62720.0 J
(C) $+627.2$ J
(D) 784.0 J

Q10. Match List – I with List – II :

List – I	**List – II**
(a) $\omega L > \dfrac{1}{\omega C}$	(i) Current is in phase with emf
(b) $\omega L = \dfrac{1}{\omega C}$	(ii) Current lags behind the applied emf
(c) $\omega L < \dfrac{1}{\omega C}$	(iii) Maximum current occurs
(d) Resonant frequency	(iv) Current leads the emf

Choose the correct answer from the options given below :

(A) (a) – (iv); (b) – (iii); (c) – (ii); (d) – (i)
(B) (a) – (ii); (b) – (i); (c) – (iv); (d) – (iii)
(C) (a) – (iii); (b) – (i); (c) – (iv); (d) – (ii)
(D) (a) – (ii); (b) – (i); (c) – (iii); (d) – (iv)

Q11. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

(A) T_0
(B) $8\pi T_0$
(C) $4T_0$
(D) $\frac{1}{4}T_0$

Q12. **Statement I :** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II : At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both **Statement I** and **Statement II** are false
(B) **Statement I** is true but **Statement II** is false
(C) **Statement I** is false but **Statement II** is true
(D) Both **Statement I** and **Statement II** are true

Q13. A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun velocity of recoil of gun are :

(A) $0.4 kg\ ms^{-1}$, $0.1 ms^{-1}$
(B) $0.2\ kg\ ms^{-1}$, $0.05\ ms^{-1}$
(C) $0.4\ kg\ ms^{-1}$, $0.05\ ms^{-1}$
(D) $0.2\ kg\ ms^{-1}$, $0.1\ ms^{-1}$

Q14. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km? The average population density around the tower is $200/km^2$ and the value of $R_e = 6.5 \times 10^6$ m.

(A) Height = 1800 m
Population Covered $= 1413 \times 10^8$

(B) Height $= 1241$ m
Population Covered $= 7 \times 10^5$

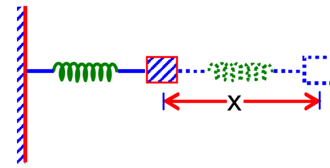
(C) Height = 1600 m
Population Covered $= 2 \times 10^5$

(D) Height $= 1731$ m
Population Covered $= 1413 \times 10^5$

Q15. Choose the correct option :

(A) True dip is not mathematically related to apparent dip.
(B) True dip is always greater than the apparent dip.
(C) True dip is always equal to apparent dip.
(D) True dip is less than the apparent dip.

Q16. The motion of a mass on a spring, with spring constant K is as shown in figure. The equation of motion is given by $x(t) = A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\frac{K}{m}}$. Suppose that at time $t = 0$, the position of mass is $x(0)$ and velocity $\upsilon(0)$, then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are :



(A) $C = \sqrt{\frac{\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{\upsilon(0)}\right)$

(B) $C = \sqrt{\frac{2\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{2\upsilon(0)}\right)$

(C) $C = \sqrt{\frac{\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{\upsilon(0)}{x(0)\omega}\right)$

(D) $C = \sqrt{\frac{2\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{\upsilon(0)}{x(0)\omega}\right)$

Q17. What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T ?

(A) $\frac{3}{2}k_B T$

(B) $\frac{1}{2}k_B T$

(C) $k_B T$

(D) $\frac{2}{3}k_B T$

Q18. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

The correct statement for this situation is :

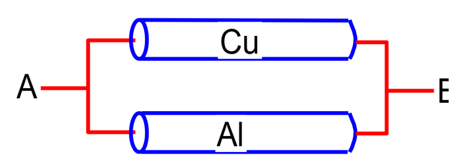
(A) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(B) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

(C) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

(D) All of them will have same velocity.

Q19. A Copper(Cu) rod of length 25cm and cross – sectional area 3 mm^2 is joined with a similar aluminium(Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.



(Take Resistivity of Copper $= 1.7 \times 10^{-8}\ \Omega m$

Resistivity of Aluminium $= 2.6 \times 10^{-8}\ \Omega m$)

(A) 0.858 mΩ

(B) 0.0858 mΩ

(C) 1.420 mΩ

(D) 2.170 mΩ

Q20. An electric dipole is placed on x – axis in proximity to a line charge of linear density 3.0×10^{-6} C/m. Line charge is placed on z –axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive charge of the dipole.

(A) 0.485 mC

(B) 8.8 μC

(C) 815.1 nC

(D) 4.44 μC

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The position of the centre of mass of a uniform semi – circular wire of radius 'R' placed in x – y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of $|x|$ is __________.

Q2. The total charge enclosed in an incremental volume of $2\times10^{-9}\,m^3$ located at the origin is ________nC, if electric flux density of its field is found as
$D = e^{-x}\sin y\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\hat{k}C/m^2$.

Q3. A ray of light passing through a prism $\left(\mu = \sqrt{3}\right)$ surface minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is ________(in degrees).

Q4. The centre of a wheel rolling on a plane surface moves with a speed υ_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}\,\upsilon_0$. Then the value of x is _________.

Q5. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of Pendulum (cm)	No. of oscillations (n)	Total for n oscillations	Time period (s)
1	64.0	8	128.0	16.0
2	64.0	4	64.0	16.0
3	20.0	4	36.0	9.0

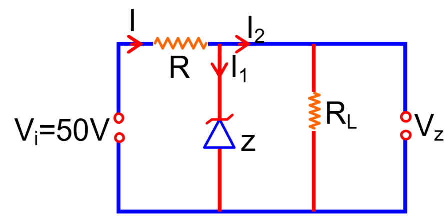
(Least count of length = 0.1 cm
least count for time = 0.1 s)
If E_1, E_2 and E_3 are the percentage errors 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no.__________.

Q6. In 5 minutes, a body cools from $75°C$ to $65°C$ at room temperature of $25°C$. The temperature of body at the end of next 5 minutes is _________ $°C$.

Q7. The area of cross – section of a railway track is $0.01\,m^2$. The temperature variation is $10°C$, Coefficient of liner expansion of material of track is $10^{-5}/°C$. The energy stored per meter in the track is __________J/m.
(Young's modulus of material of track is $10^{11}\,Nm^{-2}$)

Q8. In an electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5Ω. However, it provides a potential difference of 1 V across a load resistance of 2Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is ________.

Q9. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA be ________ Ω.



Q10. Three particles P,Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}, \vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is ________.

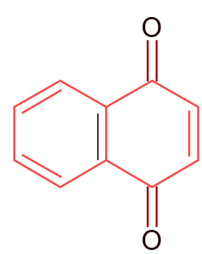
PART – B (CHEMISTRY)

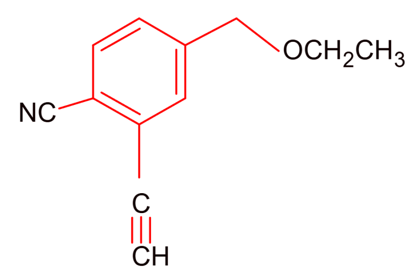
SECTION - A

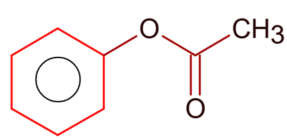
(One Options Correct Type)

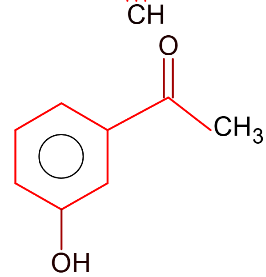
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Which one of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis?

(A)


(B)


(C)


(D)


Q2. When silver nitrate solution is added to potassium iodide solution then the sol produced is:

(A) AgI/I^-
(B) $AgNO_3/NO_3^-$
(C) KI/NO_3^-
(D) AgI/Ag^+

Q3. Isotope(s) of hydrogen which emits low energy β^- particles with $t_{1/2}$ value > 12 years is / are:

(A) Protium
(B) Tritium
(C) Deuterium
(D) Deuterium and Tritium

Q4. Which one of the following 0.06 M aqueous solutions has lowest freezing point ?

(A) KI
(B) $C_6H_{12}O_6$
(C) K_2SO_4
(D) $Al_2(SO_4)_3$

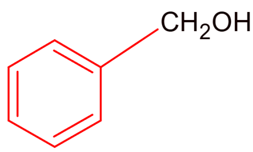
Q5. Match List – I with List – II :

List – I (Species)	**List – II (Hybrid Orbitals)**
(a) SF_4	(i) sp^3d^2
(b) IF_5	(ii) d^2sp^3
(c) NO_2^+	(iii) sp^3d
(d) NH_4^+	(iv) sp^3
	(v) sp

Choose the correct answer from the options given below :

(A) (a) –(iii) , (b) –(i), (c) –(v) and (d) –(iv)
(B) (a) –(ii), (b) –(i), (c) –(iv) and (d) –(v)
(C) (a) –(i), (b) –(ii), (c) –(v) and (d) –(iii)
(D) (a) –(iv), (b) –(iii), (c) –(ii) and (d) –(v)

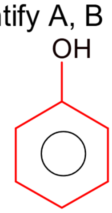
Q6. Which one of the following compounds does not exhibit resonance?

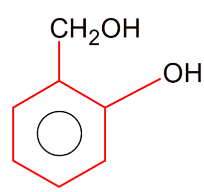
(A) $CH_3CH_2CH_2CONH_2$
(B) $CH_3CH_2CH = CHCH_2NH_2$
(C) $CH_3CH_2OCH = CH_2$
(D) 

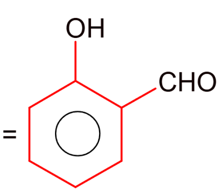
Q7. The set having ions which are coloured and paramagnetic both is :

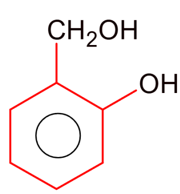
(A) $Ni^{2+}, Mn^{7+}, Hg^{2+}$
(B) Cu^{+}, Zn^{2+}, Mn^{4+}
(C) Sc^{3+}, V^{5+}, Ti^{4}
(D) Cu^{2+}, Cr^{3+}, Sc^{+}

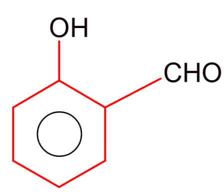
Q8. An organic compound A (C_6H_6O) gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC). Identify A, B and C.

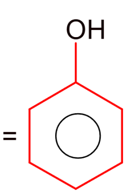
(A) A = 

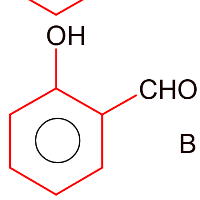
B = 

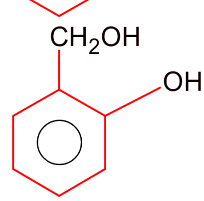
C = 

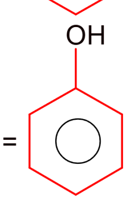
(B) A = 

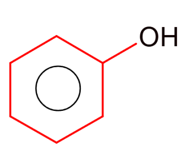
B = 

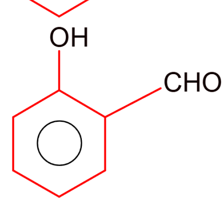
C = 

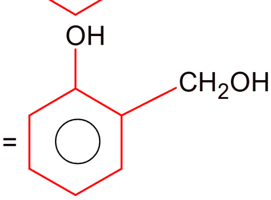
(C) A = 

B = 

C = 

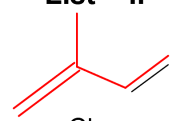
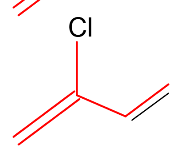
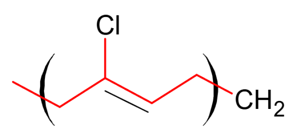
(D) A = 

B = 

C = 

Q9. Given below are the statements about diborane.
(a) Diborane is prepared by the oxidation of $NaBH_4$ With I_2.
(b) Each boron atom is in sp^2 hybridized state.
(c) Diborane has one bridged 3 centre – 2 – electron bond.
(d) Diborane is a planar molecule.
The option with correct statement(s) is :
(A) (a) only
(B) (c) only
(C) (a) and (b) only
(D) (c) and (d) only

Q10. Match List – I with List – II :

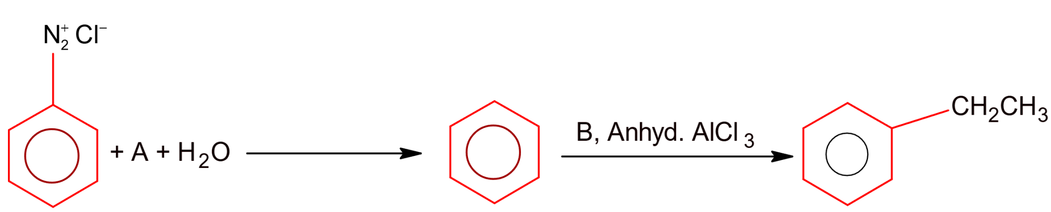
List – I	**List – II**
(a) Chloroprene	(i) 
(b) Neoprene	(ii) 
(c) Acrylonitrile	(iii) 
(d) Isoprene	(iv) $CH_2 = CH - CN$

Choose the correct answer from the options given below :
(A) (a) – (iii), (b) – (i), (c) – (iv), (d) – (ii)
(B) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)
(C) (a) – (ii), (b) – (i), (c) – (iv), (d) – (iii)
(D) (a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)

Q11. Which one of the following molecules does not show stereo isomerism ?
(A) 3 – Ethylhex – 3 – ene
(B) 3 – Methylhex – 1 – ene
(C) 3,4 – Dimethylhex – 3 – ene
(D) 4 – Methylhex – 1 – ene

Q12. Sulphide ion is soft base and its ores are common for metals.
(a) Pb
(b) Al
(c) Ag
(d) Mg
Choose the correct answer from the options given below :
(A) (c) and (d) only
(B) (a) and (c) only
(C) (a) and (d) only
(D) (a) and (b) only

Q.13.



Major Product

In the chemical reactions given above A and B respectively are:
(A) CH_3CH_2OH and H_3PO_2
(B) CH_3CH_2Cl and H_3PO_2
(C) H_3PO_2 and CH_3CH_2OH
(D) H_3PO_2 and CH_3CH_2Cl

Q14. Match List – I with List – II :

List – I (Elements)	**List – II (Properties)**
(a) Ba	(i) Organic solvent soluble compounds
(b) Ca	(ii) Outer electronic configuration $6s^2$
(c) Li	(iii) Oxalate insoluble in water
(d) Na	(iv) Formation of very strong monoacidic base

Choose the correct answer from the options given below :

(A) (a) –(ii), (b) –(iii), (c) –(i) and (d) –(iv)
(B) (a) –(iv), (b) –(i), (c) –(ii) and (d) –(iii)
(C) (a) – (i), (b) –(iv), (iii) –(ii) and (d) –(iii)
(D) (a) –(iii), (b) –(ii), (c) –(iv) and (d) –(i)

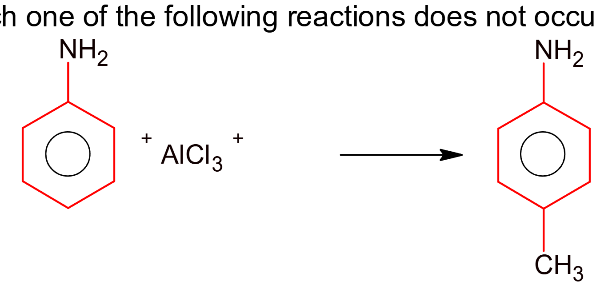
Q15. Which one of the following group – 15 hydride is the strongest reducing agent ?

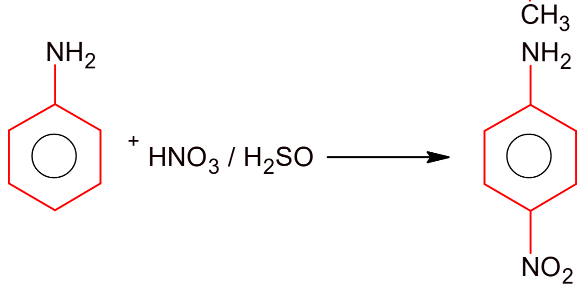
(A) AsH_3 (B) SbH_3
(C) BiH_3 (D) PH_3

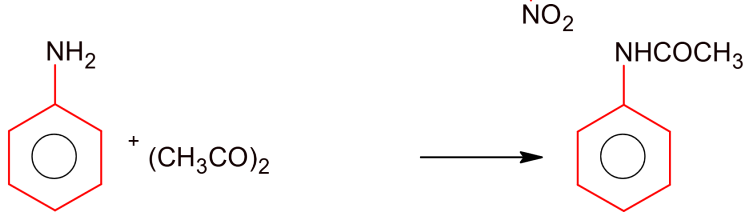
Q16. Thiamine and pyridoxine are also known respectively as :

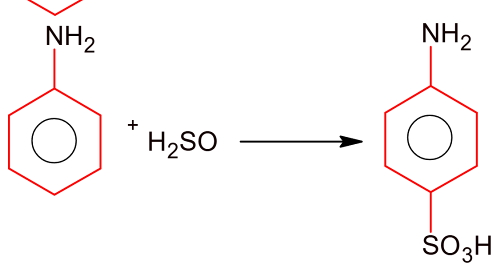
(A) Vitamin B_1 and Vitamin B_6 (B) Vitamin E and Vitamin B_2
(C) Vitamin B_6 and Vitamin B_2 (D) Vitamin B_2 and Vitamin E

Q17. Which one of the following reactions does not occur?

(A)


(B)


(C)


(D)


Q18. Which one of the following statements for D.I. Mendeleeff, is incorrect?
(A) At the time, he proposed periodic table of elements structure of atom was known.
(B) Element with atomic number 101 is named after him.
(C) He invented accurate barometer.
(D) He authored the textbook – Principles of Chemistry.

Q19. The water having more dissolved O_2 is :
(A) Water at $80^\circ C$
(B) boiling water
(C) polluted water
(D) water at $4^\circ C$

Q20. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point) ?
(A) Fractional distillation
(B) Steam distillation
(C) Simple distillation
(D) Reduced pressure distillation

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The total number of unpaired electrons present in $\left[Co(NH_3)_6\right]Cl_2$ and $\left[Co(NH_3)_6\right]Cl_3$ is -.

Q2. $N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at $318K$. The concentration of N_2O_5 after 1 hours was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at $318K$ is _________ $\times10^{-3}\ min^{-1}$. (Nearest integer)

[Given : $\log 3 = 0.477, \log 5 = 0.699$]

Q3. Number of electrons that Vanadium $(Z = 23)$ has in p-orbitals is equal to ----------.

Q4. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is _________ kJ. (Nearest integer)

Q5. Value of K_p for the equilibrium reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9. The K_c for this reaction at same temperature is __________. (Nearest integer)

$(R = 0.083$ L bar $K^{-1} mol^{-1})$

Q6. A copper complex crystallizing in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be $7.26\ g\,cm^{-3}$. The molar mass of copper complex is __________ g mol^{-1}. (Nearest integer)

[Given : $N_A = 6.022\times10^{23} mol^{-1}$]

Q7. The number of acyclic structural isomers (including geometrical isomers) for pentene is _.

Q8. Assume a cell with the following reaction

$Cu_{(s)} + 2Ag^{+}\left(1\times10^{-3}M\right) \rightarrow Cu^{2+}\left(0.250M\right) + 2Ag_{(s)}\ E^{\ominus}_{cell} = 2.97\,V$

E_{cell} for the above reaction is ____________V. (Nearest integer)

[Given : $\log 2.5 = 0.3979, T = 298K$]

Q9. Methylation of 10g of benzene give 9.2g of toluene. Calculate the percentage yield of toluene __________. (Nearest integer)

Q10. If the concentration of glucose $(C_6H_{12}O_6)$ in blood is 0.72 g L^{-1}, the molarity of glucose in blood is __________ $\times10^{-3}M$. (Nearest integer)

(Given : Atomic mass of $C = 12, H = 1, O = 16u$)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let the circle $S: 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(A) $81 < C < 156$
(B) $100 < C < 165$
(C) $100 < C < 156$
(D) $\frac{25}{9} < C < \frac{13}{3}$

Q2. If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3), \lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

(A) 3
(B) 2
(C) -1
(D) 5

Q3. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to :

(A) 1
(B) 2
(C) $\frac{3}{2}$
(D) $\frac{4}{3}$

Q4. If $\int_0^{100\pi} \frac{\sin^2 x}{{}_{E}\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)} dx = \frac{\alpha \pi^3}{1 + 4\pi^2}, \alpha \in R$, where $[x]$ is the greatest integer less than or equal to x, then the value of α is :

(A) $100(1-e)$
(B) $150(e^{-1} - 1)$
(C) $200(1 - e^{-1})$
(D) $50(e-1)$

Q5. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

(A) $[1, e)$
(B) $[\log_e 2, \log_e 3)$
(C) $[0, \log_e 2)$
(D) $[0, 1/e)$

Q6. Which of the following Boolean expressions is not a tautology?

(A) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$
(B) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$
(C) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$
(D) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Q7. Let a line $L: 2x+y=k, k>0$ be a tangent to the hyperbola $x^2-y^2=3$. If L is also a tangent to the parabola $y^2=\alpha x$, then α is equal to :

(A) 24
(B) -24
(C) -12
(D) 12

Q8. Let $y=y(x)$ be the solution of the differential equation

$\operatorname{cosec}^2 x\, dy + 2dx = (1+y\cos 2x)\operatorname{cosec}^2 x\, dx$, with $y\left(\frac{\pi}{4}\right)=0$. Then, the value of $(y(0)+1)^2$ is equal to :

(A) $e^{1/2}$
(B) $e^{-1/2}$
(C) e^{-1}
(D) e

Q9. Let $f: R \to R$ be defined as

$$f(x)=\begin{cases} -\frac{4}{3}x^3+2x^2+3x, & x>0 \\ 3xe^x & , x\le 0 \end{cases}$$

Then f is increasing function in the interval.

(A) $\left(-\frac{1}{2}, 2\right)$
(B) $\left(-1, \frac{3}{2}\right)$
(C) $(-3, -1)$
(D) $(0, 2)$

Q10. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$ and $|\vec{a}|=\sqrt{10}$. Then a possible value of $[\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{b}\vec{d}]+[\vec{a}\vec{c}\vec{d}]$ is equal to :

(A) -40
(B) -38
(C) -42
(D) -29

Q11. Let S_n denote the sum of the first n-terms of an arithmetic progression. If $S_{10}=530, S_5=140$, then $S_{20}-S_6$ is equal to :

(A) 1852
(B) 1842
(C) 1872
(D) 1862

Q12. Let $f: R \to R$ be defined as $f(x)=\begin{cases} \frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right), & x\ne 0 \\ \alpha & , x=0 \end{cases}$. If f is continuous at x = 0, then α is equal to :

(A) 0
(B) 1
(C) 2
(D) 3

Q13. The number of solutions of $\sin^7 x+\cos^7 x=1, x\in[0, 4\pi]$ is equal to :

(A) 7
(B) 11
(C) 5
(D) 9

Q14. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true?

(A) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2
(B) $[\vec{a}\vec{b}\vec{c}] + [\vec{c}\vec{a}\vec{b}] = 8$
(C) $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$
(D) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

Q15. Let L be the line of intersection of planes $\vec{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 2$ and $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) = 2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point $(1,2,0)$, then the value of $35(\alpha + \beta + \gamma)$ is equal to :

(A) 101
(B) 143
(C) 134
(D) 119

Q16. Let $A = |a_{ij}|$ be a real matrix of order 3 x 3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for $i = 1,2,3$. Then, the sum of all the entries of the matrix A^3 is equal to :

(A) 9
(B) 3
(C) 1
(D) 2

Q17. If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

(A) 1
(B) $\frac{1}{2}$
(C) 2
(D) $\frac{3}{2}$

Q18. The values of λ and μ such that the system of equations
$x + y + z = 6,\ 3x + 5y + 5z = 26,\ x + 2y + \lambda z = \mu$ has no solution, are :

(A) $\lambda = 2, \mu \neq 10$
(B) $\lambda = 3, \mu \neq 10$
(C) $\lambda = 3, \mu = 5$
(D) $\lambda \neq 2, \mu = 10$

Q19. Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

(A) $\frac{-1+\sqrt{5}}{2}$
(B) $\frac{-1+\sqrt{3}}{2}$
(C) $\frac{-1+\sqrt{8}}{2}$
(D) $\frac{-1+\sqrt{6}}{2}$

Q20. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 x 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

(A) $\frac{22}{81}$
(B) $\frac{45}{162}$
(C) $\frac{43}{162}$
(D) $\frac{23}{81}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Let $A=\{0,1,2,3,4,5,6,7\}$. Then the number of bijective functions $f:A\to A$ such that $f(1)+f(2)=3-f(3)$ is equal to……………

Q2. Let $f:R\to R$ be a function defined as $f(x)=\begin{cases}3\left(1-\frac{|x|}{2}\right) & \text{if } |x|\le 2\\ 0 & \text{if } |x|>2\end{cases}$

Let $g:R\to R$ be given by $g(x)=f(x+2)-f(x-2)$. If n and m denote the number of points in R where g is the continuous and not differentiable, respectively, then n + m is equal to…………..

Q3. Let $y=y(x)$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx=(x+2)dy,\ y(1)=1$. If the domain $y=y(x)$ is an open interval (α,β), then $|\alpha+\beta|$ is equal to………

Q4. The number of elements in the set $\left\{n\in\{1,2,3,....100\}\,\middle|\,(11)^n>(10)^n+(9)^n\right\}$ is………

Q5. The area (in sq. units) of the region bounded by the curves $x^2+2y-1=0,\ y^2+4x-4=0$ and $y^2-4x-4=0$, in the upper half plane is………

Q6. Let $A=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$. Then the number of 3 x 3 matrices B with entries from the set $\{1,2,3,4,5\}$ and satisfying AB = BA is………..

Q7. If the digits are not allowed to repeat in any number formed by using the digits 0,2,4,6,8, then the number of all numbers greater than 10,000 is equal to…….

Q8. The sum of all the elements in the set $\{n\in\{1,2,.....,100\}\,|\,\text{H.C.F. of } n \text{ and } 2040 \text{ is } 1\}$ is equal to……….

Q9. If the constant term, in binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180, then r is equal to…….

Q10. Consider the following frequency distribution :

Class :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to………………

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**B**	2.	**D**	3.	**C**	4.	**D**
5.	**B**	6.	**D**	7.	**B**	8.	**D**
9.	**A**	10.	**B**	11.	**D**	12.	**B**
13.	**B**	14.	**D**	15.	**D**	16.	**C**
17.	**A**	18.	**B**	19.	**A**	20.	**D**

SECTION - B

1.	**2**	2.	**4**	3.	**60**	4.	**2**
5.	**1**	6.	**57**	7.	**5**	8.	**15**
9.	**500**	10.	**3**				

PART – B (CHEMISTRY)

SECTION - A

1.	**C**	2.	**A**	3.	**B**	4.	**D**
5.	**A**	6.	**B**	7.	**D**	8.	**D**
9.	**A**	10.	**D**	11.	**A**	12.	**B**
13.	**D**	14.	**A**	15.	**C**	16.	**A**
17.	**A**	18.	**A**	19.	**D**	20.	**D**

SECTION - B

1.	**1**	2.	**7**	3.	**12**	4.	**21**
5.	**2**	6.	**106**	7.	**6**	8.	**3**
9.	**78**	10.	**4**				

PART – C (MATHEMATICS)

SECTION - A

1.	**C**	2.	**A**	3.	**D**	4.	**C**
5.	**C**	6.	**A**	7.	**B**	8.	**C**
9.	**B**	10.	**C**	11.	**D**	12.	**B**
13.	**C**	14.	**C**	15.	**D**	16.	**B**
17.	**D**	18.	**A**	19.	**A**	20.	**C**

SECTION – B

1.	**720**	2.	**4**	3.	**4**	4.	**96**
5.	**2**	6.	**3125**	7.	**96**	8.	**1251**
9.	**8**	10.	**4**				

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. Using conservation of energy :

$$\Rightarrow \frac{-GMm}{R}+\frac{1}{2}m\left(\sqrt{\frac{2GM}{R}}\right)^2=\frac{-GMm}{r}+\frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}mv^2=\frac{GMm}{r}$$

$$\Rightarrow V=\sqrt{\frac{2GM}{r}}$$

$$\Rightarrow \frac{dr}{dt}=\sqrt{\frac{2Gm}{r}}$$

$$\Rightarrow \int_R^{R+h} r^{1/2}dr=\sqrt{2GM}\int_0^t dt$$

$$\Rightarrow \frac{2}{3}\left[(R+h)^{3/2}-R^{3/2}\right]=\sqrt{2GM}\,t$$

$$\Rightarrow \frac{2}{3}R^{3/2}\left[\left(1+\frac{h}{R}\right)^{3/2}-1\right]=\sqrt{2GM}\,t$$

$$t=\frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(1+\frac{h}{R}\right)^{3/2}-1\right]$$

Sol2. $\lambda=\dfrac{h}{mv}=\dfrac{h}{\sqrt{2km}}=\dfrac{h}{2(ev)m}$

$$\lambda\,\alpha\,\frac{1}{\sqrt{m}}\Rightarrow\frac{\lambda_e}{\lambda_p}=\sqrt{\frac{m_p}{m_e}}$$

Sol3. Component of $\vec{A}$ an $\vec{B}$

$$=\left(\vec{A}\cdot\hat{B}\right)\hat{B}$$

$$=\left[\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}\right]\frac{\left(\hat{i}+\hat{j}\right)}{\sqrt{2}}$$

$$=\frac{1}{2}(1+1)\left(\hat{i}+\hat{j}\right)$$

$$=\hat{i}+\hat{j}$$

Sol4. $r + r' = 90^\circ$

$\Rightarrow r' = 90 - r = 90 - i$

Snell's law :

$\mu \sin i = 1 \sin r'$

$\Rightarrow \mu \sin i = \sin(90 - i)$

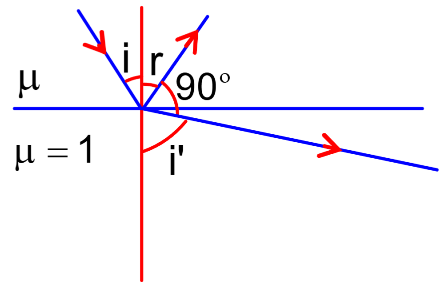
$\Rightarrow \tan i = \frac{1}{\mu}$

Critical angle :

$\mu \sin i_c = 1 \sin 90^\circ = 1$

$\Rightarrow \sin i_c = \frac{1}{\mu} = \tan i = \tan r$

$i_c = \sin^{-1}(\tan r)$



Sol5. Using conservation of momentum

$\Rightarrow 180V_1 = 4V_2$

$\Rightarrow V_2 = 45V_1$

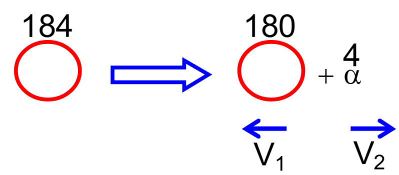
$Q = KE_{180} + KE_\alpha$

$Q = \frac{1}{2}180V_1^2 + \frac{1}{2}4V_2^2$

$= \frac{1}{2} \times 180 \times \left(\frac{V_2}{45}\right)^2 + \frac{1}{2}4V_2^2$

$= \frac{1}{2} \times 4 \times V_2^2 \left[\frac{45}{45^2} + 1\right] = KE_\alpha \left[\frac{1}{45} + 1\right]$

$KE_\alpha = \frac{5.5\text{MeV}}{\left(\frac{46}{45}\right)} = 5.38\text{MeV}$



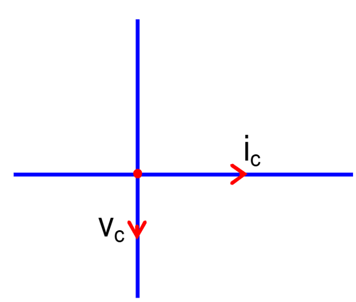
Sol6. $I = \frac{1}{2}C\varepsilon_0 E_0^2 = \frac{1}{2}C\varepsilon_0 (CB_0)^2 = \frac{1}{2}C^3\varepsilon_0 B_0^2$

$\Rightarrow B_0 = \sqrt{\frac{2I}{\varepsilon_0 C^3}}$

$= \sqrt{\frac{2 \times 0.092}{8.85 \times 10^{-12} \times 27 \times 10^{24}}}$

$= 2.27 \times 10^{-8}\,T$

Sol7. for capacitor : current leads emf by 90°.



Sol8. $E = \frac{hc}{\lambda} = \frac{1242\,eV - nm}{621\,nm} = 2eV$

$\Rightarrow$ minimum 2eV is required to emit.

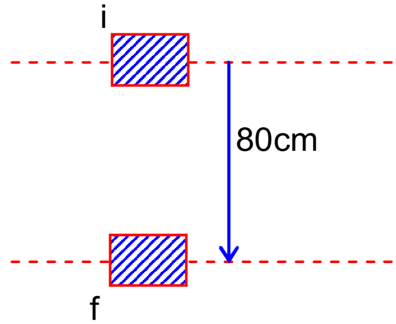
Sol9. $W = mgh$

$= 80 \times 9.8 \times \left(\frac{80}{100}\right)$

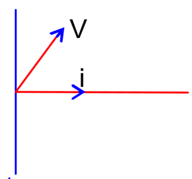
$= 627.2J$

$\Rightarrow w_f + w_g = 0$

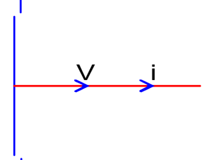
$\Rightarrow w_f = -627.2\,J$



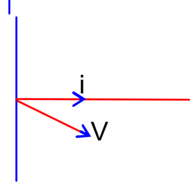
Sol10. (A) $\omega L > \frac{1}{\omega C}$



(B) $\omega L = \frac{1}{\omega C}$



(C) $\omega L < \frac{1}{\omega C}$



(D) resonant frequency — Maximum current occurs.

Sol11. $\frac{T =}{T' =} \frac{2\pi\sqrt{\frac{\ell}{g}}}{2\pi\sqrt{\frac{\frac{\ell}{16}}{g}}}$

$\Rightarrow \frac{T}{T'} = \frac{1}{\left(\frac{1}{4}\right)} = 4$

$\Rightarrow T' = \frac{T}{4}$

Sol12. I : at high temperature, ferromagnet becomes paramagnet.

II : at high temperature, domains of ferromagnet will break

Sol13. for bullet :

$I = \Delta p = mv - 0$

$= \frac{4}{1000} \times 50 = \frac{20}{100} = 0.2 kg - m/s$

Using conservation of momentum :

$\Rightarrow P_i = P_t$

$\Rightarrow \frac{4}{1000} \times 50 = 4 \times v$

$\Rightarrow v = 0.05 \; m/s$

Sol14. d = 150 km

$d = \sqrt{2hR}$

$h = \frac{d^2}{2R} = \frac{\left(150 \times 10^3\right)^2}{2 \times 6.5 \times 10^6} = 1731 \, m.$

Population covered

$= \pi r^2 \times \sigma = 3.14 \times 150^2 \times 2000 = 1413 \times 10^5$

Sol15. True dip is less than the apparent dip.

Sol16. $x(t) = A \sin \omega t + B \cos \omega t$

$\Rightarrow x(t) = \sqrt{A^2 + B^2} \sin(\omega t + \delta) = \sqrt{A^2 + B^2} \cos\left(\omega t - \left(\frac{\pi}{2} - \delta\right)\right)$

At $t = 0$:

$x(0) = \sqrt{A^2 + B^2} \sin(\delta)$

$v(0) = \omega\sqrt{A^2 + B^2} \cos(\delta)$

$\Rightarrow (x(0))^2 + \left(\frac{v(0)}{\omega}\right)^2 = A^2 + B^2$

$C = \sqrt{(x(0))^2 + \left(\frac{v(0)}{\omega}\right)^2}$

$\tan\delta = \frac{x(0)}{\frac{v(0)}{\omega}} = \frac{x(0)\omega}{v(0)}$

$\tan\phi = \tan\left(\frac{\pi}{2} - \delta\right) = \cot\delta = \frac{v(0)}{x(0)\omega}$

$\Rightarrow \phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

Sol17. Total energy of monoatomic gas at equilibrium

$= \frac{3}{2} K_B T$

Sol18. Using conservation of energy :

$mgh = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2} I_{cm} \frac{V^2}{r^2} + \frac{1}{2} mv^2$

$$v^2 = \frac{mgh}{\frac{1}{2}\frac{I_{cm}}{r^2} + mv^2}$$

So, sphere has greatest velocity of COM and ring has least.

Sol19. $R_1 = \rho_1 \frac{\ell}{A}$ $\quad R_2 = \rho_2 \frac{\ell}{A}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{A}{\ell}\left[\frac{1}{\rho_1} + \frac{1}{\rho_2}\right]$$

$$= \frac{3\times10^{-6}}{\frac{1}{4}}\left[\frac{1}{1.7\times10^{-8}} + \frac{1}{2.6\times10^{-8}}\right]$$

$$R = 0.858\times10^{-3}\,\Omega$$

Sol20. $F = \left[\frac{2k\lambda}{r_1} - \frac{2k\lambda}{r_2}\right]q = 2k\lambda q\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

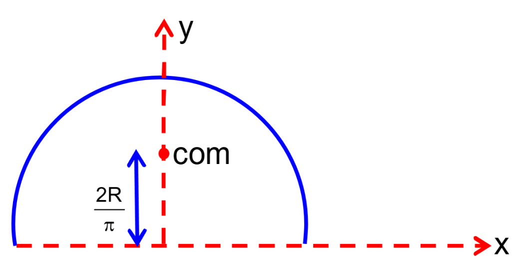
$$= 2\times9\times10^9\times3\times10^{-6}\left[\frac{1000}{10} - \frac{1000}{12}\right]q$$

$$\Rightarrow 4 = 9\times10^5 q$$

$$\Rightarrow q = 4.44\mu C$$

SECTION – B

Sol1.



Sol2. Using gauss law in differential form

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \vec{\nabla}\cdot\left(\varepsilon_0\vec{E}\right) = \rho$$

$$\Rightarrow \vec{\nabla}\cdot\vec{D} = \rho$$

$$\Rightarrow \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\vec{D} = \rho$$

$$\Rightarrow \rho = -e^{-x}\sin y + e^{-x}\sin y + 2 = 2$$

$$\Rightarrow Q = 2(\text{Volume}) = 4\times10^{-9}c = 4\,nc$$

Sol3. At minimum deviation :

$i = i'$

$r = r'$

$r + r' = A \Rightarrow r = \frac{A}{2}$

Snell's law :

$1\sin i = \sqrt{3}\sin r = \sqrt{3}\sin\frac{i}{2}$

$\Rightarrow 2\cos\frac{i}{2} = \sqrt{3}$

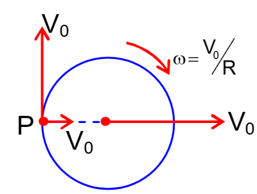
$\Rightarrow \cos\frac{i}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{i}{2} = 30^\circ$

$\Rightarrow i = 60^\circ$

$i = 2r \Rightarrow r = 30^\circ$

$\Rightarrow A = 60^\circ$

Sol4. $V_P = \sqrt{V_0^2 + V_0^2} = \sqrt{2}\,V_0$



Sol5. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow g = 4\pi^2\frac{\ell}{T^2}$

$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$

Student 1 will get least error because it has more number of oscillation

Sol6. from newtan's law of cooling :

$\frac{dT}{dt} = -k(T - T_0)$

Using average value :

$\frac{75-65}{5} = -k\left[\frac{(75+65)}{2} - 25\right]$

$\frac{(65-T)}{5} = -k\left[\frac{(65+T)}{2} - 25\right]$

$\Rightarrow \frac{10}{65-T} = \frac{45}{\frac{65+T}{2} - 25}$

$\Rightarrow T = 57^\circ C$

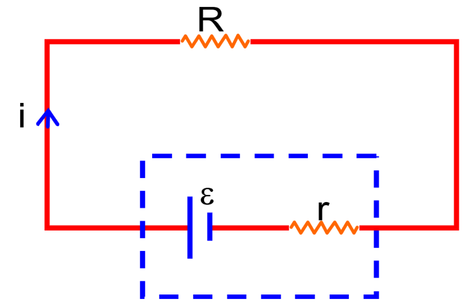
Sol7. Energy density (u) = $\frac{1}{2}\times\sigma\times\varepsilon = \frac{1}{2}\times Y\varepsilon\times\varepsilon = \frac{1}{2}\varepsilon^2 Y$

$\Rightarrow \text{energy stored}/m^2 = \frac{1}{2}\varepsilon^2 YA$

Strain $(\varepsilon) = \dfrac{\Delta \ell}{\ell_0} = \alpha \Delta T = 10^{-5} \times 10 = 10^{-4}$

$\Rightarrow$ energy stored / $m^2 = \dfrac{1}{2} \times 10^{-8} \times 10^{11} \times 10^{-2} = 5$

Sol8. $i = \dfrac{\varepsilon}{R+r}$



$V_R = \dfrac{\varepsilon R}{R+r}$

$$\frac{1.25 = \dfrac{\varepsilon(5)}{5+r}}{1 \quad = \dfrac{\varepsilon(2)}{2+r}}$$

$\Rightarrow r = 1$

Using above equ : $\varepsilon = \dfrac{3}{2} = \dfrac{15}{10} V$

Sol9. For 90mA in zener diode, current in R should be greater than 90mA.

$\dfrac{45}{R} = 90 \times 10^{-3} \Rightarrow R = \dfrac{45}{90 \times 10^{-3}} = 500\,\Omega$

Sol10. Cross product will represent a vector perpendicular to both the vector.

Dir. Of Motion of P : $= (\hat{i} + \hat{j}) \times (\hat{j} \times \hat{k})$

$= \hat{k} - \hat{j} + \hat{i}$

Dir. Of Motion of Q $= (\hat{i} + \hat{j}) \times (-\hat{i} + \hat{j})$

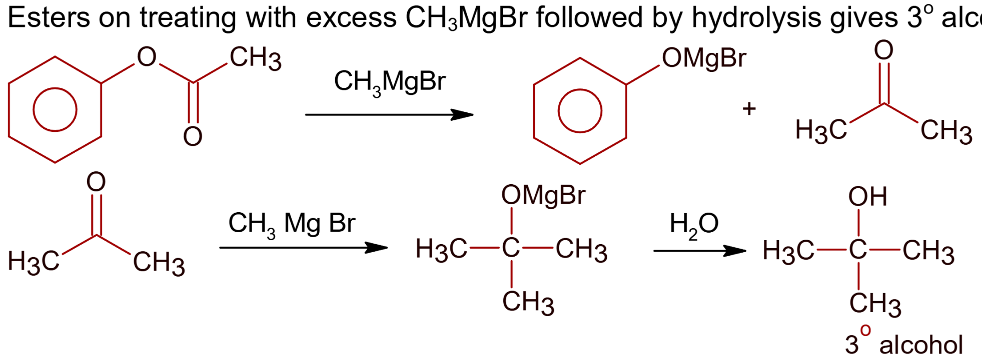
$= 2\hat{k}$

$\cos\theta = \dfrac{\vec{a}.\vec{b}}{ab} = \dfrac{2}{2\sqrt{3}} = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

PART – B (CHEMISTRY)

SECTION - A

Sol1. Esters on treating with excess CH_3MgBr followed by hydrolysis gives 3° alcohol.



Sol2. When $AgNO_3$ is added to KI solution, precipitate of AgI is formed which adsorb I^- ion from
Dispersion medium to give negatively charged sol

$AgNO_3 + KI \rightarrow AgI \downarrow + KNO_3$

AgI/I^- - negatively charged sol.

Sol3. Only tritium is the isotope of hydrogen which is radioactive in nature that emits low energy β^- particles since its $\frac{n}{p}$ ratio is above stability belt.

Sol4. Depression in freezing point will be maximum for $Al_2(SO_4)_3$ since its Van't Hoff factor is highest. So its solution will have lowest freezing point.

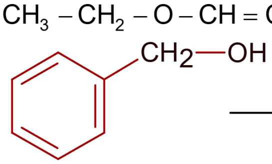
Sol5.

Species	Sigma bonds	lone pairs	Hybridisation
SF_4	4	1	Sp^3d
IF_5	5	1	Sp^3d^2
NO_2^+	2	0	Sp
NH_4^+	4	0	Sp^3

Sol6. $CH_3-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-NH_2 \longrightarrow \pi-\ell p$ conjugation

$CH_3-CH_2-CH=CH-CH_2-NH_2 \longrightarrow$ No conjugation

$CH_3-CH_2-O-CH=CH_2 \longrightarrow \pi-\ell p$ conjugation



$\longrightarrow \pi-\pi$ conjugation in benzene ring

Sol7. (A) $Ni^{2+} - 4s^0 3d^8$ – Paramagnetic & coloured

$Mn^{7+} - 4s^0 3d^0$ – Diamagnetic & colourless

$Hg^{2+} - 6s^0 4f^{14} 5d^{10}$ – Diamagnetic & colourless

(B) $Cu^{+} - 4s^0 3d^{10}$ – Diamagnetic & colourless

$Zn^{2+} - 4s^0 3d^{10}$ – Diamagnetic & colourless

$Mn^{4+} - 4s^0 3d^3$ – Paramagnetic & coloured

(C) $Sc^{3+} - 4s^0 3d^0$

$V^{5+} - 4s^0 3d^0$

$Ti^{4+} - 4s^0 3d^0$

All are diamagnetic & colourless

(D) $Cu^{2+} - 4s^0 3d^9$

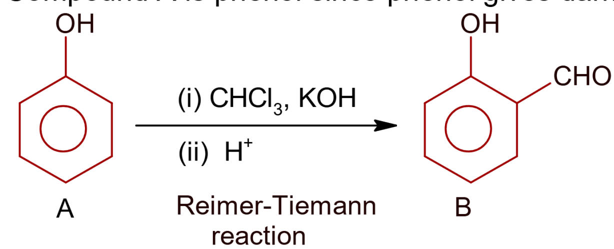
$Cr^{3+} - 4s^0 3d^3$

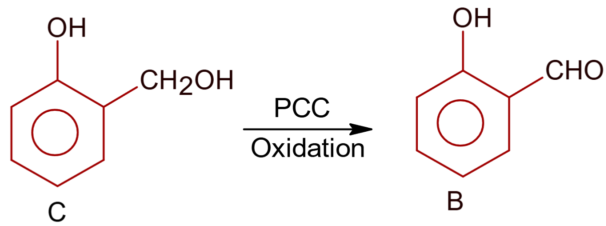
$Sc^{+} - 4s^1 3d^1$

All are paramagnetic & coloured

All d- & f block paramagnetic cations are coloured also.

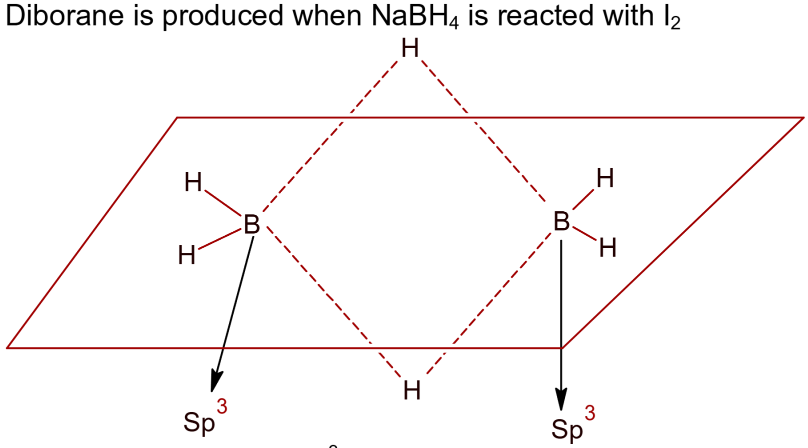
Sol8. Compound A is phenol since phenol gives dark green colour with $FeCl_3$.





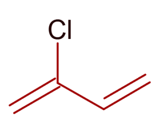
Sol9. $2NaBH_4 + I_2 \longrightarrow 2NaI + H_2 + B_2H_6$

Diborane is produced when $NaBH_4$ is reacted with I_2

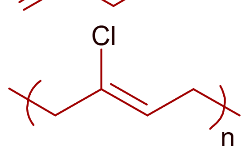


Both boron atoms are sp^3 hybridised & molecule is non- planar. Diborane as two bridged 3 centre-2- electron bonds.

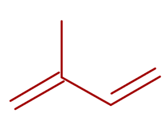
Sol10.



⟶ Chloroprene



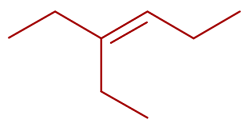
⟶ Neoprene is polymer of chloroprene



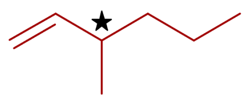
⟶ Isoprene

$CH_2{=}CH{-}CN$ ⟶ Acrylonitrile

Sol11. 3- Ethylhex-3-ene will not show stereo isomerism

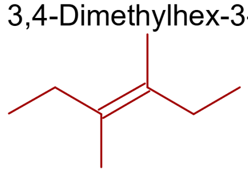


3-Methylhex-1-ene will show steroisomerism (optical isomerism)

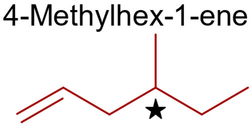


Since it has chiral carbon.

3,4-Dimethylhex-3-ene will show stereoisomerism (Geometrical isomerism)

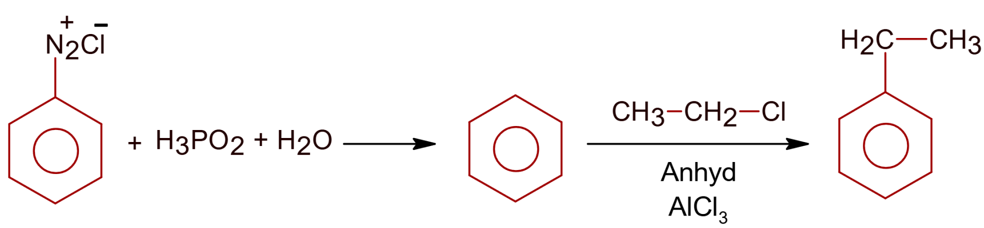


4-Methylhex-1-ene will show stereoisomerism (optical) since it has one chiral carbon



Sol12. Sulphide ion $\left(S^{2-}\right)$ form ores commonly with Pb & Ag as PbS and Ag_2S

Sol13.



So; A is H_3PO_2 & B is CH_3-CH_2-Cl

Sol14. Ba has outer electronic configuration $6s^2$.
CaC_2O_4 is insoluble in water.
Compound of Li are covalent so soluble in organic solvent.
Na forms strong monoacidic base.

Sol15. Reducing power for group-15 hydrides increases down the group, so BiH_3 is strongest reducing agent.

Sol16. Vitamin B_1 is thiamine while vitamin B_6 is pyridoxine

Sol17. Aniline being a base does not give Friedel Craft reaction because it react with $AlCl_3$ used in Friedel Craft reaction.

Sol18. At the time when Mendeleeff proposed periodic table structure of atom was not known.

Sol19. Solubility of gas in liquid increases on decreasing the temperature. So water at 4°C will have more dissolved O_2.

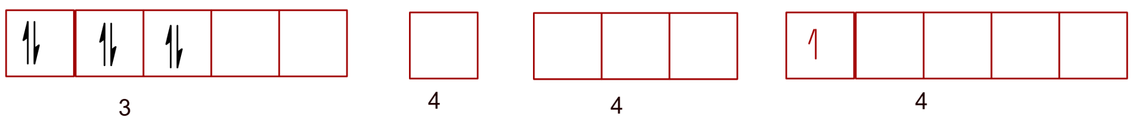
Sol20. Reduced pressure distillation technique is used for purification of high boiling organic liquid which decompose near its boiling point. By reducing pressure they boil below their normal boiling point.

SECTION - B

Sol1. NH_3 behaves as strong field ligand with Co^{2+} & Co^{3+}

$\left[Co(NH_3)_6\right]Cl_2$

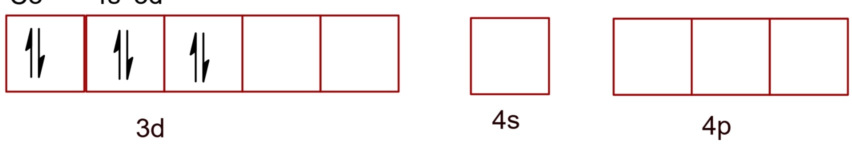
$Co^{2+} - 4s^0 3d^7$



It has one unpaired electron.

$\left[Co(NH_3)_6\right]Cl_3$

$Co^{3+} - 4s^0 3d^6$



It has no unpaired electrons

So, total unpaired electrons in both compounds is 1.

Sol2. For first order reaction, using

$K = \frac{2.303}{t}\log_{10}\frac{[R]_0}{[R]}$

$K = \frac{2.303}{1\times 60}\log_{10}\frac{2.4\times 10^{-2}}{1.6\times 10^{-2}}\,min^{-1}$

$K = \frac{2.303}{60}\log\frac{3}{2}\,min^{-1}$

$K = \frac{2.303}{60}(0.477 - 0.301)\,min^{-1}$

$K = 6.7\times 10^{-3}\,min^{-1}$

Ans. is 7 (nearest integer)

Sol3. ${}_{23}V - 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^3$

So; total electrons in p orbitals are 12.

Sol4. Molar enthalpy of combustion of graphite is -2.48×10^2 kJ/ mol, that means 2.48×10^2 kJ heat generates when combustion of 1 mol graphite takes place.

So, heat generated on combustion of 1g graphite

$= \frac{2.48\times 10^2}{12}\,kJ$

= 20.67 kJ

Ans. is 21 (nearest integer)

Sol5. $K_p = 47.9$ T = 288K

$K_C = ?$ R = 0.083 L bar / K mol

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

Using

$K_p = K_C (RT)^{\Delta ng}$

Here;

$\Delta ng = 1$

$47.9 = K_C (0.083 \times 288)^1$

$K_C = 2$

Sol6. Density, $\rho = 7.62 gcm^{-3}$

Cell edge, $a = 0.4518 nm = 0.4518 \times 10^{-7} cm$

Effective number of atoms, Z = 4 for CCP.

Using ; $\rho = \dfrac{Z \times A}{N_A \times a^3}$

$$\rho = \frac{4 \times A}{6.022 \times 10^{23} \times (0.4518 \times 10^{-7})^3}$$

$$7.62 = \frac{4 \times A}{6.022 \times (0.4518)^3 \times 10^2}$$

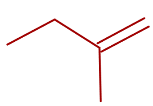
A = 105.79 g/ mol

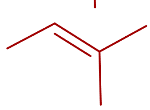
Ans. is 106 (nearest integers)

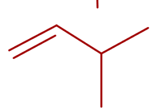
Sol7. Acyclic isomers of pentene are:-

 = 1 No Geometrical isomerism

 = 1+1 Show geometrical isomerism (Cis-trans)

 = 1 No geometrical isomerism

 = 1 No geometrical isomerism

 = 1 No geometrical isomerism

So; total isomers are 6

Sol8. Using; Nernst equation

$$E_{cell} = E^{o}_{cell} - \frac{0.06}{n} \log_{10} \frac{[Cu^{2+}]}{[Ag^{+}]^2}$$

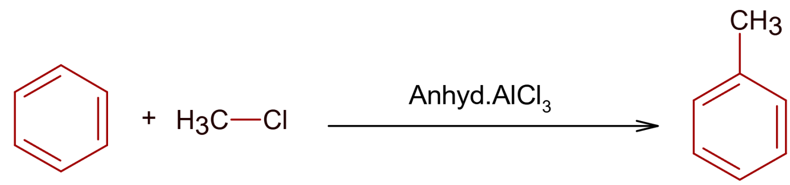
$$E_{cell} = 2.97 - \frac{0.06}{2} \log_{10} \frac{0.25}{10^{-6}} V$$

$= 2.97 - 0.03(\log 2.5 + 5) V$

$= 2.97 - 0.03(0.3979 + 5) V$

$E_{cell} = 2.8V$
Ans is 3 (nearest integer)

Sol9.



Theoretically; 1 mol benzene gives 1 mol toluene

Moles of benzene = $\frac{10}{78}$mol

Moles of toluene (Theoretical) = $\frac{10}{78}$mol

Mole of toluene (observed) = $\frac{9.2}{92} = 0.1\text{mol}$

% yield = $\frac{0.1}{\frac{10}{78}} \times 100$

= 78%

Sol10. Strength = 0.72 g/ L
Using;

Molarity = $\frac{\text{Strength}}{\text{Molar mass of solute}}$

$M = \frac{0.72}{180} M$

= $4 \times 10^{-3} M$
Ans is 4.

PART – C (MATHEMATICS)

SECTION - A

Sol1. $36x^2 + 36y^2 - 108x + 120y + c = 0$

$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{c}{36} = 0$

The circle not interesting either axes

$\therefore\ g^2 < c \;\&\; f^2 < c$

$\Rightarrow \frac{9}{4} < \frac{c}{36} \Rightarrow c > 81 - (i)$ **&** $\frac{25}{9} < \frac{c}{36} \Rightarrow c > 100$ **......(ii)**

$(i) \cap (ii) \Rightarrow c > 100$ **.......(iii)**

Point of intersection between $x - 2y - 4 = 0$ **&** $2x - y - 5 = 0$ **is** $(2 - 1)$ **lies inside the circle.**

$\therefore\ 4 + 1 - 6 - \frac{10}{3} + \frac{c}{36} < 0 \Rightarrow c < 156$ **........(iv)**

$(iii) \cap (iv) \Rightarrow 100 < c < 156$

Sol2. $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{3}$ **....... (i)**

$\frac{x-2}{1}=\frac{y-\lambda}{2}=\frac{z-3}{4}$ **.........(ii)**

$\overrightarrow{OA}=(i+2j+k)\ \overrightarrow{OB}=\left(2\hat{i}+\lambda\hat{j}+3\hat{k}\right)$

$\vec{a}=2i+j+3k\ \&\ \vec{b}=i+2j+4k$

$\vec{a}\times\vec{b}=-2i-5j+3k$

$\therefore$ **shortest distance =** $\left|\frac{\left(\overrightarrow{OB}-\overrightarrow{OA}\right).\left(\vec{a}\times\vec{b}\right)}{\vec{a}\times\vec{b}}\right|=\frac{1}{\sqrt{38}}$

$\Rightarrow |14-5\lambda|=1$

$\Rightarrow \lambda=3\ \&\ \frac{13}{5}$

Integer value of $\lambda=3$

Sol3. $z^2+3\bar{z}=0$

Let $z=x+iy$

$\Rightarrow\ \left(x^2-y^2+2ixy\right)+3(x-iy)=0$

$\Rightarrow\left(x^2-y^2+3x\right)+i(2xy-3y)=0$

$\Rightarrow 2xy-3y=0$

$\Rightarrow\ y=0$ **or** $x=\frac{3}{2}$

For $y=0$

$x^2+3x=0$

$x(x+3)=0$

$x=0,\quad x=-3.$

$\therefore\ z=0,3i$

For $x=\frac{3}{2}$

$y^2=\frac{9}{4}+\frac{9}{2}=\frac{27}{4}.$

$y=\pm\frac{3\sqrt{3}}{2}$

$z=\frac{3}{2}\left(1\pm\sqrt{3}i\right)$

$\therefore$ the equation has $n=4$ solution .

$\therefore\sum_{k=0}^{\infty}\frac{1}{n^k}=1+\frac{1}{4}+\frac{1}{4^2}+--\infty=\frac{4}{3}$

Sol4. $\int_0^{100\pi}\frac{\sin^2 x\,dx}{e^{\left\{\frac{x}{\pi}\right\}}}=I$

$f(x)=\frac{\sin^2 x}{e^{\left\{x/\pi\right\}}}$ **is periodic with period =** π

$I=100\int_0^{\pi}\frac{\sin^2 x}{e^{x/\pi}}dx.$

$=50\int_0^{\pi}\frac{1-\cos 2x}{e^{x/\pi}}$ **dx =** $50\int_0^{\pi}e^{-x/\pi}dx-50\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx$

$$= 50\frac{e^{-x/\pi}}{-1/\pi} - \int_0^{\pi} 50\left[\frac{e^{-x/\pi}\left(-\frac{1}{\pi}\cos 2x + 2\sin 2x\right)}{\frac{1}{\pi^2}+4}\right]_0^{\pi}$$

$$= -50\pi\left(e^{-1}-1\right) - \frac{50\left[e^{-1}\left(\frac{1}{\pi}\right)-1\left(-\frac{1}{\pi}\right)\right]}{\frac{1}{\pi^2}+4.}$$

$$= 50\pi\left(1-\frac{1}{e}\right) - \frac{50\pi^2}{1+4\pi^2}\times\frac{1}{\pi}\left(1-\frac{1}{e}\right)$$

$$= 50\pi\left(1-\frac{1}{e}\right)\left[1-\frac{1}{1+4\pi^2}\right] = \frac{50\left(1-\frac{1}{e}\right)4\pi^3}{1+4\pi^2}$$

$$\therefore\ \alpha = 200\left(1-\frac{1}{e}\right)$$

Sol5. $\left[e^x\right]^2 + \left[e^x\right] - 2 = 0$

Let $\left[e^x\right] = t$

$\Rightarrow t^2 + t - 2 = 0 \ \Rightarrow\ (t+2)(t-1) = 0$

$t = -2 \ \Rightarrow\ \left[e^{-x}\right] = -2$ **not possible**

$\therefore\ t = \left[e^x\right] = 1 \qquad \Rightarrow 1 \le e^x < 2$

$\Rightarrow\ 0 \le x < \ln 2$

Sol6. **Option (A)**

p	**q**	**~p**	**~q**	$\sim p \Rightarrow q$	$\sim q \Rightarrow p$	$q \Rightarrow p$	$(\sim p \Rightarrow q)\vee(\sim q \Rightarrow p)$
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**	**F**

Option (B)

p	**q**	**~p**	**~q**	$\sim p \Rightarrow q$	$\sim q \Rightarrow p$	$q \Rightarrow p$	$(q \Rightarrow p)\vee(\sim q \Rightarrow p)$
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**	**T**
F	**T**	**T**	**F**	**T**	**T**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**	**T**

Option (C)

p	**q**	**~p**	**~q**	$p \Rightarrow \sim q$	$\sim q \Rightarrow p$	$q \Rightarrow p$	$(p \Rightarrow \sim q)\vee(\sim q \Rightarrow p)$
T	**T**	**F**	**F**	**F**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**T**	**T**

F	**T**	**T**	**F**	**T**	**T**	**F**	**T**
F	**F**	**T**	**T**	**T**	**F**	**T**	**T**

Option (D)

p	**q**	**~p**	**~q**	$\sim p \Rightarrow q$	$\sim q \Rightarrow p$	$p \Rightarrow q$	$(p \Rightarrow q) \vee (\sim q \Rightarrow p)$
T	**T**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**F**	**T**
F	**T**	**T**	**F**	**T**	**T**	**T**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**	**T**

Sol7. $L : 2x + y = k.$

$y = -2x + k.$ is tangent $\frac{x^2}{3} - \frac{y^2}{3} = 1$

$\Rightarrow k^2 = 3(-2)^2 - 3 = 9 \;\Rightarrow\; k = 3$ as $k > 0$

$y = -2x + 3$ is also tangent to $y^2 = 4\left(\frac{\alpha}{4}\right)x$

$\Rightarrow\; 3 = \frac{\alpha / 4}{-2} \qquad\qquad \Rightarrow\; \alpha = -24$

Sol8. $\operatorname{cosec}^2 x\, dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x\, dx.$

$\Rightarrow \frac{dy}{dx} + 2\sin^2 x = 1 + y\cos 2x.$

$\Rightarrow \frac{dy}{dx} - y\cos 2x = 1 - 2\sin^2 x = \cos 2x.$

I.F. = $e^{-\int \cos 2x\, dx} = e^{-\frac{\sin 2x}{2}}$

$\therefore$ **Solution** $ye^{-\frac{\sin 2x}{2}} = \int e^{-\frac{\sin 2x}{2}}.\cos 2x\, dx.$ **Put** $\frac{\sin 2x}{2} = t \Rightarrow\; \cos 2x\, dx = dt$

$ye^{-\frac{\sin 2x}{2}} = \int e^{-t} dt = -e^{-t} + c.$

$ye^{-\frac{\sin 2x}{2}} = -e^{-\frac{\sin 2x}{2}} + c$

$y = -1 + c\,e^{\frac{\sin 2x}{2}}$

For $x = \frac{\pi}{4}, y = 0$

$\Rightarrow 0 = -1 + c\,e^{1/2} \Rightarrow c = \frac{1}{e^{1/2}} = e^{-1/2}$

$y = -1 + e^{-\frac{1}{2}}.e^{\frac{\sin 2x}{2}}$

$y(0) = -1 + e^{-1/2} \Rightarrow (y(0) + 1)^2 = \left(e^{-1/2}\right)^2 = e^{-1}$

Sol9. $f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3x e^x, & x \le 0 \end{cases}$

$$f'(x)=\begin{cases}-4x^2+4x+3 & x>0\\ 3 & x=0\\ 3\left(xe^x+e^x\right) & x<0\end{cases}$$

$f'(x)>0$ for $x<0$ $e^x(x+1)>0 \Rightarrow x>-1 \Rightarrow x\in(-1,0)$

for $x>0$ $-4x^2+4x+3>0$

$\Rightarrow x\in\left(0,3/2\right)$

Again at $x=0$ $f'(0)=3>0$

$\therefore$ $f(x)$ **is increasing in** $\left(-1,3/2\right)$

Sol10. $\vec{a}=\alpha\vec{b}+\beta\vec{c}=(2\alpha+\beta)i+(\alpha-\beta)J+(\alpha+\beta)k$

i**s perpendicular to** $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$

$\Rightarrow 3(2\alpha+\beta)+2(\alpha-\beta)+6(\alpha+\beta)=0$

$\Rightarrow 14\alpha+7\beta=0 \Rightarrow \beta=-2\alpha$ **.......(i)**

$\Rightarrow$ $\vec{a}=3\alpha j-\alpha k$

$|\vec{a}|=\sqrt{9\alpha^2+\alpha^2}=\sqrt{10}\Rightarrow|\alpha|=1\Rightarrow\alpha=\pm1$, **for** $\alpha=1,\vec{a}=3\hat{j}-\hat{k}$

for $\alpha=-1,\vec{a}=-3\hat{j}+\hat{k}$

$\left[\vec{a}\vec{b}\vec{c}\right]+\left[\vec{a}\vec{b}\vec{d}\right]+\left[\vec{a}\vec{c}\vec{d}\right]$

= $0+\left[\vec{d}\vec{a}\vec{b}\right]+\left[\vec{d}\vec{a}\vec{c}\right]$

= $\left(\vec{d}\times\vec{a}\right).\vec{b}+\left(\vec{d}\times\vec{a}\right).\vec{c}$

= $\left(\vec{d}\times\vec{a}\right).\left(\vec{b}+\vec{c}\right)$

$$\vec{d}\times\vec{a}=\pm\begin{vmatrix}i & j & k\\ 3 & 2 & 6\\ 0 & 3 & -1\end{vmatrix}$$

$\therefore\left(\vec{d}\times\vec{a}\right).\left(\vec{b}+\vec{c}\right)\pm\left(-20\hat{i}+3\hat{j}+9\hat{k}\right).\left(3\hat{i}+2\hat{k}\right)$

$=\pm42$

Sol11. $S_{10}=530\Rightarrow$ $5[2a+9d]=530$

$2a+9d=106$ ....(i)

$S_5=140\Rightarrow$ $\frac{5}{2}[2a+4d]=140$

$a+2d=28$ **...........(ii)**

Solving (i) & (ii) a = 88 d = 10

$\therefore$ $S_{20}-S_6=14a+175d=1862$

Sol12. $f:R\to R.$

$$f(x)=\begin{cases}\dfrac{x^3}{(1-\cos 2x)^2}\log\left(\dfrac{1+2xe^{-2x}}{\left(1-xe^{-x}\right)^2}\right) & x\neq0\\ \alpha & x=0\end{cases}$$

As f is continuous at x = 0

$\therefore\ \alpha = \lim_{x\to 0} f(x)$

= $\lim_{x\to 0} \dfrac{x^3}{4x^4\left(\dfrac{\sin x}{x}\right)^4}\left[\dfrac{\ell n\left(1+2xe^{-2x}\right)\times\left(2xe^{-2x}\right)}{2xe^{-2x}}\dfrac{-2\log\left(1-xe^{-x}\right)}{-xe^{-x}}\left(-xe^{-x}\right)\right]$

= $\text{Let}_{x\to 0} \dfrac{1}{4x}\times 2x\left[e^{-2x}+e^{-x}\right]$

= $\lim_{x\to 0} \dfrac{1}{2}\left[e^{-2x}+e^{-x}\right] = \dfrac{1}{2}\times 2 = 1$

$\therefore\ \alpha = 1$

Sol13. $\sin^7 x + \cos^7 x = 1,\ x \in [0, 4\pi]$

will satisfy for $\sin x = 1, \cos x = 0$

$\Rightarrow x = \dfrac{\pi}{2}\ \&\ \dfrac{5\pi}{2}.$

or, $\cos x = 1,\ \sin x = 0$

$x = 0, 2\pi, 4\pi$ **total 5 solutions**

Sol14. $\vec{a}\times\vec{b} = \vec{c}$ **- (i)**

$\vec{b}\times\vec{c} = \vec{a}$ **- (ii)** $|\vec{a}| = 2$

Taking dot product with $\vec{c}\ \&\ \vec{a}$ **respectively in** (i) & (ii) **we have**

$\left[\vec{c}\,\vec{a}\,\vec{b}\right] = |\vec{c}|^2$

$\left[\vec{a}\,\vec{b}\,\vec{c}\right] = |\vec{a}|^2 = 4$

$\therefore\ |\vec{c}|^2 = |\vec{a}|^2 = 4 \quad \Rightarrow |\vec{c}| = 2$

Again (i) & (ii) $\Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Opt 1. Projection of $\vec{a}$ **on** $\vec{b}\times\vec{c} = \dfrac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|} = \dfrac{4}{2} = 2$

Opt 2. (2) $\left[\vec{a}\,\vec{b}\,\vec{c}\right] + \left[\vec{c}\,\vec{a}\,\vec{b}\right] = 2\left[\vec{a}\,\vec{b}\,\vec{c}\right] = 8$

(1) $\Rightarrow \vec{b}\times\left(\vec{a}\times\vec{b}\right) = \vec{b}\times\vec{c}$

$\Rightarrow |\vec{b}|^2\vec{a} - \left(\vec{b}.\vec{a}\right)\vec{b} = \vec{b}\times\vec{c} = |\vec{b}|^2|\vec{a}| = \left|\vec{b}\times\vec{c}\right| \Rightarrow |\vec{b}| = 1$

Opt 3. $\left|3\vec{a}+\vec{b}-2\vec{c}\right|^2 = 9|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 + 2\left(3\vec{a}.\vec{b} - 6\vec{a}.\vec{c} - 2\vec{b}.\vec{c}\right)$

$= 36 + 1 + 16 + 2(0) = 53$

Opt 4. $\vec{a}\times\left(\vec{c}\times\vec{b} - \vec{b}\times\vec{c}\right)$

$= 2\vec{a}\times\left(\vec{c}\times\vec{b}\right) = 2\left\{\left(\vec{a}.\vec{b}\right)\vec{c} - \left(\vec{a}.\vec{c}\right)\vec{b}\right\} = 0$

Sol15. $\vec{r}.\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 2$

& $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=2$ are two planes the direction ratio of the line of intersection of them is collinear to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix} = -\hat{i}+5\hat{j}+3\hat{k}$

Any point on the line in given by $x-y=2$

& $2x+y=2$

$\Rightarrow x = 4/3,\ y = -2/3,\ z = 0$

$\therefore$ **equation of line** $L : \dfrac{x-4/3}{-1} = \dfrac{y+2/3}{5} = \dfrac{z}{3} = r$

Any point on this line is $P\left(-r+\dfrac{4}{3}, 5r-2/3, 3r\right)$

$A(1,2,0)$ $\therefore$ **d.r. of AP** $= -r+\dfrac{1}{3}, 5r-\dfrac{8}{3}, 3r$

AP is $\perp$ r to L $\therefore$ $-1\left(-r+\dfrac{1}{3}\right)+5\left(5r-\dfrac{8}{3}\right)+3(3r)=0$

$35r = \dfrac{41}{3}. \Rightarrow r = \dfrac{41}{3\times 35}$

$\therefore$ **point P** $\left(\dfrac{33}{35}, \dfrac{45}{35}, \dfrac{41}{35}\right) \equiv (\alpha, \beta, \gamma)$

$\therefore$ $35(\alpha+\beta+\gamma) = 119$

Sol16. $A = \left[a_{ij}\right]_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$a_{i1}+a_{i2}+a_{i3} = 1;\ i = 1,2,3$

Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ them

given $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow AX = X$ ....(i)

replace x by A x we have

$A(AX) = AX$

$\Rightarrow A^2X = AX = X$ **................(ii)**

Again replace X by AX

$A^3X = AX = X.$

As $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, **Sum of all entries in** A^3 = **sum of entries in** $X = 1+1+1 = 3$

Sol17. $f(x)=\dfrac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\dfrac{2x-1}{2}\right)}}$

For domain $0\le x^2-x+1\le 1$

& $x^2-x+1\ge 0 \quad \forall x\in R.$

$x^2-x\le 0$ **&** $x(x-1)\le 0\Rightarrow x\in[0,1]$**.........(i)**

Again $0\le\dfrac{2x-1}{2}\le 1$

$\Rightarrow \quad 0<2x-1\le 2$

$\Rightarrow \quad \dfrac{1}{2}\le x\le 3/2$ **....................(ii)**

(i) $\cap$ (ii) $x\in\left(\dfrac{1}{2},1\right]\equiv(\alpha,\beta]$

then $\alpha+\beta=3/2$

Sol18. System of equations can be written as

$$\begin{pmatrix}1&1&1\\3&5&5\\1&2&\lambda\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\26\\\mu\end{pmatrix}$$

$R'_3=R_3-R_1, R'_2=R_2-5R_1\Rightarrow$

$$\begin{pmatrix}1&1&1\\-2&0&0\\0&1&\lambda-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\-4\\\mu-6\end{pmatrix}$$

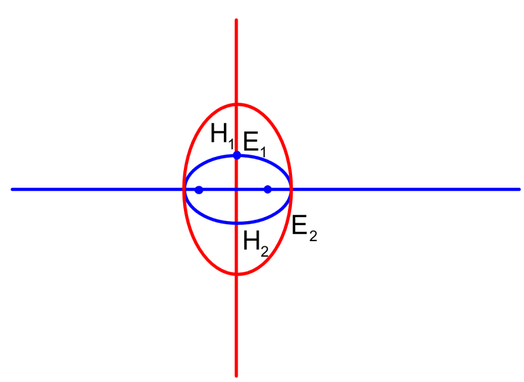
$$R''_1=R'_1+\frac{1}{2}R'_2\Rightarrow\begin{pmatrix}0&1&1\\-2&0&0\\0&1&\lambda-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\-4\\\mu-6\end{pmatrix}$$

Again $R''_3=R'_3-R'_1\Rightarrow\begin{pmatrix}0&1&1\\-2&0&0\\0&0&\lambda-2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\-4\\\mu-10\end{pmatrix}$

The system will have no solution for

$\lambda=2$ **and** $\mu\ne 10$.

Sol19. $E_1:\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$



$e_1^2=1-\dfrac{b^2}{a^2}$ - (i)

Let $E_2:\dfrac{x^2}{a^2}+\dfrac{y^2}{B^2}=1$

$B>a$

$e_2^2=1-\dfrac{a^2}{B^2}$ and given $Be_2=b$

$\Rightarrow \quad B^2e_2^2=B^2-a^2$

$\Rightarrow \quad b^2+a^2=B^2$

$$\therefore\ e_2^2 = 1-\frac{a^2}{a^2+b^2}=\frac{b^2}{a^2+b^2}=\frac{b^2/a^2}{1+b^2/a^2}$$

$$\Rightarrow e_2^2=\frac{1-e_1^2}{2-e_1^2}\quad \text{given } e_1=e_2=e\ (\text{say})$$

$$\therefore\ e^2=\frac{1-e^2}{2-e^2}\Rightarrow e^4-3e^2+1=0$$

$$e^2=\frac{3-\sqrt{5}}{2}=\left(\frac{\sqrt{5}-1}{2}\right)^2$$

$$\Rightarrow e=\frac{\sqrt{5}-1}{2}$$

Sol20. **All entries different which can be selected as $^6C_4$ ways**

there arrangement in matrix in $^6C_4\times 4!$ ways

Let $A=\begin{pmatrix} a & b\\ c & d\end{pmatrix}$ **be such matrix**

$|A|=ad-bc$

Now $|A|=0\Rightarrow\ ad-bc=0$ **cases**

1, 6	3, 2	$2\times2\times2$
2, 6	3, 4	$2\times2\times2$

as $\underbrace{\underbrace{1,6}_{2!}\ \ \underbrace{3,2}_{2!}}_{2!}$ total arrangement in one case $=2\times2\times2=8$ ways

$\therefore$ **For such non singular matrix required probability** $=\dfrac{^6C_4\times4!-8-8}{6^4}=\dfrac{43}{162}$

SECTION - B

Sol1. $f:A\rightarrow A$ **is bijective.**

where $A=\{0,1,2,3,4,5,6,7\}$

and $f(1)+f(2)+f(3)=3$

$\rightarrow$	0	1	2		3! ways
	$f(0)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$
	3	4	5	6	7 $= 5!$ ways

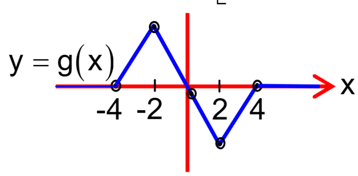
Total such required functions $=3!\times5!=720$

Sol2. $f(x)=\begin{cases}3\left(1-\dfrac{|x|}{2}\right) & ;\ |x|\le 2\\ 0 & ;\ |x|>2\end{cases}$

$$f(x+2)=\begin{cases}3\left(1-\dfrac{|x+2|}{2}\right) & ;\quad |x+2|\le 2\Rightarrow -4\le x\le 0\\ 0 & ;\quad |x+2|>2\Rightarrow x\in(-\infty,-4)\cup(0,\infty)\end{cases}$$

$$f(x-2)=\begin{cases}3\left(1-\frac{|x-2|}{2}\right) & ; \quad |x-2|\le 2 \Rightarrow 0\le x\le 4.\\ 0 & ; \quad |x-2|>2 \Rightarrow x<0 \text{ or } x>4\end{cases}$$

$$\therefore g(x)=f(x+2)-f(x-2) \quad \begin{cases}0 & x<-4\\ 3\left(1-\frac{|x+2|}{2}\right) & -4\le x\le 0\\ -3\left(1-\frac{|x-2|}{2}\right) & 0<x\le 4\\ 0 & x>4.\end{cases}$$



$g(x)$ **is continuous every where but not differentiable at** $x=-4,-2,2$ and $4.$

$\therefore \quad n=0 \quad \& m=4 \Rightarrow n+m=4$

Sol3. $\left\{(x+2)e^{\frac{y+1}{x+2}}+(y+1)\right\}dx=(x+2)dy.$

$\Rightarrow \dfrac{dy}{dx}=e^{\left(\frac{y+1}{x+2}\right)}+\left(\dfrac{y+1}{x+2}\right)$ **- (i)**

Let $y+1=v(x+2) \Rightarrow \dfrac{dy}{dx}=v+(x+2)\dfrac{dv}{dx}.$

(1) $\Rightarrow v+(x+2)\dfrac{dv}{dx}=e^v+v.$

$\Rightarrow \dfrac{dv}{e^v}=\dfrac{dx}{x+2}$ **Integrating both side**

$\Rightarrow -e^{-v}=\ell n|x+2|+c$

$\Rightarrow -e^{-\frac{y+1}{x+2}}=\ell n|x+2|+c$

$y(1)=1$

$\Rightarrow -e^{-2/3}=\ell n3+c$

$\Rightarrow c=-e^{-2/3}-\ell n3$

$\therefore \dfrac{1}{e^{\left(\frac{y+1}{x+2}\right)}}=\ell n3+e^{-2/3}-\ell n|x+2|$

$\Rightarrow e^{\frac{-2}{3}}+\ln 3-\ln|x+2|>0$ as $e^{-\left(\frac{y+1}{x+2}\right)}>0$

$\Rightarrow \ln\left|\dfrac{x+2}{3}\right|<e^{-\frac{2}{3}}$

$\Rightarrow 0<\left|\dfrac{x+2}{3}\right|<e^{e^{-2/3}}$

Let $e^{e^{-2/3}} = a \Rightarrow 0 < \left|\frac{x+2}{3}\right| < a$

$\Rightarrow \ -a < \frac{x+2}{3} < a \quad a \neq -2$

$\Rightarrow -3a - 2 < x < 3a - 2$

$\therefore \ \alpha = -3a - 2 \qquad \& \ \beta = 3a - 2$

$\Rightarrow \ |\alpha + \beta| = 4$

Sol4. $11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow 2\left[{}^{n}c_1 10^{n-1} + n_{C_3} 10^{n-3} + ---\right] > 10^{n.}$

$\Rightarrow$ **For** $n \geq 5$ **the inequality satisfies.**

For $n = 4$ **.** $\qquad 2\left[4 \times 10^3 + 4 \times 10\right] > 10^4$

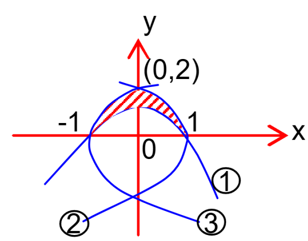
$\Rightarrow \ 8 \times 1010 > 10^4$ **which is a contradiction**

$\therefore \ n \geq 5$ all values satisfies. Hence possible values of n is 96

Sol5. $x^2 = -2\left(y - \frac{1}{2}\right)$

$y^2 = -4(x-1)$

$y^2 = 4(x+1)$



Required area $= 2\int_{-1}^{0}\left[2\sqrt{x+1} - \frac{1-x^2}{2}\right]dx$

$= 2\left[\frac{4}{3}(x+1)^{3/2} - \frac{1}{2}\left(x - \frac{x^3}{3}\right)\right]_{-1}^{0}$

$= \ 2$

Sol6. $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3}$

B is a matrix of same order with entries from $\{1,2,3,4,5\}$. and satisfying $AB = BA$.

Let B $= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

$\therefore \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

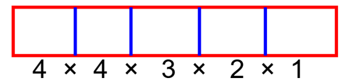
$\Rightarrow \begin{pmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} a_{12} & a_{11} & a_{13} \\ a_{22} & a_{21} & a_{23} \\ a_{32} & a_{31} & a_{33} \end{pmatrix}$

$a_{21} = a_{12},\ a_{22} = a_{11},\ a_{23} = a_{13},\ a_{31} = a_{32}$

$\therefore$ **there exist only 5 distinct entries in the matrix B so that possible case** $\quad$ **=** $5^5 = 3125$

Sol7. 1st position can be filled in 4 ways as zero cannot appear in 1^{st} position.
2^{nd} position can be filled in 4 ways and so on.
Total cases $= 4 \times 4 \times 3 \times 2 \times 1 = 96$

0, 2, 4, 6, 8



Sol8. $2040 = 2^3 \times 3 \times 5 \times 17.$

If HCF between $\{n \,\&\, 2040\} = 1$

$\therefore$ **n can not be multiple of 2, 3, 5, 17.**

Let $S(n)$ denote sum of numbers divisible by n.

$$S(2) = 2 + 4 + ----+100 = 2 \times \frac{50 \times 51}{2} = 50 \times 51$$

$$S(3) = 3 + 6 + ---+99 = 3 \times \frac{33 \times 34}{2} = 33 \times 51$$

$$S(5) = 5 + 10 ---+100 = 5 \times \frac{20 \times 21}{2} = 50 \times 21$$

$$S(17) = 17 + 34 + --+85 = 17 \times \frac{5 \times 6}{2} = 5 \times 51$$

$$S(6) = 6 + 12 + ---- = 6 \times \frac{16 \times 17}{2} = 16 \times 51$$

$$S(10) = 10 + 20 + ---- = 10 \times \frac{10 \times 11}{2} = 50 \times 11$$

$$S(34) = 34 + 68 = 102$$

$$S(15) = 15 + 30 + --- + 90 = 15 \times \frac{6 \times 7}{2} = 45 \times 7$$

$$S(51) = 51$$

$S(85) = 85, S(30) = 30 + 60 + 90 = 180$ **S(all other combinations) = 0**

$\therefore$ **Sum of all numbers which are either divisible by 2,3,5, or 17 is**

$= 50 \times 51 + 33 \times 51 + 50 \times 21 + 5 \times 51$

$-[16 \times 51 + 50 \times 11 + 102 + 45 \times 7 \times 51 + 85] + 180 \;\; = 3799$

Again sum of all number of $\{1, 2, ---100\} = \frac{100 \times 101}{2} = 5050$

$\therefore$ **Sum of required number** $= 5050 - 3799 = 1251$

Sol9. $\left(2x^r + \frac{1}{x^2}\right)^{10}$

Let the constant term is $(k+1)^{th}$ term.

$\therefore \;\; {}^{10}C_k \left(2x^r\right)^{10-k} . \, x^{-2k}$

$= {}^{10}C_k \, 2^{10-k} . x^{10r - rk - 2k}$

For constant term $10r - rk - 2k = 0.$

$\therefore \;\; k = \frac{10r}{r+2} \;\; k \in I$

$\therefore r = 3, k = 6$ **and** $r = 8, k = 8.$

For $k = 6$ ${}^{10}C_k \, 2^4 \neq 180.$

For $k = 8$ ${}^{10}C_k \, 2^2 = 180$

$\therefore \;\; r = 8$

Sol10.

Class	Frequency	x_i	$x_i f_i$	
0 – 6 6 – 12 12 – 18 18 – 24 24 - 30	a b 12 9 5 a+b+26 = N	3 9 15 21 27	3a 9b 180 189 135	Mean $\bar{x} = \frac{\sum x_i f_i}{N}$ $\Rightarrow \frac{309}{22} = \frac{3a+9b+504}{a+b+26}$ $\Rightarrow 81a + 37b = 1018$ **- (i)**

For Median $= L + \frac{\frac{N}{2} - c,}{f} xh$

$$14 = 12 + \frac{\frac{a+b}{2} + 13 - (a+b)}{12} \times 6$$

$\Rightarrow a + b = 18$ **- (ii)**

Solving (i) & (ii) $a = 8$ **&** $b = 10$

$\Rightarrow (a-b)^2 = 4$