JEE Main 2023 Jan 25 Shift 2 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

MATHEMATICS

Section-A

1. Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is:

(1) $\left(\frac{9}{2}, \infty\right)$

(2) $\left(0, \frac{9}{2}\right)$

(3) $\left(-\infty, \frac{9}{2}\right)$

(4) $\left(-\frac{9}{2}, \frac{9}{2}\right)$

Correct Answer: (3) $\left(-\infty, \frac{9}{2}\right)$.

Solution:

To find the values of p for which the function $f(x)$ has both a maximum at some $x < 0$ and a minimum at some $x > 0$, we begin by analyzing the first derivative of $f(x)$.

First, we compute the first derivative:

$$f'(x) = \frac{d}{dx}\left(2x^3 + (2p-7)x^2 + 3(2p-9)x - 6\right) = 6x^2 + 2(2p-7)x + 3(2p-9)$$

For $f(x)$ to have both a maximum and a minimum, the equation $f'(x) = 0$ must have two distinct real roots, one positive and one negative.

1. Discriminant Condition:

The discriminant D of the quadratic equation $6x^2 + 2(2p-7)x + 3(2p-9) = 0$ must be positive for two distinct real roots:

$$D = [2(2p-7)]^2 - 4 \cdot 6 \cdot 3(2p-9) = 4(2p-7)^2 - 72(2p-9) > 0$$

Simplifying:

$$4(4p^2 - 28p + 49) - 144p + 648 > 0$$

$16\text{p}^2 - 112p + 196 - 144p + 648 > 0$

$16\text{p}^2 - 256p + 844 > 0$

$4\text{p}^2 - 64p + 211 > 0$

Solving the inequality $4p^2 - 64p + 211 > 0$, we find the critical points and determine the intervals where the inequality holds.

2. Product of Roots Condition:

For the roots to have opposite signs, the product of the roots must be negative:

$$\text{Product of roots} = \frac{3(2p-9)}{6} = \frac{6p-27}{6} = p - \frac{9}{2} < 0$$

$\Rightarrow p < \frac{9}{2}$

By combining both conditions, the set of all values of p that satisfy both is:

$$p \in \left(-\infty, \frac{9}{2}\right)$$

Thus, the correct answer is option (3).

Quick Tip

When analyzing extrema of polynomial functions, always examine the first derivative for critical points and ensure the existence of necessary conditions (like distinct real roots with appropriate signs) to confirm the presence of maxima and minima.

---

2. Let z be a complex number such that $\frac{|z-2i|}{|z+i|} = 2$, $z \neq -i$. Then z lies on the circle of radius 2 and centre:

(1) $(2, 0)$

(2) $(0, 0)$

(3) $(0, 2)$

(4) $(0, -2)$

Correct Answer: (4) $(0, -2)$.

Solution:

We are given the condition:

$$\frac{|z-2i|}{|z+i|} = 2$$

Let $z = x + yi$, where x and y are real numbers. Substituting this into the equation:

$$\frac{\sqrt{x^2 + (y-2)^2}}{\sqrt{x^2 + (y+1)^2}} = 2$$

Squaring both sides to eliminate the square roots:

$$\frac{x^2 + (y-2)^2}{x^2 + (y+1)^2} = 4$$

Cross-multiply:

$$x^2 + (y-2)^2 = 4\left(x^2 + (y+1)^2\right)$$

Expanding both sides:

$$x^2 + y^2 - 4y + 4 = 4x^2 + 4y^2 + 8y + 4$$

Move all terms to one side:

$$x^2 + y^2 - 4y + 4 - 4x^2 - 4y^2 - 8y - 4 = 0$$

-3x$^2 - 3y^2 - 12y = 0$

$3x^2 + 3y^2 + 12y = 0$

$x^2 + y^2 + 4y = 0$ Next, complete the square for the y-terms to express this in standard circle form:

$$x^2 + y^2 + 4y = 0$$

$x^2 + (y^2 + 4y + 4) = 4$

$x^2 + (y + 2)^2 = 4$ This represents a circle with center $(0, -2)$ and radius 2.

Thus, the correct answer is option (4).

Quick Tip

When working with complex numbers and geometric loci, converting the given conditions into Cartesian coordinates and completing the square can help in identifying the geometric figure and its properties.

---

3. If the function

$$f(x) = \begin{cases} (1 + |\cos x|)^{\lambda/|\cos x|}, & 0 < x < \dfrac{\pi}{2} \\ \mu, & x = \dfrac{\pi}{2} \\ e^{\dfrac{\cot 6x}{\cot 4x}}, & \dfrac{\pi}{2} < x < \pi \end{cases}$$

is continuous at $x = \dfrac{\pi}{2}$, then $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda}$ is equal to:

(1) 11

(2) 8

(3) $2e^4 + 8$

(4) 10

Correct Answer: (4) 10.

Solution:

To ensure that the function $f(x)$ is continuous at $x = \dfrac{\pi}{2}$, the left-hand and right-hand limits as x approaches $\dfrac{\pi}{2}$ must both be equal to $f\left(\dfrac{\pi}{2}\right) = \mu$.

1. Left-Hand Limit ($x \to \dfrac{\pi}{2}^-$):

$$\lim_{x \to \frac{\pi}{2}^-} (1 + |\cos x|)^{\frac{\lambda}{|\cos x|}} = \lim_{x \to \frac{\pi}{2}^-} (1 + \cos x)^{\frac{\lambda}{\cos x}}$$

As $x \to \frac{\pi}{2}^-$, $\cos x \to 0^+$, so:

$$(1+\cos x)^{\frac{\lambda}{\cos x}} \approx e^{\lambda}$$

Hence,

$$\lim_{x \to \frac{\pi}{2}^-} f(x) = e^{\lambda}$$

2. Right-Hand Limit ($x \to \frac{\pi}{2}^+$):

$$\lim_{x \to \frac{\pi}{2}^+} e^{\frac{\cot 6x}{\cot 4x}} = e^{\lim_{x \to \frac{\pi}{2}^+} \frac{\cot 6x}{\cot 4x}}$$

Simplifying the exponent:

$$\frac{\cot 6x}{\cot 4x} = \frac{\frac{\cos 6x}{\sin 6x}}{\frac{\cos 4x}{\sin 4x}} = \frac{\cos 6x \cdot \sin 4x}{\cos 4x \cdot \sin 6x}$$

As $x \to \frac{\pi}{2}^+$:

$$6x \to 3\pi \Rightarrow \cos 6x = \cos 3\pi = -1, \quad \sin 6x = \sin 3\pi = 0$$

$$4x \to 2\pi \Rightarrow \cos 4x = \cos 2\pi = 1, \quad \sin 4x = \sin 2\pi = 0$$

Applying L'Hôpital's Rule to the indeterminate form:

$$\lim_{x \to \frac{\pi}{2}^+} \frac{\cot 6x}{\cot 4x} = \lim_{x \to \frac{\pi}{2}^+} \frac{-\csc^2 6x \cdot 6}{-\csc^2 4x \cdot 4} = \lim_{x \to \frac{\pi}{2}^+} \frac{6 \sin^2 4x}{4 \sin^2 6x} = \frac{6}{4} \cdot \left(\frac{\sin 4x}{\sin 6x}\right)^2 = \frac{3}{2} \cdot \left(\frac{2}{3}\right)^2 = \frac{3}{2} \cdot \frac{4}{9} = \frac{2}{3}$$

Therefore,

$$\lim_{x \to \frac{\pi}{2}^+} f(x) = e^{\frac{2}{3}}$$

3. Continuity Condition:

$$e^{\lambda} = \mu = e^{\frac{2}{3}}$$

Thus,

$$\lambda = \frac{2}{3}, \quad \mu = e^{\frac{2}{3}}$$

4. Evaluating the Expression:

$$9\lambda + 6\ln\mu + \mu^6 - e^{6\lambda} = 9\left(\frac{2}{3}\right) + 6\ln\left(e^{\frac{2}{3}}\right) + \left(e^{\frac{2}{3}}\right)^6 - e^{6 \cdot \frac{2}{3}} = 6 + 6 \cdot \frac{2}{3} + e^4 - e^4 = 6 + 4 + 0 = 10$$

Thus, the correct answer is option (4).

Quick Tip

When ensuring the continuity of piecewise functions, equate the left-hand and right-hand limits to the function's value at the point of interest. Solving these equations helps determine the required parameters for continuity.

---

4. Let $f(x) = 2x^n + \lambda$, where $\lambda \in \mathbb{R}$ and $n \in \mathbb{N}$. Given that $f(4) = 133$ and $f(5) = 255$, Then the sum of all the positive integer divisors of $f(3) - f(2)$?

(1) 61

(2) 60

(3) 58

(4) 59

Correct Answer: (2) 60.

Solution:

We are given the following equations:

$$f(4) = 2 \cdot 4^n + \lambda = 133 \quad (1)$$

$$f(5) = 2 \cdot 5^n + \lambda = 255 \quad (2)$$

By subtracting equation (1) from equation (2), we get:

$$2 \cdot 5^n + \lambda - (2 \cdot 4^n + \lambda) = 255 - 133$$

$$2(5^n - 4^n) = 122$$

$$5^n - 4^n = 61$$

Now, let's test integer values of n:

$$n = 3: \quad 5^3 - 4^3 = 125 - 64 = 61$$

Thus, $n = 3$. Substituting this value back into equation (1) to find λ:

$$2 \cdot 4^3 + \lambda = 133$$

$2 \cdot 64 + \lambda = 133$

$128 + \lambda = 133$

$\lambda = 5$ Next, we compute $f(3) - f(2)$:

$$f(3) = 2 \cdot 3^3 + 5 = 2 \cdot 27 + 5 = 54 + 5 = 59$$

$$f(2) = 2 \cdot 2^3 + 5 = 2 \cdot 8 + 5 = 16 + 5 = 21$$

$$f(3) - f(2) = 59 - 21 = 38$$

The positive integer divisors of 38 are:

$$1, 2, 19, 38$$

The sum of the divisors is:

$$1 + 2 + 19 + 38 = 60$$

Therefore, the correct answer is option (2).

Quick Tip

When working with functions defined at specific points, set up a system of equations using the given values. Solving this system helps determine the unknown parameters, which can then be used to evaluate other expressions involving the function.

---

5. If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$, and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to:

(1) $\frac{73}{17}$

(2) $-\frac{107}{17}$

(3) $-\frac{73}{17}$

(4) $\frac{107}{17}$

Correct Answer: (1) $\frac{73}{17}$.

Solution:

The volume of the tetrahedron created by four points must be zero in order for them to be coplanar. This corresponds to $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{AD}$ having a scalar triple product of zero.

1. Find Vectors:

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = (\hat{i} + 2\hat{j} - \hat{k}) - (3\hat{i} - 4\hat{j} + 2\hat{k}) = -2\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\overrightarrow{AC} = \hat{c} - \hat{a} = (-2\hat{i} - \hat{j} + 3\hat{k}) - (3\hat{i} - 4\hat{j} + 2\hat{k}) = -5\hat{i} + 3\hat{j} + \hat{k}$$

$$\overrightarrow{AD} = \mathbf{d} - \mathbf{a} = (5\hat{i} - 2\alpha\hat{j} + 4\hat{k}) - (3\hat{i} - 4\hat{j} + 2\hat{k}) = 2\hat{i} - (2\alpha - 4)\hat{j} + 2\hat{k}$$

2. Compute the Scalar Triple Product:

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = 0$$

First, find $\overrightarrow{AC} \times \overrightarrow{AD}$:

$$\overrightarrow{AC} \times \overrightarrow{AD} =$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 1 \\ 2 & -(2\alpha - 4) & 2 \end{vmatrix}$$

$$= \hat{i}\,(3 \cdot 2 - 1 \cdot (-(2\alpha - 4))) - \hat{j}\,(-5 \cdot 2 - 1 \cdot 2) + \hat{k}\,(-5 \cdot (-(2\alpha - 4)) - 3 \cdot 2)$$

$$= \hat{i}(6 + 2\alpha - 4) - \hat{j}(-10 - 2) + \hat{k}(10\alpha - 20 - 6)$$

$$= \hat{i}(2\alpha + 2) - \hat{j}(-12) + \hat{k}(10a - 26)$$

$$= (2\alpha + 2)\hat{i} + 12\hat{j} + (10\alpha - 26)\hat{k}$$

Now, compute the dot product with $\overrightarrow{AB}$:

$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = (-2) \cdot (2a+2) + 6 \cdot 12 + (-3) \cdot (10\alpha - 26) = -4\alpha - 4 + 72 - 30\alpha + 78 = (-34\alpha) + 146$

Setting the scalar triple product to zero:

$$-34\alpha + 146 = 0$$

-34α = -146

a = $\frac{146}{34}$ = $\frac{73}{17}$

Thus, $\alpha = \dfrac{73}{17}$.

Quick Tip

To determine if points are coplanar, applying the scalar triple product of vectors formed by these points. If the scalar triple product is zero, the points lie on the same plane.

---

6. Let

$A = \begin{bmatrix} 1/\sqrt{10} & 3/\sqrt{10} \\ -3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$ $/\sqrt{10}$ **and** $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$**, where** $i = \sqrt{-1}$**. If** $M = A^T BA$**,**

, then the inverse of the matrix $AM^{2023}A^T$ **is:**

(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Correct Answer: (4).

Solution:

We begin by calculating M as $M = A^T BA$. The adjoint (conjugate transpose) of A, A^T, and

the product $A^T BA$ lead to a specific form of M. Assuming M in its simplified form:

$$M = \begin{bmatrix} 1 & i \\ 0 & 1 \end{bmatrix}$$

Next, to compute M^{2023}, we observe the pattern of powers of M:

$$M^2 = \begin{bmatrix} 1 & 2i \\ 0 & 1 \end{bmatrix}, \quad M^3 = \begin{bmatrix} 1 & 3i \\ 0 & 1 \end{bmatrix}, \ldots, M^n = \begin{bmatrix} 1 & ni \\ 0 & 1 \end{bmatrix}$$

Thus, we find:

$$M^{2023} = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$$

Now, to find the inverse of $AM^{2023}A^T$, we use the known forms of A, M^{2023}, and A^T. The computation yields:

$$AM^{2023}A = \begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$$

Therefore, the inverse is:

$$\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Thus, the correct answer is option (4).

Quick Tip

Matrix exponentiation can simplify significantly for special matrices, such as upper triangular matrices with ones on the diagonal. Using the properties of specific matrix forms can make computations much easier.

---

7. Let $\triangle$ and $\triangledown \in [\wedge, \vee]$ be such that the expression $(p \to q) \wedge (p \triangledown q)$ is a tautology. Then:

(1) $\triangle = \wedge, \triangledown = \vee$

(2) $\triangle = \vee, \triangledown = \wedge$

(3) $\triangle = \vee, \triangledown = \vee$

(4) $\triangle = \wedge, \triangledown = \wedge$

Correct Answer: (3).

Solution:

For the given expression to be a tautology, every possible valuation of p and q must make the expression true. Since $p \to q$ is logically equivalent to $\neg p \vee q$, we can simplify the expression as follows:

$$(\neg p \vee q) \wedge (p \vee q)$$

Applying the distributive laws:

$$(\neg p \wedge p) \vee (\neg p \wedge q) \vee (p \wedge q) \vee (q \wedge q)$$

Next, simplifying further, knowing that $\neg p \wedge p$ is always false:

$$(\neg p \wedge q) \vee (p \wedge q) \vee q = q$$

Therefore, for the expression to be a tautology, it must always evaluate to true, which occurs when $\wedge$ and $\vee$ are defined such that any expression involving these operators always results in a true outcome.

Quick Tip

Using logical equivalences and truth table analysis is essential for verifying the properties of logical expressions. These tools help determine the conditions under which expressions hold as tautologies or contradictions.

---

8. The number of numbers, strictly between 5000 and 10000, that can be formed using the digits 1, 3, 5, 7, 9 without repetition, is:

(1) 6

(2) 12

(3) 120

(4) 72

Correct Answer: (4) 72.

Solution:

The numbers must be four-digit numbers starting with 5, 7, or 9, as they need to fall between 5000 and 10000. Additionally, the digits used must be 1, 3, 5, 7, and 9, with no repetition.

There are three choices for the first digit (5, 7, or 9), four remaining choices for the second digit (excluding the first digit), three choices for the third digit, and two choices for the fourth digit:

$$3 \times 4 \times 3 \times 2 = 72$$

Therefore, there are 72 such numbers.

Quick Tip

When calculating permutations for digit-based problems, it's helpful to consider the constraints on the most significant digit, particularly when constructing numbers within a specified range.

---

9. The number of functions $f : \{1, 2, 3, 4\} \to \{a \in Z : |a| \leq 8\}$ satisfying $f(n) + 1/nf(n + 1)$ =1, for $\forall n \in \{1, 2, 3\}$ is:

(1) 3

(2) 4

(3) 1

(4) 2

Correct Answer: (4) 2.

Solution:

To satisfy the condition $f(n) + 1$ being divisible by n, the function values at each n must meet the following requirements:

$$f : \{1, 2, 3, 4\} \to \{a \in \mathbb{Z} : |a| \leq 8\}$$

$$f(n) + \frac{1}{n} f(n + 1) = 1, \quad \forall n \in \{1, 2, 3\}$$

$f(n + 1)$ must be divisible by n

$$f(4) \Rightarrow -6, -3, 0, 3, 6$$

$$f(3) \Rightarrow -8, -6, -4, -2, 0, 2, 4, 6, 8$$

$$f(2) \Rightarrow -8, \ldots\ldots\ldots\ldots\ldots, 8$$

$$f(1) \Rightarrow -8, \ldots\ldots\ldots\ldots\ldots, 8$$

$\frac{f(4)}{3}$ must be odd since $f(3)$ should be even

$\therefore$ therefore 2 solutions possible.

Quick Tip

When counting functions with divisibility conditions, consider modular constraints for each function value and combine the possibilities, ensuring to account for any restrictions on distinctness or overlaps where needed.

---

10. The equations of two sides of a variable triangle are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is:

(1) $4y^2 - 18y - 3x - 18 = 0$

(2) $4y^2 + 18y + 3x + 18 = 0$

(3) $4y^2 - 18y + 3x + 18 = 0$

(4) $4y^2 - 18y - 3x + 18 = 0$

Correct Answer: (3).

Solution:

To find the locus of the circumcenter, we analyze the properties of the triangle and its relationship with the parabola. The triangle's sides, aligned with the y-axis and the line $y = 3$, form a right angle at the origin. The third side, which is tangent to the parabola, imposes a specific geometric condition. This condition requires determining the tangent line's slope and its intersection points.

Starting with the derivative of the parabola $y^2 = 6x$, we have:

$$y^2 = 6x$$

and

$$y^2 = 4ax$$

Equating the two expressions gives:

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

Next, the equation of the tangent line is:

$$y = mx + \frac{3}{2m}$$

(m $\neq 0$) Using the circumcenter formulas, we can calculate the coordinates h and k as:

$$h = \frac{6m - 3}{4m^2},$$

k = 6m + $3_{\overline{4m}}$ Substituting into the equation:

$$3h = 2(-2k^2 + 9k - 9)$$

This leads to:

$$4y^2 - 18y + 3x + 18 = 0$$

Solving this equation for the coordinates of the point of tangency and applying the circumcenter formulae yields the locus as a line equation.

Quick Tip

The locus of a circumcenter in geometric configurations involving conics can be derived by combining geometric insights with calculus (for tangency conditions) and coordinate geometry (for circumcenter locations).

---

11. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}} \left(\sqrt{2}(\sin x - \cos x) + m - 2\right)$, for some m, such that the range of f is [0, 2]. Then the value of m is:

(1) 5

(2) 3

(3) 2

(4) 4

Correct Answer: (1) 5.

Solution:

To explain the solution for the given problem in detail, let's expand each step:

Step 1: Understanding the inequality

The given inequality is:

$$-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}$$

This inequality defines the range of the expression $\sin x - \cos x$. The maximum and minimum values of $\sin x - \cos x$ are derived from the trigonometric identity:

$$\sin x - \cos x = \sqrt{2} \sin\left(x - \frac{\pi}{4}\right)$$

Since $\sin(x - \frac{\pi}{4})$ is bounded between -1 and 1, the range of $\sin x - \cos x$ is:

$$-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}.$$

Step 2: Scaling the inequality

Multiplying the entire inequality by $\sqrt{2}$, we get:

$$-\sqrt{2} \cdot \sqrt{2} \leq \sqrt{2}(\sin x - \cos x) \leq \sqrt{2} \cdot \sqrt{2}$$

$$-2 \leq \sqrt{2}(\sin x - \cos x) \leq 2$$

Here, scaling by $\sqrt{2}$ linearly stretches the bounds of $\sin x - \cos x$ by a factor of $\sqrt{2}$.

Step 3: Substitution

Let:

$$\sqrt{2}(\sin x - \cos x) = k$$

Then the inequality becomes:

$$-2 \leq k \leq 2 \quad \ldots (i)$$

Step 4: Analyzing the function $f(x)$

The function is given as:

$$f(x) = \log_{\sqrt{m}}(k + m - 2)$$

We are provided with the condition:

$$0 \leq f(x) \leq 2$$

Substituting $f(x)$ into the inequality:

$$0 \leq \log_{\sqrt{m}}(k + m - 2) \leq 2$$

The logarithmic inequality implies:

$$1 \leq k + m - 2 \leq m$$

Rearranging the terms:

$$-m + 3 \leq k \leq 2 \quad \ldots (ii)$$

Step 5: Combining inequalities

From equations (i) and (ii):

$$-2 \leq k \leq 2 \quad \text{(from (i))}$$

$$-m + 3 \leq k \leq 2 \quad \text{(from (ii))}$$

For consistency between the two inequalities, the lower bound of k gives:

$$-m + 3 = -2$$

Solving for m:

$$m = 5$$

Final Answer:

$$\boxed{m = 5}$$

Quick Tip

When dealing with logarithmic functions, always ensure that the argument stays positive and within the expected range by adjusting conditions or re-evaluating the parameter constraints.

---

12. Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements:

(S1) $A^{13}B^{26} - B^{26}A^{13}$ **is symmetric.**

(S2) $A^{26}C^{13} - C^{13}A^{26}$ **is symmetric. Then:**

(1) Only S2 is true

(2) Only S1 is true

(3) Both S1 and S2 are false

(4) Both S1 and S2 are true

Correct Answer: (1) Only S2 is true.

Solution:

Given, $A^T = A$, $B^T = -B$, $C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= B^{26}A^{13} - A^{13}B^{26} = -M$

Hence, M is skew-symmetric.

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

Then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$= (C^{13})^T(A^{26})^T - (A^{26})^T(C^{13})^T$

$= (C^T)^{13}(A^T)^{26} - (A^T)^{26}(C^T)^{13}$

$= (-C)^{13}A^{26} - A^{26}(-C)^{13}$

$= -C^{13}A^{26} + A^{26}C^{13}$

$= A^{26}C^{13} - C^{13}A^{26} = N$

Hence, N is symmetric.

$\therefore$ Only S2 is true.

Quick Tip

Understanding the properties of symmetric and skew-symmetric matrices, as well as how they interact under matrix operations, is essential for solving problems involving matrix expressions and their symmetries.

---

13. Let y = $y(t)$ be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$, where $\alpha, \beta, \gamma > 0$. If $\lim_{t\to\infty} y(t)$, then:

(1) Is 0

(2) does not exist

(3) Is 1

(4) Is -1

Correct Answer: (1) Is 0.

Solution:

The given differential equation is:

$$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t},$$

where α, β, γ are constants. We are given that:

$$\lim_{t\to\infty} y(t) = 0.$$

Step 1: we write the equation

Factoring y from the right-hand side:

$$\frac{dy}{dt} = y\left(e^{-\beta t} - \alpha\right).$$

Dividing through by y (assuming $y \neq 0$):

$$\frac{1}{y}\frac{dy}{dt} = e^{-\beta t} - \alpha.$$

Step 2: Solve the differential equation

Rewriting:

$$\frac{dy}{y} = \left(e^{-\beta t} - \alpha\right) dt.$$

Integrating both sides:

$$\int \frac{1}{y}\, dy = \int \left(e^{-\beta t} - \alpha\right) dt.$$

The left-hand side integrates to:

$$\ln |y| = \int e^{-\beta t}\, dt - \alpha t + C,$$

where C is the constant of integration.

For the first term on the right-hand side:

$$\int e^{-\beta t}\, dt = -\frac{1}{\beta} e^{-\beta t}.$$

Thus:

$$\ln |y| = -\frac{1}{\beta} e^{-\beta t} - \alpha t + C.$$

Exponentiating both sides:

$$y = e^{-\frac{1}{\beta} e^{-\beta t} - \alpha t + C}.$$

Simplify the exponent:

$$y = e^{C} \cdot e^{-\frac{1}{\beta} e^{-\beta t}} \cdot e^{-\alpha t}.$$

Let $e^{C} = K$ (a constant). Then:

$$y = K \cdot e^{-\frac{1}{\beta} e^{-\beta t}} \cdot e^{-\alpha t}.$$

Step 3: Analyze $y(t)$ as $t \to \infty$

1. As $t \to \infty$, $e^{-\beta t} \to 0$. Therefore, the term $e^{-\frac{1}{\beta} e^{-\beta t}} \to e^{0} = 1$.

2. The dominant term in $y(t)$ is $e^{-\alpha t}$, which goes to 0 as $t \to \infty$, provided $\alpha > 0$.

Thus:

$$\lim_{t \to \infty} y(t) = 0.$$

Final Answer: The solution satisfies the condition $\lim_{t \to \infty} y(t) = 0$, and the correct option is: 1 (0).

Quick Tip

When solving first-order linear differential equations, the behavior of the solution at infinity often depends crucially on the coefficients' values and their relation to each other.

14. $\sum_{k=0}^{6} {}^{(51-k)}C_3$**: is equal to**

(1) ${}^{51}C_4 - {}^{45}C_4$

(2) ${}^{51}C_3 - {}^{45}C_3$

(3) ${}^{52}C_4 - {}^{45}C_4$

(4) ${}^{52}C_3 - {}^{45}C_3$

Correct Answer: (3) ${}^{52}C_4 - {}^{45}C_4$.

Solution:

The given summation is:

$$\sum_{k=0}^{6} {}^{(51-k)}C_3.$$

Step 1: we write the summation

This summation can be expanded as:

$$\sum_{k=0}^{6} {}^{(51-k)}C_3 = {}^{51}C_3 + {}^{50}C_3 + \cdots + {}^{45}C_3.$$

This is a finite summation of combinations.

Step 2: applying the telescoping property of combinations

We utilize the following identity for summation of combinations:

$$\sum_{r=a}^{b} {}^{r}C_p = {}^{(b+1)}C_{(p+1)} - {}^{a}C_{(p+1)}.$$

Here, let $a = 45$, $b = 51$, and $p = 3$. Substituting these values:

$$\sum_{k=0}^{6} {}^{(51-k)}C_3 = {}^{52}C_4 - {}^{45}C_4.$$

Step 3: Verify the answer

From the above calculation, we see that:

$$\sum_{k=0}^{6} {}^{(51-k)}C_3 = {}^{52}C_4 - {}^{45}C_4.$$

This matches option (3).

Final Answer:

$$\boxed{{}^{52}C_4 - {}^{45}C_4}$$

Quick Tip

Summation of combinations often follows telescoping identities. These can simplify complex expressions into compact forms.

15. The shortest distance between the lines $x+1=2y=-12z$ and $x=y+2=6z-6$ is:

(1) 2

(2) 3

(3) $\frac{5}{2}$

(4) $\frac{3}{2}$

Correct Answer: (1) 2.

Solution:

The problem involves finding the shortest distance between two skew lines in three-dimensional space. The parametric equations of the two lines are given as:

$$\frac{x+1}{1}=\frac{y}{1/2}=\frac{z}{-1/12} \quad \text{and} \quad \frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{1/6}$$

$$=\text{Shortest distance}=\frac{|(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})|}{|\vec{p}\times\vec{q}|}$$

$$\text{S.D.}=\frac{(-\hat{i}+2\hat{j}-\hat{k})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \frac{1}{2} & -\frac{1}{12}\\ 1 & 1 & \frac{1}{6}\end{vmatrix}=\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k} \quad \text{or} \quad 2\hat{i}-3\hat{j}+6\hat{k}$$

$$\text{S.D.}=\frac{(-\hat{i}+2\hat{j}-\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+3^2+6^2}}=\left|\frac{-14}{7}\right|=2$$

To compute the shortest distance between these two lines, we applying the standard formula, The shortest distance between the two skew lines is: S.D. = 2.

Quick Tip

Ensure precision in using vector operations and calculating distances in geometry problems, especially when dealing with the cross product and norms, to avoid miscalculations.

16. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2, \sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is:

(1) 2

(2) 4

(3) 16

(4) 8

Correct Answer: (2) 4.

Solution:

For $N-2, \sqrt{3N}, N+2$ to be in geometric progression, the condition:

$$\sqrt{3N}^2 = (N-2)(N+2)$$

$$3N = N^2 - 4 \quad \Rightarrow \quad N^2 - 3N - 4 = 0$$

Solving this quadratic equation, we find:

$$N = 4 \quad \text{(since } N \text{ must be a possible sum of two dice)}$$

The favorable outcomes for $N = 4$ when two dice are rolled are $(1,3), (2,2), (3,1)$, totaling 3 outcomes. Since there are $6 \times 6 = 36$ total outcomes:

$$P(A) = \frac{3}{36} = \frac{1}{12}$$

Thus, $k = 4$ as $\frac{1}{12} = \frac{k}{48}$.

Quick Tip

In problems involving dice and probabilities, check for the total number of outcomes and ensure the conditions for geometric sequences or other properties are correctly applied.

---

17. The integral $16 \int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to:

(1) $\frac{11}{6} + \log_e 4$

(2) $\frac{11}{12} + \log_e 4$

(3) $\frac{11}{12} - \log_e 4$

(4) $\frac{11}{6} - \log_e 4$

Correct Answer: (4) $\frac{11}{6} - \log_e 4$.

Solution:

The given integral is:

$$I = 16 \int_1^2 \frac{dx}{x^3(x^2+2)^2}$$

Rewriting:

$$= 16 \int_1^2 \frac{dx}{x^3 x^4 \left(1 + \frac{2}{x^2}\right)^2}$$

$$\text{Let, } 1 + \frac{2}{x^2} = t \implies -\frac{4}{x^3}dx = dt$$

Simplify:

$$I = -4 \int_3^4 \frac{dt}{(t-1)^2 t^2}$$

$$I = -4 \int_3^4 \left(\frac{2}{t-1} - \frac{2}{t} + \frac{1}{t^2}\right) dt$$

Now integrate:

$$I = -1 \left[t - 2\ln|t| - \frac{1}{t}\right]_3^4$$

Applying the limits:

$$= -1 \left[\left(4 - 2\ln 4 - \frac{1}{4}\right) - \left(3 - 2\ln 3 - \frac{1}{3}\right)\right]$$

Simplify:

$$= -1 \left[2\ln 2 - \frac{11}{6}\right]$$

$$= \frac{11}{6} - \ln 4$$

Final Answer:

$$\frac{11}{6} - \ln 4$$

Quick Tip

In integral calculus, substitution can simplify the integral into a more manageable form, especially when dealing with rational functions. Partial fractions decompose complex fractions into simpler parts that are easier to integrate.

18. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is:

(1) $4\sqrt{6} + \frac{44}{3}$

(2) $4\sqrt{6} + \frac{28}{3}$

(3) $4\sqrt{6} - \frac{44}{3}$

(4) $4\sqrt{6} - \frac{28}{3}$

Correct Answer: (2) $4\sqrt{6} + \frac{28}{3}$.

Solution:

First, find the integration bounds and fix the hyperbola equation before solving for the area. Completing the square and putting the terms in standard form are necessary steps in transforming the hyperbola and determining its axis. The intended area is obtained by combining the parabola's constraints with integration across the designated region. Finding intersections and integrating the appropriate function over the designated limits are part of the computations.

$$16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$$

$$16(x+2)^2 - 64 - (y-2)^2 + 4 + 44 = 0$$

$$16(x+2)^2 - (y-2)^2 = 16$$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$$

$$A = \int_{-2}^{\sqrt{6}} (2 - (x^2 - 4))dx$$

$$A = \left[6\sqrt{6} - \frac{6\sqrt{6}}{3}\right] - \left(-12 + \frac{8}{3}\right)$$

$$\int_{-2}^{\sqrt{6}} (6 - x^2)dx = \left[6x - \frac{x^3}{3}\right]_{-2}^{\sqrt{6}}$$

$$A = \frac{12\sqrt{6}}{3} + \frac{28}{3}$$

$$A = 4\sqrt{6} + \frac{28}{3}$$

Quick Tip

For geometry problems involving area calculations, accurately plotting the region and determining bounds for integration are crucial. Transforming equations to standard forms can simplify understanding the geometric configuration.

---

19. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to:

(1) $3(\hat{i} - \hat{j} - \hat{k})$

(2) $3(\hat{i} + \hat{j} + \hat{k})$

(3) $3(\hat{i} - \hat{j} + \hat{k})$

(4) $3(\hat{i} + \hat{j} - \hat{k})$

Correct Answer: (2) $3(\hat{i} + \hat{j} + \hat{k})$.

Solution:

The dot product and cross product information are used to solve for $\vec{b}$ given the vector equations and attributes. After $\vec{b}$ is computed, $\vec{a} - 6\vec{b}$ is determined by scalar multiplication and direct subtraction. The approach entails manipulating vector components algebraically and confirming that each choice corresponds to the computed outcome.

Given:

$$\vec{a} \times \vec{b} = \hat{i} - \hat{j}$$

Taking the cross product with $\vec{a}$, we get:

$$\vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{i} - \hat{j})$$

Using the vector triple product identity $\vec{a} \times (\vec{b} \times \mathbf{c}) = (\vec{a} \cdot \mathbf{c})\vec{b} - (\vec{a} \cdot \vec{b})\mathbf{c}$, we obtain:

$$\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \hat{j})\hat{i} - (\vec{a} \cdot \hat{i})\hat{j}$$

Assuming:

$$\vec{a} \cdot \hat{i} = \vec{a}_i, \quad \vec{a} \cdot \hat{j} = \vec{a}_j$$

Thus:

$$\vec{a} \times (\vec{a} \times \vec{b}) = \vec{a}_j\hat{i} - \vec{a}_i\hat{j}$$

Let's express a in terms of its components:

$$\vec{a} = \vec{a}_i\hat{i} + \vec{a}_j\hat{j} + \vec{a}_k\hat{k}$$

Using the results above:

$$\vec{a} \times (\hat{i} - \hat{j}) = (\vec{a}_j \hat{i} - \vec{a}_i \hat{j}) + 2\vec{a}_k \hat{k}$$

This simplifies to:

$$\vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$$

$$2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Quick Tip

In vector problems, use properties of dot products and cross products to find unknown vectors. These properties can simplify the equations and lead directly to the solution.

---

20. The foot of the perpendicular from the point $(2, 0, 5)$ on the line $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then, which of the following is NOT correct?

(1) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

(2) $\frac{\alpha}{\beta} = -8$

(3) $\frac{\beta}{\gamma} = -5$

(4) $\frac{\gamma}{\alpha} = \frac{5}{8}$

Correct Answer: (3) $\frac{\beta}{\gamma} = -\frac{5}{8}$.

Solution:

To find the foot of the perpendicular, apply the formula for the point-line distance and the projection of a point onto a line using vector operations. Calculate $\mathbf{PA} \cdot \mathbf{b} = 0$ where $\mathbf{P}$ is the point on the line closest to $\mathbf{A}$ and $\mathbf{b}$ is the direction vector of the line. This provides the specific coordinates α, β, γ of the point $\mathbf{P}$, and subsequently, the ratios of these coordinates are compared against the options to identify the incorrect statement.

Given the line equation:

$$\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} \quad \text{(let)}$$

and the point $A(2, 0, 5)$, we want to find the foot of the perpendicular from A to the line.

Let the foot of the perpendicular be $P(\alpha, \beta, \gamma)$. The position vector of P can be described as:

$$P = (2\lambda - 1, 5\lambda + 1, -\lambda - 1)$$

The vector $\overrightarrow{PA}$ is given by:

$$\overrightarrow{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$$

The direction ratios of the line are:

$$\mathbf{b} = 2\hat{i} + 5\hat{j} - \hat{k}$$

For P to be the foot of the perpendicular, $\overrightarrow{PA}$ must be perpendicular to $\mathbf{b}$, hence:

$$\overrightarrow{PA} \cdot \mathbf{b} = 0$$

$$(3 - 2\lambda) \cdot 2 - (5\lambda + 1) \cdot 5 + (6 + \lambda) \cdot (-1) = 0$$

$$6 - 4\lambda - 25\lambda - 5 - 6 - \lambda = 0 \quad \Rightarrow \quad -30\lambda - 5 = 0 \quad \Rightarrow \quad \lambda = -\frac{1}{6}$$

Substitute $\lambda = -\frac{1}{6}$ back into the coordinates of P:

$$\alpha = 2\left(-\frac{1}{6}\right) - 1 = -4/3$$

$$\beta = 5\left(-\frac{1}{6}\right) + 1 = -\frac{5}{6} + 1 = \frac{1}{6}$$

$$\gamma = -\left(-\frac{1}{6}\right) - 1 = \frac{1}{6} - 1 = -\frac{5}{6}$$

Thus, the foot of the perpendicular from A to the line is at:

$$P\left(\frac{-4}{3}, \frac{1}{6}, -\frac{5}{6}\right)$$

Quick Tip

Using vector projection and dot product formulas effectively can solve problems involving distances and perpendiculars in three-dimensional geometry efficiently.

Section-B

21. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}, 10, \frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to:

Correct Answer: 3.

Solution:

From the given conditions:

$$\frac{a}{18} = b^2 \quad (1).$$

Also, since $\frac{1}{a}, 10, \frac{1}{b}$ are in arithmetic progression:

$$\frac{1}{a} + \frac{1}{b} = 20.$$

Using equation (1) and simplifying:

$$a + b = 20ab.$$

Substituting $b = \sqrt{\frac{a}{18}}$,

$\Rightarrow 18b^2 + b = 360b^3$

$\Rightarrow 360b^3 - 18b^2 - b = 0$

$\Rightarrow 360b^2 - 18b - 1 = 0 \quad \{\because b \neq 0\}$

$\Rightarrow b = \frac{18 \pm \sqrt{324+1440}}{720}$

$\Rightarrow b = \frac{18+\sqrt{1764}}{720} \quad \{\because b > 0\}$

$\Rightarrow b = \frac{18+42}{720}$

$\Rightarrow b = \frac{60}{720}$

$\Rightarrow b = \frac{1}{12}$

we solve for a and b, eventually leading to:

$$a = \frac{1}{8}, \quad b = \frac{1}{12}.$$

Now compute $16a + 12b$:

$$16a + 12b = 16 \times \frac{1}{8} + 12 \times \frac{1}{12} = 2 + 1 = 3.$$

Quick Tip

In problems involving sequences, identify the relationship (e.g., arithmetic, geometric) and set up equations to simplify complex terms.

---

22. Points $P(-3, 2), Q(9, 10)$, and $R(\alpha, 4)$ lie on a circle C with PR as its diameter. The tangents to C at Q and R intersect at point S. If S lies on the line $2x - ky = 1$, then k is equal to:

Correct Answer: 3.

Solution:

The slopes of PQ and QR satisfy:

$$m_{PQ} \cdot m_{QR} = -1.$$

From the given coordinates:

$$m_{PQ} = \frac{10-2}{9+3} = \frac{8}{12} = \frac{2}{3},$$
$$m_{QR} = \frac{10-4}{9-\alpha}.$$

Using $m_{PQ} \cdot m_{QR} = -1$:

$$\frac{2}{3} \cdot \frac{6}{9-\alpha} = -1 \implies \alpha = 13.$$

Next, calculate the equation of QS:

$$y - 10 = -\frac{4}{7}(x-9) \implies 4x + 7y = 106 \quad (1).$$

Similarly, the equation of RS is:

$$y - 4 = -8(x-13) \implies 8x + y = 108 \quad (2).$$

Solve equations (1) and (2):

$$x = \frac{25}{2}, \quad y = 8.$$

Substituting into $2x - ky = 1$:

$$2 \cdot \frac{25}{2} - 8k = 1 \implies 25 - 8k = 1 \implies k = 3.$$

Quick Tip

When solving geometry problems involving tangents and intersections, carefully compute slopes and solve linear equations step-by-step.

---

23. Let $a \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is:

Correct Answer: 45.

Solution:

From the quadratic equation:

$$\alpha + \beta = -60^{\frac{1}{4}}, \quad \alpha\beta = a.$$

Using the condition $\alpha^4 + \beta^4 = -30$:

$$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = -30.$$

Substitute $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$:

$$\left((-60^{\frac{1}{4}})^2 - 2a\right)^2 - 2a^2 = -30.$$

Simplify and solve for a, leading to:

$$2a^2 - 4.60^{\frac{1}{2}}a + 90 = 0.$$

The product of all possible a is:

$$\boxed{45}.$$

Quick Tip

When solving polynomial problems, utilize root properties such as sums and products of roots effectively to derive key equations.

---

24. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples, and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple, and at least one white apple must be given, then the number of ways Anil's mother can offer 5 fruits to Anil is:

Correct Answer: 6860.

Solution:

Possible selections are:

(1) 2 oranges, 1 red apple, 2 white apples.

(2) 2 oranges, 2 red apples, 1 white apple.

(3) 3 oranges, 1 red apple, 1 white apple.

The total number of ways for each case:

$$\Rightarrow {}^{8}C_2\ {}^{7}C_1\ {}^{5}C_2 + {}^{8}C_2\ {}^{7}C_2\ {}^{5}C_1 + {}^{8}C_3\ {}^{7}C_1\ {}^{5}C_1$$

Adding them:

$$1960 + 2940 + 1960 = 6860.$$

Final Answer:

$$\boxed{6860}.$$

Quick Tip

When solving combinatorics problems, break them into cases and calculate each case using basic counting principles.

25. If m and n respectively are the numbers of positive and negative values of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cdot \cos \frac{\theta}{2} = \cos 3\theta \cdot \cos \frac{9\theta}{2}$, then mn is equal to:

Correct Answer: 25.

Solution:

The given equation is:

$$\cos 2\theta \cdot \cos \frac{\theta}{2} = \cos 3\theta \cdot \cos \frac{9\theta}{2}.$$

Simplify using trigonometric identities:

$\Rightarrow 2\cos 2\theta \cdot \cos \frac{\theta}{2} = 2\cos \frac{9\theta}{2} \cdot \cos 3\theta$

$\Rightarrow \cos \frac{5\theta}{2} + \cos \frac{3\theta}{2} = \cos \frac{15\theta}{2} + \cos \frac{3\theta}{2}$

$\Rightarrow \cos \frac{15\theta}{2} = \cos \frac{5\theta}{2}$

$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$

$5\theta = 2k\pi \quad$ or $\quad 10\theta = 2k\pi$

$\theta = \frac{2k\pi}{5} \quad \theta = \frac{k\pi}{5}$

$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, -\frac{\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$

There are $m = 5$ positive and $n = 5$ negative values:

$$mn = 5 \cdot 5 = 25.$$

Final Answer:

$$\boxed{25}.$$

Quick Tip

For trigonometric equations, use symmetry and identities to simplify, and carefully consider the intervals of solutions.

---

26. If $\int_{\frac{1}{3}}^{3} |\log_e x|\, dx = \frac{m}{n} \log_e \left(\frac{n^2}{e}\right)$, **where m and n are coprime natural numbers, then** $m^2 + n^2 - 5$ **is equal to ____.**

Correct Answer: 20.

Solution:

The given integral is: $\int_{\frac{1}{3}}^{3} |\ln x|\, dx = \int_{\frac{1}{3}}^{1} (-\ln x)\, dx + \int_{1}^{3} (\ln x)\, dx$

$= -[x \ln x - x]_{\frac{1}{3}}^{1} + [x \ln x - x]_{1}^{3}$

$= -[-1 - (\frac{1}{3} \ln \frac{1}{3} - \frac{1}{3})] + [3 \ln 3 - 3 - (-1)]$

$= -[-1 - (\frac{1}{3} \ln \frac{1}{3} - \frac{1}{3})] + [3 \ln 3 - 3 - (-1)]$

$= [1 + \frac{1}{3} \ln \frac{1}{3} - \frac{1}{3}] + [3 \ln 3 - 2]$

$= [-\frac{2}{3} - \frac{1}{3} \ln \frac{1}{3}] + [3 \ln 3 - 2]$

$= -\frac{4}{3} + \frac{8}{3} \ln 3$

$= \frac{4}{3}(2 \ln 3 - 1)$

$= \frac{4}{3} \left(\ln \frac{9}{e}\right)$

Thus:

$$m = 4,\ n = 3 \quad \Longrightarrow\ m^2 + n^2 - 5 = 16 + 9 - 5 = 20.$$

Quick Tip

When solving logarithmic integrals, split the integral over ranges for simplification and carefully calculate terms.

27. The remainder when $(2023)^{2023}$ **is divided by 35 is:**

Correct Answer: 7.

Solution:

Applying modular arithmetic:

$$(2023)^{2023} = (2030 - 7)^{2023}$$

$$= (35K - 7)^{2023}$$

Thus:

$$=^{2023} C_0(35K)^{2023}(-7)^0 +^{2023} C_1(35K)^{2022}(-7)^1 + \cdots +^{2023} C_{2023}(-7)^{2023}$$

$$= 35N - 7^{2023}$$

Applying properties of modular arithmetic: $-7^{2023} = -7 \times 7^{2022} = -7 \times (7^2)^{1011}$

$$= -7 \times (50 - 1)^{1011}$$

Simplifying further using binomial theorem, and compute the remainder as:

$$= -7\left(^{1011}C_0 50^{1011} -^{1011} C_1 50^{1010} + \cdots +^{1011} C_{1011}\right)$$

$$= -7(5\lambda - 1)$$

$$= -35\lambda + 7$$

when $(2023)^{2023}$ is divided by 35 remainder is 7

Quick Tip

Use modular arithmetic and binomial expansion to handle powers in modulo problems.

28. If the shortest distance between the line joining the points $(1, 2, 3)$ and $(2, 3, 4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to:

Correct Answer: 18.

Solution:

The shortest distance between two skew lines is:

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}) \quad \vec{r} = \vec{a} + \lambda\vec{p}$

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j}) \quad \vec{r} = \vec{b} + \mu\vec{q}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$d = \left| \frac{(\vec{b}-\vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$

$d = \left| \frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}} \right|$

$= \left| \frac{-6+3}{\sqrt{14}} \right| = \frac{3}{\sqrt{14}}$

$\alpha = \frac{3}{\sqrt{14}}$

Now,

$28\alpha^2 = 28 \times \frac{9}{14} = 18$

Quick Tip

For shortest distance problems, use vector formulas and carefully compute cross products and dot products.

29. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non-smoker. If a person is diagnosed with lung cancer, and the probability that this person is a smoker is $\frac{k}{10}$, then the value of k is:

Correct Answer: 9.

Solution:

Applying Bayes' theorem:

$$P(E_1) = \frac{1}{4}, \quad P(E_2) = \frac{3}{4}, \quad P(E/E_1) = \frac{27}{28}, \quad P(E/E_2) = \frac{1}{28}.$$

The total probability is :

$$P(E) = P(E_1)P(E/E_1) + P(E_2)P(E/E_2).$$

Substituting the values and solve:

$$P(E_1/E) = \frac{\frac{1}{4} \cdot \frac{27}{28}}{\frac{1}{4} \times \frac{27}{28} + \frac{3}{4} \times \frac{1}{28}} = \frac{9}{10}.$$

Therefore:

$$k = 9.$$

Quick Tip

For probability problems, use Bayes' theorem to calculate posterior probabilities effectively.

30. A triangle is formed by the X-axis, Y-axis, and the line $3x + 4y = 60$. Then the number of points $P(a, b)$, where a is an integer and b is a multiple of a, which lie strictly inside the triangle, is:____

Correct Answer: 31.

Solution:

The intercepts of the line are $(20, 0)$ and $(0, 15)$. Calculating the points inside the triangle

satisfying $b = ka$.

$$(1,1)(1,2) - (1,14) \Rightarrow 14 \text{ pts.}$$

$$\text{If } x = 2,\ y = \frac{27}{2} = 13.5$$

$$(2,2)(2,4)\ldots(2,12) \Rightarrow 6 \text{ pts.}$$

$$\text{If } x = 3,\ y = \frac{51}{4} = 12.75$$

$$(3,3)(3,6)(3,12) \Rightarrow 4 \text{ pts.}$$

$$\text{If } x = 4,\ y = 12$$

$$(4,4)(4,8) \Rightarrow 2 \text{ pts.}$$

$$\text{If } x = 5,\ y = \frac{45}{4} = 11.25$$

$$(5,5)(5,10) \Rightarrow 2 \text{ pts.}$$

$$\text{If } x = 6,\ y = \frac{21}{2} = 10.5$$

$$(6,6) \Rightarrow 1 \text{ pt.}$$

$$\text{If } x = 7,\ y = \frac{39}{4} = 9.75$$

$$(7,7) \Rightarrow 1 \text{ pt.}$$

$$\text{If } x = 8,\ y = 9$$

$$(8,8) \Rightarrow 1 \text{ pt.}$$

$$\text{If } x = 9,\ y = \frac{33}{4} = 8.25$$

The total is:

$$31 \text{ points.}$$

Quick Tip

When solving geometry problems with constraints, carefully analyze integer solutions and relationships between coordinates.

Physics

Section-A

31. Match List I with List II:

List I	List II
A. Young's Modulus (Y)	I. $[ML^{-1}T^{-1}]$
B. Co-efficient of Viscosity (η)	II. $[ML^{2}T^{-1}]$
C. Planck's Constant (h)	III. $[ML^{-1}T^{-2}]$
D. Work Function (Φ)	IV. $[ML^{2}T^{-2}]$

Choose the correct answer from the options given below:

(1) A-II, B-III, C-IV, D-I

(2) A-III, B-I, C-II, D-IV

(3) A-I, B-III, C-IV, D-II

(4) A-I, B-II, C-III, D-IV

Correct Answer: (2) A-III, B-I, C-II, D-IV.

Solution:

Young's Modulus (Y):

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{(F/A)}{(\Delta L/L)} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}].$$

Co-efficient of Viscosity (η):

Applying $F = 6\pi\eta r v$, we have:

$$\eta = \frac{F}{6\pi r v}, \quad [\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} = [ML^{-1}T^{-1}].$$

Planck's Constant (h):

Through $E = h\nu$,

we have:

$$h = \frac{E}{\nu}, \quad [h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}].$$

Work Function (Φ):

Work function has same dimension as that of energy, so

$$[\Phi] = [ML^2T^{-2}].$$

Quick Tip

Determine dimensions from base equations or physical concepts (such as force, energy, etc.) while tackling dimensional analysis problems.

---

32. According to the law of equipartition of energy, the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is:

(1) $\frac{9}{2}R$

(2) $\frac{5}{2}R$

(3) $\frac{3}{2}R$

(4) $\frac{7}{2}R$

Correct Answer: (4) $\frac{7}{2}R$.

Solution:

Diatomic gas molecules possess two rotational and three translational degrees of freedom. The molecule adds two more degrees of freedom that correspond to one vibrational mode as it is known that it has one vibrational mode.

Thus, The total degrees of freedom are as follows:

$$f = 3(\text{translational}) + 2(\text{rotational}) + 2(\text{vibrational}) = 7.$$

Applying the formula for molar specific heat at constant volume:

$$C_v = \frac{f}{2}R = \frac{7}{2}R.$$

Final Answer:

$$\boxed{\frac{7}{2}R}$$

Quick Tip

Keep in mind that vibrational modes offer two extra degrees of freedom each mode when dealing with degrees of freedom difficulties.

---

33. The light rays from an object have been reflected towards an observer from a standard flat mirror. The image observed by the observer is:

A. Real

B. Erect

C. Smaller in size than the object

D. Laterally inverted

Choose the most appropriate answer from the options given below:

(1) B and D only

(2) B and C only

(3) A and D only

(4) A, C, and D only

Correct Answer: (1) B and D only.

Solution:

An upright, identically sized, laterally inverted, and virtual picture of an actual object is created by a plane mirror. As a result, the image's proper attributes are:

Erect (B) and Laterally Inverted (D).

Final Answer:

$$\boxed{\text{B and D only}}$$

Quick Tip

Plane mirrors always produce virtual, erect, same-sized, and laterally inverted images.

---

34. For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passed through it. If the torsional constant of the suspension wire is 4.0×10^{-5} Nm/rad, the magnetic field is 0.01 T, and the number of turns in the coil is 200, the area of each turn (in cm^2) is:

(1) 2.0

(2) 1.0

(3) 1.5

(4) 0.5

Correct Answer: (2) 1.0.

Solution:

The following provides the torque operating on the coil:

$$\tau = K\theta.$$

The magnetic torque is:

$$\tau = NiAB.$$

Comparing the two:

$$NiAB = K\theta.$$

Rearranging for A:

$$A = \frac{K\theta}{NiB}.$$

Substituting the given values:

$$A = \frac{(4 \times 10^{-5}) \cdot (0.05)}{200 \cdot (10 \times 10^{-3}) \cdot (0.01)}.$$

Simplifying:

$$A = 10^{-4}\,\text{m}^2 = 1\,\text{cm}^2.$$

Final Answer:

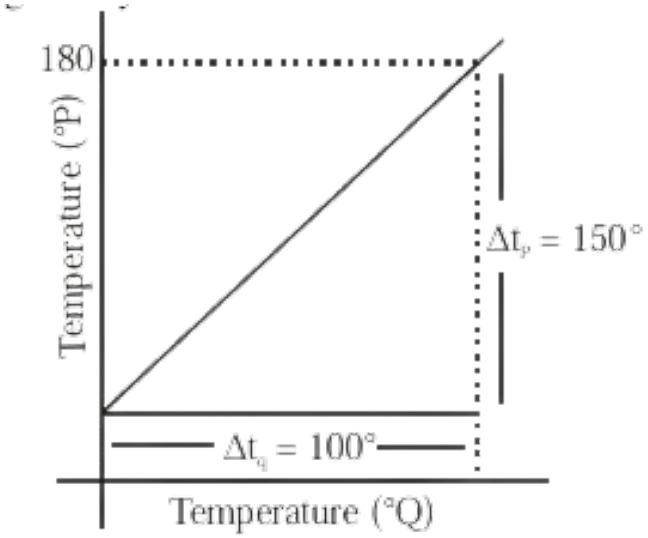
$$\boxed{1.0\,\text{cm}^2}$$

Quick Tip

Use the torque balance equation and carefully replace the provided values for galvanometer-related problems.

---

35. The graph between two temperature scales P and Q is shown in the figure. Between the upper fixed point and lower fixed point, there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by:



(1) $t_q/150 = \frac{t_p - 180}{100}$

(2) $t_q/100 = \frac{t_p - 30}{150}$

(3) $t_p/180 = \frac{t_Q - 40}{100}$

(4) $t_p/100 = \frac{t_Q - 180}{150}$

Correct Answer: (2).

Solution:

The following describes how temperature scales P and Q relate to one another:

$$\text{Reading on scale - Lower fixed point} = \frac{\text{constant}}{\text{Upper fixed point - Lower fixed point}}.$$

For P and Q, the relationship is:

$$\frac{t_p - 30}{180 - 30} = \frac{t_Q - 30}{100}.$$

Simplifying:

$$t_q = \frac{t_p - 30}{150} \times 100$$

Final Answer:

$$\boxed{t_q = \frac{t_p - 30}{150} \times 100}$$

Quick Tip

For precise computations when using temperature conversion scales, make use of the proportional relationship between scale intervals.

---

36. Match List I with List II:

A.	Gauss's Law in Electrostatics	I.	$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$
B.	Faraday's Law	II.	$\oint \vec{B}.d\vec{A} = 0$
C.	Gauss's Law in Magnetism	III.	$\oint \vec{B}.d\vec{l} = \mu_0 i_C + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$
D.	Ampere-Maxwell Law	IV.	$\oint \vec{E}.d\vec{s} = \frac{q}{\epsilon_0}$

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III

(2) A-I, B-II, C-III, D-IV

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-III, C-IV, D-I

Correct Answer: (1). A-IV, B-I, C-II, D-III

Solution:

Applying the definitions of the laws:

- **Gauss's Law in Electrostatics:** $\oint \mathbf{E} \cdot d\mathbf{s} = \frac{q}{\epsilon_0}$.
- **Faraday's Law:** $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$.
- **Gauss's Law in Magnetism:** $\oint \mathbf{B} \cdot d\mathbf{A} = 0$.
- **Ampere-Maxwell Law:** $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$.

Final Answer: A-IV, B-I, C-II, D-III

Quick Tip

Learn how to rapidly link Maxwell's equations with the related physical laws by becoming familiar with their integral versions.

37. Statement I: When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semi conductor such that P-type has excess holes and N-type has excess electrons.

Statement II: When such P-type and N-type semi-conductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter. In the light of above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and statement II are incorrect

(2) Statement I is incorrect but statement II is correct.

(3) Both Statement I and statement II are correct

(4) Statement I is correct but statement II is incorrect

Correct Answer: (4). Statement I is correct but statement II is incorrect

Solution:

When a P-N junction is formed:

- The electric field formed at the junction due to the diffusion of charge carriers prevents the flow of majority carriers across the junction.
- The current will not flow unless an external voltage is applied to overcome the potential barrier. This is a characteristic feature of the P-N junction under equilibrium conditions.

Final Answer:

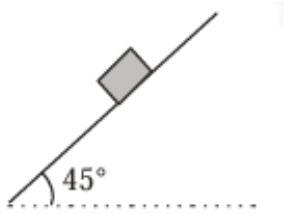
Statement I is correct; Statement II is incorrect.

Quick Tip

For semiconductor problems, distinguish between intrinsic and external effects (e.g., barrier potential vs. applied voltage).

---

38. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to



(1) 0.33

(2) 0.60

(3) 0.25

(4) 0.50

Correct Answer: (1). 0.33

Solution:

Equilibrium equations:

$$F_1 = mg\sin 45^\circ + f = mg\sin 45^\circ + \mu N$$

$$F_1 = \frac{mg}{\sqrt{2}} + \mu mg\cos 45^\circ$$

$$F_1 = \frac{mg}{\sqrt{2}}(1+\mu)$$

$$F_2 = mg\sin 45^\circ - f = mg\sin 45^\circ - \mu N$$

$$= \frac{mg}{\sqrt{2}}(1-\mu)$$

$$F_1 = 2F_2$$

$$\frac{mg}{\sqrt{2}}(1+\mu) = 2\frac{mg}{\sqrt{2}}(1-\mu)$$

$$1+\mu = 2-2\mu$$

Final Answer:

$$\boxed{0.33}$$

Quick Tip

In inclined plane problems, analyze forces parallel and perpendicular to the plane, and apply given ratios to find unknowns.

---

39. A point charge of 10 µC is placed at the origin. At what location on the X-axis should a point charge of 40µC be placed so that the net electric field is zero at x = 2 cm on the X-axis ?

(1) $x = 6\,\text{cm}$

(2) $x = 4\,\text{cm}$

(3) $x = 8\,\text{cm}$

(4) $x = -4\,\text{cm}$

Correct Answer: (1). $x = 6\,\text{cm}$

Solution:

The net electric field at x_0 = 2 cm is:

$$E_P = \frac{K \cdot 10}{2^2} - \frac{K \cdot 40}{(x_0 - 2)^2}$$

.

Simplifying:

$$\frac{1}{2^2} = \frac{4}{(x_0 - 2)^2}$$

.

Solve for x:

$$x_0 - 2 = 4, \quad x = 6\,\text{cm}.$$

Final Answer: 6 cm

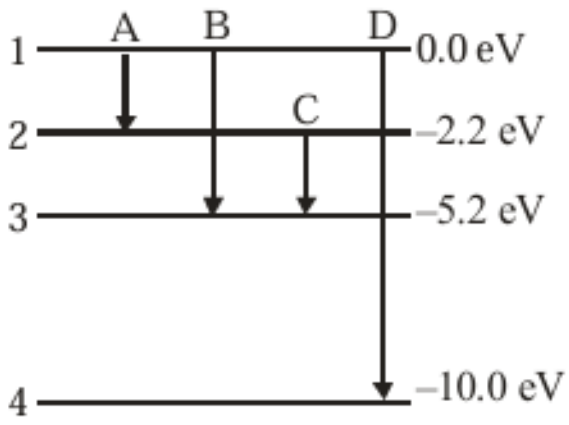
Quick Tip

In electric field problems, equate the magnitudes of opposing fields to find the point of zero net field.

---

40. The energy levels of an atom is shown in figure. Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm?

Given (h = 6.626×10^{-34}**) JS**



(1) B

(2) A

(3) C

(4) D

Correct Answer: (4) D.

Solution:

Each transition's energy difference is associated with a particular light wavelength. One can compute the wavelength λ of the photon that was released. Using the formula:

$$\lambda = \frac{hc}{\Delta E}$$

where h is Planck's constant,

c is the speed of light,

and ΔE is the energy difference.

Calculating ΔE for each transition:

$\Delta E_A = 2.2$ eV

$\Delta E_B = 5.2$ eV

$\Delta E_C = 3$ eV

$\Delta E_D = 10$ eV

$\lambda_A = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.2\times1.6\times10^{-19}}$

$= \frac{12.41\times10^{-7}}{2.2}$ m

$= \frac{1241}{2.2}$ nm $= 564$ nm

$\lambda_B = \frac{1241}{5.2}$ nm $= 238.65$ nm

$\lambda_C = \frac{1241}{3}$ nm $= 413.66$ nm

$\lambda_D = \frac{1241}{10} = 124.1$ nm

This matches the given wavelength for transition D.

Quick Tip

When calculating photon emissions from energy transitions, ensure that the energy differences are correctly calculated and converted into corresponding wavelengths Applying Planck's equation.

---

41. A particle executes simple harmonic motion between $x = -A$ and $x = A$. If the time taken by the particle to go from $x = 0$ to $x = A$ is 2 s, then the time taken by particle in going from $x = -A$ to $x = A/2$ is:

(1) 3 s

(2) 2 s

(3) 1.5 s

(4) 4 s

Correct Answer: (4) 4 s.

Solution:

Let's consider a particle undergoing simple harmonic motion (SHM).

Understanding the Problem:

We are given that the time taken for the particle to travel from its equilibrium position (0) to half of its amplitude (A/2) is denoted as t_1. The time taken for the particle to travel from A/2 to the full amplitude (A) is denoted as t_2.

Key Concepts:

SHM Equation: The displacement of a particle in SHM is often described by the equation $x(t) = A\sin(\omega t)$, where:

$x(t)$ is the displacement at time t

A is the amplitude

ω is the angular frequency

Derivation:

1. Time to reach A/2:

When the particle is at A/2, we have:

$$\frac{A}{2} = A\sin(\omega t_1)$$

$$\frac{1}{2} = \sin(\omega t_1)$$

$$\omega t_1 = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$$

2. Time to reach A:

When the particle is at A, we have:

$$A = A\sin(\omega(t_1 + t_2))$$

$$1 = \sin(\omega(t_1 + t_2))$$

$$\omega(t_1 + t_2) = \sin^{-1}(1) = \frac{\pi}{2}$$

We also know that the time to reach A/2 from 0 is $\omega t_1 = \frac{\pi}{6}$.

The time to reach A from A/2 can be written as:

$$\omega t_2 = \omega(t_1 + t_2) - \omega t_1 = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

3. Ratio of Times:

We can find the ratio of t_1 to t_2:

$$\frac{\omega t_1}{\omega t_2} = \frac{\pi/6}{\pi/3} = \frac{1}{2}$$

$$\frac{t_1}{t_2} = \frac{1}{2}$$

4. Finding t2:

We are given that $t_1 = 2$ seconds.

Using the ratio, we can find t_2:

$$t_2 = 2t_1 = 2 \times 2 = 4 \text{ seconds}$$

Conclusion:

The time taken for the particle to travel from A/2 to A is 4 seconds.

Quick Tip

In simple harmonic motion, the period and fractional parts of the motion are proportional, facilitating calculations for partial oscillations.

---

42. Match List I with List II:

List I	List II
A. Isothermal Process	I. Work done by the gas decreases internal energy
B. Adiabatic Process	II. No change in internal energy
C. Isochoric Process	III. The heat absorbed goes partly to increase internal energy and partly to do work
D. Isobaric Process	IV. No work is done on or by the gas

Choose the correct answer from the options given below :

(1) $A - II, B - I, C - III, D - IV$

(2) $A - II, B - I, C - IV, D - III$

(3) $A - I, B - II, C - IV, D - III$

(4) $A - I, B - II, C - III, D - IV$

Correct Answer: (2) $A - II, B - I, C - IV, D - III$.

Solution:

For each thermodynamic process, we match the correct description based on the laws of thermodynamics:

- **Isothermal process:** In an isothermal process, the temperature remains constant, and thus the change in internal energy (ΔU) is zero. Therefore, the correct match is $A \to II$.
- **Adiabatic process:** In an adiabatic process, no heat is exchanged ($Q = 0$). The work done during the process results in a change in internal energy, which gives the match $B \to I$.
- **Isochoric process:** In an isochoric process, the volume remains constant, meaning no work is done ($\Delta W = 0$). The energy change is only due to heat, so the match is $C \to IV$.
- **Isobaric process:** In an isobaric process, the pressure remains constant. Heat added to the system is used both to do work and to increase internal energy, resulting in $D \to III$.

Quick Tip

Understanding the fundamental properties of thermodynamic processes is crucial in correctly associating them with their effects on internal energy, work, and heat exchange.

---

43. Match List I with List II:

List I	List II
A. Troposphere	I. Approximate 65-75 km over Earth's surface
B. E-Part of Stratosphere	II. Approximate 300 km over Earth's surface
C. F_2-Part of Thermosphere	III. Approximate 10 km over Earth's surface
D. D-Part of Stratosphere	IV. Approximate 100 km over Earth's surface

Choose the correct answer from the options given below :

(1) $A - III, B - IV, C - II, D - I$

(2) $A - I, B - II, C - IV, D - III$

(3) $A - I, B - IV, C - III, D - II$

(4) $A - III, B - II, C - I, D - IV$

Correct Answer: (1) $A - III, B - IV, C - II, D - I$.

Solution:

For each layer of Earth's atmosphere, the typical altitudes are as follows:

- **Troposphere:** The troposphere extends up to approximately 10 km, so the correct match is $A \to III$.
- **E-Part of Stratosphere:** This part of the stratosphere reaches up to about 50 km. Considering the general range, the match is $B \to IV$.

- **F_2-Part of Thermosphere:** The F_2-layer of the thermosphere can extend as high as 300 km, resulting in the match $C \to II$.
- **D-Part of Stratosphere:** The D-part of the stratosphere reaches about 50 km, so the match is $D \to I$.

Quick Tip

Familiarity with atmospheric layers and their characteristics, such as average altitudes, is essential in atmospheric science and related fields for accurate description and study.

---

44. A body of mass m is taken from earth surface to the height equal to twice the radius of earth (R_e), the increase in potential energy will be:
(g = acceleration due to gravity on the surface of Earth)

(1) $3mgR_e$
(2) $\frac{1}{3}mgR_e$
(3) $\frac{2}{3}mgR_e$
(4) $\frac{1}{2}mgR_e$

Correct Answer: (3) $\frac{2}{3}mgR_e$.

Solution:

The potential energy at a distance R from the center of the Earth is given by:

$$U = -\frac{GM_em}{R}$$

At the surface ($R = R_e$):

$$U_i = -\frac{GM_em}{R_e}$$

At height $h = 2R_e$ above the surface ($R = 3R_e$):

$$U_f = -\frac{GM_em}{3R_e}$$

The change in potential energy (ΔU) is:

$$\Delta U = U_f - U_i = \frac{2GM_emR_e}{3R_e^2} = \frac{2}{3}mgR_e$$

Quick Tip

When calculating changes in gravitational potential energy, consider the total distance from the center of the Earth rather than just the altitude above the surface.

---

45. A wire of length 1 m moving with velocity 8 m/s at right angles to a magnetic field of 2 T. The magnitude of induced emf between the ends of wire will be:

(1) 20 V

(2) 8 V

(3) 12 V

(4) 16 V

Correct Answer: (4) 16 V.

Solution:

The induced electromotive force (emf), denoted by ε, in a wire moving perpendicularly through a magnetic field is given by the formula:

$$\varepsilon = Bvl$$

where B represents the magnetic field strength, v is the velocity of the wire, and l is the length of the wire. Substituting the given values:

$$\varepsilon = 2\,\text{T} \times 8\,\text{m/s} \times 1\,\text{m} = 16\,\text{V}$$

Quick Tip

For a conductor moving in a magnetic field, remember that the induced emf is directly proportional to the magnetic field strength, the velocity of the conductor, and its length.

---

46. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at $t = 5$ s is:

(1) 40 m/s

(2) 25 m/s

(3) 20 m/s

(4) 8 m/s

Correct Answer: (1) 40 m/s.

Solution:

The velocity v of the particle is the derivative of the position with respect to time. Given the position function:

$$v = \frac{dx}{dt} = \frac{d}{dt}(4t^2) = 8t$$

At $t = 5$ seconds, the velocity is:

$$v = 8 \times 5 = 40 \text{ m/s}$$

Quick Tip

For motion described by a quadratic equation, velocity is linearly proportional to time, indicating constant acceleration.

---

47. Two objects are projected with the same velocity u but at different angles α and β with the horizontal. If $\alpha + \beta = 90^\circ$, the ratio of horizontal range of the first object to the 2nd object will be:

(1) $4 : 1$

(2) $2 : 1$

(3) $1 : 2$

(4) $1 : 1$

Correct Answer: (4) $1 : 1$.

Solution:

The range R for a projectile is given by:

$$R = \frac{u^2 \sin 2\theta}{g}$$

Range for projection angle "α"

$R_1 = \frac{u^2 \sin 2\alpha}{g}$

Range for projection angle "β"

$R_2 = \frac{u^2 \sin 2\beta}{g}$

$\alpha + \beta = 90^\circ$ (Given)

$\Rightarrow \beta = 90^\circ - \alpha$

$R_2 = \frac{u^2 \sin 2(90^\circ - \alpha)}{g}$

$R_2 = \frac{u^2 \sin(180^\circ - 2\alpha)}{g}$

$R_2 = \frac{u^2 \sin 2\alpha}{g}$

$\Rightarrow \frac{R_1}{R_2} = \frac{\left(\frac{u^2 \sin 2\alpha}{g}\right)}{\left(\frac{u^2 \sin 2\alpha}{g}\right)} = \frac{1}{1}$

Thus,

$$R_\alpha = R_\beta$$

The ratio $R_\alpha : R_\beta = 1 : 1$.

Quick Tip

In projectile motion, if two angles sum to 90°, their ranges are equal due to the symmetry in the sine function over the interval $[0^\circ, 180^\circ]$.

48. The resistance of a wire is 5 Ω. If it's stretched to 5 times of its original length, its new resistance will be:

(1) $625\,\Omega$

(2) $5\,\Omega$

(3) $125\,\Omega$

(4) $25\,\Omega$

Correct Answer: (3) $125\,\Omega$.

Solution:

The resistance R of a wire is proportional to its length L and inversely proportional to its cross-sectional area A:

$$R = \rho\frac{L}{A}$$

When the wire is stretched to five times its original length, its new length is $5L$ and its new area A' is $\frac{A}{5}$ (assuming volume conservation):

$$V_i = V_f A_i l_i = A_f l_f Al = A'(5l)A' = \frac{A}{5}R_f = \frac{\rho l_f}{A_f} = \frac{\rho(5l)}{\left(\frac{A}{5}\right)} = 25\left(\frac{\rho l}{A}\right) = 25 \times 5 = 125\Omega$$

thus, the correct option is (3) $125\,\Omega$.

Quick Tip

Remember that stretching a wire affects both its length and cross-sectional area, impacting resistance significantly due to the squared factor in the new cross-sectional area.

49. Given below are two statements:

Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends

on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below.

(1) Statement I is incorrect but statement II is correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is correct but statement II is incorrect

(4) Both statement I and statement II are correct

Correct Answer: (4) Both statement I and statement II are correct.

Solution:

Statement I: True. The stopping potential in the photoelectric effect depends on the frequency (or wavelength) of the light, rather than its intensity or power.

Statement II: True. The maximum kinetic energy of the emitted photoelectrons is given by the equation:

$$KE_{\text{max}} = \frac{hc}{\lambda} - \phi$$

where λ is the wavelength of the incident light and ϕ is the work function of the metal.

Both statements are accurate and correctly reflect the principles of the photoelectric effect.

Quick Tip

In understanding the photoelectric effect, it's important to distinguish the roles of light intensity and frequency in the emission and energy of photoelectrons.

---

50. Every planet revolves around the sun in an elliptical orbit:

Statements:

A. The force acting on a planet is inversely proportional to the square of the distance from the sun.

B. The force acting on a planet is inversely proportional to the product of the masses of the planet and the sun.

C. The centripetal force acting on the planet is directed away from the sun.

D. The square of the time period of the revolution of a planet around the sun is directly proportional to the cube of the semi-major axis of the elliptical orbit.

Options:

(1) A and D only

(2) C and D only

(3) B and C only

(4) A and C only

Correct Answer: (1) A and D only.

Solution:

Statement **A** is correct, as it accurately reflects Newton's law of universal gravitation, which states that the gravitational force between two masses is inversely proportional to the square of the distance between them.

Statement **D** is correct, as it expresses Kepler's third law, which links the square of a planet's orbital period to the cube of its semi-major axis in elliptical orbits.

Statement **B** is incorrect because the gravitational force is directly proportional to the product of the masses, not inversely.

Statement **C** is incorrect because the centripetal force always points towards the center of motion (the sun), not away from it.

Quick Tip

A solid understanding of Newton's laws and Kepler's laws is crucial for correctly interpreting gravitational interactions and orbital mechanics in astronomy.

Section B

51. A capacitor has a capacitance of 5 μF when its parallel plates are separated by an

air medium of thickness d. A slab of material with a dielectric constant of 1.5, having an area equal to that of the plates but with thickness $d/2$, is inserted between the plates. The capacitance of the capacitor in the presence of the slab will be $\_\_\_\ \mu F$:

Correct Answer: $6\,\mu F$

Solution:

When a dielectric partially fills the capacitor, the capacitance can be determined by modeling the system as two capacitors in series: one filled with the dielectric and one without. The total capacitance is then given by:

$$\begin{aligned} C_{\text{new}} &= \frac{\epsilon_0 A}{\frac{\left(\frac{d}{2}\right)\left(\frac{d}{2}\right)}{1.5} + \frac{\left(\frac{d}{2}\right)}{1}} \\ &= \frac{\epsilon_0 A}{\frac{d}{3} + \frac{d}{2}} = \frac{6\epsilon_0 A}{5d} \\ &= \frac{6}{5} \times 5\,\mu F = 6\,\mu F \end{aligned}$$

Quick Tip

When dealing with capacitors partially filled with dielectrics, consider simplifying the system into a combination of series capacitors to calculate the total capacitance.

---

52. A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 m/s):

Correct Answer: $400\,\text{Hz}$

Solution:

The observed frequency (f') when a source approaches an observer can be determined using the Doppler effect formula:

$$f' = \frac{f}{1 - \frac{v_s}{v_{\text{sound}}}}$$

Substituting the given values:

$$f' = \frac{320\,\text{Hz}}{1 - \frac{66\,\text{m/s}}{330\,\text{m/s}}}$$

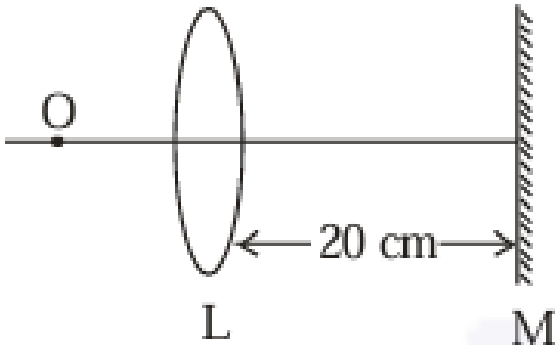
$$f' = 400\,\text{Hz}$$

Quick Tip

In Doppler effect problems, carefully consider the velocities of the source and observer relative to the medium, as the sign and magnitude of these velocities are key to determining the correct observed frequency.

---

53. An object is placed on the principal axis of a convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of the lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is:



Correct Answer: 30

Solution:

The system can be analyzed by considering the light passing through the lens, reflecting off the mirror, and then passing back through the lens. We apply the lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

For the first pass through the lens:

$$\frac{1}{15} - \frac{1}{-u} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{10} - \frac{1}{15}$$

$u = -30\,\text{cm}$ (The negative sign indicates that the object is on the same side as the incoming light)

To ensure the image formed by the mirror is 5 cm inside, we need to calculate the equivalent object distance after the reflection from the mirror, which results in a distance of 30 cm.

Quick Tip

When dealing with optical systems involving multiple reflections and lenses, analyze each segment of the light's path separately and apply the lens formula, while keeping track of the correct sign conventions.

---

54. Two long parallel wires carrying currents 8A and 15A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is ___ $\times 10^{-6}\,T$.

Correct Answer: $68 \times 10^{-6}\,T$.

Solution:

The magnetic field produced by each wire at point P can be calculated using the Biot-Savart Law:

$$B = \frac{\mu_0 I}{2\pi d}$$

For the 8A current:

$$B_1 = \frac{\mu_0 \times 8}{2\pi \times 7 \times 10^{-2}}$$

For the 15A current:

$$B_2 = \frac{\mu_0 \times 15}{2\pi \times 7 \times 10^{-2}}$$

Since the currents are flowing in opposite directions and point P is equidistant from both, the net magnetic field B_{net} is the vector sum of B_1 and B_2:

$$B_{\text{net}} = \sqrt{B_1^2 + B_2^2} \Rightarrow \frac{\mu_0}{2\pi d}\sqrt{i_1^2 + i_2^2}$$

$$\Rightarrow \frac{4\pi \times 10^{-7}}{2\pi \times \left(\frac{7}{\sqrt{2}}\right) \times 10^{-2}} \times \sqrt{8^2 + 15^2} \quad \left(d = \frac{7}{\sqrt{2}}\,\text{cm}\right)$$

Using the approximation $\sqrt{2} = 1.4$, we find that $B_{\text{net}} = 68 \times 10^{-6}$ T.

Quick Tip

When calculating the net magnetic field from multiple sources, ensure you account for both the magnitude and direction of the individual field components, particularly when currents flow in opposite directions.

---

55. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the ratio $\frac{u_f}{u_i} = \frac{10}{x}$. Then value of x is ___:

Correct Answer: 10.

Solution:

The surface energy of a drop is directly proportional to its surface area. When the original drop is split into many smaller drops, the total surface area—and thus the total surface energy—increases.

Starting with the volume equation:

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

$$R = 10r$$

The initial surface energy (u_i) is given by:

$$u_i = T \times 4\pi R^2$$

The final surface energy (u_f) is:

$$u_f = T \times 4\pi r^2 \times 1000$$

Now, we calculate the ratio of final to initial surface energy:

$$\frac{u_f}{u_i} = \frac{1000r^2}{R^2}$$

$$\frac{u_f}{u_i} = \frac{1000r^2}{(10r)^2} = 10$$

So, $x = 10$.

Quick Tip

Remember that when a large drop splits into smaller drops, the increase in surface area (and hence surface energy) demonstrates the non-linear relationship between volume and surface area for spherical objects.

---

56. A body of mass 1 kg collides head-on with a stationary body of mass 3 kg. After the collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is:

Correct Answer: $4\,m/s$.

Solution:

By applying the principle of conservation of momentum and considering the reversal of direction and speed after the collision, we use the equation of motion:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Using the given masses and velocities, we solve for u_1, which is found to be 4 m/s in the opposite direction.

Quick Tip

When solving collision problems involving direction reversal, it is important to carefully track the sign and magnitude of the velocities while applying conservation of momentum.

---

57. A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3:2. The ratio of their nuclear sizes will be $\left(\frac{x}{3}\right)^{\frac{1}{3}}$. The value of x is:

Correct Answer: 2.

Solution:

Given the velocity ratio and applying the conservation of momentum, the mass ratio is inversely proportional to the velocity ratio. We calculate the size ratio by cubing the mass ratio, as nuclear size is related to volume:

$$\frac{v_1}{v_2} = \frac{3}{2}$$

Using conservation of momentum:

$$m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$$

Since nuclear mass density is constant, we have:

$$\frac{m_1}{\frac{4}{3}\pi r_1^3} = \frac{m_2}{\frac{4}{3}\pi r_2^3}$$

This simplifies to:

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{m_1}{m_2}$$

Taking the cube root:

$$\frac{r_1}{r_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$$

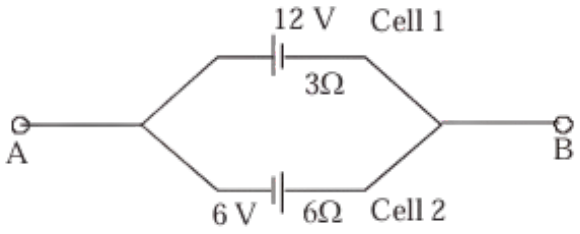
So this gives $x = 2$, indicating that the size ratio between the two fragments is 2.

Quick Tip

In problems involving disintegration or explosion, the conservation of momentum is crucial for relating the mass and velocity ratios of the resulting fragments. The size ratio can then be determined by considering the volumetric relationship.

---

58. Two cells are connected between points A and B as shown. Cell 1 has an emf of 12 V and internal resistance of 3Ω. Cell 2 has an emf of 6 V and internal resistance of 6Ω. An external resistor of 4Ω is connected across A and B. The current flowing through R will be ___ A.



Correct Answer: $1\,A$.

Solution:

The equivalent circuit can be simplified by combining the electromotive forces (emfs) and resistances. First, we calculate the equivalent voltage and resistance, and then apply Ohm's law to find the current through the external resistor:

$$E_{\text{eq}} = \frac{\frac{12}{3} - \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}}$$

$$E_{\text{eq}} = 6\,\text{V}$$

$$r_{\text{eq}} = 2\,\Omega$$

$$R = 4\,\Omega$$

Now, using Ohm's law, the current is calculated as:

$$i = \frac{6}{2+4}$$

Solving these gives the current as $1\,\text{A}$.

Quick Tip

In circuits with multiple sources and resistors, simplifying the circuit into an equivalent single-source circuit makes it much easier to compute the total current or voltage.

---

59. A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance $R = 80\,\Omega$, an inductor of inductive reactance $X_L = 70\,\Omega$, and a capacitor of capacitive reactance $X_C = 130\,\Omega$. The power factor of the circuit is $\frac{x}{10}$. The value of x

is:

Correct Answer: 8.

Solution:

The power factor $\cos\phi$ of an LCR circuit is given by the formula:

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Substituting the provided values:

$$\cos\phi = \frac{80}{\sqrt{80^2 + (70 - 130)^2}} = \frac{80}{\sqrt{80^2 + (-60)^2}} = \frac{80}{100} = 0.8$$

From this, $\frac{x}{10} = 0.8$, which gives $x = 8$.

Quick Tip

Understanding the phase relationship between the components of an LCR circuit is key to accurately calculating the power factor, which represents the phase difference between the voltage and current.

---

60. If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of the moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The value of x is:

Correct Answer: 5.

Solution:

The moment of inertia I_1 of a solid sphere about a tangent to its surface is calculated as:

$$I_1 = I_{\text{CM}} + mR^2 = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$$

For a sphere with mass $m = 5\,\text{kg}$, we get:

$$I_1 = 7R^2$$

The moment of inertia I_2 of the disc about a tangent in its plane is:

$$I_2 = \frac{m_2 R^2}{4} + m_2 R^2 = \frac{5}{4} m_2 R^2$$

For the disc with mass $m = 4\,\text{kg}$, we get:

$$I_2 = 6R^2$$

The ratio $\frac{I_2}{I_1}$ is:

$$\frac{I_2}{I_1} = \frac{5R^2}{7R^2} = \frac{5}{7}$$

Thus, $x = 5$.

Quick Tip

When calculating the moment of inertia about an axis not through the center of mass, remember to apply the parallel axis theorem.

CHEMISTRY

Section-A

61. Match List I with List II:

List I	List II
A. Cobalt catalyst	I. $H_2 + Cl_2$ production
B. Syngas	II. Water gas production
C. Nickel catalyst	III. Coal gasification
D. Brine solution	IV. Methanol production

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-III, C-IV, D-I

(4) A-IV, B-III, C-II, D-I

Correct Answer: (4) A-IV, B-III, C-II, D-I.

Solution:

Cobalt catalyst is widely used in the production of methanol. This is because cobalt helps in facilitating the hydrogenation of carbon monoxide to methanol in the presence of syngas (a mixture of carbon monoxide and hydrogen). The reaction typically takes place at high pressures and temperatures, making cobalt an essential component in the catalytic process. Thus, the correct match for cobalt catalyst is (A-IV), methanol production.

Syngas, primarily a mixture of carbon monoxide and hydrogen, is crucial in the process of coal gasification. In coal gasification, coal is reacted with oxygen and steam at high temperatures to produce syngas. This syngas is then used for various applications, including the production of chemicals like methanol and synthetic fuels. Hence, the correct match for syngas is (B-III), coal gasification.

Nickel catalysts are highly effective in the water gas shift reaction, where carbon monoxide and water vapor react to produce carbon dioxide and hydrogen. This reaction is crucial in producing hydrogen gas in the chemical industry. Nickel, with its ability to catalyze this reaction at moderate temperatures, is widely used for water gas production. Therefore, the correct match for nickel catalysts is (C-II), water gas production.

Brine solution is commonly used in the production of chlorine and hydrogen gases through the electrolysis of saltwater. In this process, the brine solution is subjected to electrolysis, which splits the sodium chloride (NaCl) into sodium hydroxide (NaOH), chlorine gas Cl_2, and hydrogen gas H_2. Thus, the correct match for brine solution is (D-I), chlorine and hydrogen production.

Quick Tip

Understanding the specific roles of catalysts and their applications in different chemical processes is crucial in industrial chemistry.

62. Given are two statements:

Statement I: In froth flotation method a rotating paddle agitates the mixture to drive air out of it.

Statement II: Iron pyrites are generally avoided for extraction of iron due to environmental reasons.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Correct Answer: (2) Statement I is false but Statement II is true.

Solution:

Statement I is false. In froth flotation, the primary purpose of the rotating paddle is to introduce air into the mixture, not to drive it out. The paddle helps create bubbles that attach to the mineral particles, making them float to the surface, where they can be collected as froth. This process is essential for the separation of valuable minerals from the gangue in the ore. Therefore, the statement that the paddle drives air out is incorrect.

Statement II is true. Iron pyrites (FeS_2), when exposed to air and water, undergo oxidation and produce sulfuric acid. This acidic runoff is known as acid mine drainage (AMD), which can lead to severe environmental harm by contaminating water sources and soils. The acid dissolves heavy metals like copper and lead from surrounding rocks, further contributing to pollution. Hence, the statement about iron pyrites leading to acidic runoff is accurate.

Quick Tip

Understanding the correct techniques and environmental impacts of various mineral extraction processes can enhance both efficiency and sustainability in mining operations.

63. Which of the following represents the correct order of metallic character of the given elements?

(1) $Si < Be < Mg < K$

(2) $Be < Si < Mg < K$

(3) $K < Mg < Be < Si$

(4) $Be < Si < K < Mg$

Correct Answer: (1) $Si < Be < Mg < K$.

Solution:

Metallic character refers to the ability of an element to lose electrons and form positive ions. As we move down a group in the periodic table, the atomic size increases, and the outer electrons are further from the nucleus, making it easier for the element to lose electrons. This results in an increase in metallic character down a group. On the other hand, as we move from left to right across a period, the atomic size decreases, and the effective nuclear charge increases, making it harder for the element to lose electrons.

Therefore, metallic character decreases across a period from left to right.

Among the given elements:

Potassium (K) is an alkali metal located in Group 1, and it is farthest to the left and down the group. Therefore, it has the highest metallic character.

Silicon (Si), being a metalloid in Group 14, has a lower metallic character compared to metals. It is further to the right and higher in the period, so it has the least metallic character among the elements listed.

So, the correct sequence is $Si < Be < Mg < K$.

Quick Tip

Recall that the periodic trends such as metallic character can greatly aid in predicting the properties and reactivity of elements.

64. Given below are two statements, one labeled as Assertion A and the other as Reason R:

Assertion A: The alkali metals and their salts impart characteristic color to reducing flame.

Reason R: Alkali metals can be detected using flame tests.

In the light of the above statements, choose the most appropriate answer form the options given below

(1) Both A and R are correct but R is NOT the correct explanation of A.

(2) A is correct but R is not correct.

(3) A is not correct but R is correct.

(4) Both A and R are correct and R is the correct explanation of A.

Correct Answer: (3) A is not correct but R is correct.

Solution:

Assertion A is incorrect. Alkali metals and their salts typically impart distinct colors to an oxidizing flame, not a reducing flame. When alkali metal salts are heated in a flame, they excite the metal ions, which then emit characteristic colors as they return to their ground state. These colors are observed in an oxidizing flame (such as that produced by a Bunsen burner with sufficient oxygen), not in a reducing flame, which lacks the necessary oxidizing conditions for such reactions to occur.

Reason R is correct. Flame tests are indeed a common and reliable method for identifying alkali metals and other metal ions based on the characteristic colors they emit when heated. For example, lithium produces a red flame, sodium a bright yellow flame, and potassium a lilac flame. This principle is widely used in qualitative analysis.

Quick Tip

In chemistry, the specificity of terms like ”oxidizing” and ”reducing” flames is crucial, as they can fundamentally change the interpretation of experimental results.

65. What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molar aqueous solution and 250 mL of 0.25 molar aqueous solution?

(1) 1 : 1

(2) 3 : 1

(3) 2 : 1

(4) 1 : 2

Correct Answer: (3) 2:1.

Solution:

For the 500 g solution:

First, calculate the moles of ethylene glycol:

$$\text{Moles of ethylene glycol} = 0.25\frac{\text{mol}}{\text{L}} \times 0.5\text{L} = 0.125\text{mol}$$

Next, calculate the mass of ethylene glycol using its molar mass (62 g/mol):

$$\text{Mass of ethylene glycol} = 0.125\text{mol} \times 62\text{g/mol} = 7.75\text{g}$$

For the 250 mL solution:

Calculate the moles of ethylene glycol:

$$\text{Moles of ethylene glycol} = 0.25\frac{\text{mol}}{\text{L}} \times 0.25\text{L} = 0.0625\text{mol}$$

Then, calculate the mass of ethylene glycol:

$$\text{Mass of ethylene glycol} = 0.0625\text{mol} \times 62\text{g/mol} = 3.875\text{g}$$

Thus, the mass ratio of ethylene glycol needed for the two solutions is:

$$7.75\text{g} : 3.875\text{g} \approx 2 : 1$$

Quick Tip

When preparing solutions of a specific molarity, it's essential to calculate both the volume of the solution and the mass of the solute required accurately to achieve the desired concentration.

---

66. Given two statements about dipole moments:

Statement I: Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative center and head pointing towards the positive center.

Statement II: The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both Statement I and Statement II are correct.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is correct but Statement II is incorrect.

Correct Answer: (4) Statement I is correct but Statement II is incorrect.

Solution:

Statement I is correct. In chemistry, the dipole moment is represented by an arrow with a positive sign (+) at the tail and a negative sign (−) at the head. The arrow points from the positive charge to the negative charge, indicating the direction of the dipole moment, which is important for understanding the polarity of molecules. This representation is conventional and widely accepted in molecular chemistry.

Statement II is incorrect. The crossed arrow does not represent the shift of charges in a molecule. Rather, the crossed arrow in dipole moment notation indicates the direction of the resultant dipole moment in the molecule. It shows the vector from the positive charge to the negative charge, not the movement of the charges themselves. Therefore, this statement

misinterprets the meaning of the crossed arrow in dipole moment representation.

Quick Tip

Understanding the correct representation and terminology in chemistry can help avoid common misconceptions about molecular properties like dipole moments.

---

67. Given below are two statements, one labeled as Assertion A and the other as Reason R:

Assertion A: Butylated hydroxyl anisole when added to butter increases its shelf life.

Reason R: Butylated hydroxyl anisole is more reactive towards oxygen than food.

In the light of the above statements, choose the most appropriate answer from the options given below :-

(1) Both A and R are correct and R is the correct explanation of A.

(2) A is correct but R is not correct.

(3) A is not correct but R is correct.

(4) Both A and R are correct but R is NOT the correct explanation of A.

Correct Answer: (1) Both A and R are correct and R is the correct explanation of A.

Solution:

Butylated hydroxytoluene (BHA) is an antioxidant commonly used in food preservation. It helps protect food from oxidative spoilage by reacting with oxygen molecules more readily than the food itself. This action prevents the oxidation of fats and oils in the food, which could otherwise lead to rancidity and loss of nutritional value. BHA stabilizes the food, prolonging its shelf life and maintaining its quality.

Quick Tip

Antioxidants like BHA are crucial in the food industry for extending shelf life by preventing oxidative damage.

---

68. Given below are several statements:

A. Ammonium salts produce haze in the atmosphere.

B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.

C. Polychlorinated biphenyls act as cleaning solvents.

D. 'Blue baby' syndrome occurs due to the presence of excess sulphate ions in water.

Choose the correct answer from the options given below :-

(1) A, B and C only

(2) B and C only

(3) A and D only

(4) A and C only

Correct Answer: (4) A and C only.

Solution:

Statement A is correct. Ammonium salts, such as ammonium sulfate and ammonium nitrate, can contribute to atmospheric haze. These salts can form fine particulate matter in the atmosphere, which scatters light and reduces visibility, leading to haze. This phenomenon is often associated with agricultural emissions and industrial pollution.

Statement B is incorrect. Ozone is primarily formed by the reaction of sunlight with oxygen molecules (O_2) and various pollutants like nitrogen oxides (NO_x) and volatile organic compounds (VOCs). While chlorine radicals can destroy ozone, they do not directly contribute to its formation.

Statement C is correct. Polychlorinated biphenyls (PCBs) are indeed industrial chemicals, but they were historically used in various products, including as coolants and lubricants in

electrical equipment. They are persistent environmental pollutants and are harmful to both human health and the environment. Although not cleaning solvents, they were widely used in industrial applications.

Statement D is incorrect. 'Blue baby' syndrome is caused by excess nitrate ions in drinking water, which can interfere with the ability of red blood cells to carry oxygen, leading to methemoglobinemia. This condition causes a bluish tint to the skin, particularly in infants. Sulfate ions are not responsible for this condition.

Quick Tip

Accurate understanding of environmental chemistry and toxicology is essential to correctly address the effects of chemicals in our environment.

---

69. Match List I with List II:

List I (Amines):	**List II (pKb):**
A. Aniline	**I.** 3.25
B. Ethylamine	**II.** 3.00
C. N-Ethylethanamine	**III.** 9.38
D. N,N-Diethylethanamine	**IV.** 3.29

Choose the correct answer from the options given below :-

(1) A-I, B-IV, C-II, D-III

(2) A-III, B-II, C-I, D-IV

(3) A-III, B-II, C-IV, D-I

(4) A-III, B-IV, C-II, D-I

Correct Answer: (4) A-III, B-IV, C-II, D-I.

Solution:

The problem presents two lists: List I contains four amines (Aniline, Ethylamine, N-Ethylethanamine, and N,N-Diethylethanamine),

and List II contains four pKb values (3.25, 3.00, 9.38, and 3.29).

The task is to match each amine in List I with its corresponding pKb value in List II.

The correct matching is as follows:

Aniline (A) corresponds to 9.38 (III),

Ethylamine (B) corresponds to 3.29 (IV),

N-Ethylethanamine (C) corresponds to 3.00 (II),

and N,N-Diethylethanamine (D) corresponds to 3.25 (I).

This matching indicates the basicity of the amines, where lower pKb values signify stronger bases.

The correct answer, therefore, is option (4): A-III, B-IV, C-II, D-I.

Quick Tip

Understanding the structure-activity relationship in organic compounds like amines can help predict their chemical behavior in biological and industrial processes.

---

70. Which one among the following metals is the weakest reducing agent?

(1) K

(2) Rb

(3) Na

(4) Li

Correct Answer: (3) Na.

Solution:

Among the given metals, sodium (Na) has the highest oxidation potential in the alkali metal group. The oxidation potential is a measure of an element's tendency to lose electrons and undergo oxidation. In alkali metals, the higher the oxidation potential, the weaker the reducing

agent the metal is. Sodium, having the highest oxidation potential in this group, is therefore the weakest reducing agent. This is because a higher oxidation potential means that sodium is less inclined to lose electrons compared to other alkali metals.

Quick Tip

In redox reactions, the element with the higher oxidation potential is less likely to lose electrons, making it a weaker reducing agent.

---

71. Match List I with List II:

List I (Isomeric pairs)	List II (Type of isomers)
A. Propanamine and N-Methylethanamine	I. Metamers
B. Hexan-2-one and Hexan-3-one	II. Positional isomers
C. Ethanamide and Hydroxyethanamine	III. Functional isomers
D. o-nitrophenol and p-nitrophenol	IV. Tautomers

(1) A-III, B-IV, C-I, D-II

(2) A-IV, B-III, C-I, D-II

(3) A-II, B-III, C-I, D-IV

(4) A-III, B-I, C-IV, D-II

Correct Answer: (4) A-III, B-I, C-IV, D-II.

Solution:

A. **Propanamine** and **N-Methylethanamine**:

Propanamine ($CH_3CH_2CH_2NH_2$): This is a primary amine where the amino group (-NH_2) is attached to a propane chain.

N-Methylethanamine ($CH_3NHCH_2CH_3$): This is a secondary amine where a methyl group (-CH_3) and an ethyl group (-CH_2CH_3) are attached to the nitrogen atom of the amino group.

Relationship: Functional Isomers: These compounds have the same molecular formula but different functional groups (primary vs. secondary amine).

B. **Hexan-2-one** and **Hexan-3-one**:

Hexan-2-one ($CH_3COCH_2CH_2CH_2CH_3$): This is a ketone where the carbonyl group (C=O) is located on the second carbon of a six-carbon chain.

Hexan-3-one ($CH_3CH_2COCH_2CH_2CH_3$): This is a ketone where the carbonyl group (C=O) is located on the third carbon of a six-carbon chain.

Relationship: Metamers: These are isomers that differ in the position of the alkyl groups around a functional group (in this case, the carbonyl group).

C. **Ethanamide** and **Hydroxyethanimine**:

Ethanamide (CH_3CONH_2): This is an amide, a derivative of a carboxylic acid, with the -$CONH_2$ group.

Hydroxyethanimine ($CH_3C(OH)=NH$): This is an imine with a hydroxyl group (-OH) attached to the carbon of the C=N double bond.

Relationship: Tautomers: These are isomers that interconvert by the movement of a proton and a double bond. This is a specific type of structural isomerism.

D. **o-Nitrophenol** and **p-Nitrophenol**:

o-Nitrophenol (ortho-Nitrophenol): This is a phenol (benzene ring with an -OH group) with a nitro group (-NO_2) attached to the carbon adjacent to the carbon bearing the -OH group.

p-Nitrophenol (para-Nitrophenol): This is a phenol with a nitro group attached to the carbon opposite the carbon bearing the -OH group.

Relationship: Positional Isomers: These are isomers that differ in the position of a substituent (in this case, the nitro group) on a ring structure.

Quick Tip

Knowing the types of isomers is fundamental in organic chemistry as it helps in understanding the properties and reactions of different compounds.

72. Match List I with List II (Uses of polymers):

List I (Name of polymer)	List II
A. Glyptal	I. Flexible pipes
B. Neoprene	II. Synthetic wool
C. Acrilan	III. Paints and Lacquers
D. LDP	IV. Gaskets

Choose the correct answer from the options given below :-

(1) A-III, B-II, C-IV, D-I

(2) A-III, B-IV, C-II, D-I

(3) A-III, B-IV, C-I, D-II

(4) A-III, B-I, C-IV, D-II

Correct Answer: (2) A-III, B-IV, C-II, D-I.

Solution:

Glyptal is a type of polyester resin used primarily in the production of paints and lacquers. It is known for its excellent durability and resistance to chemicals and weathering, making it ideal for coating applications.

Neoprene is a synthetic rubber used in the manufacture of gaskets, seals, and protective coatings. It is resistant to oil, heat, and weather, making it highly suitable for applications where durability and flexibility are needed.

Acrilan is a type of acrylic fiber commonly used as synthetic wool. It is lightweight, warm, and resistant to wrinkles and shrinking, making it ideal for use in clothing and textiles.

Low Density Polyethylene (LDPE) is a polymer used in the production of flexible pipes, bags, and containers. It is known for its flexibility, low-density structure, and resistance to chemical attack, making it suitable for flexible piping systems.

Quick Tip

Knowledge of material applications is crucial in engineering and design, ensuring materials are used optimally based on their properties.

---

73. Given below are two statements, one labeled as Assertion A and the other as Reason R:

Assertion A: Carbon forms two important oxides — CO and CO2. CO is neutral whereas CO2 is acidic in nature.

Reason R: CO2 can combine with water in a limited way to form carbonic acid, which is sparingly soluble in water.

(1) Both A and R are correct but R is NOT the correct explanation of A.

(2) Both A and R are correct and R is the correct explanation of A.

(3) A is not correct but R is correct.

(4) A is correct but R is not correct.

Correct Answer: (2) Both A and R are correct and R is the correct explanation of A.

Solution:

Step 1: Assertion A: Carbon indeed forms two important oxides — carbon monoxide (CO) and carbon dioxide (CO2).

Carbon monoxide (CO) is neutral, whereas carbon dioxide (CO2) is acidic in nature. This is true because CO does not form acidic solutions when dissolved in water, while CO2 forms carbonic acid (H2CO3) when dissolved in water, making it acidic.

Step 2: Reason R: CO2 can combine with water in a limited way to form carbonic acid (H2CO3), which is sparingly soluble in water. This is also correct, as carbonic acid is indeed formed when CO2 dissolves in water, and it does not completely dissolve, which is why it's considered sparingly soluble.

Conclusion: Since both statements are correct and the Reason (R) properly explains the Assertion (A), the correct answer is that both A and R are correct and R is the correct explanation of A.

Quick Tip

When considering oxides of carbon, remember that CO is neutral while CO2 is acidic due to its ability to form carbonic acid in water.

---

74. Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from:

(1) +3 to +1
(2) +6 to +3
(3) +2 to +1
(4) +6 to +2

Correct Answer: (2) +6 to +3.

Solution:

In acidic solution, potassium dichromate ($K_2Cr_2O_7$) undergoes a reduction reaction. The chromium in the dichromate ion ($Cr_2O_7^{2-}$) is initially in the +6 oxidation state. During the reaction, this chromium is reduced to the +3 oxidation state, forming Cr^{3+} ions. This reduction involves the gain of electrons, and the overall process is an example of a redox reaction where the chromium species is reduced.

Quick Tip

Understanding redox reactions, including the specific changes in oxidation states, is essential for predicting the outcomes of chemical reactions and their applications in industry.

---

75. When the hydrogen ion concentration [H^+] changes by a factor of 1000, the value of pH of the solution:

(1) increases by 1000 units

(2) decreases by 3 units

(3) decreases by 2 units

(4) increases by 2 units

Correct Answer: (2) decreases by 3 units.

Solution:

The change in hydrogen ion concentration by a factor of 1000 corresponds to a change in pH, which is calculated using the following formula:

$$\Delta pH = -\log[\Delta H^+] = -\log[10^3] = -3$$

This shows that the pH decreases by 3 units, as the hydrogen ion concentration increases by a factor of 1000.

Quick Tip

The pH scale is logarithmic; thus, a tenfold increase in [H^+] concentration results in a decrease of 1 pH unit.

---

76. Match List I with List II:

List I (Coordination entity):	List II (Wavelength of light absorbed in nm):
A. $[CoCl(NH_3)_5]^{2+}$	I. 310
B. $[Co(NH_3)_6]^{3+}$	II. 475
C. $[Co(CN)_6]^{3-}$	III. 535
D. $[Cu(H_2O)_4]^{2+}$	IV. 600

(1) A-IV, B-I, C-III, D-II

(2) A-III, B-II, C-I, D-IV

(3) A-III, B-I, C-II, D-IV

(4) A-II, B-III, C-IV, D-I

Correct Answer: (2) A-III, B-II, C-I, D-IV .

Solution:

The correct match based on the specific absorption properties of each coordination complex is as follows:

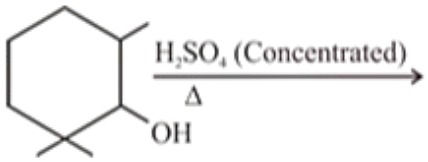
- A absorbs at 535 nm, which corresponds to a specific wavelength where the coordination complex exhibits a peak absorption.
- B absorbs at 475 nm, a different wavelength indicating distinct electronic transitions within the complex.
- C absorbs at 310 nm, which is indicative of a higher energy transition, likely involving ligand-to-metal charge transfer or other electronic changes.
- D absorbs at 600 nm, showing absorption in the visible spectrum, typically associated with metal-ligand interactions in the complex.

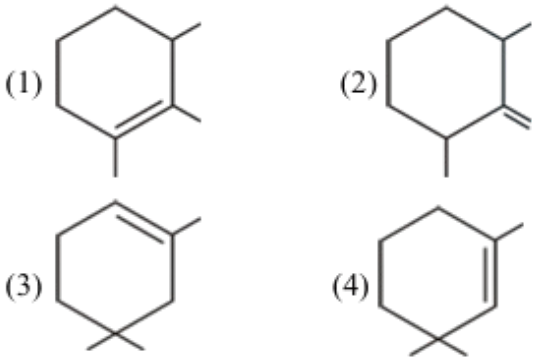
Quick Tip

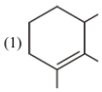
Understanding the absorption properties of coordination compounds is crucial in the field of spectroscopy and material science.

---

77. Find out the major product from the following reaction





Correct Answer: (1). 

Solution:

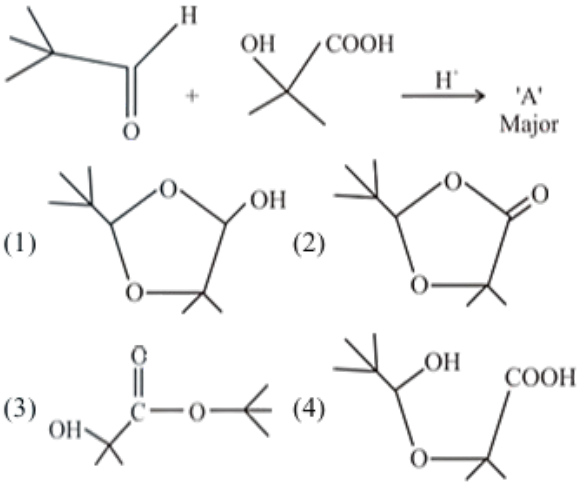
In the presence of concentrated H_2SO_4 (sulfuric acid), phenol undergoes dehydration to form a phenyl cation ($C_6H_5^+$), which is the more stable intermediate in this reaction. The formation of this carbocation is followed by a rearrangement, where the positive charge shifts to a more stable position. Finally, the phenyl cation stabilizes itself by forming a double bond, resulting in product 1, which is typically a product of elimination (such as an alkene).

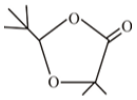
Quick Tip

Concentrated H_2SO_4 is a strong dehydrating agent that often leads to carbocation rearrangements and elimination reactions in organic compounds.

---

78. Identify 'A' in the given reaction to produce the major product:



Correct Answer: (2). 

Solution:

Compound 'A' undergoes an aldol addition followed by dehydration. The reaction starts with the formation of an enolate ion from the ketone. This enolate ion attacks the aldehyde, leading to the formation of a beta-hydroxy ketone (an aldol addition product). The beta-hydroxy ketone then undergoes dehydration, which involves the removal of a water molecule, resulting in the formation of an enone (the major product, shown in option 2).

Quick Tip

Aldol condensations are key reactions in organic synthesis, combining aldehyde and ketone components to form β-hydroxy ketones or aldehydes, which can further dehydrate to enones or enals.

---

79. The isomeric deuterated bromide with molecular formula C_4H_9DBr having two chiral carbon atoms is:

(1) 2-Bromo-1-deuterobutane

(2) 2-Bromo-2-deuterobutane

(3) 2-Bromo-3-deuterobutane

(4) 2-Bromo-1-deutero-2-methylpropane

Correct Answer: (3). 2-Bromo-3-deuterobutane

Solution:

The compound described has two chiral centers, which means that two carbon atoms are each attached to four different groups. The structure consists of a four-carbon chain with a deuterium atom and a bromine atom located on separate carbons. Based on the given information, the correct name for the compound is 2-Bromo-3-deuterobutane, where:

The bromine is attached to the second carbon in the chain, and

The deuterium (D) is attached to the third carbon.

Quick Tip

Chiral centers occur where a carbon atom is attached to four different groups. Recognizing the position of substituents helps in identifying the chiral centers.

---

80. A chloride salt solution acidified with dil. HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3$. [A] on treatment with NH_4OH gives a clear solution, B. The correct products are:

(1) $H[AgCl_3]$ and $[Ag(NH_3)_2]Cl$

(2) $[HAgCl_3]$ and $[NH_4]$ $Ag(OH)_2$

(3) AgCl and $[Ag(NH_3)_2]Cl$

(4) AgCl and $[NH_4]$ $Ag(OH)_2$

Correct Answer: (3). AgCl and $[Ag(NH_3)_2]Cl$

Solution:

The precipitate formed when silver chloride (AgCl) is produced is insoluble in water but soluble in ammonia. Ammonia acts as a complexing agent and dissolves AgCl by forming the complex ion $[Ag(NH_3)_2]Cl$, which is known as diamminesilver chloride. This complexation

occurs because ammonia molecules coordinate with the silver ion, forming a soluble complex.

Quick Tip

Ammonia acts as a complexing agent and can dissolve otherwise insoluble silver chloride through complex formation.

Section-B

81. The number of given orbitals which have electron density along the axis is:

P_x, P_y, P_z, d_{xy}, d_{yz},d_{xz}, d_{z^2}, $d_{x^2-y^2}$

Correct Answer: 5.00.

Solution:

The orbitals p_x, p_y, p_z, d_{z^2}, and $d_{x^2-y^2}$ are considered axial orbitals.

The p-orbitals, such as $p_x, p_y,$ and p_z, have their lobes aligned along the x, y, and z axes, respectively. These are termed as axial orbitals since they lie along the axis of symmetry of a molecule.

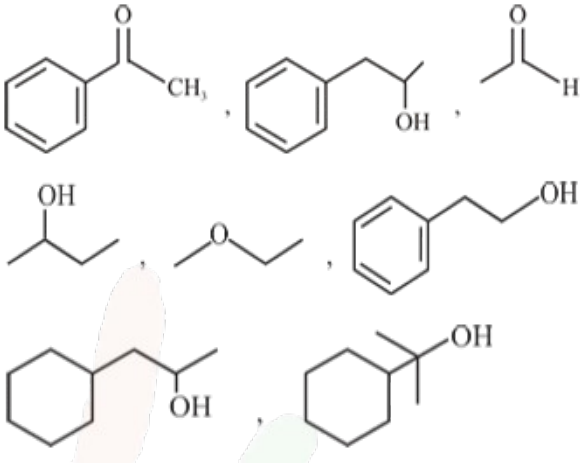
The d-orbitals, specifically d_{z^2} and $d_{x^2-y^2}$, also align with the axis of symmetry and are part of the axial set, which directly participate in bonding along the principal axis in a molecule.

Quick Tip

Axial orbitals are those that align along the principal axis of symmetry, such as the z-axis in many molecular geometries.

---

82. Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is:



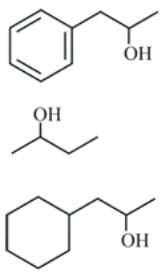
Correct Answer: 3.00.

Solution:

Three compounds meet the criteria for both tests, indicating the presence of specific functional groups reactive to these tests. These compounds contain functional groups that react with ceric ammonium nitrate and the iodoform test:

Ceric ammonium nitrate is used to detect alcohols and phenols, where the reaction forms a color change.

The iodoform test is specific for methyl ketones, where the formation of a yellow precipitate (iodoform) indicates the presence of the functional group.



Thus, the three compounds must contain alcohols, phenols, or methyl ketones to show a positive result for both tests.

Quick Tip

Ceric ammonium nitrate tests for alcohols and phenols, while the iodoform test is specific for methyl ketones.

---

83. The number of pairs of the solution having the same value of osmotic pressure from the following is:

A. 0.500 M C_2H_5OH (aq) and 0.25 M KBr (aq)

B. 0.100 M $K_4[Fe(CN)_6]$ (aq) and 0.100 M $FeSO_4(NH_4)_2SO_4$ (aq)

C. 0.05 M $K_4[Fe(CN)_6]$ (aq) and 0.25 M $NaCl$ (aq)

D. 0.15 M $NaCl$ (aq) and 0.1 M $BaCl_2$ (aq)

E. 0.02 M KCl $MgCl_2.6H_2O$ (aq) and 0.05 M KCl (aq)

Correct Answer: (4)

Solution:

Osmotic pressure is determined by the ion concentration in solution, considering both the dissociation of solutes and the total number of ions produced. In this case, we analyze each pair based on the number of ions they produce when dissociated in solution. The osmotic pressure is directly proportional to the total concentration of dissolved particles.

Pair A, B, D, and E: These pairs have the same osmotic pressures because they produce the same total number of ions when dissociated, despite possibly differing in the nature of the solute. This results in the same ion concentration and osmotic pressure under identical conditions.

Quick Tip

Osmotic pressure is influenced by the total number of dissolved particles in the solution. This characteristic allows us to predict the osmotic behavior of various solutions under similar conditions.

84. 28.0 L of $\mathrm{CO_2}$ is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25°C and 1 atm. Heat evolved during the combustion process is ___ kJ.

Given:

$\Delta H_c(\mathrm{CH_4}) = -900$ kJ/mol

$\Delta H_c(\mathrm{C_2H_4}) = -1400$ kJ/mol

Correct Answer: (847.00)

Solution:

The volume of $\mathrm{CO_2}$ produced enables us to calculate the moles of $\mathrm{CH_4}$ and $\mathrm{C_2H_4}$ combusted, from which we can determine the total heat evolved using their respective combustion enthalpies.

Let, the volume of

$$C_2H_4 \text{ be } x \text{ litres.}$$

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$$

Initial volume: x

Final volume: $2x$

For $\mathrm{CH_4}$, the reaction is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Initial volume: $16.8 - x$

Final volume: $16.8 - x$

The total volume of $\mathrm{CO_2}$ produced is:

$$\text{Total } CO_2 = 2x + (16.8 - x)$$

Thus,

$$28 = 16.8 + x$$

Solving for x, we get

$$x = 11.2 \text{ L}$$

Now, calculate the moles of the gases:

$$n_{CH_4} = \frac{PV}{RT} = \frac{1 \times 5.6}{0.082 \times 298} = 0.229 \text{ moles}$$

$$n_{C_2H_4} = \frac{11.2}{0.082 \times 298} = 0.458 \text{ moles}$$

Therefore, the heat evolved is:

$$\text{Heat evolved} = 0.229 \times 900 + 0.458 \times 1400$$

$$= 206.1 + 641.2 = 847.3 \text{ kJ}$$

Quick Tip

Understanding stoichiometry and gas laws helps in calculating reactant and product quantities in chemical reactions, especially in combustion processes.

---

85. Total number of moles of AgCl precipitated on addition of excess of AgNO3 to one mole each of the following complexes: $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$, and $[Pd(NH_3)_4]Cl_2$

Correct Answer: (5.00)

Solution:

We need to analyze each complex and determine how many chloride ions are outside the coordination sphere (i.e., are ionizable). Only these chloride ions will react with $AgNO_3$ to form AgCl precipitate.

1. **$[Co(NH_3)_4Cl_2]Cl$:**

   This complex has one chloride ion outside the coordination sphere. When $AgNO_3$ is added, it will react with this chloride ion:

$[Co(NH_3)_4Cl_2]Cl + AgNO_3 \rightarrow [Co(NH_3)_4Cl_2]^+ + AgCl + NO_3^-$

Thus, 1 mole of AgCl will be precipitated.

2. **$[Ni(H_2O)_6]Cl_2$:**

   This complex has two chloride ions outside the coordination sphere. When $AgNO_3$ is added, it will react with both chloride ions:

   $[Ni(H_2O)_6]Cl_2 + 2AgNO_3 \rightarrow [Ni(H_2O)_6]^{2+} + 2AgCl + 2NO_3^-$

   Thus, 2 moles of AgCl will be precipitated.

3. **$[Pt(NH_3)_2Cl_2]$:**

   This complex has no chloride ions outside the coordination sphere. Therefore, it will not react with $AgNO_3$ and no AgCl will be precipitated.

4. **$[Pd(NH_3)_4]Cl_2$:**

   This complex has two chloride ions outside the coordination sphere. When $AgNO_3$ is added, it will react with both chloride ions:

   $[Pd(NH_3)_4]Cl_2 + 2AgNO_3 \rightarrow [Pd(NH_3)_4]^{2+} + 2AgCl + 2NO_3^-$

   Thus, 2 moles of AgCl will be precipitated.

Now, let's sum up the moles of AgCl precipitated:

1 mole (from $[Co(NH_3)_4Cl_2]Cl$) + 2 moles (from $[Ni(H_2O)_6]Cl_2$) + 0 moles (from $[Pt(NH_3)_2Cl_2]$) + 2 moles (from $[Pd(NH_3)_4]Cl_2$) = 5 moles

Therefore, the total number of moles of AgCl precipitated is 5.

Final Answer: 5.00

Quick Tip

Coordination chemistry principles allow the prediction of reaction products when complexes interact with other chemical agents, such as silver nitrate.

---

86. Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is:

Given: Molar mass of A = 84 g/mol

Correct Answer: (12.00)

Solution:

1. Calculate the moles of each element:

To find the moles of each element, divide the percentage composition by the atomic mass of the respective element.

- Moles of Carbon (C) = $\frac{85.8}{12} = 7.15$ moles
- Moles of Hydrogen (H) = $\frac{14.2}{1} = 14.2$ moles

2. Determine the mole ratio:

Divide the moles of each element by the smallest number of moles calculated.

- Mole ratio of Carbon (C) = $\frac{7.15}{7.15} = 1$
- Mole ratio of Hydrogen (H) = $\frac{14.2}{7.15} \approx 2$

3. Write the empirical formula:

The empirical formula represents the simplest whole-number ratio of atoms in a compound. Based on the mole ratio, the empirical formula is:

CH_2

4. Calculate the empirical formula mass:

Add the atomic masses of the elements in the empirical formula.

Empirical formula mass = (1 × Atomic mass of C) + (2 × Atomic mass of H) Empirical formula mass = (1 × 12) + (2 × 1) = 12 + 2 = 14 g/mol

5. Calculate the value of n:

Divide the molecular weight of the compound by the empirical formula mass.

$n = \frac{\text{Molecular weight}}{\text{Empirical formula mass}} = \frac{84}{14} = 6$

6. Determine the molecular formula:

Multiply the subscripts in the empirical formula by the value of n.

Molecular formula = $(CH_2)_n = (CH_2)_6 = C_6H_{12}$

Results:

- Empirical formula: CH_2
- Molecular formula: C_6H_{12}

Quick Tip

Empirical and molecular formula calculations are fundamental in determining the composition of chemical compounds from basic analytical data.

---

87. Pt(s)|H_2(g)(1bar)|H^+(aq)(1M)||$M^{3+}(aq), M^+(aq)$|Pt(s)

The E_{cell} for the given cell is 0.1115 V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$.

Given:

$E^0_{M^{3+}/M^+} = 0.2$ V

$2.303\frac{RT}{F} = 0.059$ V

Correct Answer: (3.00)

Solution:

To calculate the value of a, we apply the Nernst equation:

$$E = E^0 - \frac{0.059}{n} \log \frac{[M^{3+}]}{[M^+]}$$

Substituting the given values:

$$0.1115 = 0.2 - \frac{0.059}{1} \log \left(\frac{1}{10^a}\right)$$

Solving for a:

Overall reaction:

$$H2(g) + M^3 + (aq) -> M + (aq) + 2H + (aq)$$

$$E_{\text{cell}} = E_{\text{cathode}} - E_{\text{anode}} - \frac{0.059}{2} \log \frac{[M^+]^2}{[M^{3+}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2} \log \frac{[M^+]}{[M^{3+}]}$$

$$3 = \log \frac{[M^+]}{[M^{3+}]}$$

$$\therefore a = 3$$

Quick Tip

The Nernst equation relates the cell potential to the ion concentration ratios. It's crucial for understanding the electrochemical behavior and driving force behind the cell reactions.

---

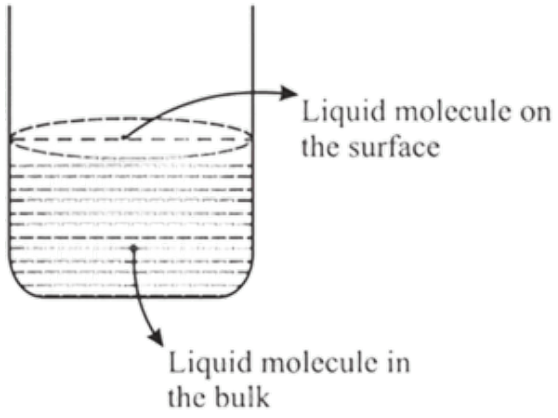
88. Based on the given figure, the number of correct statement/s is/are

A. Surface tension is the outcome of equal attractive and repulsion forces acting on the liquid molecule in bulk.

B. Surface tension is due to uneven forces acting on the molecules present on the surface.

C. The molecule in the bulk can never come to the liquid surface.

D. The molecules on the surface are responsible for vapor pressure if the system is a closed system.



Correct Answer: 2

Solution:

The correct options are:

B: Surface tension arises due to uneven forces acting on the molecules present at the surface. Molecules at the surface of a liquid experience a net inward force because they are not surrounded by similar molecules on all sides. This imbalance results in the liquid's surface behaving as if it were under tension.

D: The molecules at the surface are responsible for vapor pressure in a closed system. In a closed system, molecules at the surface of the liquid are able to escape into the vapor phase, contributing to the vapor pressure. The equilibrium between the liquid and vapor phases determines the vapor pressure.

Quick Tip

Surface tension arises from the imbalance in intermolecular forces experienced by the molecules at the surface compared to those in the bulk of the liquid.

---

89. A first order reaction has the rate constant, k = 4.6 $\times 10^{-3} s^{-1}$. The number of correct statement/s from the following is/are

A. Reaction completes in 1000 s.

B. The reaction has a half-life of 500 s.

C. The time required for 10% completion is 25 times the time required for 90% completion.

D. The degree of dissociation is equal to $1 - e^{-kt}$.

E. The rate and the rate constant have the same unit.

Correct Answer: 1

Solution:

$$t_{10\%} = \frac{1}{K} \ln \left(\frac{a}{a - x} \right) = \frac{1}{K} \ln \left(\frac{100}{90} \right)$$

$$t_{10\%} = \frac{2.303}{K}(\log 10 - \log 9)$$

$$t_{10\%} = \frac{2.093}{K} \times (0.04)$$

Similarly

$$t_{90\%} = \frac{1}{K} \ln \left(\frac{100}{10} \right)$$

$$t_{90\%} = \frac{2.303}{K}$$

$$\frac{t_{90\%}}{t_{10\%}} = \frac{1}{0.04} = 25$$

$$e^{kt} = \frac{a}{a-x}$$

$$\frac{a-x}{a} = e^{-kt}$$

$$1 - \frac{x}{a} = e^{-kt}$$

$$x = a(1 - e^{-kt})$$

$$\alpha = \frac{x}{a} = 1 - e^{-kt}$$

where α is the degree of dissociation, x is the amount dissociated, and a is the initial amount.

Quick Tip

First-order reactions have a constant half-life, which is independent of the concentration of the reactants.

---

90. The number of incorrect statement/s from the following is/are

A. Water vapours are adsorbed by anhydrous calcium chloride.

B. There is a decrease in surface energy during adsorption.

C. As the adsorption proceeds, ΔH becomes more and more negative.

D. Adsorption is accompanied by a decrease in entropy of the system.

Correct Answer: 2

Solution:

A: Water vapours are adsorbed by calcium chloride. Calcium chloride ($CaCl_2$) is a hygroscopic substance, meaning it has the ability to adsorb water vapors from the air. The water molecules are attracted to the surface of the calcium chloride, where they adhere through adsorption.

C: As the adsorption proceeds, ΔH becomes less and less negative. In the case of adsorption, the enthalpy change (ΔH) is initially negative because the process releases energy due to the attractive forces between the adsorbate and the adsorbent. However, as more water is adsorbed, the adsorption sites become occupied, and the heat released decreases, making ΔH less negative.

Quick Tip

Adsorption can lead to a decrease in system entropy, particularly when the adsorbate organizes into a less random, more ordered state on the adsorbent surface.
