Solutions to JEE(Main) -2021

Test Date: 25^{th} July 2021 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

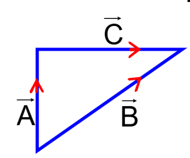
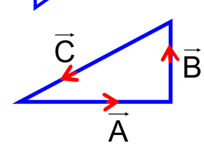
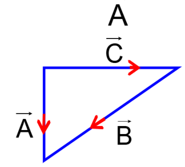
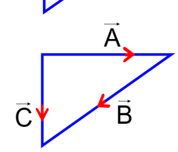
Assertion A : Moment of inertia of a circular disc 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y & I_z respectively. The respective radii of gyration about all the three axes will be the same.

Reason R : A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) **A** is correct but **R** is not correct .
(B) Both **A** and **B** are correct and **R** is the correct explanation of **A** .
(C) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.
(D) **A** is not correct but **R** is correct.

Q2. Match **List I** with **List II**.

List I		**List II**	
List – I		**List – II**	
(a)	$\vec{C}-\vec{A}-\vec{B}=0$	(i)	
(b)	$\vec{A}-\vec{C}-\vec{B}=0$	(ii)	
(c)	$\vec{B}-\vec{A}-\vec{C}=0$	(iii)	
(d)	$\vec{A}+\vec{B}=-\vec{C}$	(iv)	

Choose the correct answer from the options given below :

(A) $(a)\to(iv), (b)\to(i), (c)\to(iii), (d)\to(ii)$
(B) $(a)\to(iv), (b)\to(iii), (c)\to(i), (d)\to(ii)$
(C) $(a)\to(i), (b)\to(iv), (c)\to(ii), (d)\to(iii)$
(D) $(a)\to(iii), (b)\to(ii), (c)\to(iv), (d)\to(i)$

Q3. In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin\omega t$; where d_0, ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as :

(A) $\frac{\lambda D}{d_0 + a_0}$ (B) $\frac{\lambda D}{d_0^2} a_0$

(C) $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$ (D) $\frac{2\lambda D(d_0)}{(d_0^2 - a_0^2)}$

Q4. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

(A) $\frac{v_0 x_1^2}{x_2^2}$ (B) $\frac{v_0 x_2^2}{x_1^2}$

(C) $\frac{v_0 x_2}{x_1}$ (D) $\frac{v_0 x_1}{x_2}$

Q5. Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as :
(where λ is the decay constant)

(A) $\frac{\ell n \frac{3}{2}}{\lambda}$ (B) $\frac{\ell n 2}{\lambda}$

(C) $\frac{2 \ell n 2}{\lambda}$ (D) $\frac{1}{2} \frac{\ell n 2}{\lambda}$

Q6. A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond- air interface. It is going from diamond to air. The refractive index diamond is 2.42 and that of air is 1. Choose the correct option.

(A) angle of refraction is 53.4° (B) angle of refraction is 30°

(C) angle of refraction is 24.41° (D) refraction is not possible

Q7. What should be the order of arrangement of de-Broglie wavelength of electron (λ_e), an α – particle(λ_α) and proton (λ_p) given that all have the same kinetic energy ?

(A) $\lambda_e = \lambda_p = \lambda_\alpha$ (B) $\lambda_e < \lambda_p < \lambda_\alpha$

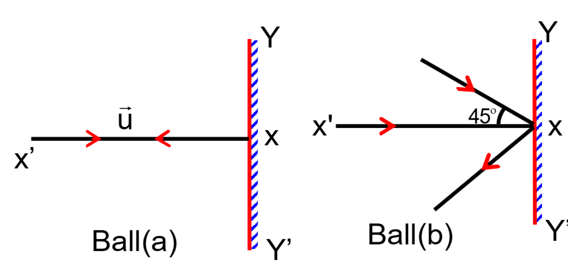
(C) $\lambda_e = \lambda_p > \lambda_\alpha$ (D) $\lambda_e > \lambda_p > \lambda_\alpha$

Q8. Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4^{th} second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap ? (Take g = 9.8 m/s^2)

(A) 1 drop / 7 seconds (B) 3 drops / 2 seconds

(C) 1 drop / second (D) 2 drops / second

Q9. Two billiard balls of equal mass 30g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball 'a' and ball 'b' by the wall along 'X' direction is :



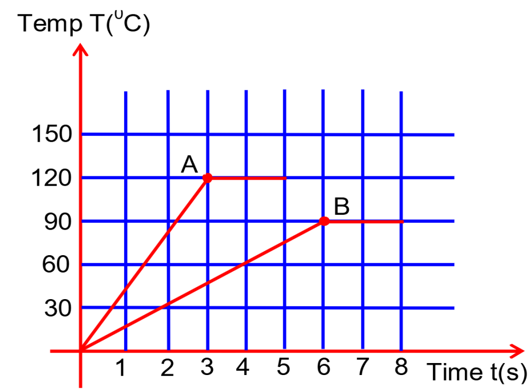
(A) $\sqrt{2} : 1$

(B) $1 : \sqrt{2}$

(C) 1 : 1

(D) 2 : 1

Q10. Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is :



(A) $\frac{3}{8}$

(B) $\frac{8}{3}$

(C) $\frac{4}{3}$

(D) $\frac{3}{4}$

Q11. A linearly polarized electromagnetic wave in vacuum is
$E = 3.1\cos\left[(1.8)z - (5.4\times10^6)t\right]\hat{i}$ N/C
Is incident normally on a perfectly reflecting wall at $z = a$. Choose the correct option

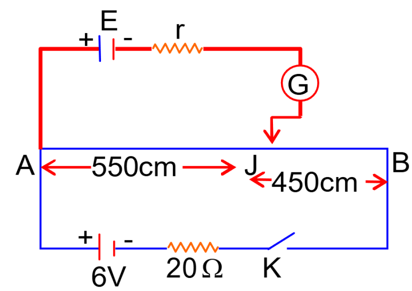
(A) The frequency of electromagnetic wave is 54×10^4 Hz.

(B) The transmitted wave will be $3.1\cos\left[(1.8)z - (5.4\times10^6)t\right]\hat{i}$ N/C

(C) The reflected wave will be $3.1\cos\left[(1.8)z + (5.4\times10^6)t\right]\hat{i}$ N/C

(D) The wavelength is 5.4 m

Q12. In the given figure, there is a circuit of potentiometer of length $AB = 10$ m. The resistance per unit length is 0.1Ω per cm. Across AB, a battery of emf E and internal resistance 'r' is connected. The maximum value of emf measured by this potentiometer is :



(A) 2.75 V
(B) 6 V
(C) 5 V
(D) 2.25 V

Q13. A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If ℓ_1 and ℓ_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$ will be :

(A) $\frac{\ell_2}{\ell_1}$

(B) $\left(\frac{\ell_1}{\ell_2}\right)^{\frac{2}{3}}$

(C) $\frac{\ell_1}{\ell_2}$

(D) $\left(\frac{\ell_2}{\ell_1}\right)^{\frac{2}{3}}$

Q14. For a gas $C_P - C_V = R$ in a state P and $C_P - C_V = 1.10\,R$ in a state Q, T_P and T_Q are the temperatures in two different states P and Q respectively. Then

(A) $T_P < T_Q$
(B) $T_P = 0.9\,T_Q$
(C) $T_P > T_Q$
(D) $T_P = T_Q$

Q15. In Amplitude Modulation, the message signal $V_m(t) = 10\sin(2\pi \times 10^5 t)$ volts and carrier signal $V_c(t) = 20\sin(2\pi \times 10^7 t)$ volts. The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is α kHz. The value of α is :

(A) 200 kHz
(B) 100 kHz
(C) 0
(D) 50 kHz

Q16. Two wires of same length and radius are joined end to end and loaded. The Young's modulii of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire then its Young's modulus is :

(A) $Y = \frac{Y_1 Y_2}{2(Y_1 + Y_2)}$
(B) $Y = \frac{2Y_1 Y_2}{3(Y_1 + Y_2)}$
(C) $Y = \frac{Y_1 Y_2}{Y_1 + Y_2}$
(D) $Y = \frac{2Y_1 Y_2}{Y_1 + Y_2}$

Q17. A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as:

$$\varepsilon(x) = \varepsilon_0 + kx, \text{for } \left(0 < x \le \frac{d}{2}\right)$$

$$\varepsilon(x) = \varepsilon_0 + k(d - x), \text{for} \left(\frac{d}{2} \le x \le d\right)$$

(A) 0
(B) $\frac{kA}{2}\ell n\left(\frac{2\varepsilon_0}{2\varepsilon_0 - kd}\right)$
(C) $\frac{kA}{2\,\ell n\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}$
(D) $\left(\varepsilon_0 + \frac{kd}{2}\right)^{2/kA}$

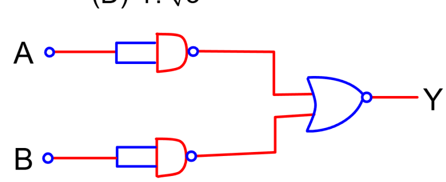
Q18. The half-life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g /mol then the activity of 2 mg of ^{198}Au is [in disintegration / second] :

(A) 6.06×10^{18}
(B) 2.67×10^{12}
(C) 16.18×10^{12}
(D) 32.36×10^{12}

Q19. A particle of mass 4M at rest disintegrates into two particles of mass M and 3M respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass 3M will be :

(A) 3 : 1
(B) 1 : 1
(C) 1 : 3
(D) $1 : \sqrt{3}$

Q20. Identify the logic operation carried out.



(A) OR
(B) NOR
(C) AND
(D) NAND

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

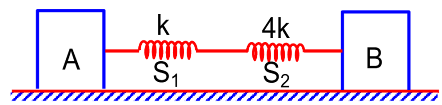
Q1. A particle of mass 'm' is moving in time 't' on a trajectory given by

$\vec{r} = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

Where α and β are dimensional constants.

The angular momentum of the particle become the same as it was for $t = 0$ at time $t =$ _________seconds.

Q2. In the reported figure, two bodies A and B of masses 200g and 800g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be ______rad /s when k = 20 N / m.



Q3. A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at a distance '2 m' when each is carrying same change 'q'. If the free charged particle is displaced from its equilibrium position through distance 'x' (x << 1m). The particle executes SHM. Its angular frequency of oscillation will be _______$\times 10^5$ rad / s if $q^2 = 10C^2$.

Q4. An inductor of 10 mH is connected to a 20 V battery through a resistor of $10k\Omega$ and a switch . After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then x is equal to ________.

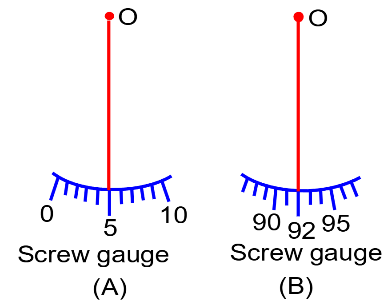
(Take $e^{-1} = 0.37$)

Q5. A pendulum bob has a speed of 3 m / s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be $\left(g = 10 m/s^2\right)$_________m/s.

Q6. A body of mass 2 kg moving with a speed of 4 m /s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body centre of mass is $\frac{x}{10}$ m / s. Then the value of x is __________.

Q7. An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance 'R' that must be put in series with the bulb so that the bulb delivers the same power is_________Ω .

Q8. Student A and Student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is ___________.
[Figure shows position of reference 'O' when jaws of screw gauge are closed]
Given pitch = 0.1 cm.



Q9. The value of aluminium susceptibility is 2.2×10^{-5}. The percentage increase in the magnetic field if space within a current carrying toroid is filled with Aluminium is $\frac{x}{10^4}$. Then the value of x is _______.

Q10. A circular conducting coil of radius 1 m being heated by the change of magnetic field $\vec{B}$ passing perpendicular to the plane in which the coil is laid. The resistance of the coil is $2\mu\Omega$. The magnetic field is slowly switched off such that its magnitude changes in time as
$B = \frac{4}{\pi} \times 10^{-3} T \left(1 - \frac{t}{100}\right)$
The energy dissipated by the coil before the magnetic field is switched off completely is
E = ________mJ

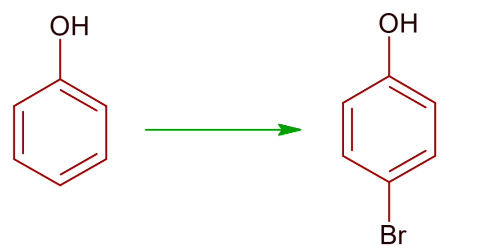
PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1.



(Major Product)

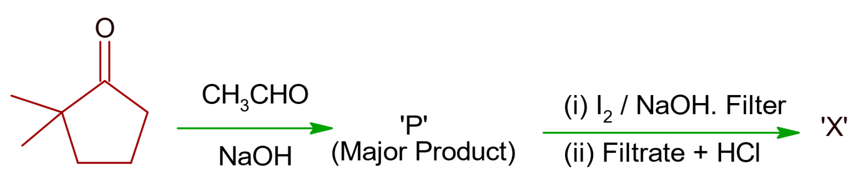
The given reaction can occur in the presence of:

(a) Bromine water (b) Br_2 in CS_2, 273 K (c) Br_2 / $FeBr_3$ (d) Br_2 in $CHCl_3$, 273 K

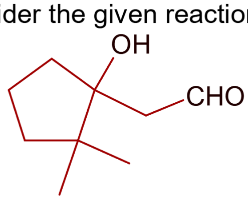
Choose the correct answer from the options given below:

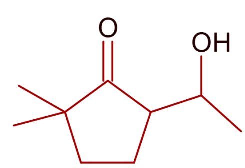
(A) (a) and (c) only
(B) (b), (c) and (d) only
(C) (b) and (d) only
(D) (a), (b) and (d) only

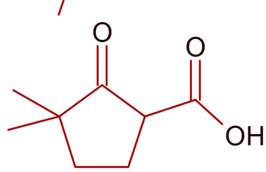
Q2.

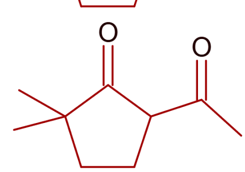


Consider the given reaction, the product 'X' is:

(A) 

(B) 

(C) 

(D) 

Q3. Given below are two statements, one is labelled as Assertion (A) and other is labelled as Reason (R)

Assertion (A): Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

Reason (R): Aryl halides do not undergo nucleophilic substitution reaction.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both **(A)** and **(R)** are true and **(R)** is correct explanation of **(A).**
(B) **(A)** is false but **(R)** is true.
(C) **(A)** is true but **(R)** is false
(D) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.

Q4. At 298.2 K the relationship between enthalpy of bond dissociation (in kJ mol^{-1}) for hydrogen (E_H) and its isotope, deuterium (E_D), is best described by:

(A) $E_H = E_D$
(B) $E_H = \frac{1}{2}E_D$
(C) $E_H = 2E_D$
(D) $E_H \simeq E_D - 7.5$

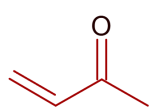
Q5. An organic compound 'A' C_4H_8 on treatment with $KMnO_4$ / H^+ yields compound 'B' C_3H_6O. Compound 'A' also yields compound 'B' on ozonolysis. Compound 'A' is:

(A) 2-Methylpropene
(B) 1-Methylcyclopropane
(C) Cyclobutane
(D) But-2-ene

Q6. Which one of the following compounds of Group-14 elements is not known?

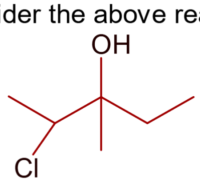
(A) $[GeCl_6]^{2-}$
(B) $[Sn(OH)_6]^{2-}$
(C) $[SiF_6]^{2-}$
(D) $[SiCl_6]^{2-}$

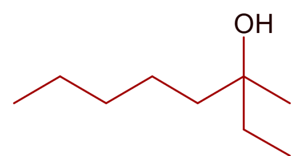
Q7.

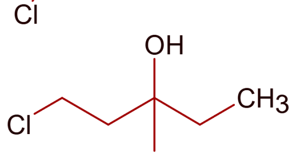


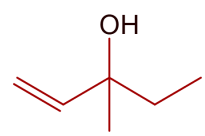
(i) C_2H_5MgBr, dry ether
(ii) H_2O, HCl
→ P (Major Product)

Consider the above reaction, the major product 'P' is:

(A)


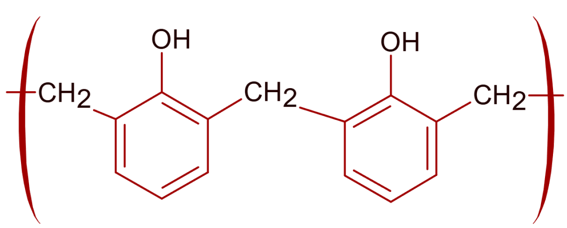
(B)


(C)


(D)


Q8. In the leaching of alumina from bauxite, the ore expected to leach out in the process by reacting with NaOH is:

(A) ZnO
(B) Fe_2O_3
(C) SiO_2
(D) TiO_2

Q9.


is a repeating unit for

(A) Neoprene
(B) Novolac
(C) Buna-N
(D) Acrilan

Q10. The ionic radii of K^+, Na^+, Al^{3+} and Mg^{2+} are in the order:

(A) $K^+ < Al^{3+} < Mg^{2+} < Na^+$
(B) $Al^{3+} < Mg^{2+} < K^+ < Na^+$
(C) $Al^{3+} < Mg^{2+} < Na^+ < K^+$
(D) $Na^+ < K^+ < Mg^{2+} < Al^{3+}$

Q11. The water soluble protein is:

(A) Myosin
(B) Collagen
(C) Fibrin
(D) Albumin

Q12. Which one of the following species responds to an external magnetic field?

(A) $\left[Co(CN)_6\right]^{3-}$ (B) $\left[Ni(CN)_4\right]^{2-}$

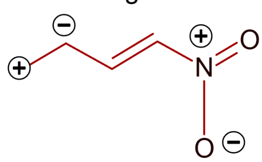
(C) $\left[Ni(CO)_4\right]$ (D) $\left[Fe(H_2O)_6\right]^{3+}$

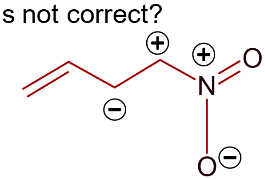
Q13. Which one of the following compounds will liberate CO_2, when treated with $NaHCO_3$?

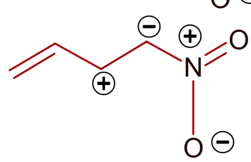
(A) $(CH_3)_4\overset{\oplus}{N}\ \overset{\ominus}{O}H$ (B) CH_3NH_2

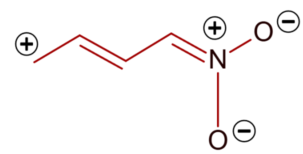
(C) $(CH_3)_3\overset{\oplus}{N}\ H\overset{\ominus}{Cl}$ (D) $CH_3-\overset{O}{\overset{||}{C}}-NH_2$

Q14. Which one among the following resonating structure is not correct?

(A) 

(B) 

(C) 

(D) 

Q15. The correct order of following 3d metal oxides, according to their oxidation numbers is:

(a) CrO_3 (b) Fe_2O_3 (c) MnO_2 (d) V_2O_5
(e) Cu_2O

(A) (a) > (d) > (c) > (b) > (e) (B) (c) > (a) > (d) > (e) > (b)
(C) (d) > (a) > (b) > (c) > (e) (D) (a) > (c) > (d) > (b) > (e)

Q16. Sodium stearate $CH_3(CH_2)_{16}COO^{-}Na^{+}$ is an anionic surfactant which forms micelles in oil. Choose the correct statement for it from the following:

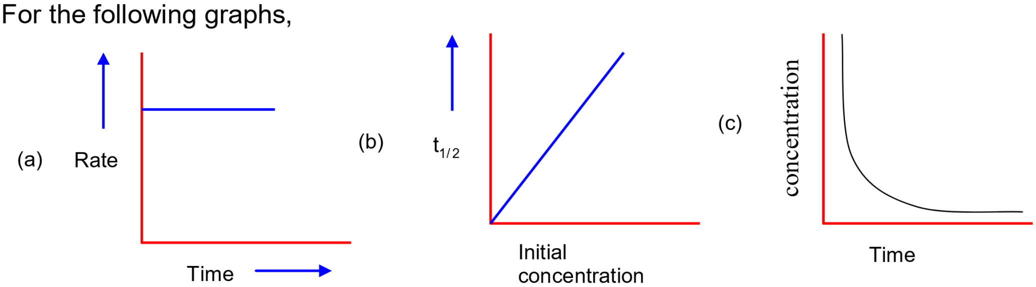
(A) It forms spherical micelles with $CH_3(CH_2)_{16}-$ group pointing outwards on the surface of sphere.

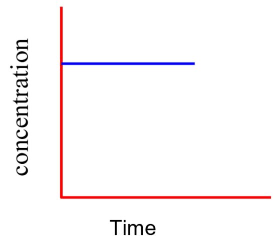
(B) It forms non-spherical micelles with $-COO^{\ominus}$ group pointing outwards on the surface

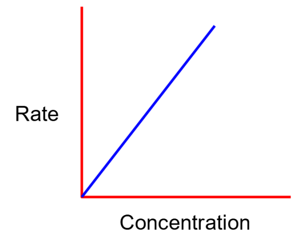
(C) It forms spherical miclelles with $CH_3(CH_2)_{16}-$ group pointing towards the centre of sphere.

(D) It forms non- spherical micelles with $CH_3(CH_2)_{16}-$ group pointing towards the centre.

Q17. For the following graphs,



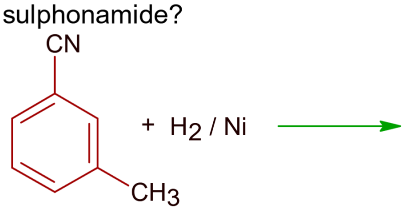
(d) 

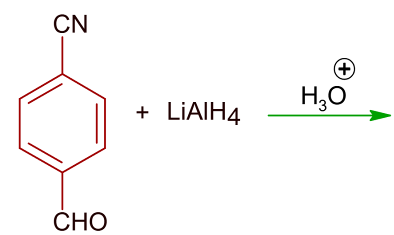
(e) 

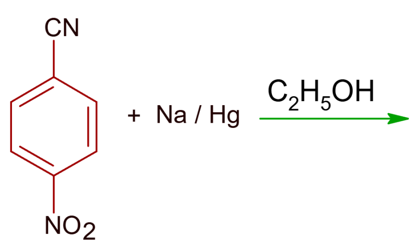
Choose from the options given below, the correct one regarding order of reaction is:
(A) (a) and (b) zero order (e) First order
(B) (b) and (d) zero order (e) First order
(C) (a) and (b) zero order (c) and (e) First order
(D) (b) zero order (c) and (e) First order

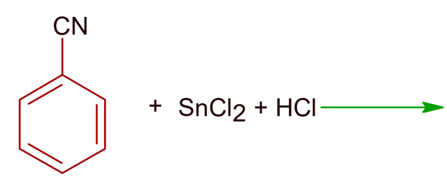
Q18. Which one of the following chemical agent is not being used for dry- cleaning of clothes?
(A) CCl_4
(B) $Cl_2C{=}CCl_2$
(C) Liquid CO_2
(D) H_2O_2

Q19. Which one of the products of the following reactions **does not** react with Hingberg reagent to form sulphonamide?

(A) 

(B) 

(C) 

(D) 

Q20. Given below are two statements:
Statement I: None of the alkaline earth metal hydroxides dissolves in alkali.
Statement II: Solubility of alkaline earth metal hydroxides in water increases down the group.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(A) **Statement I** and **Statement II** both are incorrect.
(B) **Statement I** is incorrect but **Statement II** is correct.
(C) **Statement I** is correct but **Statement II** is incorrect.
(D) **Statement I** and **Statement II** both are correct.

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The number of sigma bonds in

$H_3C-\underset{|}{\overset{}{C}}=CH-C\equiv C-H$ is____________________.
$\quad\quad\;\; H$

Q2. At 298 K, the enthalpy of fusion of a solid (X) is 2.8kJ mol^{-1} and the enthalpy of vaporization of the liquid (X) is 98.2 kJ mol^{-1}. The enthalpy of sublimation of the substance (X) in kJ mol^{-1} is __________. (in nearest integer)

Q3. Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is __________ $\times 10^{-1}$g, (in nearest integer)

Q4. CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is__________(Nearest integer)
(Henry's law constant for CO_2 at 298 K is 1.67×10^3 bar)

Q5. When 10 ml of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO_4 using diphenylamine indicator, 15 mL of 0.02 M solution of $K_2Cr_2O_7$ was required to get the end point. The molarity of the solution containing Fe^{2+} ions is x $\times 10^{-2}$M. The value of x is___. (Nearest integer)

Q6. Consider the cell at 25°C
$Zn \mid Zn^{2+}(aq), (1M) \parallel Fe^{3+}(aq), Fe^{2+}(aq) \mid Pt(s)$
The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500V is x $\times 10^{-2}$. The value of x is _____. (Nearest integer)
(Given: $E^0_{Fe^{3+}/Fe^{2+}} = 0.77V, E^0_{Zn^{2+}/Zn} = -0.76V$)

Q7. For the reaction
$A + B \rightleftharpoons 2C$
The value of equilibrium constant is 100 at 298K. If the initial concentration of all the three species is 1M each, then the equilibrium concentration of C is $x\times10^{-1}$M. The value of x is__________. (Nearest integer)

Q8. A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, x $\times 10^{20}$ electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is_______(Nearest integer)
(h = 6.626 $\times 10^{-34}$Js)

Q9. A home owner uses $4.00\times10^3 m^3$ of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is $x\times10^5$g. The value of x is ___________. (Nearest integer)
(Given R = 0.083 L atm $K^{-1}mol^{-1}$)

Q10. Three moles of AgCl get precipitated when one mole of an octahedral co-ordination compound with empirical formula $CrCl_3 \cdot 3NH_3 \cdot 3H_2O$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is__________.

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The area (in sq. units) of the region, given by the set $\left\{(x,y)\in R\times R \,\middle|\, x\ge 0,\, 2x^2\le y\le 4-2x\right\}$ is :

(A) $\frac{17}{3}$ (B) $\frac{8}{3}$

(C) $\frac{7}{3}$ (D) $\frac{13}{3}$

Q2. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

(A) $\{x\in R : |x-2|\le 1\}$ (B) $\{x\in R : |x-5|\le 1\}$

(C) $\{x\in R : |x-3|< 1\}$ (D) $\{x\in R : |x-1|< 1\}$

Q3. The number of real roots of the equation $e^{6x}-e^{4x}-2e^{3x}-12e^{2x}+e^x+1=0$ is :

(A) 1 (B) 4

(C) 2 (D) 6

Q4. The values of a and b, for which the system of equations

$2x+3y+6z=8$

$x+2y+az=5$

$3x+5y+9z=b$

Has no solution, are :

(A) $a\ne 3, b=3$ (B) $a\ne 3, b\ne 13$

(C) $a=3, b\ne 13$ (D) $a=3, b=13$

Q5. Let $f:R\to R$ be defined as

$$f(x)=\begin{cases}\dfrac{\lambda\left|x^2-5x+6\right|}{\mu\left(5x-x^2-6\right)}, & x<2\\ e^{\frac{\tan(x-2)}{x-[x]}}, & x>2\\ \mu, & x=2\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then $\lambda+\mu$ is equal to :

(A) $e(-e+1)$ (B) $e(e-2)$
(C) $2e-1$ (D) 1

Q6. Let the foot of perpendicular from a point $P(1,2,-1)$ to the straight line $L:\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x+y+2z=0$ which meets L and point Q. If α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to……

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2\sqrt{3}}$

Q7. The Boolean expression $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to:

(A) q (B) p
(C) ~ p (D) ~ q

Q8. Let an ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2>b^2$, passes through $\left(\sqrt{\frac{3}{2}},1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha,0), \alpha>0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

(A) $\frac{16}{3}$ (B) 3
(C) $\frac{4}{3}$ (D) $\frac{8}{3}$

Q9. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

(A) $8\left(2+2\sqrt{3}+\sqrt{2}\right)$ (B) $8\left(\sqrt{6}+\sqrt{2}+2\right)$
(C) $8\left(\sqrt{6}-\sqrt{2}+2\right)$ (D) $8\left(\sqrt{2}+2+\sqrt{3}\right)$

Q10. Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to :

(A) 32 (B) $8\sqrt{2}$
(C) 16 (D) $16\sqrt{2}$

Q11. The locus of the centroid of the triangle formed by any point P on this hyperbola $16x^2-9y^2+32x+36y-164=0$, and its foci is :

(A) $16x^2-9y^2+32x+36y-144=0$ (B) $16x^2-9y^2+32x+36y-36=0$
(C) $9x^2-16y^2+36x+32y-144=0$ (D) $9x^2-16y^2+36x+32y-36=0$

Q12. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is ;

(A) 2
(B) 6
(C) 8
(D) 4

Q13. Let $f:[0,\infty)\to[0,\infty)$ be defined as

$f(x)=\int_0^x [y]dy$

where [x] is the greatest integer less than or equal to x. Which of the following is true?

(A) f is differentiable at every point in $[0,\infty)$

(B) f is continuous everywhere except at the integer points in $[0,\infty)$.

(C) f is continuous at every point in $[0,\infty)$ and differentiable except at the integer points.

(D) f is both continuous and differentiable except at the integer points in $[0,\infty)$.

Q14. Let the vectors

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k}$,

$(1+b)\hat{i}+2bj-b\hat{k}$ and $(2+b)\hat{i}+2bj+(1-b)\hat{k}, a,b,c \in R$ be co-planar. Then which of the following is true?

(A) $2a=b+c$
(B) $2b=a+c$
(C) $a=b+2c$
(D) $3c=a+b$

Q15. Let $g:N\to N$ be defined as

$g(3n+1)=3n+2,$

$g(3n+2)=3n+3,$

$g(3n+3)=3n+1,$ for all $n\geq 0$

Then which of the following statements is true?

(A) There exists a function $f:N\to N$ such that $gof=f$

(B) $gogog=g$

(C) There exists a one-one function $f:N\to N$ such that fog = f

(D) There exists on onto function $f:N\to N$ such that $fog=f$

Q16. Let $f(x)=3\sin^4 x+10\sin^3 x+6\sin^2 x-3,\ x\in\left[-\frac{\pi}{6},\frac{\pi}{2}\right]$. Then, f is :

(A) decreasing in $\left(0,\frac{\pi}{2}\right)$
(B) increasing in $\left(-\frac{\pi}{6},\frac{\pi}{2}\right)$
(C) decreasing in $\left(-\frac{\pi}{6},0\right)$
(D) increasing in $\left(-\frac{\pi}{6},0\right)$

Q17. The value of the definite integral

$\int_{\pi/24}^{5\pi/24}\frac{dx}{1+\sqrt[3]{\tan 2x}}$ is :

(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{12}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{18}$

Q18. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :

(A) 11π (B) 9π

(C) 8π (D) 12π

Q19. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity

$$\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+....+\frac{1}{a-nb}=\alpha n+\beta n^2+\gamma n^3,$$

(A) $\frac{a+b}{3a^2}$ (B) $\frac{a^2+b}{3a^3}$

(C) $\frac{b^2}{3a^3}$ (D) $\frac{a+b^2}{3a^3}$

Q20. Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx}=1+xe^{y-x}, -\sqrt{2}<x<\sqrt{2}, y(0)=0$$

then, the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to :

(A) $(1+\sqrt{3})-\log_e(\sqrt{3}-1)$ (B) $(2+\sqrt{3})+\log_e 2$

(C) $(1-\sqrt{3})-\log_e(\sqrt{3}-1)$ (D) $(2-\sqrt{3})-\log_e 2$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to.......

Q2. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector : $\vec{r} = \left(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}\right)$ is perpendicular to each of the vectors $\left(\vec{p} + \vec{q}\right)$ and $\left(\vec{p} - \vec{q}\right)$, and $\left|\vec{r}\right| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to......

Q3. the term independent of 'x' in the expression of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to.......

Q4. Let $y = y(x)$ be solution of the following differential equation $e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0,\ y\left(\frac{\pi}{2}\right) = 0$.
If $y(0) = \log_e\left(\alpha + \beta e^{-2}\right)$, then $4(\alpha + \beta)$ is equal to.......

Q5. Let $M = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \{\pm 3, \pm 2, \pm 1, 0\}\right\}$. Define $f : M \to Z$, as $f(A) = \det(A)$, for all $A \in M$, where Z is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to..........

Q6. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is.......

Q7. Let $S = \left\{n \in N \left| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right. \forall\, a,b,c,d \in R\right\}$, where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is ....

Q8. Consider the following frequency distribution :

Class :	10-20	20-30	30-40	40-50	50-60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to.......

Q9. If α, β are roots of the equation $x^2 + 5\left(\sqrt{2}\right)x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\dfrac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to………

Q10. If the value of $\left(1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + ...\text{upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ...\text{upto } \infty\right)}$ is l, then l^2 is equal to……..

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**D**	2.	**B**	3.	**C**	4.	**C**
5.	**A**	6.	**D**	7.	**D**	8.	**C**
9.	**A**	10.	**A**	11.	**C**	12.	**C**
13.	**D**	14.	**C**	15.	**A**	16.	**D**
17.	**C**	18.	**C**	19.	**B**	20.	**C**

SECTION - B

1.	**10**	2.	**10**	3.	**6000**	4.	**74**
5.	2	6.	25	7.	50	8.	13
9.	**22**	10.	**80**				

PART – B (CHEMISTRY)

SECTION - A

1.	**B**	2.	**C**	3.	**A**	4.	**D**
5.	**A**	6.	**D**	7.	**A**	8.	**C**
9.	**B**	10.	**C**	11.	**D**	12.	**D**
13.	**C**	14.	**B**	15.	**A**	16.	**C**
17.	**C**	18.	**B**	19.	**D**	20.	**B**

SECTION - B

1.	**10**	2.	**101**	3.	**464**	4.	**25**
5.	**18**	6.	**24**	7.	**25**	8.	**2**
9.	**26**	10.	**0**				

PART – C (MATHEMATICS)

SECTION - A

1.	**C**	2.	**C**	3.	**C**	4.	**C**
5.	**A**	6.	**C**	7.	**C**	8.	**A**
9.	**B**	10.	**C**	11.	**B**	12.	**B**
13.	**C**	14.	**B**	15.	**D**	16.	**C**
17.	**B**	18.	**B**	19.	**C**	20.	**C**

SECTION – B

1.	**238**	2.	**3**	3.	**210**	4.	**4**
5.	**16**	6.	**1**	7.	**11**	8.	**164**
9.	**1**	10.	**3**				

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. $I_X = I_Y = \frac{MR^2}{4} = MK_1^2 \Rightarrow K_1 = \frac{R}{2}$

$I_Z = \frac{MR^2}{2} = MK_2^2 \Rightarrow K_2 = \frac{R}{\sqrt{2}}$

Clearly, $K_1 \neq K_2$

Sol2. From triangle law of addition, we can clearly write answer.

Sol3. $\beta = \frac{\lambda D}{d_0 + a_0 \sin \omega t}$

$\beta_1 - \beta_2 = \frac{\lambda D}{d_0 - a_0} - \frac{\lambda D}{d_0 + a_0}$

$= \lambda D\left[\frac{(d_0 + a_0) - (d_0 - a_0)}{d_0^2 - a_0^2}\right]$

$= \frac{2\lambda D a_0}{d_0^2 - a_0^2}$

Sol4. At closest distance, speed will be maximum.
From Conservation of Angular Momentum, $mv_0 x_2 = mv\, x_1$

$v = \frac{v_0 x_2}{x_1} [x_2 > x_1]$

Sol5. $\frac{3N_0}{4} = N_0 e^{-\lambda t_1}$

$\Rightarrow e^{\lambda t_1} = \frac{4}{3}$

$t_1 = \frac{1}{\lambda} \ell n\left(\frac{4}{3}\right)$

$t_2 = \frac{\ell n 2}{\lambda}$

$t_2 - t_1 = \frac{\ell n 2}{\lambda} - \frac{\ell n\left(\frac{4}{3}\right)}{\lambda}$

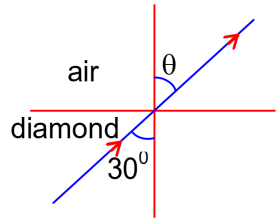
$= \frac{\ell n 2\left(\frac{2}{4/3}\right)}{\lambda} = \frac{\ell n(3/2)}{\lambda}$

Sol6. $\mu_1 \sin 30^\circ = \mu_2 \sin\theta$

$\Rightarrow 2.42 \times \frac{1}{2} = (1)\sin\theta$

$\sin\theta = 1.21 > 1$

Refraction is not possible, total inte



Sol7. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$

For same K :

$\lambda \times \frac{1}{\sqrt{m}}$

$\lambda_e : \lambda_p : \lambda_\alpha = \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{4m_p}}$

As $m_p > m_e$,

$\lambda_e > \lambda_p > \lambda_\alpha$

Sol8. Displacement in 4th second

$= S_4 - S_3$

$= \frac{1}{2} g\,[2n - 1]$

$= \frac{1}{2} g[2 \times 4 - 1]$

$= \frac{1}{2} \times 9.8 \times 7$

$= 34.3\,m$

As this distance matches with data given in question for position of next drop. So drops are falling at the rate of 1 drop/second.

Sol9. For ball(a) : $I_1 = \Delta p_1 = 2mu$

For ball(b) : $I_2 = \Delta p_2 = 2mu\cos 45^\circ$

$\frac{I_1}{I_2} = \frac{2mu}{2mu\cos 45^\circ} = \sqrt{2}$

Sol10. $dQ_A = m_A C_A dT_A$

$dQ_B = m_B C_B dT_B$

$\frac{dQ_A}{dt} = \frac{dQ_B}{dt}$

$\Rightarrow m_A C_A \frac{dT_A}{dt} = m_B C_B \frac{dT_B}{dt}$

$\Rightarrow \frac{dT_A / dt}{dT_B / dt} = \frac{C_B}{C_A}$

$\frac{C_A}{C_B} = \frac{dT_B / dt}{dT_A / dt} = \frac{90/6}{120/3} = \frac{3}{8}$

Sol11. Reflected wave will be $3.1\cos\left[1.8z+\left(5.4\times10^{6}\right)t\right]\hat{i}\ N/C$

$K=\frac{2\pi}{\lambda}$

$\Rightarrow 1.8=\frac{2\pi}{\lambda}$

$\lambda=\frac{2\pi}{1.8}=\frac{\pi}{0.9}=\frac{10\pi}{9}=\frac{10}{9}\times3.14$

$=3.48\,m$

Sol12. Maximum value of emf is measured when pointer is at B with $I_G = 0$.

So, $I_{AB}=\frac{6}{20+0.1\times1000}=\frac{6}{120}=\frac{1}{20}A$

$V_{AB}=E=I_{AB}\times R_{AB}$

$=\frac{1}{20}\times100$

$=5V$

Sol13. For adiabatic process

$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$

$\Rightarrow T_1\left(A\ell_1\right)^{\gamma-1}=T_2\left(A\ell_2\right)^{\gamma-1}$

$\therefore \frac{T_1}{T_2}=\left(\frac{\ell_2}{\ell_1}\right)^{\gamma-1}=\left(\frac{\ell_2}{\ell_1}\right)^{5/3-1}=\left(\frac{\ell_2}{\ell_1}\right)^{2/3}$

Sol14. $C_p=\frac{dQ}{n\,dt}=\frac{dU+pdV}{ndT}$

$=C_V+\frac{pdv}{ndt}$

$C_p-C_v=\frac{pdV}{ndT}=R$ - For ideal Gas $\begin{bmatrix}PV=nRT\\ pdV=nRdT\end{bmatrix}$

$C_p-C_v=1.1R$ - For Non – Idea gas (for Real gas)

And Real gas behaves as ideal gas at high temperature & low pressure.

$\therefore T_p>T_Q$

Sol15. Bandwidth

$=\left(\omega_C+\omega_m\right)-\left(\omega_C-\omega_m\right)$

$=2\omega_m$

$=2\left(2\pi f_m\right)$

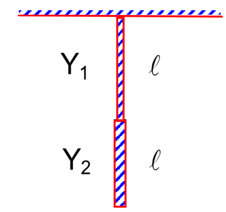
$=4\pi\ f_m$

$=2\pi\left(2f_m\right)$

$=2\pi\left(2\times10^{5}\right)rad/s$

$=2\times10^{5}Hz$

$=200\,kHz$



Sol 16. $Y_1 = \dfrac{F/A}{\Delta\ell_1/\ell}$

$$Y_2 = \frac{F/A}{\Delta\ell_2/\ell}$$

$$Y = \frac{F/A}{\left(\dfrac{\Delta\ell_1+\Delta\ell_2}{2\ell}\right)}$$

$$= \frac{F/A}{\dfrac{1}{2}\left[\dfrac{\Delta\ell_1}{\ell}+\dfrac{\Delta\ell_2}{\ell}\right]}$$

$$= \frac{F/A}{\dfrac{1}{2}\times\left[\dfrac{F}{AY_1}+\dfrac{F}{AY_2}\right]}$$

$$= \frac{2Y_1Y_2}{Y_1+Y_2}$$

Sol17. $dC_1 = \dfrac{(\varepsilon_0+kx)A}{dx}$ $\left[\text{For } 0 < x \le \dfrac{d}{2}\right]$

$$\int\frac{1}{dC_1} = \int\frac{dx}{(\varepsilon_0+kx)A}$$

$$\frac{1}{C_1} = \frac{1}{A}\frac{\left[\ell n(\varepsilon_0+kx)\right]_0^{d/2}}{k}$$

$$= \frac{1}{kA}\ell n\left(1+\frac{kd}{2\varepsilon_0}\right)$$

$$C_1 = \frac{kA}{\ell n\left(1+\dfrac{kd}{2\varepsilon_0}\right)}$$

Similarly $dC_2 = \dfrac{\left[\varepsilon_0+k(d-x)\right]A}{dx}$ $\left[\text{For } \dfrac{d}{2} < x \le d\right]$

$$\int\frac{1}{dC_2} = \int_{d/2}^{d}\frac{dx}{\left[\varepsilon_0+k(d-x)\right]A} = \frac{\left[\ell n\{\varepsilon_0+k(d-x)\}\right]_{d/2}^{d}}{-kA}$$

$$C_2 = \frac{kA}{\ell n\left(1+\dfrac{kd}{2\varepsilon_0}\right)}$$

Clearly, $C_1 = C_2 = C$

Clearly,

$$= \frac{C_1C_2}{C_1+C_2} = \frac{C}{2} = \frac{kA}{2\ell n\left(\dfrac{2\varepsilon_0+kd}{2\varepsilon_0}\right)}$$

Sol18. $t_{1/2} = 3 \text{ days} = 72 \text{hours}$

$$\left|\frac{dN}{dt}\right| = \lambda N = \frac{\ell n2}{t_{1/2}} \times \frac{6.02\times10^{23}}{198} \times 2\times10^{-3}$$

$$= \frac{0.693\times6.02\times10^{23}\times2\times10^{-3}}{72\times3600\times198}$$

$$= 1.618\times10^{13}$$

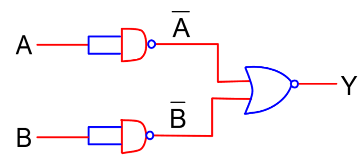
$$= 16.18\times10^{12} \text{ disintegration/second}$$

Sol19. $MV_1 = 3MV_2 = p$

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{1}$$

Sol20. $\overline{\overline{A}+\overline{B}} = \overline{\overline{A}}.\overline{\overline{B}} = A\cdot B$



SECTION - B

Sol1. $\vec{r} = 10\alpha t^2\hat{i} + 5\beta(t-5)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 20\alpha t\hat{i} + 5\beta\hat{j}$$

As $\vec{L} = m(\vec{r}\times\vec{v})$

So, at $t = 0, L = 0$

given $\vec{L}_{t=t} = \vec{L}_{t=0}$

$\Rightarrow \vec{r}\times\vec{v} = \vec{0}$

$\Rightarrow 50\alpha\beta t^2\hat{k} - 100\alpha\beta t(t-5)\hat{k} = \vec{0}$

$\Rightarrow t = 10 \text{ second}$

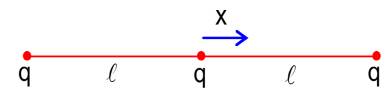
Sol2. $\omega = \sqrt{\frac{k_{eq}}{\mu}}$ $\left[\mu = \frac{m_1m_2}{m_1m_2}(\text{Reduced mass})\right]$

$$= \sqrt{\frac{\left(\frac{k\times4k}{k+4k}\right)}{\left(\frac{m_1m_2}{m_1+m_2}\right)}}$$

$$= \sqrt{\frac{4\times20/5}{\left(\frac{0.2\times0.8}{0.2+0.8}\right)}}$$

$$= 10 \text{ rad/s}$$

Sol3. $F_{net} = \frac{kq^2}{(\ell+x)^2} - \frac{kq^2}{(\ell-x)^2}$



$= kq^2\left[\frac{(\ell-x)^2-(\ell+x)^2}{(\ell^2-x^2)^2}\right]$

$ma = \frac{kq^2}{\ell^4}(-4\ell x) \qquad [x << \ell]$

$= -\frac{4kq^2}{\ell^3}x$

$a = -\frac{4kq^2}{m\ell^3}x$

$\omega = \sqrt{\frac{4kq^2}{m\ell^3}} = \sqrt{\frac{4\times9\times10^9\times10}{1\times10^{-6}\times1}}$

$= 6\times10^8 \text{ rad/s}$

$= 6000\times10^5 \text{ rad/s}$

Sol4. $I = I_0e^{-t/\tau}$

$= \left(\frac{20}{10000}\right)e^{-\left[\frac{1\times10^{-6}}{\left(\frac{10\times10^{-3}}{10\times10^3}\right)}\right]}$

$= 2\times10^{-3}e^{-1}$

$= 2e^{-1}\text{mA}$

$= 2\times0.37\text{mA}$

$= \frac{74}{100}\text{mA}$

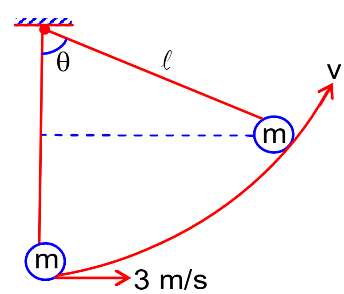
Sol5. $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell(1-\cos\theta)$

$\Rightarrow v^2 = u^2 - 2g\ell(1-\cos\theta)$

$\Rightarrow v^2 = 3^2 - 2\times10\times0.5\times\left(1-\frac{1}{2}\right)$

$\Rightarrow v^2 = 4$

$v = 2\text{m/s}$



Sol6. $2\times4 = 2v_1 + mv_2$

$8 = 2\times\frac{4}{4} + mv_2$

$mv_2 = 6 \qquad \ldots(i)$

$-1 = \frac{v_2 - v_1}{0-4}$

$\Rightarrow -1 = \frac{v_2 - 1}{0-4}$

$v_2 = 5 \qquad \ldots(ii)$

Put (2) in (1), $m = 1.2\text{kg}$

$$V_{cm}=\frac{m_1u_1+m_2u_2}{m_1+m_2}=\frac{2\times 4}{2+1.2}$$
$$=\frac{8}{3.2}=2.5=\frac{25}{10}\text{ m / s}$$

Sol7. $$I=\frac{200}{R+\frac{(100)^2}{200}}=\frac{200}{R+50}$$

Now, $I^2R_B=200$

$$\Rightarrow\left(\frac{200}{R+50}\right)^2\times 50=200$$
$$R=50\Omega$$

Sol8. Difference in Reading = Positive Zero Error – Negative Zero Error

$$=(+5)-[-(100-92)]$$
$$=13$$

Sol9. We know, $\mu_r=1+x_m$

$$B_0\propto\mu_0$$
$$B\propto\mu$$
$$\frac{\Delta B}{B}=\frac{\mu-\mu_0}{\mu_0}=\frac{\mu}{\mu_0}-1=\mu_r-1=x_m$$
$$\%\frac{\Delta B}{B}=x_m\times 100=2.2\times 10^{-5}\times 100=\frac{22}{10^4}$$

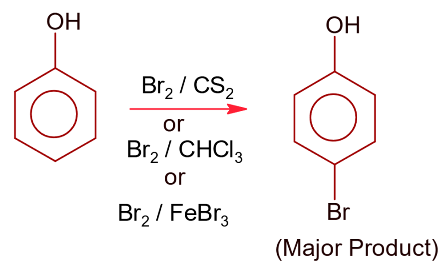
Sol10. $$\varepsilon=\left|\frac{d\phi}{dt}\right|=A\frac{dB}{dt}=\pi\times 1^2\times\frac{4}{\pi}\times 10^{-3}\times\frac{1}{100}$$

$$=4\times 10^{-5}\text{ v}$$
$$B=0$$
$$\Rightarrow\frac{4}{\pi}\times 10^{-3}\left(1-\frac{t}{100}\right)=0$$
$$\Rightarrow t=100\text{ second}$$
$$E=I^2Rt==\frac{\varepsilon^2}{R}t=\frac{\left(4\times 10^{-5}\right)^2}{2\times 10^{-6}}\times 100=80\text{mJ}$$

PART – B (CHEMISTRY)

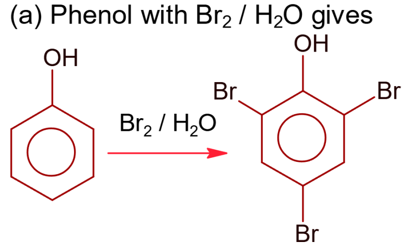
SECTION - A

Sol1.



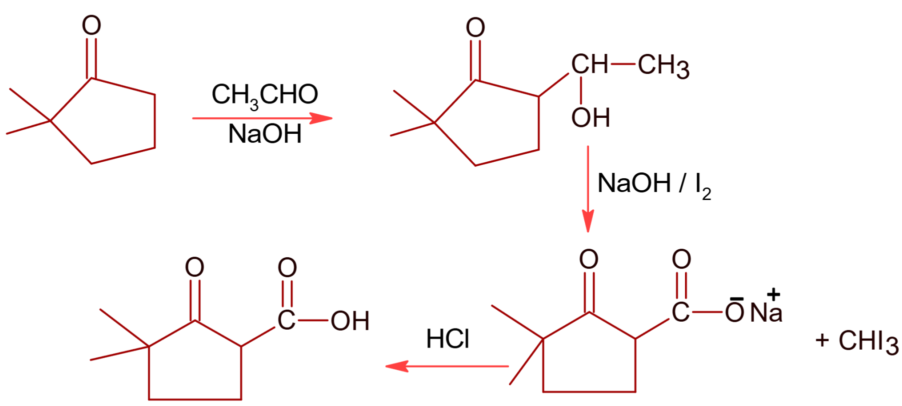
(less ionisation due to non- polar solvent)

(a) Phenol with Br_2 / H_2O gives



(High ionisation due to polar solvent and high activation of ring)

Sol2.



(filtrate)

Sol3. Gabriel phthalimide is used for the preparation of 1° aliphatic amine not 1° aromatic amines since 1° aromatic amines do not undergo nucleophilic substitution reaction.

Sol4. $B.D.E_{D-D}$ bond > $B.D.E_{H-H}$ bond

$B.D.E_{H-H} = 435.9$ kJ / mole

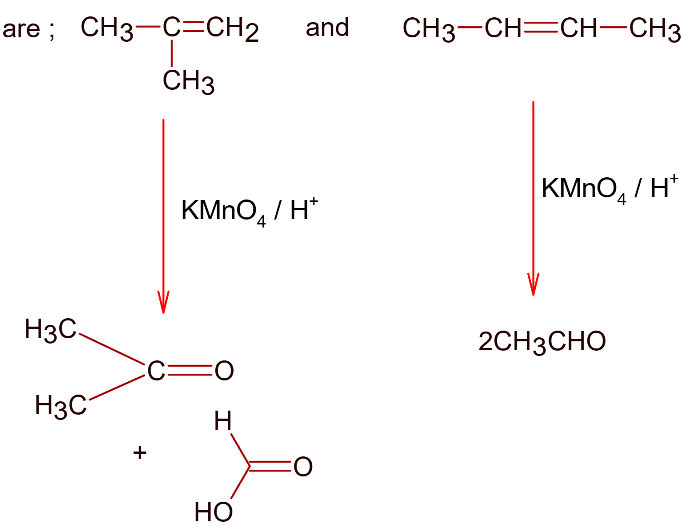
$B.D.E_{D-D} = 443.4$ kJ / mole

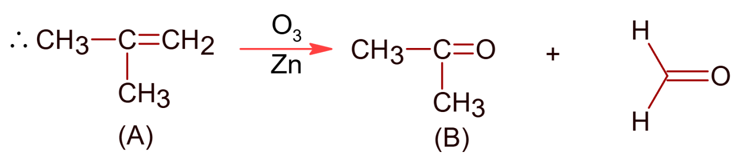
$E_H \approx E_D - 7.5$

Sol5.

$C_4H_8 \longrightarrow$ possible structure of C_4H_8 are ;

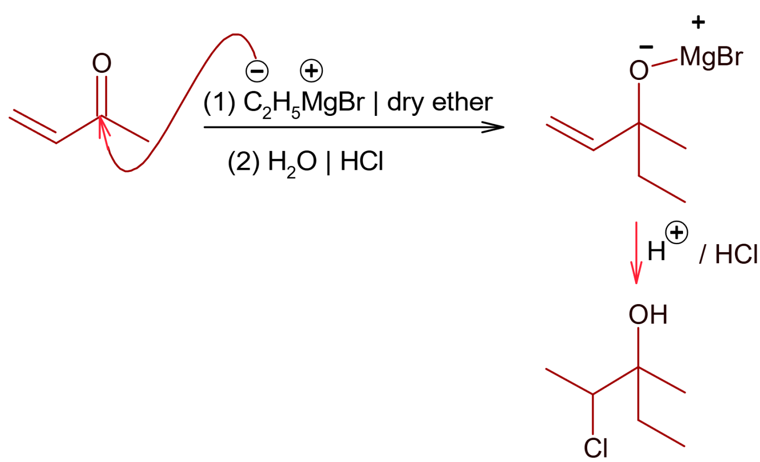
(Unsaturation factor = 1)





Sol6. $[SiCl_6]^{-2}$ is not known because six larger chloride ions can't be accomodated around Si^{+4} due to its small size.

Sol7.



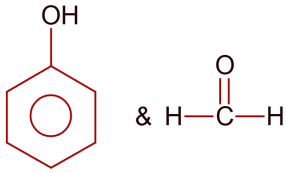
(P)

Sol8. Bauxite contains SiO_2, TiO_2 & Fe_2O_3 as impurities. SiO_2 being acidic is leached with NaOH as Na_2SiO_3.

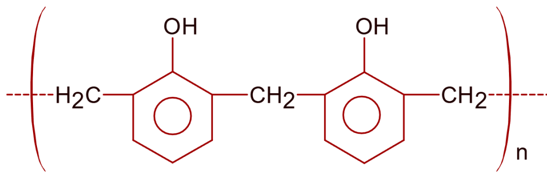
$$Al_2O_3 + 2NaOH \longrightarrow 2NaAlO_2 + H_2O$$

$$SiO_2 + 2NaOH \longrightarrow Na_2SiO_3 + H_2O$$

Sol9. Novolac is a linear polymer obtained from



by condensation reaction

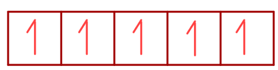


Sol10. When positive charge increases, ionic radii decreases in isoelectronic ions

$A\ell^{+3} < Mg^{+2} < Na^{+} < K^{+}$

Sol11. Albumin is the water soluble protein

Sol12.

$Fe^{+3} = 4s^0\ 3d^5$  (no pairing of electrons due to weak field ligand)

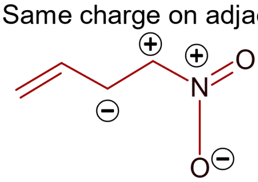
It has five unpaired electrons thus attracted in external magnetic field

$Co^{+3} = 4s^0 3d^6$ $Ni^{+2} = 4s^0 3d^8$ $Ni = 4s^2 3d^8$

While in all other central atoms electrons get paired up due to strong field ligand.

Sol13. $(CH_3)_3\overset{\oplus}{N}H\overset{\ominus}{Cl}$ produces H^+ which reacts with $NaHCO_3$ and CO_2 gas evolved, because HCl is stronger acid than H_2CO_3

Sol14. Same charge on adjacent atom is not stable. Hence the incorrect resonating structure is,

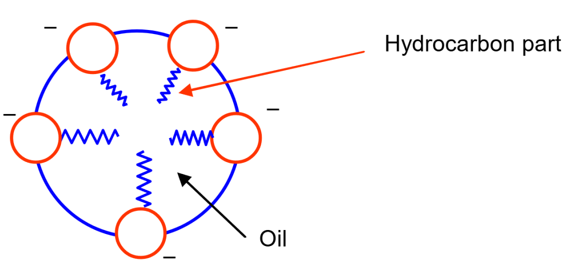


Sol15.

+6	+3	+4	+5	+1
CrO_3	Fe_2O_3	MnO_2	V_2O_5	Cu_2O
(a)	(b)	(c)	(d)	(e)

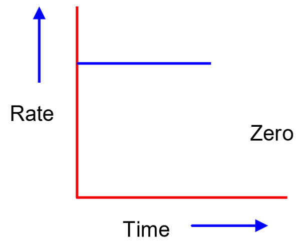
$Cu_2O < Fe_2O_3 < MnO_2 < V_2O_5 < CrO_3$. Hence correct order is; $e < b < c < d < a$

Sol16.

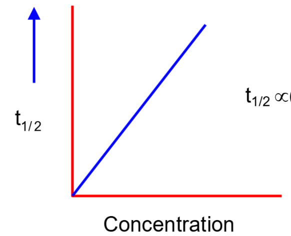


Spherical micelles forms and hydrocarbon part are water repelling and attracted by oily part.

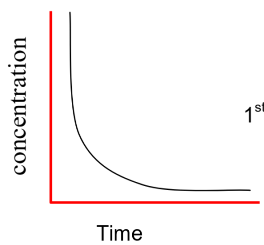
Sol17.

(a) 

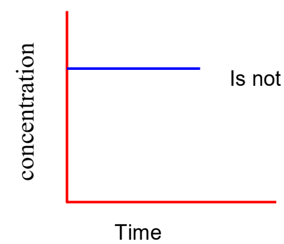
Zero order, because Rate = const.

(b) 

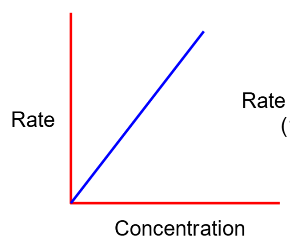
$t_{1/2} \propto (C_o)$ for zero order

(c) 

1st order $C_t = C_o e^{-kt}$

(d) 

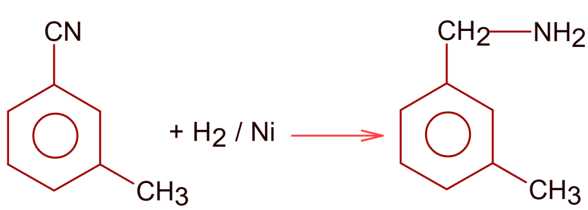
Is not correct graph

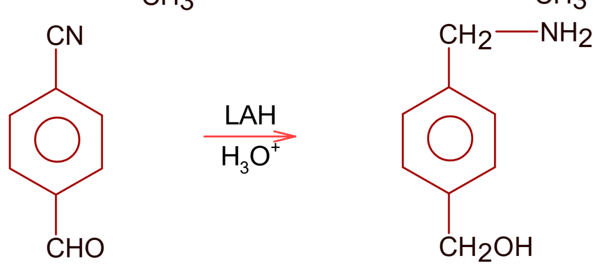
(e) 

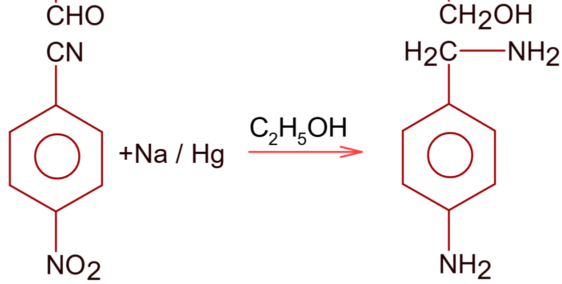
Rate = K [Conc.]^1
(1st order)

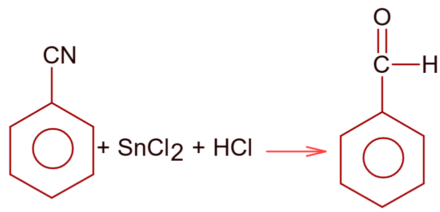
Sol18. $Cl_2C = CCl_2$ is not used for dry cleaning of clothes due to safety purpose.

Sol19.

(1) 

(2) 

(3) 

(4) 

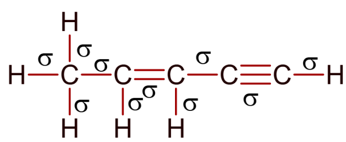
(does not react due to absence of $-NH_2$ group)

Products of reaction 1, 2 and 3 are 1^{o} amines, so react with Hinsberg's reagent to form sulphonamide.

Sol20. Solubility of 2^{nd} group hydroxide increases down the group.

SECTION - B

Sol1.



Number of sigma bonds are 10

Sol2. $\Delta H_{sublimation} = \Delta H_{fusion} + \Delta H_{vaporisation}$

= 2.8 + 98.2 = 101 kJ / mole

Sol3. $C_4H_{10} + \frac{13}{2}O_2 \longrightarrow 4CO_2 + 5H_2O$

1 mole C_4H_{10} produces 5 mole H_2O

$\therefore$ 5 mole H_2O = 5×18 = 90 gm

$\because$ 90 g H_2O obtained from 58 g C_4H_{10}

$\therefore$ 72g H_2O obtained from $\frac{58}{90} \times 72g$

$= \frac{58}{90} \times 72g = 46.4$

$= 464 \times 10^{-1}g$

Sol4 $P_{CO_2} = K_H X_{CO_2}$

$\therefore X_{CO_2} = \frac{P_{CO_2}}{K_H} = \frac{0.835}{1.67 \times 10^{3}} = 0.5 \times 10^{-3}$

$n_{H_2O} = \frac{900}{18} = 50$

$\frac{n_{CO_2}}{n_{CO_2} + n_{H_2O}} = 0.5 \times 10^{-3} \quad (n_{CO_2} << 50)$

$\therefore \frac{n_{CO_2}}{n_{H_2O}} = 0.5 \times 10^{-3}$

$n_{CO_2} = 0.5 \times 10^{-3} \times 50 = 25 \times 10^{-3}$ moles

Sol5. $Cr_2O_7^{--} + Fe^{+2} \longrightarrow Fe^{+3} + Cr^{+3}$

$m.e_{Cr_2O_7^{--}} = m.e_{Fe^{+2}}$

$15 \times 0.02 \times 6 = 10 \times M \times 1$

$\therefore M = \dfrac{15 \times 0.02 \times 6}{10} = 0.18 = 18 \times 10^{-2} M$

Sol6. $Fe^{+3} + e^{-} \longrightarrow Fe^{+2} \qquad E^{\circ} = 0.77V$

$Zn(s) \longrightarrow Zn^{+2} + 2e^{-} \qquad E^{\circ} = 0.76V$

$$\therefore E_{cell} = E^{o}_{cell} - \frac{0.059}{2} \log \frac{\left[Zn^{+2}\right]\left[Fe^{+2}\right]^2}{\left[Fe^{+3}\right]^2}$$

$$1.5 = 1.53 - \frac{0.06}{2} \log \frac{1 \times \left[Fe^{+2}\right]^2}{\left[Fe^{+3}\right]^2}$$

$$\therefore 0.03 \log \left\{\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]}\right\}^2 = 0.03$$

$$\therefore \left\{\frac{\left[Fe^{+2}\right]}{\left[Fe^{+3}\right]}\right\}^2 = 10$$

$$\frac{\left[Fe^{+3}\right]}{\left[Fe^{+2}\right]} = \frac{1}{\sqrt{10}} = 0.316$$

$$\therefore \text{ fraction of } \left[Fe^{+3}\right] = \frac{\left[Fe^{+3}\right]}{\left[Fe^{+2}\right] + \left[Fe^{+3}\right]} = \frac{0.316}{1.316}$$

$= 0.24 = 24 \times 10^{-2}$

Sol7.

	A	+	B	$\rightleftharpoons$	2C
Initially	1		1		1
At eq.	$1-x$		$1-x$		$(1+2x)$

$$\therefore 100 = \frac{(1+2x)^2}{(1-x)(1-x)}$$

$$\left(\frac{1+2x}{1-x}\right) = 10$$

$\therefore x = 3/4$

$$\therefore [C] = (1+2x) = 1 + 2 \times \frac{3}{4} = 1 + \frac{3}{2} M$$

$= 2.5M = 25 \times 10^{-1} M$

Sol8. In 10 sec provides 1000 J energy

In 1 sec provides 100 J energy

$\therefore \quad E = E_0 + K.E$

$\lambda = \quad$ 400 nm is sufficient for ejection of e^-

$$\therefore \quad E = E_0 = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{400 \times 10^{-9}}$$

$= \quad \dfrac{6.626\times 3}{4}\times 10^{-34+15}$

$= \quad \dfrac{6.626\times 3}{4}\times 10^{-19}\,J$

$= \quad 4.965\times 10^{-19}\,J$

Number of e^- ejected $= \dfrac{100}{4.965\times 10^{-19}} = 20.14\times 10^{19}$

$= \quad 2.01\times 10^{20}$

Sol9. Volume of CH_4 gas = $4\times 10^3 m^3 = 4\times 10^6$ lit
P of gas = 1 atm
Temperature = 300 K

$\therefore PV = nRT = \dfrac{W}{M}RT$

$W = \dfrac{PVM}{RT} = \dfrac{1\times 4\times 10^6\times 16}{0.083\times 300}\,g$

$= \dfrac{4\times 16\times 10^4}{0.083\times 3}\,g = 257.02\times 10^4\,g$

$= 25.70\times 10^5\,g \approx 26\times 10^5\,g$

Sol10. $CrCl_3\cdot 3NH_3\cdot 3H_2O$

$\downarrow$

$\left[Cr(NH_3)_3(H_2O)_3\right]Cl_3 \xrightarrow{AgNO_3} \underset{(\text{white ppt})}{3AgCl}$

The 3 chloride ion satisfy only primary valency.
$\therefore$ secondary valency satisfied by chloride ion = 0

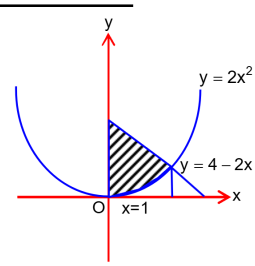
PART – C (MATHEMATICS)

SECTION - A

Sol1. $\{(x,y)\in R\times R, x\ge 0, 2x^2\le y\le 4-2x\}$.

Required area $= \int_0^1 (4-2x-2x^2)dx$

$= \left(4x - x^2 - \dfrac{2}{3}x^3\right)\Big|_0^1 = \dfrac{7}{3}$



Sol2. Three balls can be given to B_3 in ${}^9C_3$ ways. Now remaining 6 balls can be distributed into 3 boxes in 3^6 ways.

Total no. of favourable events $= {}^9C_3\times 3^6$

$= 28\times 3^7$

Total no. of events = 9 balls distributed into 4 boxes in 4^9 ways.

$\therefore$ probability $= 28\times \dfrac{3^7}{4^9} = \dfrac{28}{9}\left(\dfrac{3}{4}\right)^9 \Rightarrow k = \dfrac{28}{9}$.

$k \in |x-3| < 1$

Sol3. $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$

$e^{6x} - 2e^{3x} + 1 - e^{x}\left[e^{3x} + 12e^{x} - 1\right] = 0$

$\left(e^{3x} - 1\right)^{2} - e^{x}\left(e^{3x} - 1\right) - 12e^{2x} = 0$

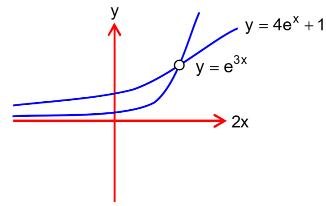
$\left(e^{3x} - 1\right)^{2} - 4e^{x}\left(e^{3x} - 1\right) + 3e^{x}\left(e^{3x} - 1\right) - 12e^{2x} = 0$

$\left(e^{3x} - 1\right)\left\{e^{3x} - 1 - 4e^{x}\right\} + 3e^{x}\left\{e^{3x} - 1 - 4e^{x}\right\} = 0$

$\left(e^{3x} - 4e^{x} - 1\right)\left(e^{3x} - 1 + 3e^{x}\right) = 0$

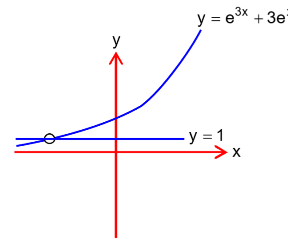
Either $e^{3x} = 4e^{x} + 1$

One Solution



OR $\left(e^{3x} = 1 - 3e^{x}\right)$

One Solution



$\therefore$ the equation has total 2 solutions.

Sol4. System of equations can be written as

$$\begin{pmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ b \end{pmatrix}$$

$$R_1 - 2R_2, R_3 - 3R_2 \Rightarrow \begin{pmatrix} 0 & -1 & 6-2a \\ 1 & 2 & a \\ 0 & -1 & 9-3a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ b-15 \end{pmatrix}$$

$$R_3 - R_1 \Rightarrow \begin{pmatrix} 0 & -1 & 6-2a \\ 1 & 2 & a \\ 0 & 0 & 3-a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ b-13 \end{pmatrix}$$

the system will have no solution as for $a = 3\ \&\ b \neq 13$

Sol5.

$$f(x)\left[\begin{array}{ll} \dfrac{\lambda\left|x^2 - 5x + 6\right|}{\mu\left(5x - x^2 - 6\right)} & x < 2 \\ e^{\dfrac{\tan(x-2)}{x - [x]}} & x > 2 \\ \mu & x = 2 \end{array}\right.$$

LHL : $\displaystyle \mathop{Lt}_{x\to 2} \frac{\lambda\left|(x-2)(x-3)\right|}{\mu\left\{-(x-2)(x-3)\right\}}$

$$= \mathop{Lt}_{x\to 2} \frac{\lambda(x-2)(x-3)}{-\mu(x-2)(x-3)} = -\frac{\lambda}{\mu}$$

RHL:

$$\mathop{Lt}_{x\to 2} e^{\frac{\tan(x-2)}{x-[x]}}$$

$$= \underset{h\to 0}{\text{Lt}} \quad e^{\frac{\tan h}{2+h-2}}$$

$$= \underset{h\to 0}{\ell t} \quad e^{\frac{\tan h}{h}} = e$$

As $f(x)$ is continuous at x = 2

$\therefore -\frac{\lambda}{\mu} = e = \mu$

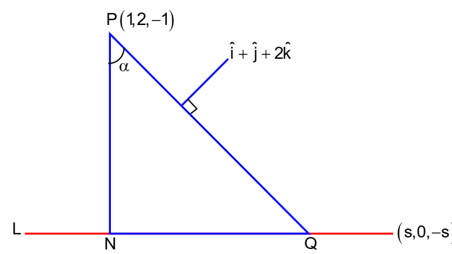
$\Rightarrow \mu = e$

$-\lambda = \mu^2$

$\lambda = -e^2$

$\therefore \lambda + \mu = -e^2 + e = e(1-e)$

Sol6. $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ any point on it



$(r, 0, -r)$

$\therefore$ direction ratio of PN $r-1, -2, -r+1$

perpendicular to $1, 0, -1$.

$\therefore \ 1(r-1) - 1(-r+1) = 0$

$2r = 2 \quad \Rightarrow r = 1$

$\therefore \ N(1, 0, -1)$

Again for PQ direction ratio of PQ is $s-1, -2, -s+1$

It is perpendicular to $\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \ 1(s-1) + (-2) + 2(-s+1) = 0$

$\Rightarrow s = -1$

$\therefore \ Q(-1, 0, 1)$

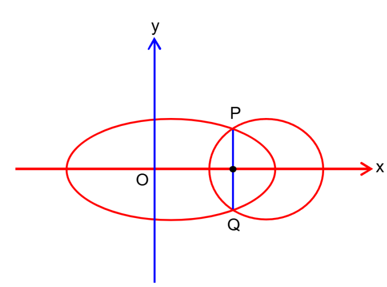
$\overrightarrow{PN} = -2\hat{j} \qquad \overrightarrow{PQ} = -2\hat{i} - 2\hat{j} + 2\hat{k}$.

$$\therefore \cos\alpha = \left|\frac{4}{\sqrt{4}\sqrt{4+4+4}}\right| = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Sol7.

P	q	~ P	$P \Rightarrow q$	$q \Rightarrow \sim P$	$(P \Rightarrow q) \wedge (q \Rightarrow \sim P)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Sol8. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad -(i)$



(i) Passes $\left(\sqrt{\frac{3}{2}}, 1\right)$

$\therefore \ \frac{3}{2a^2} + \frac{1}{b^2} = 1$, From (i)

$\frac{1}{3} = 1 - \frac{b^2}{a^2}$ ....(ii)

$\Rightarrow\ b^2 = 2 \quad \& \ a^2 = 3$

$\therefore$ Ellipse $\frac{x^2}{3} + \frac{y^2}{2} = 1$, From (i)

focus $F\left(\sqrt{3} \times \frac{1}{\sqrt{3}}, 0\right)$

$\equiv\ F(1,0)$

Circle $(x-1)^2 + y^2 = \frac{4}{3}$..........(iii)

Solving (i) & (iii), we get $y = \pm \frac{2}{\sqrt{3}}$ $\quad \therefore PQ = \frac{4}{\sqrt{3}}$.

$PQ^2 = \frac{16}{3}$

Sol9. $\frac{OQ}{OA} = \sin 30^\circ = \frac{1}{2}$ $\quad \Rightarrow OA = 32\text{m}$.

Again $ON = OA \sin 75^\circ$

$= 32 \sin(45^0 + 30^0)$

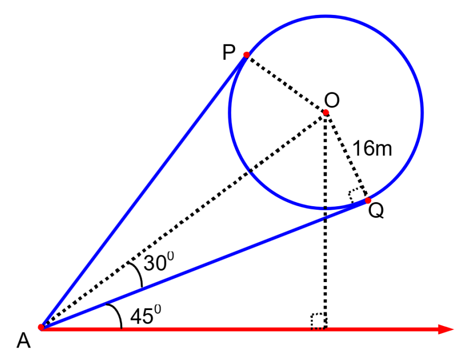
$= 32\left[\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right] = \frac{32(\sqrt{3}+1)}{2\sqrt{2}}$

$= 8(\sqrt{6}+\sqrt{2}) = 8(\sqrt{6}+\sqrt{2})$

Height of top most point

$= 8(\sqrt{6}+\sqrt{2}) + 16$

$= 8(\sqrt{6}+\sqrt{2}+2)$



Sol10. $a = VF = 2$

$P : y^2 = 8(x-2)$

Equation of line SR is T = 0

$\Rightarrow\ y.0 = 4(x+0) - 16$

$\Rightarrow\ x = 4 \quad \therefore\ y^2 = 16$

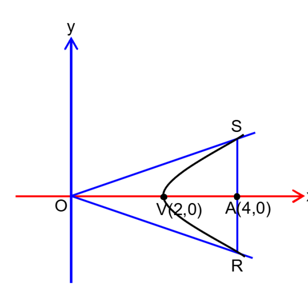
$\Rightarrow\ y = \pm 4$

$\therefore\ < SOR = \frac{\pi}{2} \qquad \therefore SR = 8$

$\Rightarrow\ OS = OR = 4\sqrt{2}$

$\therefore\ \therefore$ area of $\Delta SOR = \frac{1}{2}(4\sqrt{2})(4\sqrt{2})$

$= 16$ sq.unit



Sol11. $16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) = 16 - 36 + 164$

$\Rightarrow \quad \frac{(x+1)^2}{9}-\frac{(y-2)^2}{16}=1$

$e=\sqrt{1+\frac{16}{9}}=\frac{5}{3}$ focii $x+1=\pm 5$ & $y-2=0$

$\therefore S(4,2), S'(-6,2)$ $\Rightarrow s(4,2)$ $s^1(-6,2)$

Any point on the hyperbola is $x+1=3\sec\theta$ & $y-2=4\tan\theta$

$\Rightarrow \quad P(3\sec\theta-1, 4\tan\theta+2)$

$\therefore$ centroid of $\Delta PSS'(h,K)\equiv\left(\frac{3\sec\theta-3}{3}, \frac{4\tan\theta+6}{3}\right)$

$\equiv\left(\sec\theta-1, \frac{4\tan\theta}{3}+2\right)$

$\therefore$ required locus $(x+1)^2-\left(\frac{3}{4}(y-2)\right)^2=1$

$\Rightarrow 16(x+1)^2-9(y-2)^2=16$

$\Rightarrow 16x^2-9y^2+32x+36y-36=0$

Sol12. $S_{3n}=3S_{2n}$

$\Rightarrow \quad \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]}=3$

$\Rightarrow \quad \frac{2a+(3n-1)d}{2a+(2n-1)d}=2$

$\Rightarrow \quad 2a+(3n-1)d=4a+2(2n-1)d$

$\Rightarrow \quad 2a+(n-1)d=0$

$2a=-(n-1)d$

$\frac{S_{4n}}{S_{2n}}=\frac{\frac{4n}{2}[2a+(4n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]}$

$=\frac{2[-(n-1)d+(4n-1)d]}{-(n-1)d+(2n-1)d} = 6$

Sol13. $f:[0,\infty)\to[0,\infty)$

$f(x)=\int_0^x [y]dy$

Let $x=n+f$ where $n=[x]$

$f(x)=\int_0^1 0\,dy+\int_1^2 1dy\,dy+\int_2^3 2dy+\ldots+\int_{n-1}^{n}(n-1)dy+\int_n^{n+f} ndy$

$=(2-1)+2(3-2)+\ldots.+(n-1)(n-n+1)+n(n+f-n)$

$=1+2+\ldots.+(n-1)+nf.$

$=\frac{n(n-1)}{2}+nf$

$= \frac{[x]([x]-1)}{2} + [x](x-[x])$

$= x[x] - \frac{[x]([x]+1)}{2}$

$f(x) = \begin{cases} [x]\left(\frac{[x]-1}{2}\right) & \text{for } x \in I \\ x[x]-[x]\left(\frac{[x]+1}{2}\right) & \text{for } x \notin I \end{cases}$

For continuity at $x = n \quad n \in I$

$f(n) = \frac{n(n-1)}{2}$

$\text{LHL} = \lim_{h\to 0}\left[(n-h)[n-h]-[n-h]\left(\frac{[n-h]+1}{2}\right)\right]$

$= n(n-1)-(n-1)\left(\frac{n-1+1}{2}\right) = \frac{n(n-1)}{2}$

$\text{RHL} = \lim_{h\to 0}\left[(n+h)[n+h]-[n+h]\left(\frac{[n+h]+1}{2}\right)\right]$

$= n(n)-n\left(\frac{n+1}{2}\right) = \frac{n(n-1)}{2}$

$f(x)$ is continuous at all integers.

$f'(x) = [x]$ which in discontinuous at all integer

Sol14. The given three vectors are coplanar then

$(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k} = \alpha\left[(1+b)\hat{i}+2b\hat{j}-b\hat{k}\right]+\beta\left[(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}\right]$

$2+a+b = \alpha(1+b)+\beta(2+b)$ ....(i)

$a+2b+c = 2b\alpha+2b\beta$ ....(ii)

$b+c = b\alpha-(1-b)\beta$..........(iii)

From (ii) & (iii) $2+a+2b+c = (\alpha+\beta)(1+2b)$ ....(iv)

From (iii) & (iv) $2+2b(\alpha+\beta) = (\alpha+\beta)(1+2b)$

$2 = \alpha+\beta-5$

From (ii) $a+2b+c = 2b(\alpha+\beta) = 4b$

$\Rightarrow a+c = 2b$

Sol15. $g: N \to N$ be defined as

$g(3n+1) = 3n+2,$

$g(3n+2) = 3n+3,$

$g(3n+3) = 3n+1,$ for all $n \geq 0$

(A) : $f: N \to N$ is and one one function and f(g(x))=f(x)$\Rightarrow g(x) = x$ which is not correct

(B) :

$gogog(3n+1) = gog(3n+2) = g(3n+3) = 3n+1$

$\Rightarrow g(f(x)) \neq f(x)$

(C) : not correct

(D) : $f : N \to N$ is an onto function and f(g(x))=f(x) which occur for f(x) being onto

$$f(x) = \begin{cases} a & x = 3n+1 \\ a & x = 3n+2 \\ a & x = 3n+3 \end{cases} \quad a \in N \Rightarrow f(g(x)) = f(x) \ \forall x \in N$$

Sol16. $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$

$f'(x) = 12\sin^3 x\cos x + 30\sin^2 x\cos x + 12\sin x\cos x.$

$= 3\sin 2x \underset{(+)ve}{(2\sin x + 1)} \underset{(+)ve}{(\sin x + 2)}$

$\sin x + 2 > 0 \quad \forall x \in R$

$2\sin x + 1 > 0 \quad \forall x \in \left(-\frac{\pi}{6}, \frac{\pi}{2}\right]$

$\sin 2x < 0 \ \forall x \in \left(-\frac{\pi}{6}, 0\right)$

$\therefore$ f is decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Sol17. $I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+(\tan 2x)^{1/3}}$

$= \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\left(\tan\left(\frac{\pi}{2} - 2x\right)\right)}$, applying $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$= \int_{\pi/24}^{5\pi/24} \frac{dx}{1+(\cot 2x)^{1/3}}$

$= \int_{\frac{\pi}{24}}^{5\pi/24} \frac{(\tan 2x)^{1/3}\, dx}{(\tan 2x)^{1/3} + 1}$

$I + I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{1+(\tan 2x)^{1/3}}{1+(\tan 2x)^{1/3}} dx = \frac{5\pi}{24} - \frac{\pi}{24} = \frac{\pi}{6}$

$\Rightarrow \quad I = \frac{\pi}{12}$

Sol18. $\sin x + \sin 2x + \sin 3x + \sin 4x = 0 \quad x \in [0, 2\pi]$

$2\sin 2x \cos x + 2\sin 3x \cos x = 0$

$4\cos x \sin\frac{5x}{2} \cos\frac{x}{2} = 0$

$\cos x = 0 \quad \Rightarrow \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin\frac{5x}{2}=0 \Rightarrow \frac{5x}{2}=0,\ \pi,\ 2\pi,\ 3\pi, 4\pi, 5\pi \Rightarrow x=0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

$\cos\frac{x}{2}=0 \Rightarrow \frac{x}{2}=\frac{\pi}{2}, \Rightarrow x=\pi$

$\therefore$ sum of all values of x $=2\pi+6\pi+\pi=9\pi$

Sol19. $\left(\frac{b}{a}\right)^3=\left(\frac{b}{a}\right)^4 --- =\left(\frac{b}{a}\right)^n \to 0$

$\frac{1}{a-rb}=\frac{1}{a}\left(1-r\frac{b}{a}\right)^{-1}$

$=\frac{1}{a}\left[1+\left(r\frac{b}{a}\right)+\left(\frac{rb}{a}\right)^2+.......\right]$

$\therefore \sum_{r=1}^{n}\frac{1}{a-rb} \quad =\frac{1}{a}\left[\sum_{r=1}^{n}1+\frac{b}{a}\sum_{r=1}^{n}r+\frac{b^2}{a^2}\sum_{r=1}^{n}r^2\right]$

$=\frac{1}{a}\left[n+\frac{b}{a}.\frac{n(n+1)}{2}+\frac{b^2}{a^2}.\frac{n(n+1)(2n+1)}{6}\right]=\alpha n+\beta n^2+\gamma n^3$

$\gamma=$ coeff. of n^3

$=\frac{2}{6a}\times\frac{b^2}{a^2}=\frac{b^2}{3a^3}$

Sol20. $\frac{dy}{dx}=1+x.e^{y-x}$ ....(i), $|x|<\sqrt{2}$ & $y(0)=0$

$\frac{d(y-x)}{dx}=x.e^{y-x}$

$e^{-(y-x)}d(y-x)=x\ dx$

Integrating

$-e^{-(y-x)}=\frac{x^2}{2}+c$

$y(0)=0 \Rightarrow \quad c=-1$

$e^{x-y}=1-\frac{x^2}{2}$

$x-y=\ell n\left(1-\frac{x^2}{2}\right)$

$\therefore\ y=x-\ell n\left(1-\frac{x^2}{2}\right) \quad -(2)$

$\frac{dy}{dx}=1-\frac{-x}{1-\frac{x^2}{2}}=\frac{2-x^2+2x}{2-x^2}$

$\frac{dy}{dx}=0 \Rightarrow x=\pm\sqrt{3}+1= 1-\sqrt{3}$ as $|x|<\sqrt{2}$

y minimum for $x=1-\sqrt{3}$

y min $=(1-\sqrt{3})-\ell n(\sqrt{3}-1)$

SECTION - B

Sol1.

Class	No. of students	Namber of possible cases		
10	5	2	3	2
11	6	2	2	3
12	8	6	5	5
		(I)	(II)	(III)

Total cases

$= {}^5C_2 \times {}^6C_2 \times {}^8C_6 + {}^5C_3 \times {}^6C_2 \times {}^8C_5 + {}^5C_2 \times {}^6C_3 \times {}^8C_5$

$= 23,800 = 100K$

$\therefore K = 238$

Sol2. $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}, \quad \vec{q} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{r} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ is $\perp$r to

$\vec{p} + \vec{q}$ and $\vec{p} - \vec{q}$

$\therefore \quad \vec{r}$ is collinear with $(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}) = -2\vec{p} \times \vec{q}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$$

$\therefore \quad \vec{r} = \lambda(\hat{i} - \hat{j} + \hat{k}) \quad |\vec{r}| = \sqrt{3} \Rightarrow \lambda = 1$

$\therefore \quad \vec{r} = \hat{i} - \hat{j} + \hat{k} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\Rightarrow \qquad \alpha = 1, \quad \beta = -1, \quad \gamma = 1$

$|\alpha| + |\beta| + |\gamma| = 3$

Sol3. $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}\right)^{10}$

$= \left((x^{1/3} + 1) - (1 + x^{-1/2})\right)^{10}$

$= \left(x^{1/3} - x^{-1/2}\right)^{10}$

general term $= {}^{10}C_r x^{\frac{10-r}{3}} . x^{-\frac{r}{2}}$

$= {}^{10}C_r \, x^{\frac{20-5r}{6}}$

For independent of x, $20 - 5r = 0 \Rightarrow r = 4$

$\therefore$ Coefficient $= {}^{10}C_4 = 210$

Sol4. $e^y \dfrac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0$

$\dfrac{d}{dx}(e^y) - (2\sin x)e^y = -\sin x \cos x^2$

I.F. $= e^{-\int 2\sin x \, dx} = e^{2\cos x}$

Solution. $\quad e^y . e^{2\cos x} = -\int e^{2\cos x} \sin x \cos^2 x \, dx.$

Let $\cos x = t. \Rightarrow -\sin x \, dx = dt.$

$$e^y e^{2\cos x} = \int e^{2t} t^2 \, dt = \frac{e^{2t} t^2}{2} - \int 2t.\frac{e^{2t}}{2} \, dt$$

$$e^y.e^{2\cos x} = \frac{t^2 e^{2t}}{2} - \left(t\frac{e^{2t}}{2} - \frac{e^{2t}}{4}\right) + c$$

$$e^{y+2\cos x} = \frac{e^{2\cos x}\cos^2 x}{2} - \frac{\cos x.e^{2\cos x}}{2} + \frac{e^{2\cos x}}{4} + c$$

$$y\left(\frac{\pi}{2}\right) = 0$$

$$e^0 = 0 + \frac{e^0}{4} + c \Rightarrow c = \frac{3}{4}$$

$$e^y = \frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4}e^{-2\cos x}$$

$$y = \ell n\left[\frac{1}{4} + \frac{1}{2}\left(\cos^2 x - \cos x\right) + \frac{3}{4}e^{-2\cos x}\right]$$

$$y(0) = \ell n\left(\frac{1}{4} + \frac{3}{4}e^{-2}\right) = \log_e^{\left(\alpha + \beta e^{-2}\right)}$$

$$\alpha = \frac{1}{4} \ \& \ \beta = \frac{3}{4} \Rightarrow 4(\alpha + \beta) = 4$$

Sol5. $M = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad a,b,c,d \in \{\pm 3, \pm 2, \pm 1, 0\}\right\}$

$f(A) = \det(A) = ad - bc = 15$

Case 1. $ad = 9$ which can occur in 2 ways as 3×3 or $(-3) \times (-3)$

then $bc = -6$

$(3,-2)(-3,2)(2,-3)(-2,3)$ 4 ways

total cases **=** $2 \times 4 = 8$

Case 2.

ad = 6 can occur in 4 ways as

$(3,2) \quad (2,3) \ (-3,-2) \ (-2,-3)$

Then $bc = -9$ in 2 ways

as $3x - 3$ or -3×3

total cases $= 4 \times 2 = 8$

Total no. of possible such cases $= 8 + 8 = 16$

Sol6. $\dfrac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \dfrac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

Sol7. $S = \left\{n \in \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad a,b,c,d, \in R\right\}$

$$\begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$$

$$\begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}^4=\begin{pmatrix}i & 0\\ 0 & i\end{pmatrix}^2=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$$

$$\begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}^8=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I.$$

$$\therefore \begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}^8\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}a & b\\ c & d\end{pmatrix} \quad \forall\ a,b,c,d\in R.$$

$\therefore$ n must be multiple of 8

$\therefore$ number such 2 digit numbers are

$n\ \{16,24,....96\}=11$

Sol8. $\alpha+110+54+30+\beta=584$

$\Rightarrow \alpha+\beta=390 \quad -(i)$

$$\text{Median }=L+\left(\frac{\frac{N}{2}-C}{f}\right)\times h=45$$

$L=40,\ \ N=584,\ C=\alpha+146, f=30,\ h=10$

$$\therefore\ 45=40+\left(\frac{292-\alpha-164}{30}\right)\times 10$$

$$5=\frac{128-\alpha}{3}. \qquad \Rightarrow \alpha=113$$

$(1)\Rightarrow \beta=277.$

$\therefore\ \ |\alpha-\beta|=164$

Sol9. α,β are roots of $x^2+\left(5\sqrt{2}\right)x+10=0$

$\alpha+\beta=-5\sqrt{2} \qquad \alpha\beta=10$

$$\frac{P_{17}P_{20}+5\sqrt{2}\,P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}\,P_{18}^2}$$

$$=\frac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)}$$

$$=\frac{P_{17}}{P_{18}}\ \frac{\alpha^{20}-\beta^{20}-(\alpha+\beta)\left(\alpha^{19}-\beta^{19}\right)}{\alpha^{19}-\beta^{19}-(\alpha+\beta)\left(\alpha^{18}-\beta^{18}\right)}$$

$$=\frac{P_{17}}{P_{18}}\times\frac{\alpha\beta\left(\beta^{18}-\alpha^{18}\right)}{\alpha\beta\left(\beta^{17}-\alpha^{17}\right)}$$

$$=\frac{P_{17}}{P_{18}}\times\frac{-P_{18}}{-P_{17}}=\ \ 1$$

Sol10. $\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+.........\infty\right)=\frac{1}{2}$

$$S=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+........\infty.$$

$$S = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3} + ...\infty$$

$$\frac{S}{3} = \frac{2}{3^2} + \frac{6}{3^3} + ...\infty$$

$$S\left(1 - \frac{1}{3}\right) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + ...\infty$$

$$S\left(\frac{2}{3}\right) = \frac{2}{3} + \frac{4}{3^2}\left(1 + \frac{1}{3} + \frac{1}{3^2} + ...\infty\right)$$

$$= \frac{2}{3} + \frac{4}{3^2} \times \frac{1}{1 - 1/3} = \frac{4}{3}$$

$$\therefore S = 2 \quad \therefore S = 1 + 2 = 3$$

$$\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + ...\infty\right)$$

$$= \quad 3^{1/2} = \ell$$

$$\therefore \quad \ell^2 = 3$$