Solutions to JEE(Main) -2021

Test Date: 25th July 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

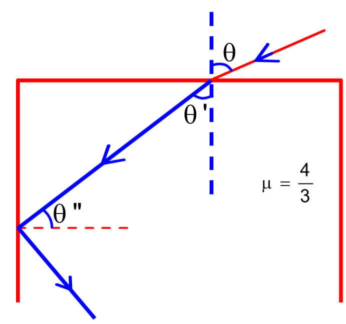
PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

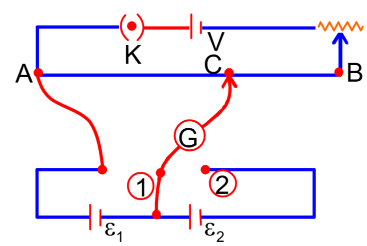
Q1. A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to :



(A) $\sin^{-1}\frac{\sqrt{5}}{4}$ (B) $\sin^{-1}\frac{\sqrt{7}}{3}$

(C) $\sin^{-1}\frac{\sqrt{7}}{4}$ (D) $\sin^{-1}\frac{\sqrt{5}}{3}$

Q2. In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the emf of two cells, $\frac{\varepsilon_1}{\varepsilon_2}$ is:



(A) $\frac{4}{3}$ (B) $\frac{8}{5}$

(C) $\frac{3}{2}$ (D) $\frac{5}{3}$

Q3. A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around :
(takes the value of g as 10 m /s^2)

(A) 250 m (B) 300 m
(C) 200 m (D) 125 m

Q4. An electron moving with speed v and a photon moving with speed c, have same D-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is :

(A) $\frac{v}{2c}$ (B) $\frac{2c}{v}$

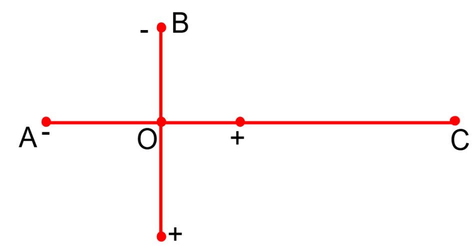
(C) $\frac{v}{3c}$ (D) $\frac{3c}{v}$

Q5. Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $\left(\vec{X}-\vec{Y}\right)$ is n times the magnitude of $\left(\vec{X}+\vec{Y}\right)$. The angle between $\vec{X}$ and $\vec{Y}$ is :

(A) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$ (B) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$

(C) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$ (D) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$

Q6. Two ideal electric dipoles A and B, having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37° with the axis.



The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is :

(take $\sin 37^\circ = \frac{3}{5}$)

(A) $\frac{3}{8}$

(B) $\frac{3}{2}$

(C) $\frac{4}{3}$

(D) $\frac{2}{3}$

Q7. A $10\,\Omega$ resistance is connected across $220\,V - 50\,Hz$ AC supply. The time taken by the current to change from its maximum value to the rms value is :

(A) 2.5 ms
(B) 4.5 ms
(C) 3.0 ms
(D) 1.5 ms

Q8. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is : (Where v stands for velocity)

(A) $2n^2v^2$
(B) $2mnv^3$
(C) $2mv^3$
(D) $2nv^3$

Q9. A prism of refractive index μ and angle of prism A is placed in the position of minimum angle of deviation. If minimum angle of deviation is also A, then in terms of refractive index value of A is :

(A) $2\cos^{-1}\left(\frac{\mu}{2}\right)$
(B) $\cos^{-1}\left(\frac{\mu}{2}\right)$
(C) $\sin^{-1}\left(\frac{\mu}{2}\right)$
(D) $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$

Q10. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be :

(A) 2 W
(B) W
(C) $\sqrt{2}$ W
(D) $2^{\frac{1}{3}}$ W

Q11. A force $\vec{F} = \left(40\hat{i} + 10\hat{j}\right)N$ acts on a body of mass 5 kg. If the body starts from rest, its position vector $\vec{r}$ at time $t = 10\,s$, will be :

(A) $\left(100\hat{i} + 100\hat{j}\right)$ m
(B) $\left(400\hat{i} + 100\hat{j}\right)$ m
(C) $\left(400\hat{i} + 400\hat{j}\right)$ m
(D) $\left(100\hat{i} + 400\hat{j}\right)$ m

Q12. Two ions having same mass have charges in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2 : 3. The ratio of the radii of their circular trajectories is :

(A) 3 : 1
(B) 2 : 3
(C) 4 : 3
(D) 1 : 4

Q13. If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant k placed between the capacitor plates, then bound charge q_b can be expressed as :

(A) $q_b = q_f\left(1+\frac{1}{\sqrt{k}}\right)$
(B) $q_b = q_f\left(1-\frac{1}{\sqrt{k}}\right)$
(C) $q_b = q_f\left(1-\frac{1}{k}\right)$
(D) $q_b = q_f\left(1+\frac{1}{k}\right)$

Q14. Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has radius equal to :

(A) $\sqrt{r_1 r_2}$
(B) $\sqrt{r_1^2 + r_2^2}$
(C) $\frac{r_1 r_2}{r_1 + r_2}$
(D) $\frac{r_1 + r_2}{2}$

Q15. A heat engine has an efficiency of $\frac{1}{6}$. When the temperature of sink is reduced by $62°C$, its efficiency get doubled. The temperature of the source is :

(A) 124°C
(B) 99°C
(C) 37°C
(D) 62°C

Q16. The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is :

(A) $\frac{\alpha}{2}+\frac{\beta}{3}$
(B) $\frac{3}{2}\alpha+\frac{7}{3}\beta$
(C) $\frac{3}{2}\alpha+\frac{7}{2}\beta$
(D) $3\alpha + 7\beta$

Q17. The force is given in terms of time t and displacement x by the equation
$F = A\cos Bx + C\sin Dt$

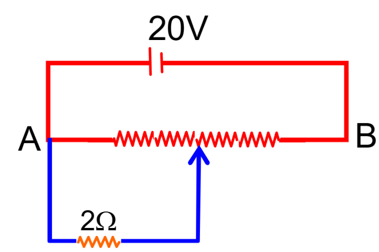
The dimensional formula of $\frac{AD}{B}$ is :

(A) $[M^2 L^2 T^{-3}]$
(B) $[M^1 L^1 T^{-2}]$
(C) $[M L^2 T^{-3}]$
(D) $[M^0 L T^{-1}]$

Q18. When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V. If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V. The threshold wavelength of the metal is:

(A) 6λ
(B) 8λ
(C) 2λ
(D) 4λ

Q19. The given potentiometer has its wire of resistance 10Ω. When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2Ω resistor is :



(A) $\frac{40}{11}$ V (B) 10 V

(C) $\frac{40}{9}$ V (D) 5 V

Q20. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$

(C) $\frac{1}{3}$ (D) $\frac{1}{4}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. From the given data, the amount of energy required to break the nucleus of aluminium $^{27}_{13}Al$ is _________ $x \times 10^{-3}$ J.

Mass of neutron $= 1.00866u$

Mass of proton $= 1.00726$ u

Mass of Aluminium nucleus $= 27.18846$ u

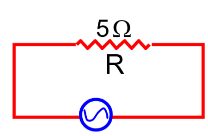
(Assume 1 u corresponds to x J of energy)
(Round off to the nearest integer)

Q2. A force of $F = (5y + 20)\hat{j}$ N acts on a particle. The work done by this force when the particle is moved from $y = 0$m to $y = 10$m is _________ J.

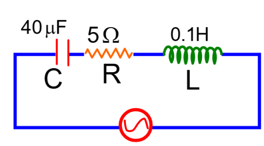
Q3. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10s is __________ $\pi \times 10^{-1}$ Nm.

Q4. A $16\,\Omega$ wire is bend to form a square loop. A 9V supply having internal resistance of $1\,\Omega$ is connected across one of its sides. The potential drop across the diagonals of the square loop is __________ $\times 10^{-1}$ V.

Q5. Two circuits are shown in the figure (a) & (b). At a frequency of ______ rad /s the average power dissipated in one cycle will be same in both the circuits.



220V
figure(a)



220V
figure(b)

Q6. A light beam of wavelength 500 nm is incident on a metal having work function of 1.25 eV, placed in a magnetic field of intensity B. The electrons emitted perpendicular to the magnetic field B, with maximum kinetic energy are bent into circular arc of radius 30cm. The value of B is ___________ $\times 10^{-7}$ T.

Given $hc = 20 \times 10^{-26}$ J – m, mass of electron $= 9 \times 10^{-31}$ kg

Q7. A message single of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt. The modulation index will be ________.

Q8. A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25} / m^3$. The diameter of A and B are $10\,\mathring{A}$ and $5\,\mathring{A}$ respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is __________ $\times 10^{-2}$.

Q9. In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16}\ /\text{m}^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22}\ /\text{m}^3$. The electron density in the doped semiconductor is ________$\times 10^9\ /\text{m}^3$.

Q10. The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{\text{th}}$ of its initial value in 30 years. The half –life of radioactive element is __________years.

PART – B (CHEMISTRY)

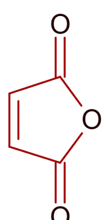
SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below are two statements:
Statement I : Chlorofluoro carbons breakdown by radiation in the visible energy region and release chlorine gas in the atmosphere which then reacts with stratospheric ozone.
Statement II : Atmospheric ozone reacts with nitric oxide to give nitrogen and oxygen gases, which add to the atmosphere.
For the above statements choose the correct answer from the options given below:
(A) **Statement I** is incorrect but **Statement II** is true
(B) Both **Statement I** and **II** are correct
(C) Both **Statement I** and **II** are false
(D) **Statement I** is correct but **Statement II** is false

Q2.



Maleic anhydride
Maleic anhydride can be prepared by:
(A) Heating cis-but-2-enedioic acid
(B) Treating cis-but-2-enedioic acid with alcohol and acid
(C) Heating trans- but-2- enedioic acid
(D) Treating trans-but-2-enedioic acid with alcohol and acid

Q3. The ionic radii of F^- and O^{2-} respectively are 1.33 $\mathring{A}$ and 1.4 $\mathring{A}$, while the covalent radius of N is 0.74 $\mathring{A}$.
The correct statement for the ionic radius of N^{3-} from the following is:
(A) It is smaller than O^{2-} and F^-, but bigger than of N
(B) It is bigger than F^- and N, but smaller than of O^{2-}
(C) It is smaller than F^- and N
(D) It is bigger than O^{2-} and F^-

Q4. The spin only magnetic moments (in BM) for free Ti^{3+}, V^{2+} and Sc^{3+} ions respectively are (At. No. Sc: 21; Ti:22; V:23)
(A) 1.73,0,3.87
(B) 0,3.87,1.73
(C) 3.87,1.73,0
(D) 1.73, 3.87,0

Q5. Which one of the following metal complexes is most stable?

(A) $\left[Co(en)_2(NH_3)_2\right]Cl_2$ (B) $\left[Co(NH_3)_6\right]Cl_2$

(C) $\left[Co(en)_3\right]Cl_2$ (D) $\left[Co(en)(NH_3)_4\right]Cl_2$

Q6. Match List – I with List – II:

List – I **Elements**		**List – II** **Properties**
(a) Li	(i)	Poor water solubility of I^- salt
(b) Na	(ii)	Most abundant element in cell fluid
(c) K	(iii)	Bicarbonate salt used in fire extinguisher
(d) Cs	(iv)	Carbonate salt decomposes easily on heating

Choose the correct answer from the options given below:

(A) (a) – (iv), (b) – (ii), (c) – (iii), (d) – (i) (B) (a) – (i), (b) –(ii), (c) – (iii), (d) – (iv)
(C) (a) –(i), (b) – (iii), (c) – (ii), (d) – (iv) (D) (a) – (iv), (b) –(iii), (c) –(ii), (d) –(i)

Q7. In the following the correct bond order sequence is:

(A) $O_2 > O_2^- > O_2^{2-} > O_2^+$ (B) $O_2^+ > O_2 > O_2^- > O_2^{2-}$

(C) $O_2^{2-} > O_2^+ > O_2^- > O_2$ (D) $O_2^+ > O_2^- > O_2^{2-} > O_2$

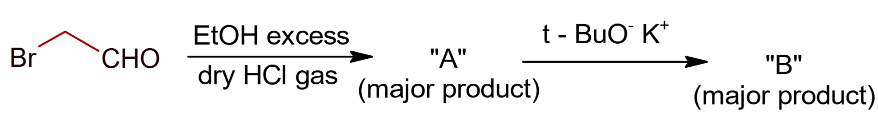
Q8. Match List – I with List – II:

List – I **Example of Colloids**		**List – II** **Classification**
(a) Cheese	(i)	dispersion of liquid in liquid
(b) Pumice stone	(ii)	dispersion of liquid in gas
(c) Hair cream	(iii)	dispersion of gas in solid
(d) Cloud	(iv)	dispersion of liquid in solid

Choose the most appropriate answer from the option given below:

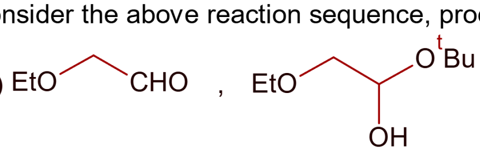
(A) (a) – (iii), (b) –(iv), (c) –(i), (d) –(ii) (B) (a) –(iv), (b) –(iii), (c) –(ii), (d) –(i)
(C) (a) –(iv), (b) –(iii), (c) –(i), (d) –(ii) (D) (a) –(iv), (b) –(i), (c) –(iii), (d) –(ii)

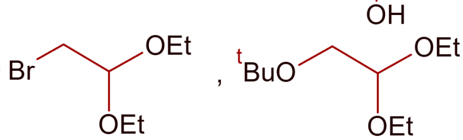
Q9.

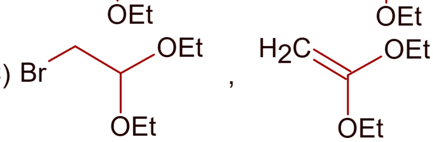


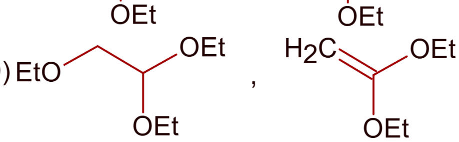
[where $Et \Rightarrow C_2H_5$, $t-Bu \Rightarrow (CH_3)_3C-$]

Consider the above reaction sequence, product "A" and product "B" formed respectively are:

(A) 

(B) 

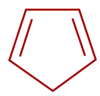
(C) 

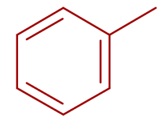
(D) 

Q10. Which among the following is the strongest acid?

(A) $CH_3CH_2CH_2CH_3$

(B) 

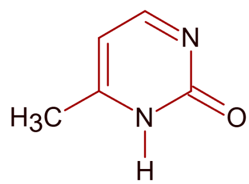
(C) 

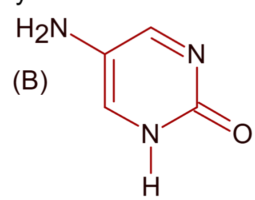
(D) 

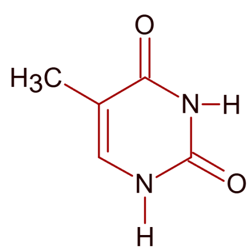
Q11. Which one of the following metals forms interstitial hydride easily?

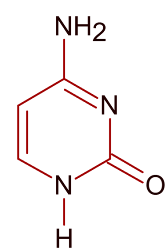
(A) Fe
(B) Cr
(C) Mn
(D) Co

Q12. Which one of the following is correct structure for cytosine?

(A) 

(B) 

(C) 

(D) 

Q13. Identify the species having one π-bond and maximum number of canonical forms from the following:

(A) SO_2
(B) CO_3^{2-}
(C) O_2
(D) SO_3

Q14. A biodegradable polyamide can be made from:

(A) Glycine and aminocaproic acid
(B) Hexamethylene diamine and adipic acid
(C) Glycine and isoprene
(D) Styrene and caproic acid

Q15. Match List – I with List – II:

List – I	List – II
(a) Concentration of Ag ore	(i) Reverberatory furnace
(b) Blast furnace	(ii) Pig iron
(c) Blister copper	(iii) Leaching with dilute NaCN solution
(d) Froth floatation method	(iv) Sulfide ores

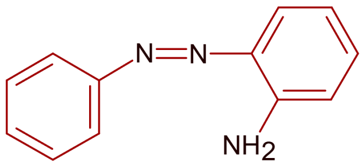
Choose the correct answer from the options given below:

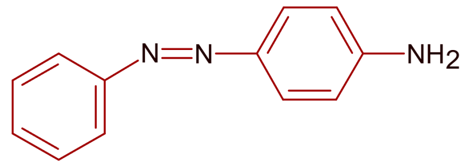
(A) (a) –(iii), (b) –(ii), (c) –(i), (d) –(iv)
(B) (a) –(iii), (b) – (iv), (c) –(i), (d) – (ii)
(C) (a) – (iv), (b) – (i), (c) – (iii), (d) – (ii)
(D) (a) –(iv), (b) – (iii), (c) – (ii), (d) –(i)

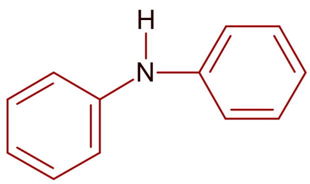
Q16.

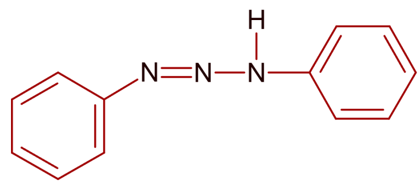
$$C_6H_5NO_2 \xrightarrow{Sn\ +\ HCl} \text{"A"} \xrightarrow[H^{\oplus}]{C_6H_5\overset{\oplus}{N_2}\overset{\ominus}{Cl}} \underset{\text{(Yellow coloured Compound)}}{P}$$

Consider the above reaction, the product "P" is:

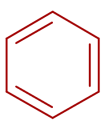
(A) 

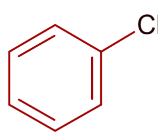
(B) 

(C) 

(D) 

Q17. The correct decreasing order of densities of the following compounds is:


(A)



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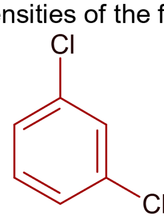
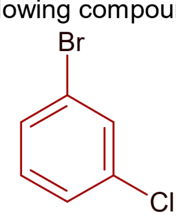


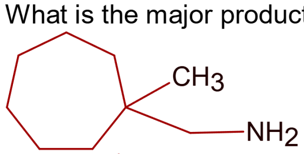
(C)



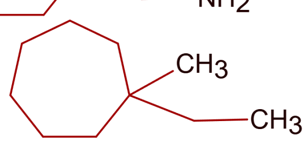
(D)

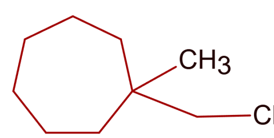
(A) (C) > (B) > (A) > (D)
(B) (A) > (B) > (C) > (D)
(C) (D) > (C) > (B) > (A)
(D) (C) > (D) > (A) > (B)

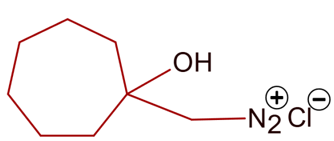
Q18. What is the major product "P" of the following reaction?

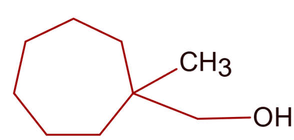


$$\xrightarrow[\text{(ii) } H_2O]{\text{(i) } NaNO_2,\ HCl,\ 278\ K} \underset{\text{(major product)}}{P}$$

(A) 

(B) 

(C) 

(D) 

Q19. Identify the process in which change in the oxidation state is five:

(A) $CrO_4^{2-} \rightarrow Cr^{3+}$
(B) $MnO_4^{-} \rightarrow Mn^{2+}$
(C) $Cr_2O_7^{2-} \rightarrow 2Cr^{3+}$
(D) $C_2O_4^{2-} \rightarrow 2CO_2$

Q20. A reaction of benzonitrile with one equivalent CH_3MgBr followed by hydrolysis produces a yellow liquid "P". The compound "P" will give positive_______.

(A) Iodoform test
(B) Schiff's test
(C) Ninhydrin's test
(D) Tollen's test

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

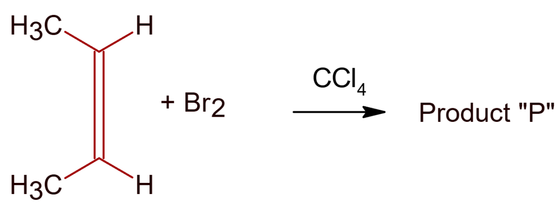
Q1. The number of significant figure in 0.00340 is______________.

Q2. An LPG cylinder contains gas at a pressure of 300 kPa at 27°C. The cylinder can withstand the pressure of 1.2×10^{6} Pa. The room in which the cylinder is kept catches fire. The minimum temperature at which the bursting of cylinder will take place is______ °C. (Nearest integer)

Q3. An accelerated electron has speed of $5\times10^{6}\ ms^{-1}$ with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is $x\times10^{-9}\,m$. The value of x is_______.
(Nearest integer)
[Use mass of electron = 9.1×10^{-31} kg, h = 6.63×10^{-34}Js, π=3.14]

Q4. Number of electrons present in 4f orbital of Ho^{3+} ion is_______________.
(Given atomic number of Ho = 67)

Q5.



Consider the above chemical reaction. The total number of stereoisomers possible for product ‘P’ is__________.

Q6. Assuming that $Ba(OH)_2$ is completely ionized in aqueous solution under the given conditions the concentration of H_3O^+ ions in 0.005 M aqueous solution of $Ba(OH)_2$ at 298 K is ___________ $\times 10^{-12}$ mol L^{-1}. (nearest integer)

Q7. For a chemical reaction $A\rightarrow B$, it was found that concentration of B is increased by 0.2 mol L^{-1} in 30 min. The average rate of the reaction is ________$\times10^{-1}$ mol L^{-1} h^{-1} . (in nearest integer)

Q8. When 3.00 g of substance ‘X” is dissolved in 100 g of CCl_4, it raises the boiling point by 0.60K. The molar mass of the substance ‘X” is _________g mol^{-1}. (nearest integer)
[Given Kb for CCl_4 is 5.0 K kg mol^{-1}]

Q9. A system does 200 J of work and at the same time absorbs 150 J of heat. The magnitude of the change in internal energy is ________J. (Nearest integer)

Q10. 0.8 g of an organic compound was analysed by Kjeldahl’s method for the estimation of nitrogen. If the percentage of nitrogen in the compound was found to be 42%, then _________ ml. of 1 M H_2SO_4 would have been neutralized by the ammonia evolved during the analysis.

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to :

(A) 4 (B) -2
(C) 2 (D) 0

Q2. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is:

(A) 2 (B) 4
(C) 1 (D) 3

Q3. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to :

(A) $\dfrac{2}{\frac{1}{a}+\frac{1}{b}}$ (B) $\dfrac{a+b}{2}$

(C) $\frac{1}{a}+\frac{1}{b}$ (D) $\sqrt{ab}$

Q4. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :

(A) $(1,1)$ (B) $(\sqrt{2},0)$

(C) $(\sqrt{3},0)$ (D) $(-1,1)$

Q5. If $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then P^{50} is :

(A) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

Q6. The sum of all those terms which are rational numbers in the expansion of $\left(2^{\frac{1}{3}} + 3^{\frac{1}{4}}\right)^{12}$ is :

(A) 89 (B) 35
(C) 27 (D) 43

Q7. Let the equation of the pair of lines, y = px and y = qx, can be written as $(y-px)(y-qx)=0$. Then the equation of the pair of the angle bisectors of the lines $x^2-4xy-5y^2=0$ is :

(A) $x^2-3xy+y^2=0$ (B) $x^2+3xy-y^2=0$
(C) $x^2-3xy-y^2=0$ (D) $x^2+4xy-y^2=0$

Q8. The number of real solutions of the equation, $x^2-|x|-12=0$ is :

(A) 1 (B) 4
(C) 3 (D) 2

Q9. The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is.........

(A) 3 (B) 2
(C) 4 (D) 1

Q10. If $|\vec{a}|=2, |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|=8$, then $|\vec{a}\,.\,\vec{b}|$ is equal to :

(A) 5 (B) 4
(C) 6 (D) 3

Q11. The value of $\cot\frac{\pi}{24}$ is :

(A) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$ (B) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$
(C) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (D) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$

Q12. Let X be a random variable such that the probability function of a distribution is given by $P(X=0)=\frac{1}{2}, P(X=j)=\frac{1}{3^j}(j=1,2,3,....,\infty)$. Then the mean of the distribution and P(X is positive and even) respectively are :

(A) $\frac{3}{4}$ and $\frac{1}{9}$ (B) $\frac{3}{4}$ and $\frac{1}{16}$
(C) $\frac{3}{4}$ and $\frac{1}{8}$ (D) $\frac{3}{8}$ and $\frac{1}{8}$

Q13. Consider functions $f:A\rightarrow B$ and $g:B\rightarrow C\,(A,B,C\subseteq R)$ such that $(gof)^{-1}$ exists, then :

(A) f and g both are onto (B) f is onto and g is one-one
(C) f is one-one and g is onto (D) f and g both are one-one

Q14. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :

(A) 4 (B) 6
(C) 5 (D) 8

Q15. Let $y=y(x)$ be the solution of the differential equation $xdy=(y+x^3\cos x)dx$ with $y(\pi)=0$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(A) $\frac{\pi^2}{4}-\frac{\pi}{2}$ (B) $\frac{\pi^2}{4}+\frac{\pi}{2}$
(C) $\frac{\pi^2}{2}-\frac{\pi}{4}$ (D) $\frac{\pi^2}{2}+\frac{\pi}{4}$

Q16. The value of the integral $\int_{-1}^{1} \log\left(x+\sqrt{x^2+1}\right)dx$ is :

(A) -1 (B) 1
(C) 2 (D) 0

Q17. If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to :

(A) -1 (B) 2
(C) -2 (D) 1

Q18. If $f(x) = \begin{cases} \int_0^x \left(5+|1-t|\right)dt, & x > 2 \\ 5x+1, & x \le 2 \end{cases}$, then

(A) f(x) is not continuous at x = 2
(B) f(x) is not differentiable at x = 1
(C) f(x) is continuous but not differentiable at x = 2
(D) f(x) is everywhere differentiable

Q19. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to :

(A) 4 (B) 1
(C) 3 (D) 2

Q20. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following :
(A) The match will not be played or weather is good and ground is not wet.
(B) The match will not be played or weather is not good and ground is wet.
(C) If the match will not be played, then either weather is not good or ground is wet.
(D) The match will be played and weather is not good or ground is wet.

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Consider the function $f(x) = \dfrac{P(x)}{\sin(x-2)}, \quad x \neq 2$

$= 7, \quad x = 2$

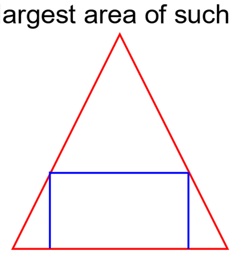
where P(x) is a polynomial such that P"(x) is always a constant and P(3) = 9. If (x) is continuous at x = 2, then P(5) is equal to.......

Q2. If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to.....

Q3. If the co-efficients of x^7 and x^8 in the expansion of $\left(2 + \dfrac{x}{3}\right)^n$ are equal, then the value of n is equal to ....

Q4. If the lines $\dfrac{x-k}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$ and $\dfrac{x+1}{3} = \dfrac{y+2}{2} = \dfrac{z+3}{1}$ are co-planar, then the value of k is......

Q5. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ....



Q6. If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is.......

Q7. Let $n \in N$ and [x] denote the greatest integer less than or equal to x. If the sum of (n+1) terms ${}^nC_0, 3.{}^nC_1, 5.{}^nC_2, 7.{}^nC_3, ...$ is equal to $2^{100}.101$, then $2\left[\dfrac{n-1}{2}\right]$ is equal to

Q8. A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is...........

Q9. Let a curve $y = f(x)$ pass through the point $\left(2, (\log_e 2)^2\right)$ and have slope $\dfrac{2y}{x \log_e x}$ for all positive real value of x. Then the value of $f(e)$ is equal to..........

Q10. The equation of a circle is $\mathrm{Re}(z^2) + 2(\mathrm{Im}(z))^2 + 2\mathrm{Re}(z) = 0$, where $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to.........

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**B**	2.	**D**	3.	**D**	4.	**A**
5.	**C**	6.	**D**	7.	**A**	8.	**C**
9.	**A**	10.	**D**	11.	**B**	12.	**C**
13.	**C**	14.	**B**	15.	**B**	16.	**B**
17.	**C**	18.	**D**	19.	**C**	20.	**B**

SECTION - B

1.	**27**	2.	**450**	3.	**4**	4.	**45**
5.	**500**	6.	**125**	7.	**1**	8.	**25**
9.	**5**	10.	**10**				

PART – B (CHEMISTRY)

SECTION - A

1.	**C**	2.	**A**	3.	**D**	4.	**D**
5.	**C**	6.	**D**	7.	**B**	8.	**C**
9.	**C**	10.	**C**	11.	**B**	12.	**D**
13.	**B**	14.	**A**	15.	**A**	16.	**B**
17.	**C**	18.	**D**	19.	**B**	20.	**A**

SECTION - B

1.	**3**	2.	**927**	3.	**58**	4.	**10**
5.	**2**	6.	**1**	7.	**4**	8.	**250**
9.	**50**	10.	**12**				

PART – C (MATHEMATICS)

SECTION - A

1.	**A**	2.	**C**	3.	**D**	4.	**C**
5.	**A**	6.	**D**	7.	**B**	8.	**D**
9.	**A**	10.	**C**	11.	**A**	12.	**C**
13.	**C**	14.	**A**	15.	**B**	16.	**D**
17.	**B**	18.	**C**	19.	**D**	20.	**D**

SECTION – B

1.	**39**	2.	**13**	3.	**55**	4.	**1**
5.	**3**	6.	**60**	7.	**98**	8.	**4**
9.	**1**	10.	**1**				

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. By shell's law

$$\frac{\sin\theta}{\sin\theta'} = \frac{4}{3} \text{ ....(i)}$$

For TIR on second surface

$$\sin\theta'' > \sin\theta_c \qquad (\theta' + \theta'' = 90)$$

$$\sin(90 - \theta') > \sin\theta_c$$

$$\cos\theta' > \sin\theta_c$$

$$1 - \left(\frac{3}{4}\sin\theta\right)^2 > \sin^2\theta_c \qquad \text{(By equation (i))}$$

$$1 - \frac{9}{16}\sin^2\theta > \sin^2\theta_c \qquad \left[\sin\theta_c = \frac{3}{4}\right]$$

$$1 - \frac{9}{16}\sin^2\theta > \frac{9}{16}$$

$$1 - \frac{9}{16} > \frac{9}{16}\sin^2\theta$$

$$\frac{7}{16} > \frac{9}{16}\sin^2\theta$$

$$\sin\theta < \frac{\sqrt{7}}{3}$$

Sol2. $\varepsilon_1 = 250 \times \text{Potential Gradient}$

$\varepsilon_1 + \varepsilon_2 = 400 \times \text{Potential Gradient}$

$$\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2} = \frac{250}{400} = \frac{5}{8}$$

By solving above

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{5}{3}$$

Sol3. Body is dropped from height 75m with initial velocity (upward) 10m/s

So, $s = ut + \frac{1}{2}at^2$

$$-75 = 10t - \frac{1}{2}at^2$$

$$5t^2 - 10t - 75 = 0$$

By solving $t = 5\,\text{sec}$.

In 5sec. balloon covered

$h = 10 \times 5 = 50\,m$

Now height of balloon = 75 + 50 = 125m

Sol4. For electron,

$\lambda_e = \frac{h}{p} \Rightarrow (KE)_e = \frac{1}{2}mv^2 = \frac{pv}{2}$

$(K.E)_p = \frac{hc}{\lambda} = pc \qquad \left(p = \frac{h}{\lambda}\right)$

$\frac{KE_e}{KE_p} = \frac{pv/2}{pc} = \frac{v}{2c}$

Sol5. According to question

$|\vec{x}| = |\vec{y}|$ and

$|\vec{x} - \vec{y}| = n|\vec{x} + \vec{y}|$

$x^2 + y^2 - 2\vec{x}\cdot\vec{y} = n^2\left(x^2 + y^2 + 2\vec{x}\cdot\vec{y}\right)$

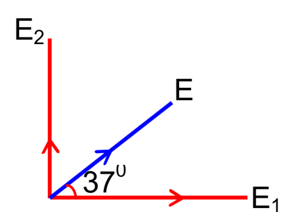
$\left(1-n^2\right)\left(x^2+y^2\right) = \left(1+n^2\right)2\vec{x}\cdot\vec{y}$

$\left(1-n^2\right)\left(x^2+y^2\right) = \left(1+n^2\right)2xy\cos\theta$

$\cos\theta = \frac{1-n^2}{1+n^2}$

$\theta = \cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

Sol6. $\tan 37^\circ = \frac{E_2}{E_1}$



$E_1 = k\frac{2p_1}{r^3}$ axial direction

$E_2 = k\frac{p_2}{r^3}$ equatorial direction

$\tan 37^\circ = \frac{p_2}{2p_1} = \frac{3}{4}$

$\frac{p_1}{p_2} = \frac{2}{3}$

Sol7. $I = I_0 \cos\omega t$

Current is changing from its maximum value to rms value $\left(I_0/\sqrt{2}\right)$

$\frac{I_0}{\sqrt{2}} = I_0\cos\omega t$

$\cos\omega t = \frac{1}{\sqrt{2}}$

$2\pi \times 50t = \frac{\pi}{4}$

$t = 2.5\,ms$

Sol8. By $t = mx^2 + nx$

Now $\frac{dt}{dx} = 2mx + n$

$v = \frac{1}{2mx+n}$

$a = v\frac{dv}{dx} = v\frac{-2m}{(2mx+n)^2} = -2mv^3$

So, Retardation will be $(2mv^3)$

Sol9. $\mu = \frac{\sin\left(\frac{(A+\delta_m)}{2}\right)}{\sin(A/2)}$

Since, $\delta_m = A$

$\mu = \frac{\sin A}{\sin\left(\frac{A}{2}\right)} = \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\mu = 2\cos\left(\frac{A}{2}\right)$

$A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

Sol10. $g = \frac{4}{3}\pi R\rho G$ ....(i) $(\rho = \text{density})$

Now $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$

$R^3 \alpha \left(\frac{M}{\rho}\right)$

$R\ \alpha \left(\frac{M}{\rho}\right)^{1/3}$

By equation (i)

$g \alpha \frac{4}{3}\left(\frac{M}{\rho}\right)^{1/3} \pi\rho G$

$g \alpha M^{1/3} \Rightarrow W \alpha g$

$W' = 2^{1/3} W$

Sol11. $\vec{a} = \frac{\vec{F}}{m} = 8\hat{i} + 2\hat{j}$

$\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{r} = 0 + \frac{1}{2}\left(8\hat{i} + 2\hat{j}\right)10^2$

$\vec{r} = 400\,\hat{i} + 100\,\hat{j}$

Sol12. $2q_1 = q_2$ and $\frac{v_1}{v_2} = \frac{2}{3}$, $m_1 = m_2$

$r = \frac{mv}{qB}$

$\frac{r_1}{r_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{q_2}{q_1}\right)\left(\frac{v_1}{v_2}\right)$

$\frac{r_1}{r_2} = 2 \times \frac{2}{3} = \frac{4}{3}$

Sol13. $E_{net} = \frac{E_0}{k}$, $\left[\frac{q_f}{E_0} - \frac{q_b}{E_0} = \frac{q_f}{kE_0}\right]$

$q_b = q_f\left(1 - \frac{1}{k}\right)$

Sol14. Pressure outside is 0.

Here, $P_{in} = \frac{4T}{r}$

By, $P_1V_1 + P_2V_2 = PV$

$\frac{4T}{r_1} \times \frac{4}{3}\pi r_1^3 + \frac{4T}{r_2} \times \frac{4}{3}\pi r_2^3 = \frac{4T}{r} \times \frac{4}{3}\pi r^3$

$r_1^2 + r_2^2 = r^2$

$r = \sqrt{r_1^2 + r_2^2}$

Sol15. $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6}$ …………(i)

$1 - \frac{T_2 - 62}{T_1} = \frac{1}{3}$ ………..(ii)

By (i) and (ii)

$\frac{62}{T_1} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

$T_1 = 62 \times 6 = 372K = 99^\circ C$

Sol16. $v = \alpha t + \beta t^2$

$\frac{ds}{dt} = \alpha t + \beta t^2$

$\int_0^r ds = \int_1^2 \left(\alpha t + \beta t^2\right) dt$

$s = \left[\frac{\alpha t^2}{2} + \frac{\beta t^3}{3}\right]_1^2$

$s = \left[2\alpha + \frac{8\beta}{3}\right] - \left[\frac{\alpha}{2} + \frac{\beta}{3}\right]$

$$=\frac{3\alpha}{2}+\frac{7\beta}{3}$$

Sol17. Dimension of $B=\left[L^{-1}\right]$

Dimension of $D=\left[T^{-1}\right]$

Dimension of $A=\left[MLT^{-2}\right]$

The dimensional formula of $\frac{AD}{B}$

$$=\left[MLT^{-2}\right]\frac{\left[T^{-1}\right]}{\left[L^{-1}\right]}=\left[ML^{2}T^{-3}\right]$$

Sol18. By Einstein's equation of photoelectric effect,

$$\frac{hc}{\lambda}=\frac{hc}{\lambda_0}+eV_s$$

Now $\frac{hc}{\lambda}=\frac{hc}{\lambda_0}+e\times 4.8$ ....(i)

$\frac{hc}{2\lambda}=\frac{hc}{\lambda_0}+e\times 1.6$ ....(ii)

By dividing (i) by (ii)

$$\frac{2(\lambda_0-\lambda)}{(\lambda_0-2\lambda)}=3$$

$$2\lambda_0-2\lambda=3\lambda_0-6\lambda$$

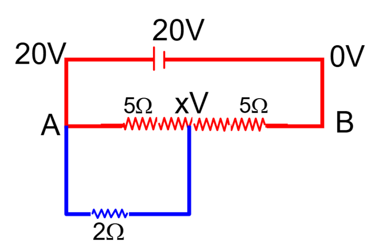
$$\lambda_0=4\lambda$$

Sol19. $\frac{x-0}{5}+\frac{x-20}{5}+\frac{x-20}{2}=0$

$$\Rightarrow x=\frac{140}{9}V$$

Potential drop across 2Ω

$$=20-x=\frac{40}{9}V$$



Sol20. $v=\omega\sqrt{A^2-x^2}$

K.E. $=\frac{1}{2}m\omega^2\left(A^2-x^2\right)$ $\qquad x=\frac{1}{2}A$

$$=\frac{1}{2}m\omega^2A^2\left(\frac{3}{4}\right)$$

$$\frac{\text{K.E.}}{\frac{1}{2}m\omega^2A^2}=\frac{3}{4}$$

SECTION - B

Sol1. Binding Energy $=(\Delta m)c^2$

$$=\left[Zm_P+(A-Z)m_n-M_{Al}\right]c^2$$

$$=\left[(13.09438+14.12124)-27.18846\right]u$$

$$=\left[27.21562-27.1884\right]u$$

$$=0.02716x$$

$$=27.16x\times10^{-3}$$

Sol2. $W=\int_0^{10}F_y dy$

$$W=\int_0^{10}(5y+20)dy$$

$$W=\left[\frac{5y^2}{2}+20y\right]_0^{10}$$

$$W=450\,J$$

Sol3. $\tau=I\alpha$

$$=I\left[\frac{\omega_2-\omega_1}{\Delta t}\right]$$

$$=I\left[\frac{0-600\times\frac{2\pi}{60}}{10}\right]$$

$$=\frac{MR^2}{2}\left[\frac{0-2\pi\times10}{10}\right]$$

$$=\frac{10\times400\times10^{-4}}{2}(2\pi)$$

$$=4\pi\times10^{-1}\,Nm$$

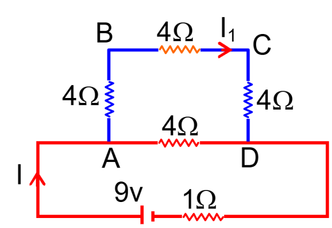
Sol4. $R_{eff}=4\Omega$

$$I=\frac{9}{4}A \text{ for}$$

$$I_1=I\times\frac{4}{16}=\frac{9}{16}A$$

$$V_{AC}=4\times\frac{9}{16}+4\times\frac{9}{16}$$

$$=\frac{9}{16}\times8=4.5V=45\times10^{-1}V$$



Sol5. Power dissipated is same

$$P_1=P_2$$

$$\frac{220\times220}{5}=220\times\frac{220}{z}\times\frac{5}{z}$$

$z^2=25,\ z=5$ (Resonance condition)

$z = R = 5$

$\omega = \dfrac{1}{\sqrt{LC}}$

$\omega = \dfrac{1}{\sqrt{0.1 \times 40 \times 10^{-6}}} = \dfrac{1}{2 \times 10^{-3}}$

$\omega = 500\,\text{rad/s}$

Sol6. $K = \dfrac{hc}{\lambda} - W_0$ ....(i)

$R = \dfrac{\sqrt{2mK}}{Be} \Rightarrow B = \dfrac{\sqrt{2mK}}{Re}$ ....(ii)

By (i) equation

$K = \dfrac{20 \times 10^{-26}}{500 \times 10^{-9}} - 1.25 \times 1.6 \times 10^{-19}$

$\Rightarrow 2 \times 10^{-19}\,J$

By equation (2)

$B = \dfrac{\sqrt{2 \times 9 \times 10^{-31} \times 2 \times 10^{-19}}}{30 \times 10^{-2} \times 1.6 \times 10^{-19}}$

$B = 125 \times 10^{-7}\,T$

Sol7. Modulation Index

$\mu = \dfrac{A_m}{A_c} = \dfrac{20}{20} = 1$

Sol8. Mean free path :

$\lambda \, \alpha \, \dfrac{1}{d^2}$

$\dfrac{\lambda_1}{\lambda_2} = \dfrac{d_2^2}{d_1^2} = \dfrac{25}{100} = 25 \times 10^{-2}$

Sol9. $N^2 = N_h \times N_e$

$\left(1.5 \times 10^{16}\right)^2 = \left(4.5 \times 10^{22}\right) \times N_e$

$N_e = \dfrac{1.5 \times 1.5 \times 10^{32}}{4.5 \times 10^{22}}$

$= \dfrac{10^{10}}{2} = 0.5 \times 10^{10}$

$= 5 \times 10^{9}$

Sol10. $A = \dfrac{A_0}{2^n}$

$A = \dfrac{A_0}{8} \Rightarrow n = 3$

$t = nT_{1/2}$

$30 = 3T_{1/2}$

$T_{1/2} = 10\text{ years}$

PART – B (CHEMISTRY)

SECTION - A

Sol 1. CFC breakdown by visible light to give Cl radical which react with stratospheric ozone.

$CFC,\ CF_2Cl_2 \xrightarrow{hv} \dot{C}l + \dot{C}F_2Cl$

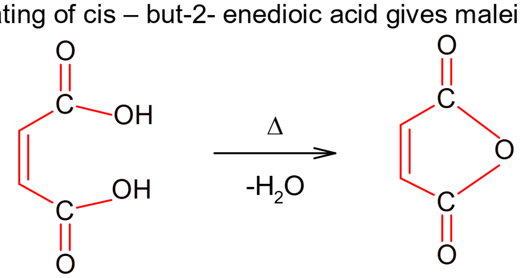
$\dot{C}l(g) + O_3 \longrightarrow \dot{C}lO + O_2$

$\dot{C}lO + O \longrightarrow \dot{C}l + O_2$

Atmospheric ozone reacts with NO to give NO_2 and O_2.

$O_3 + NO \longrightarrow NO_2 + O_2$

Sol 2. Heating of cis – but-2- enedioic acid gives maleic anhydride as shown.



Cis - but-2- enedioic acid Maleic anhydride

Sol 3. Anions have larger radii than atoms. Also, higher the $\frac{e}{p}$ ratio higher the ionic radii. So,

$r_{N^{-3}} > r_{O^{-2}} > r_{F^-}$

Sol 4. Using, $\mu = \sqrt{n(n+2)}\,B.M$, n = number of unpaired electrons.

${}_{22}Ti^{+3} = 4s^0\,3d^1$, unpaired electron = 1 $\quad \mu = 1.73\,B.M$

${}_{23}V^{+2} = 4s^0\,3d^3$, unpaired electron = 3 $\quad \mu = 3.87\,B.M$

${}_{21}Sc^{+3} = 4s^0\,3d^0$, unpaired electron = 0 $\quad \mu = 0\,B.M$

Sol 5. Chelation increases stability of complex. Also more the chelation more the stability. Here, $[Co(en)_2(NH_3)_2]Cl_2$, $[Co(en)_3]Cl_2$ and $[Co(en)(NH_3)_4]Cl_2$ have chelation due to ethylenediamine. Therefore $[Co(en)_3]Cl_2$ has highest chelation so highest stability.

Sol 6. Li_2CO_3 decomposes easily on heating as;

$Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2 \uparrow$

$NaHCO_3$ is used in dry fire extinguishers.

K is most abundant element in cell fluid.

CsI is least soluble due to smaller hydration energy of Cs^+ & I^-.

Sol 7. $O_2 = \sigma_{1s}^2\,\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2pz}^2\pi_{2px}^2 \equiv \pi_{2py}^2\pi_{2px}^{*1} \equiv \pi_{2py}^{*1}$

$O_2^- = \sigma_{1s}^2\,\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2pz}^2\pi_{2px}^2 \equiv \pi_{2py}^2\pi_{2px}^{*2} \equiv \pi_{2py}^{*1}$

$O_2^{2-} = \sigma_{1s}^2\,\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2pz}^2\pi_{2px}^2 \equiv \pi_{2py}^2\pi_{2px}^{*2} \equiv \pi_{2py}^{*2}$

$$O_2^{+} = \sigma_{1s}^2\,\sigma_{1s}^{*2}\sigma_{2s}^2\sigma_{2s}^{*2}\sigma_{2pz}^2\pi_{2px}^2 \equiv \pi_{2py}^2\pi_{2px}^{*1} \equiv \pi_{2py}^{*0}$$

$$B.O = \frac{\text{Bonding}e^{-} - \text{Antibonding}e^{-}}{2}$$

$$B.O \text{ of } O_2 = \frac{10-6}{2} = 2$$

$$B\cdot O \text{ of } O_2^{+} = \frac{10-5}{2} = 2.5$$

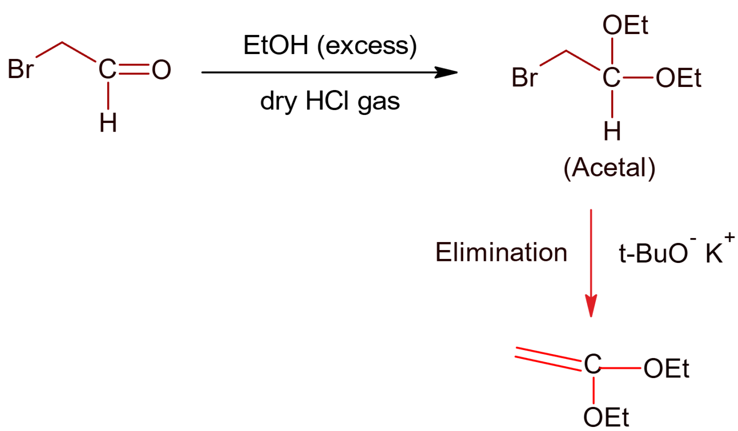
$$B\cdot O \text{ of } O_2^{-} = \frac{10-7}{2} = 1.5$$

$$B\cdot O \text{ of } O_2^{--} = \frac{10-8}{2} = 1$$

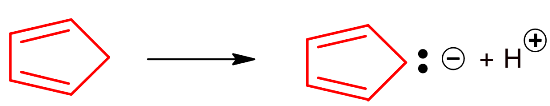
Sol 8.

	Phase	**Medium**	
(1) Cheese	Liquid	Solid	Gel
(2) Pumice stone	gas	Solid	Solid sol
(3) Hair cream	liquid	liquid	Emulsion
(4) Cloud	liquid	gas	Aerosol

Sol 9.



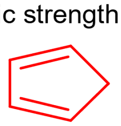
Sol 10.



(Aromatic)

Conjugate base is highly stable.

Acidic strength $\propto$ stability of conjugate base.

So,  is most acidic.

Sol 11. Metal of group 7, 8, & 9 dose not form interstitial hydride this is called hydride gap.

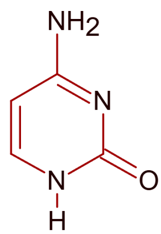
$Mn \rightarrow$ group -7

$Fe \rightarrow$ group -8

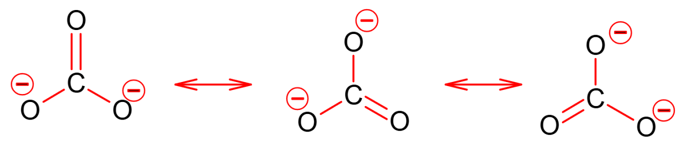
$Co \rightarrow$ group -9

So, Cr will forms interstitial hydride.

Sol 12. Cytosine is



Sol 13.

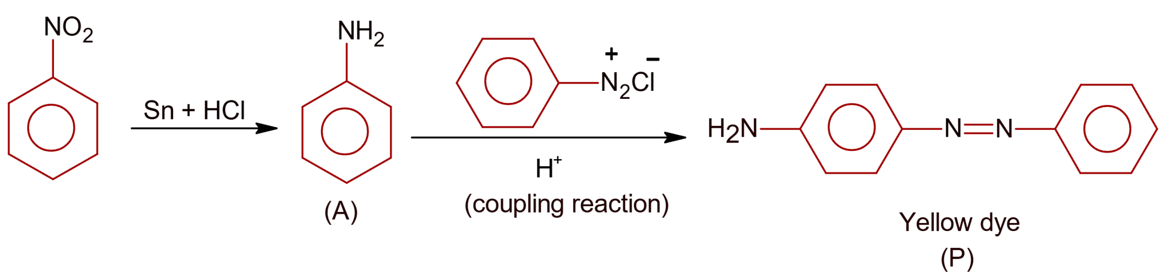


It has 1 π – bond & 3 canonical structure.

Sol14. Polyamide copolymers are biodegradable polymers.
PolymersNylon-2 Nylon-6 are polyamides and monomers of Nylon-2 nylon-6 are:
H_2N-CH_2-COOH (Glycine) & $H_2N-(CH_2)_5-COOH$ (Aminocaproic acid)

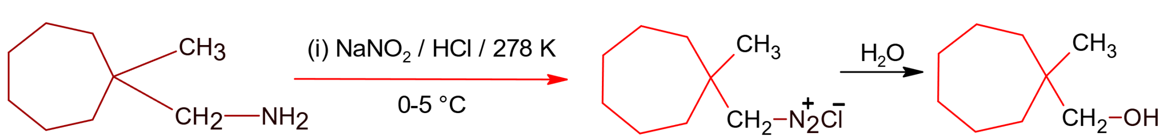
Sol15. (A) Concentration of Ag & Au done by NaCN. It is known as Mac – Arthur Process (cyanide process)
(B) Blast furnace produces pig Fe.
(C) Blister Cu $\rightarrow$ Reverberatory furnace. In this furnace smelting & roasting of ore take place
$CuFeS_2 + O_2 \rightarrow Cu_2O + FeO$
(D) Froth floatation process is used for concentration of sulphide ores.

Sol16.



Sol17. Density $\propto$ molar mass, as size of given molecules is nearly same.

Sol18.



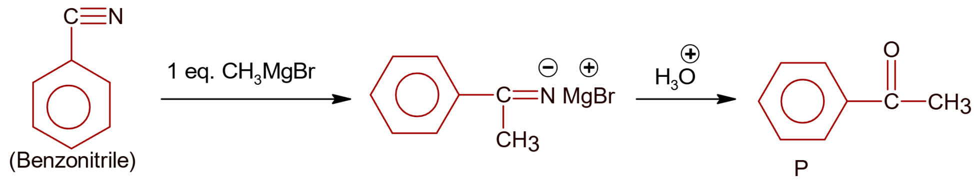
Sol19. $\overset{+6}{Cr}O_4^{-2} \longrightarrow Cr^{+3}$, Total oxidation number change = 3

$\overset{+7}{Mn}O_4^{-} \longrightarrow Mn^{+2}$, Total oxidation number change = 5

$\overset{+6}{Cr_2}O_7^{2-} \longrightarrow 2Cr^{+3}$, Total oxidation number change = 6

$\overset{+3}{C_2}O_4^{-2} \longrightarrow 2\overset{+4}{C}O_2$, Total oxidation number change = 2

Sol20.



P will give positive iodoform test.

SECTION - B

Sol1. Number of significant figure in 0.00340 are 3.

Sol2. $P_1 = 300\,kPa = 3 \times 10^5 Pa = 3\,bar$

$T_1 = 300\,k$

$P_2 = 1.2 \times 10^6 Pa = 12 \times 10^5 Pa = 12\,bar$

$T_2 = ?$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \frac{3}{300} = \frac{12}{T_2}$$

$$T_2 = \frac{12 \times 300}{3} K$$

$= 1200K$

$\therefore$ T in $^\circ C = 1200 - 273 = 927^\circ C$

Sol3. $$\Delta x . \Delta P = \frac{h}{4\pi}$$

$$\Delta x . \Delta v = \frac{h}{4\pi m}$$

$$\Delta v = 5 \times 10^6 \times \frac{0.02}{100} = 5 \times 2 \times 10^2 = 1000\,m/s$$

$$\therefore\ \Delta x = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 1000} m$$

$= 5.8 \times 10^{-8}\,m$

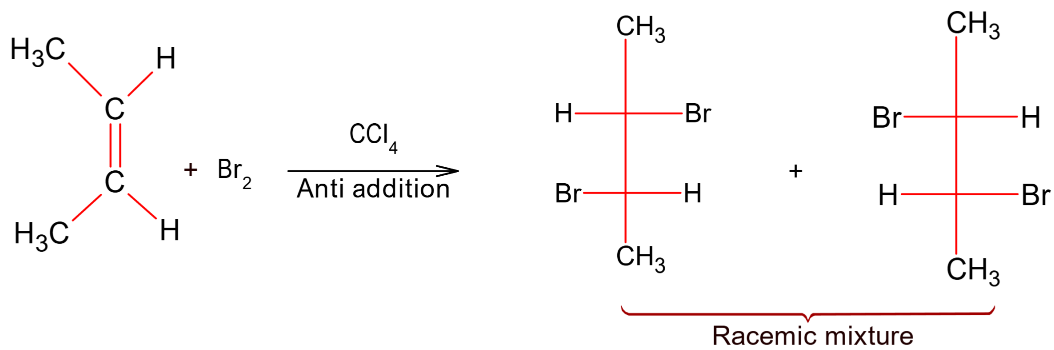
$= 58 \times 10^{-9} m.$

Sol4. $_{67}Ho = [Xe]4f^{11}6s^2$

$\therefore\quad Ho^{+3} = [Xe]4f^{10}6s^0$

Number of electrons in 4f- subshell = 10

Sol5.



Number of products = 2.

Sol6. $Ba(OH)_2 \rightleftharpoons Ba^{+2} + 2(OH)^-$

0.005M	0	0
0	0.005	0.010

$\left[OH^-\right] = 0.01M = 10^{-2}M$

$K_w = \left[H^+\right]\left[OH^-\right]$

$\left[H^+\right]or\left[H_3O^+\right] = \frac{K_w}{\left[OH^-\right]} = \frac{10^{-14}}{10^{-2}} = 10^{-12}M$

$\therefore \left[H_3O^+\right] = 1\times10^{-12}M$

Sol7. $A \longrightarrow B$

$t=0$	a	0
$t = 30\,min$	$(a-x)$	x

given $x = 0.2$ mole / lit

$$\text{Average rate} = +\frac{\Delta[B]}{\Delta t} = \frac{0.2\,mole/lit}{(30/60)h} = \frac{0.2}{0.5}mole\,lit^{-1}h^{-1}$$

$= 0.4\,mole\ lit^{-1}h^{-1}$

$= 4\times10^{-1}mole\ lit^{-1}h^{-1}$

Sol8. $\Delta T_b = 0.6K$

$$\Delta T_b = K_b \times m = K_b \times \frac{wt\cdot\times1000}{mol\,wt.\times W_{solvent}(gm)}$$

$$0.6 = 5\times\frac{3\times1000}{mol\,wt.\times100}$$

$$\therefore mol\,wt. = \frac{15\times10}{0.6} = \frac{15\times100}{6}$$

$= 5\times50 = 250\,g/mole$

Sol9. $q = +150\,J$

$w = -200\,J$

$\therefore\ \Delta U = q + w$

$= 150 - 200 = -50\,J$

So, magnitude of ΔU is 50J.

Sol10. $\%\text{ of N} = \dfrac{1.4 \times \text{Normality of acid} \times \text{volume of acid used}}{\text{mass of organic compound}}$

$42 = \dfrac{1.4 \times (1 \times 2) \times \text{volume of acid used}}{0.8}$

Volume of acid used $= \dfrac{42 \times 0.8}{1.4 \times 2}$ ml

$= \dfrac{21 \times 0.8}{1.4}$ ml $= \dfrac{3 \times 8}{2}$ ml = 12 ml

$= 12\,\text{ml}$

PART – C (MATHEMATICS)

SECTION - A

Sol1. $\sum_{n=8}^{100} \left[\dfrac{(-1)^n . n}{2}\right]$

$= \left[\dfrac{8}{2}\right] + \left[\dfrac{-9}{2}\right] + \left[\dfrac{10}{2}\right] + \left[\dfrac{-11}{2}\right] + ------- + \left[\dfrac{-99}{2}\right] + \left[\dfrac{100}{2}\right]$

$= 4 - 5 + 5 - 6 + 6 ------ + (-50) + 50$

$= 4$

Sol2. $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \qquad -\pi/4 \le x \le \pi/4$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$

$\sin x = \cos x$

$\tan x = 1 \Rightarrow x = \pi/4$

or

$\sin x = -2\cos x$

$\tan x = -2$

Not within given range.

Sol 3. $a\hat{i}+a\hat{j}+c\hat{k},\ \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co –planar,

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$c^2 = ab$$

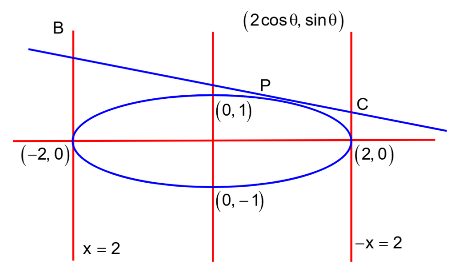
$$c = \sqrt{ab}$$

Sol4. Equation of tangent of P is

$(\cos\theta)x + (2\sin\theta)y = 2$ ……(i)

Solving equation of tangent with equation of

$\therefore x = -2$



$$-2\cos\theta + 2\sin\theta\ y = 2$$

$$y = \frac{1+\cos\theta}{\sin\theta}$$

Therefore point 'B' is

$$B\left(-2, \frac{1+\cos\theta}{\sin\theta}\right)$$

$$B\left(-2, \cot\frac{\theta}{2}\right)$$

For point 'C'

$x = 2$

$$y = \frac{1-\cos\theta}{\sin\theta}$$

$$c\left(2, \frac{1-\cos\theta}{\sin\theta}\right)$$

$$c\left(2, \tan \theta/2\right)$$

Now B C is the diameter of circle

$$\therefore (x+2)(x-2) + \left(y - \cot \theta/2\right)\left(y - \tan \theta/2\right) = 0$$

$\left(\sqrt{3}, 0\right)$ satisfying equation of circles

Sol5. $P = \begin{bmatrix} -1 & 0 \\ 1/2 & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Similarly

$$P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

Sol6. $\left(2^{1/3} + 3^{1/4}\right)^{12}$

$T_{r+1} = {}^{12}C_r \left(2^{1/3}\right)^{12-r} \left(3^{1/4}\right)^r$

$= {}^{12}C_r \; 2^{\frac{12-r}{3}} \; 3^{\frac{r}{4}}$

$r = 0, 12$ for term to be rational

$T_1 + T_{13} = 12_{C_0} \; 2^{12/3} . + {}^{12}C_{12} \; 3^3$

$= 2^4 + 3^3 = 43$

Sol7. $x^2 - 4xy - 5y^2 = 0$

Equation of pair of straight line $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$

$\dfrac{x^2 - y^2}{1 - (-5)} = \dfrac{xy}{-2}$

$\dfrac{x^2 - y^2}{6} = \dfrac{xy}{-2}$

$x^2 - y^2 = -3xy$

$x^2 + 3xy - y^2 = 0$

Sol8. $x^2 - |x| - 12 = 0$

$|x|^2 - 4|x| + 3|x| - 12 = 0$

$(|x| - 4)(|x| + 3) = 0$

$|x| = 4 \Rightarrow x = \pm 2, |x| = -3$ not possible

Sol9. Let $P = \left(1 + \dfrac{1}{10^{100}}\right)^{10^{100}}$

Let $n = 10^{100}$

$P = \left(1 + \dfrac{1}{n}\right)^n$

$P = n_{C_0} + n_{C_1}\left(\dfrac{1}{n}\right) + n_{C_2}\left(\dfrac{1}{n}\right)^2 + ------ + n_{C_n}\left(\dfrac{1}{n}\right)^n$

$P = 1 + 1 + \dfrac{n!}{2.!(n-2)!\, n^2} + \dfrac{n!}{3!(n-3)!\, n^3} + ------$

$\Rightarrow P = 2 + \dfrac{n(n-1)}{2!} . \dfrac{1}{n^2} + \dfrac{n(n-1)(n-2)}{3!} . \dfrac{1}{n^3} + ------$

$\Rightarrow P = 2 + \dfrac{n^2 - n}{2!(n^2)} + \dfrac{n^3 - 3n^2 + 2n}{n^3\, 3!} + -------$

$\Rightarrow P = 2 + \left[\dfrac{1}{2!} - \dfrac{1}{2!(n)}\right] + \left[\dfrac{1}{3!} - \dfrac{1}{2n} + \dfrac{1}{3n^2}\right] + ------$

$\dfrac{1}{n} \approx 0$

$$\Rightarrow P = 2 + \underbrace{\left(\frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} - - - - - - - -\right)}_{\text{positive value less than}}$$

$$\Rightarrow P = 1 + \frac{1}{1!} + \frac{1}{3!} + \frac{1}{4!} + - - - - -$$

$\Rightarrow P \approx e$

$\therefore$ Lowest integer greater than P is 3.

Sol10. $\left|\vec{a}\right| = 2,\ \left|\vec{b}\right| = 5\left|\vec{a}\times\vec{b}\right| = 8$

$\left|\vec{a}\,.\,\vec{b}\right| = \left|\left|\vec{a}\right|.\left|\vec{b}\right|.\cos\theta\right|$

$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|.\left|\vec{b}\right|.\sin\theta$

$8 = 2.5\ \sin\theta$

$\sin\theta = \frac{+8}{10}$

$\sin\theta = 4/5$

$\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|\cos\theta$

$= 2.5 \cdot \left(\frac{3}{5}\right)$

$= 6$

$\left|\vec{a}.\,\vec{b}\right| = 6$

Sol11. $\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta}$

$$\cot\frac{\pi}{24} = \frac{1+\cos\frac{\pi}{12}}{\sin\pi/12}$$

As, $\cos\theta = \sqrt{\frac{\cos 2\theta + 1}{2}}$

$$\cos\frac{\pi}{12} = \sqrt{\frac{\cos\pi/6 + 1}{2}}$$

$\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}$,

$\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\cot\left(\frac{\pi}{24}\right) = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$

Sol12. Mean = $\Sigma x_i p_i = \frac{1}{2}(0) + \sum_{r=0}^{\infty} r\frac{1}{3^r} = \frac{3}{4}$

$P(x \text{ is even})$

$$=\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}.....=\frac{\frac{1}{9}}{1-\frac{1}{9}}=\frac{1}{8}$$

Sol13. If $(gof)^{-1}$ exist

then $g(f(x))$ must be bijective.

$f(x)$ must one – one

$g(x)$ must onto.

Sol14. 1^{st} sample

$\Sigma x_i = 100\times 15 = 1500$

$\sigma_x^2 = \frac{\Sigma x_i^2}{100}-(15)^2 = 9$

$\Rightarrow \Sigma x_i{}^2 = 23400$

2^{nd} sample

$\Sigma y_i = 150(\overline{y})$

$\sigma_y^2 = \frac{\Sigma y_i^2}{150}-(\overline{y})^2$

Combined sample

$\sigma^2 = \frac{\Sigma x_i^2+\Sigma y_i^2}{250}-(15.6)^2 = 13.44$

$\Rightarrow \Sigma y_i^2 = 40,800$

$\overline{y} = 16$

$\sigma_y^2 = \frac{\Sigma y_i^2}{150}-(16)^2$

$\sigma_y = 4$

Sol15. $x\,dy = (y+x^2\cos x)dx$

$\frac{x.dy-y\,dx}{x^2}=\frac{x^3.\cos x\,dx}{x^2}$

$\int d\left(y/x\right)=\int x\cdot\cos x\ dx$

$y/x = x\cdot\sin x+\cos x+c$

$y = x^2.\sin x + x\cos x + c\,x$

$y(\pi)=\pi^2\sin\pi+\pi.\cos\pi+\pi.c$

$y(\pi)=0+(-\pi)+\pi c$

$0=\pi(-1+c)$

$c=1$

$y\left(\pi/2\right)=\left(\pi/2\right)^2.\sin\left(\pi/2\right)+\pi/2.\cos\pi/2+1.\pi/2$

$y\left(\pi/2\right)=\frac{\pi^2}{4}+\frac{\pi}{2}$

Sol16. $\int_{-1}^{1} \log\left(x+\sqrt{x^2+1}\right)dx$

$\log\left(x+\sqrt{x^2-1}\right)$ is a odd function

$\therefore \int_{-1}^{1} \log\left(x+\sqrt{x^2+1}\right)dx = 0$

Sol17. $T_{r+1} = {}^{10}C_{C_r}\,(x.\sin\alpha)^{(10-r)}\left(\dfrac{a.\cos\alpha}{x}\right)^r$

$= 10_{C_r}\, x^{10-r}\sin^{10-r}\alpha.a^r.\cos^r\alpha.x^{-r}$

$T_{r+1} = {}^{10}C_r x^{10-2r}.\sin^{10-r}\alpha.\cos^r.\alpha.a^r$

For Independent of x

$10-2r=0$

$r=5$

$T_6 = {}^{10}C_5 x^{o}.\sin^5\alpha.\cos^5\alpha.a^5$

$T_6 = {}^{10}C_5.\dfrac{1}{2^5}(\sin 2\alpha)^5.\,a^5$

Now term will be greatest when

$\sin 2\alpha = 1$

$= {}^{10}C_5.\dfrac{1}{2^5}.a^5 = \dfrac{10!}{(5!)^2}$

$\therefore a = 2$

Sol18. $f(x) = \begin{cases} \int_0^x (5+|1-t|)dt & x>2 \\ 5x+1 & x\le 2 \end{cases}$

Now

$|1-t| = \begin{cases} -(1-t) & , \quad t>1 \\ (1-t) & , \quad t\le 1 \end{cases}$

$\therefore \int_0^x (5+|1-t|)dt$

$= \int_0^1 (5+(1-t))dt + \int_1^x (5+t-1)\,dt$

$= 6 - \tfrac{1}{2} + \left[4t+\dfrac{t^2}{2}\right]_1^x$

$= \dfrac{11}{2}+4x+\dfrac{x^2}{2}-4-\dfrac{1}{2}$

$f(x) = \dfrac{x^2}{2}+4x+1 \qquad x>2$

$f(2^+)=11, \quad f(2^-)=11, f(2)=11$

$\therefore$ continuous at x = 2

$f'(x) = \begin{cases} x+4, & x>1 \\ 5, & x>2 \end{cases}$

$\left.\begin{aligned} f'(2^+) &= 6 \\ f'(2^-) &= 5 \end{aligned}\right\}$

Differentiable at x = 1
Not differentiable at x = 2

Sol19. ${}^{n}P_{r} = {}^{n}P_{r+1}$

$$\frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$(n-r) = 1$ ....(i)

${}^{n}C_{r} = {}^{n}C_{r-1} \Rightarrow 2r-1 = n$ ....(ii)

(i) & (ii) $\Rightarrow r = 2$

Sol20. P : weather is good
q : ground is not wet

$\sim(p \wedge q) = \sim p \vee \sim q$

$\equiv$ weather is not good or ground is wet.

SECTION - B

Sol1. $f(x) = \begin{cases} \dfrac{P(x)}{\sin(x-2)} & x \neq 2 \\ 7 & x = 2 \end{cases}$

$P''(x)$ = constant

$\Rightarrow P(x)$ is 2 degree polynomial

$f(x)$ is cont at x = 2

$$\therefore \lim_{x\to 2} \frac{P(x)}{\sin(x-2)} = 7$$

$$\lim_{x\to 2} \frac{(x-2)(ax+b)}{\sin(x-2)} = 7$$

Let $P(x) = (x-2)(ax+b)$

$\therefore 2a + b = 7$ ....(i)

$P(3) = (3a+b)$

$(3a+b) = 9$ ....(ii)

Solving (i) and (ii),

$a = 2, b = 3$

$P(x) = (x-2)(2x+3)$

$P(5) = 39$

Sol2. $a + b + c = 1$

$ab + bc + ca = 2$

abc = 3

$a^4 + b^4 + c^4 = (a^2+b^2+c^2)^2 - 2\Sigma(ab)^2$

$(a^2+b^2+c^2)^2 = \left((a+b+c)^2 - 2\Sigma ab\right)^2 = 9$

$a^4 + b^4 + c^4 = 9 - 2(-2) = 13$

Sol3. $T_{r+1} = {}^nC_r.(2)^{n-r}.\left(\frac{x}{3}\right)^r \Rightarrow$ Coefficient of $x^r = \frac{{}^nC_r\, 2^{n-r}}{3^r}$

Coefficient of x^7 = coefficient of x^8

$\Rightarrow \frac{{}^nC_7.2^{n-7}}{3^7} = \frac{{}^nC_8\, 2^{n-8}}{3^8}$

$\Rightarrow n-7=48$

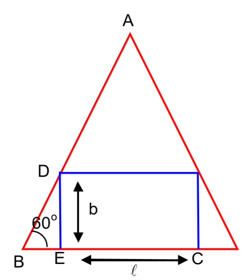
n = 55

Sol4. $\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$(k+1)(2-6)-4(1-9)+6(2-6)=0$

$k=1$

Sol5.



Area of rectangle $= DE \times EC$

In Δ DEB

$\tan 60^\circ = \frac{DE}{BE}$

$\because \quad BE = \frac{2\sqrt{2}-\ell}{2}$

$\sqrt{3} = \frac{2.DE}{2\sqrt{2}-\ell}$

$\frac{\sqrt{3}\left(2\sqrt{2}-\ell\right)}{2} = DE$

A = Area of rectangle = $\ell \times \frac{\sqrt{3}}{2}\left(2\sqrt{2}-\ell\right)$

$\frac{dA}{d\ell} = 0$ (for A to be largest)

$\Rightarrow \ell = \sqrt{2}\,,\ \frac{d^2A}{d\ell^2} = -\sqrt{3} < 0$

then, $A = \sqrt{2} \times \frac{\sqrt{3}}{2}\left(\sqrt{2}\right) = \sqrt{3}$

$A^2 = 3$

Sol6. $\left(\vec{a}+3\vec{b}\right) \perp \left(7\vec{a}-5\vec{b}\right)$

$\left(\vec{a}+3\vec{b}\right).\left(7\vec{a}-5\vec{b}\right)=0$

$7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\,\vec{a}.\vec{b} = 0$ ....(i)

$(\vec{a} - 4\vec{b}) \perp (7\vec{a} - 2\vec{b})$

$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\,\vec{a}.\vec{b} = 0$ ......(ii)

Solving (i) and (ii)

$|\vec{a}| = |\vec{b}|$

$\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|} \qquad = \dfrac{|\vec{a}|^2}{2|\vec{a}||\vec{a}|}$

$\cos\theta = 1/2$

$\theta = 60^o$

Sol7. $n_{C_0} + 3.{}^nC_1 + 5.n_{C_2} + 7.\,n_{C_3} + - - - - - - - + (2n+1)\,n_{C_n}$

$T_r = (2r+1)\,n_{C_r}$

$S = \sum_{r=0}^{n}(2r+1)n_{C_r}$

$= \sum_{r=0}^{n} 2.r.n_{C_r} + \sum_{r=0}^{n} n_{C_r}$

$= 2\sum_{r=0}^{n} r.\dfrac{n}{r}.{}^{n-1}C_{r-1} + \sum_{r=0}^{n} n_{C_r}$

$= 2n.(2^{n-1}) + 2^n$

$= 2^n(n+1)$

Now

$2^n(n+1) = 2^{100}.101$

$\Rightarrow n = 100$

$2\left[\dfrac{n-1}{2}\right] = 98$

Sol8. $P(H) = 1/2$

$1 - \left(\dfrac{1}{2}\right)^n \geq 0.9$

$\left(\dfrac{1}{2}\right)^n \leq \dfrac{1}{10}$

Minimum value of $n = 4$

Sol9. $\dfrac{dy}{dx} = \dfrac{2y}{x.\ell n x}$

$\dfrac{dy}{y} = \dfrac{2.dx}{x.\ell n x}$

$\ln y = 2.\ell n\,|\,\ell n\, x| + c$

At $x = 2,\ y = (\ell n 2)^2$ given

$\ln(\ell n\,2)^2 = 2\ln(\ell n\,2) + c$

$C = 0$

$\ln y = \ln(\ell n x)^2$

$f(x) = y = (\ell n\ x)^2$

$f(e) = 1$

Sol10. Equation of circle

$(x^2 - y^2) + 2y^2 + 2x = 0$

$x^2 + y^2 + 2x = 0$

Centre $= (-1, 0)$

Equation of parabola

$x^2 - 6x - y + 13 = 0$

$(x-3)^2 = (y-4)$

Vertex $= (3, 4)$

Equation of line passing through centre and vertex

$$\frac{(y-0)}{(x+1)} = \frac{4-0}{3+1}$$

$y = x + 1$

y – intercept = 1