Solutions to JEE(Main) -2021

Test Date: 27th July 2021 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Two identical tennis balls each having mass 'm' and charge 'q' are suspended from a fixed point by threads of length 'ℓ'. What is the equilibrium separation when each thread makes a small angle 'θ' with the vertical ?

(A) $x = \left(\frac{q^2 \ell}{2\pi\varepsilon_0 mg}\right)^{1/2}$ (B) $x = \left(\frac{q^2 \ell}{2\pi\varepsilon_o mg}\right)^{1/3}$

(C) $x = \left(\frac{q^2 \ell^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{1/3}$ (D) $x = \left(\frac{q^2 \ell^2}{2\pi\varepsilon_0 m^2 g}\right)^{1/3}$

Q2.

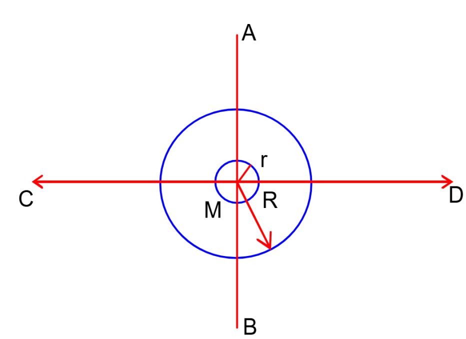
	List – I		**List – II**
(a)	MI of the rod (length L, Mass M, about an axis $\perp$ to the rod Passing through the midpoint)	(i)	$8\,ML^2/3$
(b)	MI of the rod (length L, Mass 2M, about an axis $\perp$ to the rod Passing through one of its end)	(ii)	$ML^2/12$
(c)	MI of the rod (length 2L, Mass M, about an axis $\perp$ to the rod Passing through its midpoint)	(iii)	$ML^2/12$
(d)	MI of the rod (length 2L, Mass 2M, about an axis $\perp$ to the rod Passing through one of its end)	(iv)	$2\,ML^2/3$

Choose the correct answer from the options given below :

(A) (a) – (ii), (b) – (i), (c) – (iii), (d) – (iv) (B) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)

(C) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i) (D) (a) – (ii), (b) – (iii), (c) – (i), (d) – (iv)

Q3. The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (which is $\perp$ to the plane of the disc and passing through its centre) to MI of smaller disc around one of its diameters lying on its plane? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia)



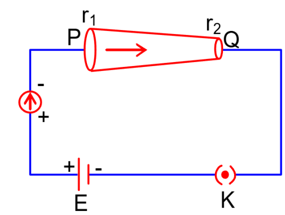
(A) $R^2 : r^2$

(B) $2r^4 : R^4$

(C) $2R^4 : r^4$

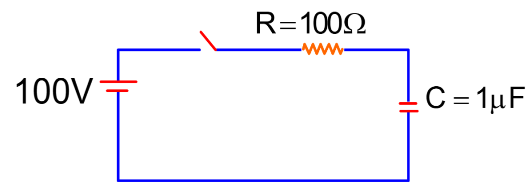
(D) $2R^2 : r^2$

Q4. In the given figure, a battery of emf E is connected across a conductor PQ of length 'ℓ' and different area of cross- sections by having radii r_1 and r_2 $(r_2 < r_1)$.



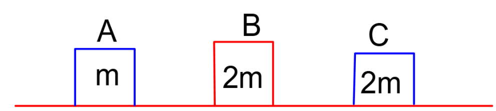
Choose the correct option as one moves from P to Q :
(A) All of these
(B) Electron current decreases.
(C) Drift velocity of electron increases.
(D) Electric field decreases.

Q5. A capacitor of capacitance $C = 1\mu F$ is suddenly connected to a battery of 100 volt through a resistance $R = 100\Omega$. The time taken for the capacitor to be charged to get $50V$ is :
[Take $\ln 2 = 0.69$]



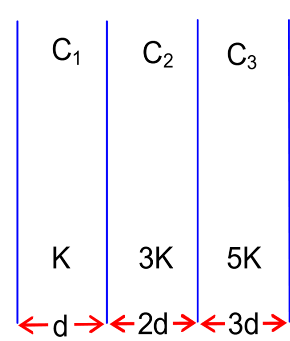
(A) 3.33×10^{-4} s
(B) 1.44×10^{-4} s
(C) 0.30×10^{-4} s
(D) 0.69×10^{-4} s

Q6. Three objects A, B and c are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, 2 and 2 m respectively. A moves towards B with a speed of $9 m/s$ and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. all motions occur along same straight line. The final speed of C is :



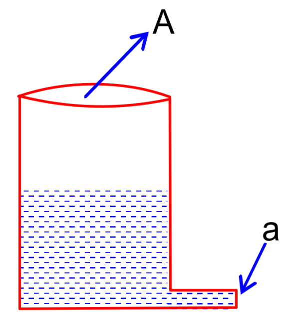
(A) 4 m /s
(B) 6 m /s
(C) 9 m /s
(D) 3 m /s

Q7. In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be : (Given area of plate = A)



(A) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$
(B) $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$
(C) $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$
(D) $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

Q8. A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a << A):



(A) None of these
(B) $\frac{2a}{A}$
(C) $\frac{A}{2a}$
(D) $\frac{a}{A}$

Q9. The relative permittivity of distilled water is 81. The velocity of light in it will be : (Given $\mu_r = 1$)

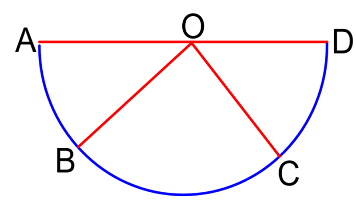
(A) 4.33×10^7 m/s
(B) 2.33×10^7 m/s
(C) 5.33×10^7 m/s
(D) 3.33×10^7 m/s

Q10. **Assertion A :** If A, B, D are four points on a semi-circular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$$

Reason R : Polygon law of vector addition yields

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$$

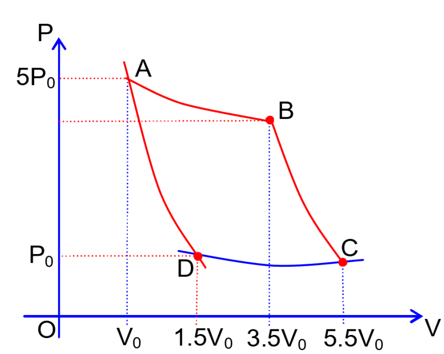


In the light of the above statements, choose the most appropriate answer from the options given below :
(A) **A** is not correct but **R** is correct.
(B) **A** is correct but **R** is not correct.
(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**.
(D) Both **A** and **R** are correct but **R** is not the correct explanation of A.

Q11. The number of molecules in one litre of an ideal gas at 300 K and 2 atmospheric pressure with mean kinetic energy 2×10^{-9} J per molecule is :

(A) 0.75×10^{11}
(B) 3×10^{11}
(C) 6×10^{11}
(D) 1.5×10^{11}

Q12. In the reported figure, there is a cyclic process ABCDA on a sample of 1mol of a diatomic gas. The temperature of the gas during the process $A \to B$ and $C \to D$ are T_1 and T_2 $(T_1 > T_2)$ respectively.
Choose the correct option out of the following for work done if processes BC and DA are adiabatic.



(A) $W_{AB} = W_{DC}$
(B) $W_{AD} = W_{BC}$
(C) $W_{AB} < W_{CD}$
(D) $W_{BC} + W_{DA} > 0$

Q13. If 'f' denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at $t = 0\ (N_0)$ then for a collection of radioactive nuclei, the rate of change of 'f' with respect to time is given as : [λ is the radioactive decay constant]

(A) $\lambda e - \lambda t$ (B) $\lambda\left(1-e^{-\lambda t}\right)$

(C) $-\lambda\left(1-e^{-\lambda t}\right)$ (D) $-\lambda e^{-\lambda t}$

Q14. **Assertion A :** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R : $\text{Least Count} = \dfrac{\text{Pitch}}{\text{Total divisions on circular scle}}$

In the light of the above statement, choose the most appropriate answer from the options given below :

(A) **A** is correct but **R** is not correct.
(B) **A** is not correct but **R** is correct.
(C) Both **A** and **R** are correct and **R** is NOT the correct explanation of **A** .
(D) Both **A** and **R** are correct and **R** is the correct explanation of **A .**

Q15. Two capacitors of capacities 2C and C are joined in parallel and charge up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The potential difference across the capacitors will now be :

(A) $\dfrac{3V}{K}$ (B) $\dfrac{V}{K}$

(C) $\dfrac{V}{K+2}$ (D) $\dfrac{3V}{K+2}$

Q16. In Young's double slit experiment, if the source of light changes from orange to blue then:

(A) the distance between consecutive fringes will decrease.
(B) the distance between consecutive fringes will increase.
(C) the central bright fringe will become a dark fringe.
(D) the intensity of the minima will increase.

Q17. A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\dfrac{h}{3}$ in both the directions.

(A) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (B) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(C) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ (D) $\dfrac{1}{3}$

Q18. A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\dfrac{3E}{4}$ then its displacement 'y' is given by :

(A) $y = \dfrac{a}{2}$ (B) $y = \dfrac{a}{\sqrt{2}}$

(C) $y = \dfrac{a\sqrt{3}}{2}$ (D) y = a

Q19. A 0.07 H inductor and a 12Ω resistor are connected in series to a 220 V, 5 Hz ac source. The approximate current in the circuit and the phase angle between current and source voltage are respectively. [Take π as $\frac{22}{7}$]

(A) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(B) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(C) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(D) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

Q20. A body takes 4 min. to cool from $61^\circ C$ to $59^\circ C$. If the temperature of the surroundings is $30^\circ C$, the time taken by the body to cool from $51^\circ C$ to $49^\circ C$ is :

(A) 6 min.

(B) 3 min.

(C) 4 min.

(D) 8 min.

SECTION - B

(Numerical Answer Type)

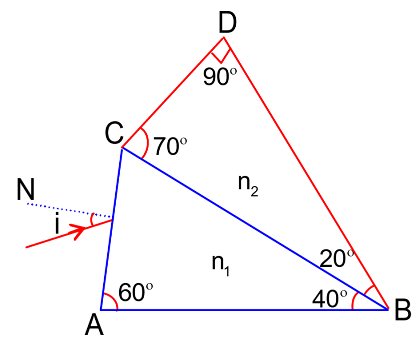
This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. A transistor is connected in common emitter circuit configuration, then collector supply voltage is 10 V and the voltage drop across a resistor of 1000Ω in the collector is 0.6 V. If the current gain factor (β) is 24, then the base current is ________ μA. (Round off to the Nearest Integer)

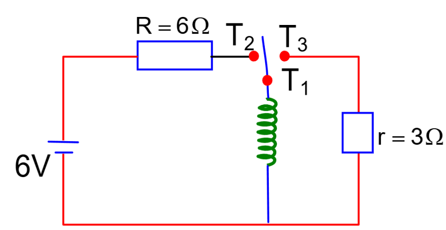
Q2. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ, the wavelength of light, according to the relation

$$n_1 = 1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2} \text{ and } n_2 = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}$$

The wavelength for which rays incident at any angle on the interface BC pass through without bending at that interface will be ________nm.



Q3. Consider an electrical circuit containing a two way switch 'S'. Initially S is open and then T_1 is connected to T_2. As the current in $R = 6\Omega$ attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3. Potential drop across $r = 3\Omega$ resistor immediately after T_1 is connected to T_3 is __________V. (Round off to the Nearest Integer)



Q4. A stone of mass 20g is projected from a rubber catapult of length 0.1m and area of cross section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is ________ m /s. (Young's modulus of rubber $= 0.5 \times 10^9 \, N/m^2$)

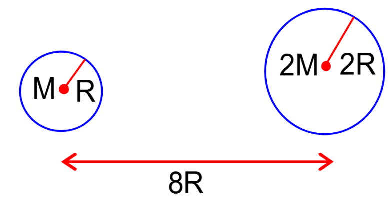
Q5. A particle of mass 9.1×10^{-31} kg travels in a medium with a speed of 10^6 m/s and a photon of a radiation of linear momentum 10^{-27} kg m/s travels in vacuum. The wavelength of photon is ________times the wavelength of the particle.

Q6. The amplitude of upper and lower side bands of A.M. wave where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulate by a 7.7 kHz sine wave of 5 V amplitude are $\frac{a}{10}$ V and $\frac{b}{10}$ V respectively. Then the value of $\frac{a}{b}$ is ____________.

Q7. In a uniform magnetic field, the magnetic needle has a magnetic moment $9.85 \times 10^{-2} \, A/m^2$ and moment of inertia 5×10^{-6} kg m^2. If it performs 10 complete oscillations in 5 seconds then the magnitude of the magnetic field is ____________mT. [Take π^2 as 9.85]

Q8. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'υ' required for the satellite to reach the surface of the second plant is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is ________.

[Given : The two planets are fixed in their position]



Q9. In Bohr's atomic model, the electron is assumed to revolve in a circular orbit of radius 0.5 $\overset{o}{A}$. If the speed of electron is $2.2\times 10^{6}\ m/s$, then the current associated with the electron will be __________ $\times 10^{-2}$ mA. [Take π as $\frac{22}{7}$]

Q10. A radioactive sample has an average life of 30ms and is decaying. A capacitor of capacitance $200\ \mu F$ is first charged and later connected with resistor 'R'. If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time then the value of 'R' should be ____________ Ω.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below are two statements:
Statement I: Rutherford's gold foil experiment cannot explain the line spectrum of hydrogen atom.
Statement II: Bohr's model of hydrogen atom contradicts Heisenberg's uncertainty principle.
In the light of the above statements, choose the most appropriate answer from the options given below:
(A) **Statement I** is true but **Statement II** is false.
(B) Both **Statement I** and **Statement II** are false.
(C) Both **Statement I** and **Statement II** are true.
(D) **Statement I** is false but **Statement II** is true.

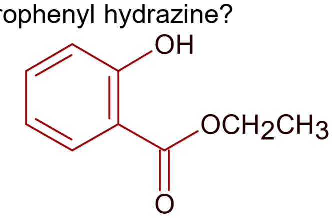
Q2. For a reaction of order n, the unit of the rate constant is:
(A) $mol^{1-n}\ L^{n-1}\ s^{-1}$
(B) $mol^{1-n}\ L^{1-n}\ s^{-1}$
(C) $mol^{1-n}\ L^{2n}\ s^{-1}$
(D) $mol^{1-n}\ L^{1-n}\ s$

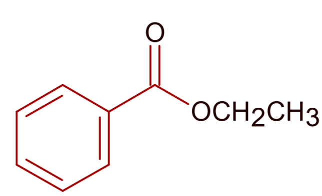
Q3. Which one among the following chemical tests is used to distinguish monosaccharide from disaccharide?
(A) Tollen's test
(B) Seliwanoff's test
(C) Iodine test
(D) Barfoed test

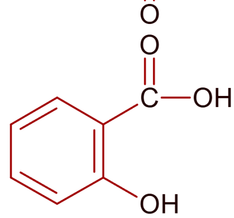
Q4. Staggered and eclipsed conformers of ethane are:
(A) Mirror images
(B) Polymers
(C) Enantiomers
(D) Rotamers

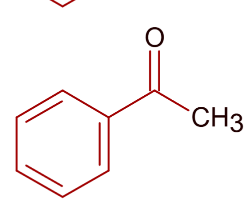
Q5. The number of geometrical isomers found in the metal complexes $\left[PtCl_2(NH_3)_2\right]$, $\left[Ni(CO)_4\right]$, $\left[Ru(H_2O)_3Cl_3\right]$ and $\left[CoCl_2(NH_3)_4\right]^+$ respectively are
(A) 2,1,2,2
(B) 2,0,2,2
(C) 1,1,1,1
(D) 2,1,2,1

Q6. Which one of the following compounds will give orange precipitate when treated with 2,4-Dinitrophenyl hydrazine?

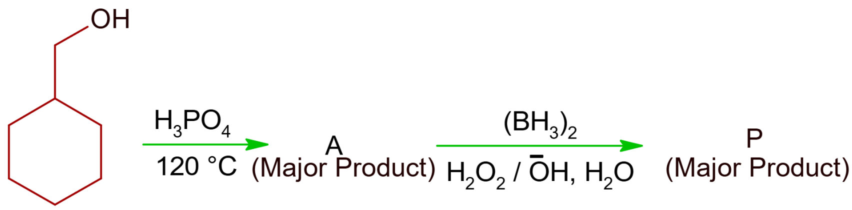
(A) 

(B) 

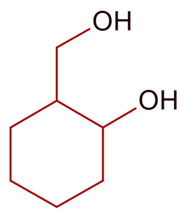
(C) 

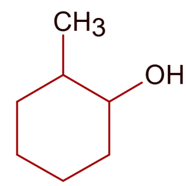
(D) 

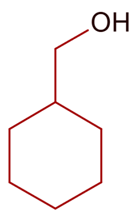
Q7.

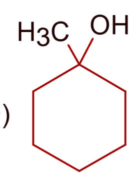


Consider the above reaction and identify the product P:

(A) 

(B) 

(C) 

(D) 

Q8. Match List- I with List-II:

List – I (Drug)		**List – II (Class of Drug)**
(a) Furacin	(i)	Antibiotic
(b) Arsphenamine	(ii)	Tranquilizers
(c) Dimetone	(iii)	Antiseptic
(d) Valium	(iv)	Synthetic antihistamines

Choose the most **appropriate** match:

(A) (a) – (ii), (b) –(i), (c) –(iii), (d) –(iv)
(B) (a) –(i), (b) –(iii), (c) –(iv), (d) –(ii)
(C) (a) –(iii), (b) – (i), (c) –(iv), (d) –(ii)
(D) (a) –(iii), (b) –(iv), (c) –(ii), (d) –(i)

Q9. The parameters of the unit cell of substance are a = 2.5, b= 3.0, c=4.0, $\alpha = 90^\circ$, $\beta = 120^\circ$, $\gamma = 90^\circ$. The crystal system of the substance is:

(A) Monoclinic
(B) Triclinic
(C) Orthorhombic
(D) Hexagonal

Q10. The oxidation states of 'P' in $H_4P_2O_7$, $H_4P_2O_5$ and $H_4P_2O_6$, respectively are:

(A) 5,3 and 4
(B) 5,4 and 3
(C) 6,4 and 5
(D) 7,5 and 6

Q11. The type of hybridisation and magnetic property of the complex $[MnCl_6]^{3-}$, respectively, are:

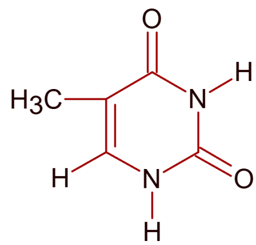
(A) sp^3d^2 and paramagnetic
(B) d^2sp^3 and paramagnetic
(C) d^2sp^3 and diamagnetic
(D) sp^3d^2 and diamagnetic

Q12. Presence of which reagent will affect the reversibility of the following reaction, and change it to a irreversible reaction:

$$CH_4 + I_2 \underset{\text{Reversible}}{\overset{h\nu}{\rightleftharpoons}} CH_3I + HI$$

(A) HOCl
(B) Liquid NH_3
(C) dilute HNO_2
(D) Concentrated HIO_3

Q13.



The compound 'A' is a complementary base of ______________ in DNA strands.

(A) Adenine
(B) Cytosine
(C) Guanine
(D) Uracil

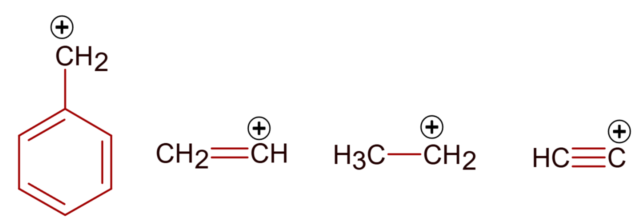
Q14. Match List- I with List-II:

List – I	**List – II**
(a) NaOH	(i) Acidic
(b) $Be(OH)_2$	(ii) Basic
(c) $Ca(OH)_2$	(iii) Amphoteric
(d) $B(OH)_3$	
(e) $Al(OH)_3$	

Choose the **most appropriate** answer from the options given below:

(A) (a) – (ii), (b) – (i), (c) –(ii), (d) –(iii), (e) – (iii)
(B) (a) – (ii), (b) – (iii), (c) –(ii), (d) –(i), (e) – (iii)
(C) (a) – (ii), (b) – (ii), (c) –(iii), (d) –(ii), (e)– (iii)
(D) (a) – (ii), (b) – (ii), (c) –(iii), (d) –(i), (e) – (iii)

Q15.



A B C D

The correct order of stability of given carbocations is:

(A) A > C > B > D
(B) D > B > C > A
(C) C > A > D > B
(D) D > B > A > C

Q16. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion R: Lithium halides are some what covalent is nature.

Reason R: Lithium possess high polarisation capability.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(A) **A** is true but **R** is false
(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(C) **A** is false but **R** is true
(D) Both **A** and **R** are true and **R** is the correct explanation of **A**

Q17. Which one of the following statements is **NOT** correct?

(A) Eutrophication indicates that water body is polluted
(B) Eutrophication leads to increase in the oxygen level in water
(C) Eutrophication leads to anaerobic conditions
(D) The dissolved oxygen concentration below 6 ppm inhibits fish growth

Q18. The product obtained from the electrolytic oxidation of acidified sulphate solutions, is:

(A) HO_3SOSO_3H (B) HSO_4^-
(C) HO_2SOSO_2H (D) HO_3SOOSO_3H

Q19. The statement that is INCORRECT about Ellingham diagram is:
(A) Provides idea about the reaction rate.
(B) provides idea about changes in the phases during the reaction
(C) provides idea about free energy change.
(D) provides idea about reduction of metal oxide.

Q20. Given below are two statements:
Statement I: Aniline is less basic than acetamide.
Statement II: In aniline, the lone pair of electrons on nitrogen atom is delocalised over benzene ring due to resonance and hence less available to a proton.
Choose the most appropriate option:
(A) **Statement I** is true but **statement II** is false.
(B) **Statement I** is false but **Statement II** is true
(C) Both **Statement I** and **Statement II** are true.
(D) Both **Statement I** and **Statement II** are false.

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is 2.0×10^{-5} S cm^{-1}. If $\wedge_m^o(HA) = 190 S\ cm^2 mol^{-1}$, the ionization constant (K_a) of HA is equal to _______ $\times10^{-6}$ (Round off to the nearest integer)

Q2. The number of geometrical isomers possible in triamminetrinitrocobalt(III) is X and in trioxalatochromate(III) is Y. Then the value of X + Y is____________.

Q3. In gaseous triethyl amine the "$-C-N-C-$" bond angle is __________ degree.

Q4. An organic compound is subjected to chlorination to get compound A using 5.0 g of chlorine. When 0.5 g of compound A is reacted with $AgNO_3$ [Carius Method], the percentage of chlorine in compound A is _________ when it forms 0.3849 g of AgCl
(Round off to the Nearest integer)
(Atomic masses of Ag and Cl are 107.87 and 35.5 respectively)

Q5. 1.46 g of a biopolymer dissolved in a 100 mL water at 300 K exerted an osmotic pressure of 2.42×10^{-3} bar.
The molar mass of the biopolymer is _________________ $\times10^4$ g mol^{-1}.
(Round off to the Nearest integer)
[Use : R = 0.083 L bar mol^{-1} K^{-1}]

Q6. CO_2 gas adsorbs on charcoal following Freundlich adsorption isotherm. For a given amount of charcoal, the mass of CO_2 adsorbed becomes 64 times when the pressure of CO_2 is doubled. The value of n in the Freundlich isotherm equation is ______ $\times10^{-2}$.
(Round off to the Nearest integer)

Q7. $PCl_5 \rightleftharpoons PCl_3 + Cl_2$ $K_C = 1.844$
3.0 moles of PCl_5 is introduced in a 1L closed reaction vessel at 380K. The number of moles of PCl_5 at equilibrium is________ $\times10^{-3}$. (Round off to the Nearest integer)

Q8. The difference between bond orders of CO and $NO^{\oplus}$ is $\frac{x}{2}$ where x_________
(Round off to the Nearest integer)

Q9. The density of NaOH solution is 1.2 gm cm^{-3}. The molality of this solution is ________m.
(Round off to the Nearest integer)
[Use : Atomic masses : Na 23.0u O: 16.0 u H:1.0 u Density of H_2O: 1.0g cm^{-3}]

Q10. For water at 100°C and 1 bar
$\Delta_{vap}H - \Delta_{vap}U =$ ______ $\times10^2$ J mol^{-1}
(Round off to the Nearest integer)
[Use: R = 8.31 J mol^{-1} K^{-1}]
[Assume volume of $H_2O(l)$ is much smaller than volume of $H_2O(g)$. Assume $H_2O(g)$ can be treated as an ideal gas]

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let C be the set of all complex numbers. Let

$S_1 = \{z \in C \mid |z-3-2i|^2 = 8\}$,

$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \mid |z-\bar{z}| \geq 8\}$.

Then the number of element in $S_1 \cap S_2 \cap S_3$ is equal to:

(A) 0
(B) 2
(C) 1
(D) infinite

Q2. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta)+\cos(4\theta)+\sin(6\theta))$ is equal to :

(A) 27
(B) −23
(C) −27
(D) 23

Q3. Let $\vec{a} = \hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = -\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product

$(\vec{a}+\vec{b})\times\left(\left(\vec{a}\times\left((\vec{a}-\vec{b})\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to :

(A) $5(34\hat{i}-5\hat{j}+3\hat{k})$
(B) $5(30\hat{i}-5\hat{j}+7\hat{k})$
(C) $7(30\hat{i}-5\hat{j}+7\hat{k})$
(D) $7(34\hat{i}-5\hat{j}+3\hat{k})$

Q4. Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$ be $ax+by+cz+8=0$. Then the value of $a+b+c$ is equal to :

(A) 4
(B) 8
(C) 5
(D) 3

Q5. Let $f:\left(-\frac{\pi}{4},\frac{\pi}{4}\right)\rightarrow R$ be defined as

$$f(x) = \begin{cases} (1+|\sin x|)^{\frac{3a}{|\sin x|}} & , -\frac{\pi}{4} < x < 0 \\ b & , x = 0 \\ e^{\cot 4x/\cot 2x} & , 0 < x < \frac{\pi}{4} \end{cases}$$

If f is continuous at x = 0, then the value of $6a+b^2$ is equal to :

(A) 1 – e
(B) e
(C) 1 + e
(D) e – 1

Q6. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to :

(A) $P \vee Q$
(B) $P \wedge \sim Q$
(C) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$
(D) $\sim (P \Rightarrow Q)$

Q7. Let

$A = \{(x, y) \in R \times R \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$,

$B = \{(x, y) \in R \times R \mid 4x^2 + 4y^2 - 16y + 7 = 0\}$ and

$C = \{(x, y) \in R \times R \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to :

(A) $1 + \sqrt{5}$
(B) $\frac{3 + 2\sqrt{5}}{2}$
(C) $\frac{3 + \sqrt{10}}{2}$
(D) $\frac{2 + \sqrt{10}}{2}$

Q8. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A, \alpha, \beta \in R$. I is a 2 x 2 identify matrix, then $4(\alpha - \beta)$ is equal to :

(A) 2
(B) 5
(C) 4
(D) $\frac{8}{3}$

Q9. Let $y = y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with $y(0) = 0$.

If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to :

(A) $\frac{1}{4}$
(B) $-\frac{1}{4}$
(C) 2
(D) $-\frac{1}{2}$

Q10. A ray of light through (2, 1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of any ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be :

(A) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$
(B) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$
(C) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$
(D) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$

Q11. The value of $\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} \frac{(2j - 1) + 8n}{(2j - 1) + 4n}$ is equal to :

(A) $2 - \log_e\left(\frac{2}{3}\right)$
(B) $1 + 2\log_e\left(\frac{3}{2}\right)$
(C) $5 + \log_e\left(\frac{3}{2}\right)$
(D) $3 + 2\log_e\left(\frac{2}{3}\right)$

Q12. If the area of the bounded region $R=\left\{(x, y): \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\right\}$ is, $\alpha(\log_e 2)^{-1}+\beta(\log_e 2)+\gamma$, then the value of $(\alpha+\beta-2\gamma)^2$ is equal to :

(A) 4 (B) 8
(C) 2 (D) 1

Q13. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b \ne 0$ are equal, then the value of b is equal to :

(A) -1 (B) 1
(C) 2 (D) -2

Q14. Two tangents are drawn from the point $P(-1, 1)$ to the circle $x^2+y^2-2x-6y+6=0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to :

(A) $3(\sqrt{2}-1)$ (B) $(3\sqrt{2}+2)$
(C) 4 (D) 2

Q15. The probability that a randomly selected 2-digit number belongs to the set $\{n \in N : (2^n-2) \text{ is a multiple of } 3\}$ is equal to :

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$
(C) $\frac{1}{6}$ (D) $\frac{1}{3}$

Q16. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set $\{P, Q\}$ is equal to :

(A) $\{(-1, 5), (5, 1)\}$ (B) $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})\}$
(C) $\{(2+2\sqrt{2}, 3-\sqrt{5}), (2-2\sqrt{2}, 3+\sqrt{5})\}$ (D) $\{(4, 0), (0, 6)\}$

Q17. The value of the definite integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$ is equal to :

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{2\sqrt{2}}$
(C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{\sqrt{2}}$

Q18. Let $f: R \to R$ be a function such that $f(2)=4$ and $f'(2)=1$. Then the value of $\lim_{x\to 2}\frac{x^2 f(2)-4f(x)}{x-2}$ is equal to :

(A) 12 (B) 4
(C) 16 (D) 8

Q19. Let α, β be two roots of the equation $x^2 + (20)^{\frac{1}{4}} x + (5)^{\frac{1}{2}} = 0$ Then $\alpha^8 + \beta^8$ is equal to :

(A) 50 (B) 100
(C) 10 (D) 160

Q20. If the mean and variance of the following data :

6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to :

(A) 32 (B) 24
(C) 12 (D) 16

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right]$ is the solution of the differential equation

$\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$, with $y(0) = 0$, then $5y'\left(\frac{\pi}{2}\right)$ is equal to________.

Q2. For real numbers α and β, consider the following system of linear equations:
$x + y - z = 2,\ x + 2y + \alpha z = 1,\ 2x - y + z = \beta$.
If the system has infinite solutions, then $\alpha + \beta$ is equal to________.

Q3. Let a plane P pass through the point $(3, 7, -7)$ and contain the line, $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to________.

Q4. Let $F : [3, 5] \to R$ be a twice differentiable function on $(3, 5)$ such that $F(x) = e^{-x}\int_3^x \left(3t^2 + 2t + 4F'(t)\right)dt$.

If $F'(4) = \frac{\alpha e^{\beta} - 224}{\left(e^{\beta} - 4\right)^2}$, then $\alpha + \beta$ is equal to________.

Q5. Let $f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}, x \in [0, \pi]$.

Then the maximum value of $f(x)$ is equal to________.

Q6. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is ℓ, then the value of $3\ell^2$ is equal to________.

Q7. Let $f : [0, 3] \to R$ be defined by
$f(x) = \min\{x - [x], 1 + [x] - x\}$
Where [x] is the greatest integer less than or equal to x.
Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to________.

Q8. Let $S = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of possible function $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to________.

Q9. Let the domain of the function

$f(x)=\log_4\left(\log_5\left(\log_3\left(18x-x^2-77\right)\right)\right)$ be (a,b).

Then the value of the integral $\int_a^b \frac{\sin^3 x}{\left(\sin^3 x+\sin^3(a+b-x)\right)}dx$ is equal to________.

Q10. If $\log_3 2, \log_3\left(2^x-5\right), \log_3\left(2^x-\frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to________.

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**B**	2.	**C**	3.	**C**	4.	**C**
5.	**D**	6.	**D**	7.	**A**	8.	**B**
9.	**D**	10.	**D**	11.	**D**	12.	**B**
13.	**A**	14.	**B**	15.	**D**	16.	**A**
17.	**B**	18.	**A**	19.	**B**	20.	**A**

SECTION - B

1.	**25**	2.	**600**	3.	**3**	4.	**20**
5.	**910**	6.	**1**	7.	**8**	8.	**4**
9.	112	10.	**150**				

PART – B (CHEMISTRY)

SECTION - A

1.	**C**	2.	**A**	3.	**D**	4.	**D**
5.	**B**	6.	**D**	7.	**B**	8.	**C**
9.	**A**	10.	**A**	11.	**A**	12.	**D**
13.	**A**	14.	**B**	15.	**A**	16.	**D**
17.	**B**	18.	**D**	19.	**A**	20.	**B**

SECTION - B

1.	**12**	2.	**2**	3.	**108**	4.	**19**
5.	**15**	6.	**17**	7.	**1396**	8.	**0**
9.	**5**	10.	**31**				

PART – C (MATHEMATICS)

SECTION - A

1.	**C**	2.	**B**	3.	**D**	4.	**A**
5.	**C**	6.	**C**	7.	**B**	8.	**C**
9.	**B**	10.	**C**	11.	**B**	12.	**C**
13.	**B**	14.	**C**	15.	**B**	16.	**A**
17.	**B**	18.	**A**	19.	**A**	20.	**D**

SECTION – B

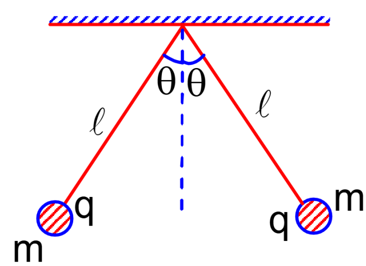
1.	**2**	2.	**5**	3.	**3**	4.	**16**
5.	**6**	6.	**2**	7.	**5**	8.	**490**
9.	**1**	10.	**3**				

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. $T\sin\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2\ell\sin\theta)^2}$



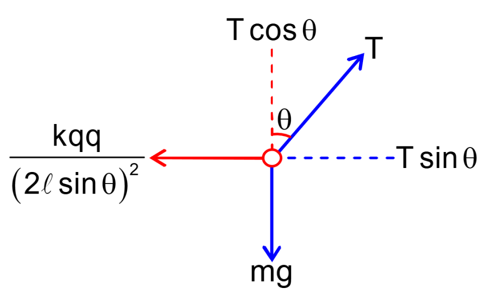
$T\cos\theta = mg$

$\therefore \tan\theta = \frac{q^2}{4\pi\varepsilon_0 mg 4\ell^2\theta^2}$

$[\tan\theta \approx \theta$, for small angle$]$

So, $\theta^3 = \frac{q^2}{16\pi\varepsilon_0 mg\ell^2}$

$\theta = \left(\frac{q^2}{16\pi\varepsilon_0 mg\ell^2}\right)^{1/3}$



Also separation $= 2\ell\sin\theta$

$\cong 2\ell\theta$

$2\ell\left(\frac{q^2}{16\pi\varepsilon_0 mg\ell^2}\right)^{1/3}$

$= \left(\frac{8q^2\ell^3}{16\pi\varepsilon_0 mg\ell^2}\right)^{1/3}$

$= \left(\frac{q^2\ell}{2\pi\varepsilon_0 mg}\right)^{1/3}$

Sol2. (a) $I = \frac{ML^2}{12}$

(b) $I = \frac{(2M)L^2}{3}$

(c) $I = \frac{M(2L)^2}{12} = \frac{ML^2}{3}$

(d) $I = \frac{(2M)(2L)^2}{3} = \frac{8ML^2}{3}$

Sol3. $\dfrac{I_1}{I_2} = \dfrac{\left(\dfrac{MR^2}{2}\right)}{\left(\dfrac{mr^2}{4}\right)} = \dfrac{2M}{m}\left(\dfrac{R}{r}\right)^2 \qquad \begin{bmatrix} M = \sigma\pi R^2 \\ m = \sigma\pi r^2 \end{bmatrix}$

$= 2\left(\dfrac{M}{m}\right)\left(\dfrac{R}{r}\right)^2$

$= 2 \times \left(\dfrac{R}{r}\right)^2\left(\dfrac{R}{r}\right)^2$

$= \dfrac{2R^4}{r^4}$

Sol4. i = constant

$v_d \,\alpha\, E \ \&\ E \,\alpha\, \dfrac{1}{r^2}$ So, E increases with decrease in the radius.

also $V_d \,\alpha\, E$

So, drift speed increases.

Sol5. $v = \varepsilon\left(1 - e^{-t/\tau}\right)$

Where $\tau = Rc = 100 \times 10^{-6} = 10^{-4}$ sec

$50V = 100V\left(1 - e^{\frac{-t}{10^{-4}}}\right)$

$\dfrac{1}{2} = 1 - e^{-10^4 t}$

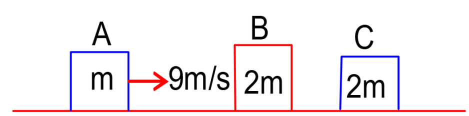
$\dfrac{1}{2} = e^{-10^4 t}$

$-\ell n 2 = -10^4 t$

$\dfrac{\ell n 2}{10^4} = t$

$0.693 \times 10^{-4}\ \text{sec} = t$

Sol6. Let vel of A and B just after collision be v_A & v_B respectively.



$m \times 9 + 0 = m \times v_A + 2mv_B$

$9 = v_A + 2v_B$

again $e = \dfrac{v_B - v_A}{9 - 0}$

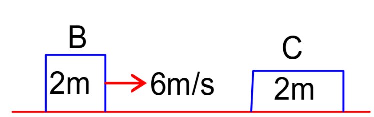
$1 = \dfrac{v_B - v_A}{9 - 0}$

$9 = v_B - v_A$

From (i) &

$v_B = 6$ m/s & $v_A = -3$ m/s

Now,



$2m \times 6 + 0 = (2m + 2m)v_C$

$12m = 4mv_C$

$3\,m/s = V_C$

Sol7. $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$

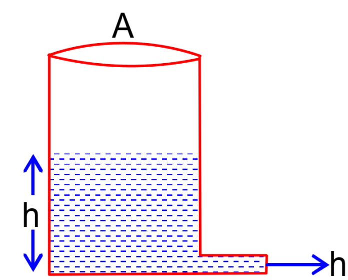
$$\frac{1}{C_{eq}}=\frac{1}{\left(\frac{kA\varepsilon_0}{d}\right)}+\frac{1}{\left(\frac{3kA\varepsilon_0}{2d}\right)}+\frac{1}{\left(\frac{5kA\varepsilon_0}{3d}\right)}$$

$$\frac{1}{C_{eq}}=\frac{d}{kA\varepsilon_0}+\frac{2d}{3kA\varepsilon_0}+\frac{3d}{5kA\varepsilon_0}$$

$$\frac{1}{C_{eq}}=\frac{(15+10+9)d}{15kA\varepsilon_0}$$

$$C_{eq}=\frac{15kA\varepsilon_0}{34}$$

Sol8. $v=\sqrt{2gh}\rightarrow$ velocity of efflux.



$$F=v\frac{dm}{dt}=v(a\rho v)=a\rho 2gh$$

$f_r=\mu R=\mu Ah\rho g$

For just sliding,

$f_r = F$

$\mu Ah\rho g=2a\rho gh$

or $\mu=\frac{2a}{A}$

Sol9. $v=\frac{1}{\sqrt{\mu\varepsilon}}$

$$=\frac{1}{\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}}$$

$$=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\cdot\frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

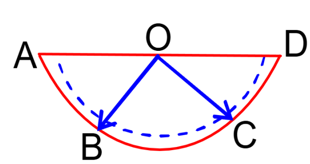
$$=\frac{3\times10^8}{\sqrt{1\times81}}$$

$$v=\frac{3\times10^8}{9}\ m/s$$

$$=0.33\times10^8\ m/s$$

$$=3.33\times10^7\ m/s$$

Sol10. Polygon law is applicable in both the situation given but the equation given in the reason is not useful in explaining the assertion.



Sol11. 1 litre, $T=300k$, $P=2\,atm$, $KE=2\times10^{-9}\ J/molecule$,

no of molecule = ?

No. of molecules $=(\text{no of moles})\times N_A$

$=nN_A$

Also, $n = \frac{PV}{RT} = \frac{PV}{N_A kT}$

$KE = 2\times10^{-9}\,J \quad [\text{Given}]$

$\therefore \frac{3}{2}kT = 2\times10^{-9}$

$kT = \frac{4\times10^{-9}}{3}$

$P = 2\,atm = 2\times1.013\times10^{5}\,N/m^2$

$vol = 1\,lit = 10^{-3}m^3$

$\therefore$ No. of molecules

$= \frac{PV}{N_A kT}\times N_A$

$= \frac{PV}{kT}$

$= \frac{2\times1.013\times10^{5}\times10^{-3}}{\frac{4}{3}\times10^{-9}}$

$\approx \frac{3}{2}\times10^{11}$

Sol12. $A\to B$ & $D\to C$ (iso thermal process)

So, $T_A = T_B$ & $T_C = T_D$ Now $B\to C$ & $D\to A$ (adiabatic process)

$\therefore |W_{BC}| = \frac{nR}{\gamma-1}(T_B - T_C)$

$|W_{AD}| = \frac{nR}{\gamma-1}(T_A - T_D) - \frac{nR}{\gamma-1}(T_B - T_C)$

$\therefore W_{BC} = W_{AD}$

Sol13. $f = \frac{N_0 - N_0\left(e^{-\lambda t}\right)}{N_0}$

$f = 1 - e^{-\lambda t}$

$\frac{df}{dt} = 0 - e^{-\lambda t}\times -\lambda = \lambda e^{-\lambda t}$

Sol14. Least count $= \frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In 5 revolution, distance travelled = 5mm

$\therefore$ In 1 revolution, distance travelled = 1mm

So least count $= \frac{1}{50} = 0.02\,mm = 0.002\,cm$

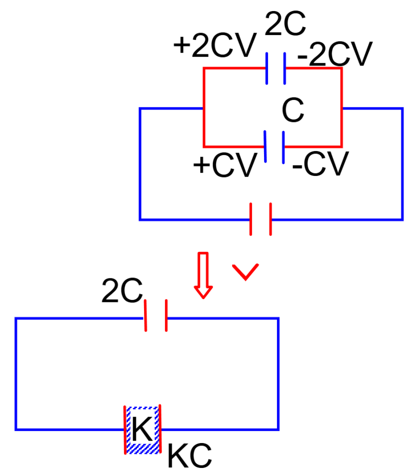
So, Assertion is not correct but reason is correct.

Sol15. As charge remains same. So,

$$V_C = \frac{2CV + CV}{KC + 2C}$$

$$= \frac{3CV}{(K+2)C}$$

$$V_C = \frac{3V}{K+2}$$



Sol16. $\beta = \frac{\lambda D}{d}$ or $\beta \propto \lambda$

Also, we know that

$\lambda_{Blue} < \lambda_{Orange}$

So, $\beta_{orange} > \beta_{Blue}$

So, dist b/w consecutive fringes will decrease.

Sol17. $h = \frac{u^2}{2g}, u = \sqrt{2gh}$

Now,

$$S = \frac{h}{3}$$

$$S = ut + \frac{1}{2}at^2$$

$$\frac{h}{3} = \sqrt{2gh}\,t - \frac{1}{2}gt^2$$

$$0 = t^2\left(\frac{g}{2}\right) - \sqrt{2gh}\,t + \frac{h}{3}$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}}$$

$$\frac{t_1}{t_2} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

Sol18. $E = \frac{1}{2}m\omega^2 a^2$ ...(i)

When $KE = \frac{3E}{4}, PE = \frac{E}{4}$

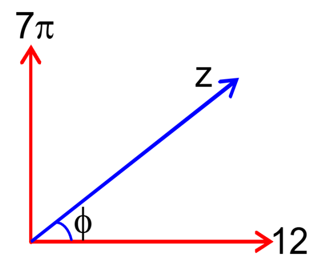
$\therefore \frac{E}{4} = \frac{1}{2}m\omega^2 y^2$ ...(ii)

Divide eq^n (i) by eq^n (ii) we get

$$4 = \frac{a^2}{y^2} \Rightarrow y = \pm\frac{a}{2}$$

Sol19. $L = 0.07H$ and $R = 12\Omega$

$V_S = 220V, 50Hz$



$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{12^2 + (7\pi)^2}$$

$$\approx 25\Omega$$

$$\therefore\ i = \frac{v}{z} = \frac{220v}{25\Omega} = 8.8A$$

$$\tan\phi = \frac{7\pi}{12} \Rightarrow \phi = \tan^{-1}\left(\frac{11}{6}\right)$$

Sol20. $\frac{-dT}{dt} = K[T - T_S]$

$$\frac{-[61^\circ C - 59^\circ C]}{4\,min} = K\left[\left(\frac{61^\circ C + 59^\circ C}{2}\right) - 30^\circ C\right]$$

So, $K = \frac{-1}{60\,min}$

Again

$$\frac{-[51^\circ C - 49^\circ C]}{t} = K\left[\left(\frac{51^\circ C + 49^\circ C}{2}\right) - 30^\circ C\right]$$

$$\frac{-2^\circ C}{t} = \frac{-1}{60\,min} \times 20^\circ C$$

$$\therefore\ t = 6\,min$$

SECTION - B

Sol1. As we know,

$$\beta = \frac{I_C}{I_B} = 24\ (\text{Given})$$

$$R = 1000\Omega,\ \Delta v = 0.6V,\quad I_C = \frac{0.6}{1000} = 6 \times 10^{-4} A$$

$$\text{So, } I_B = \frac{I_C}{\beta} = \frac{6 \times 10^{-4}}{24}$$

$$= 25 \times 10^{-6} A$$

or $I_B = 25\mu A$

Sol2. Light rays travel undeviated if the refractive index of the medium does not change for any value of angle of incidence.

ie $n_1 = n_2$

$$\Rightarrow 1.2 + \frac{10.8 \times 10^{-14}}{\lambda_0^2} = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda_0^2}$$

$\Rightarrow \lambda_0 = 600\,\text{nm}$

Sol3. When T_1 & T_2 are connected, at steady state current, I becomes

$I = \frac{6v}{6\Omega} = 1A$

Now when T_1 & T_3 are connected, the current through the inductor, just after connecting, remains same. So

$V_{3\Omega} = 1A \times 3\Omega = 3\text{volt}.$

Sol4. Loss in elastic potential energy = Gain in KE

$$\frac{1}{2}\frac{YA}{L}\ell^2 = \frac{1}{2}mv^2$$

$$\frac{0.5\times10^9\times10^{-6}}{0.1}\times(0.04)^2 = 20\times10^{-3}v^2$$

$\therefore \quad v = 20\,m/s$

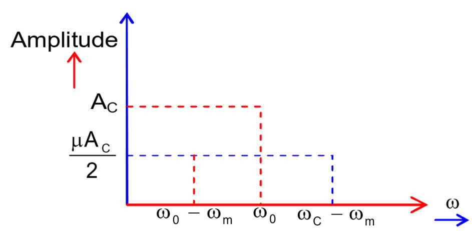
Sol5. For photon, $\lambda_1 = \frac{h}{p} = \frac{6.6\times10^{-34}}{10^{-27}}$

For a particle, $\lambda_2 = \frac{h}{mv} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^6}$

$\therefore \quad \frac{\lambda_1}{\lambda_2} = 910$

Sol6. $\frac{a}{10} = \frac{b}{10} = \frac{\mu A_C}{2}$

$\Rightarrow \frac{a}{b} = 1$



Sol7. $M = 9.85\times10^{-2}\,A/m^2$

$I = 5\times10^{-6}\,kg - m^2$

$T = 0.5\,\text{sec}$

$B = ?$

$T = 2\pi\sqrt{\frac{I}{MB_H}}$

$\therefore B_H = \frac{4\pi^2 I}{MT^2} = \frac{4\pi^2\times5\times10^{-6}}{9.85\times10^{-2}\times0.5\times0.5}$

$= \frac{4\times9.85\times5\times10^{-6}}{9.85\times10^{-2}\times0.5\times0.5}$

$= 8\times10^{-3}$

$= 8\,mT$

Sol8. We have to find the point where the gravitational field must be zero.

$E_G = 0$

$$\frac{GM}{x^2} = \frac{G9M}{(8R-x)^2}$$

$$\frac{1}{x^2} = \frac{9}{(8R-x)^2}$$

$$\frac{8R-x}{x} = 3$$

$x = 2R$

Potential at A, V_A

$$V_A = -\frac{GM}{R} - \frac{G9M}{7R} = \frac{-16}{7}\frac{GM}{R}$$

Potential at point x dist away from A

$$V_x = \frac{-GM}{2R} - \frac{G9M}{6R} = -\frac{12GM}{6R}$$

$$= -\frac{2GM}{R}$$

$$\therefore \Delta V = \frac{-2GM}{R} - \left(\frac{-16}{7}\frac{GM}{R}\right) = \frac{2}{7}\frac{GM}{R}$$

Using conservation of mechanical energy

$\Delta KE = \Delta U$

$$\frac{1}{2}mv^2 = m\Delta V$$

$$\frac{1}{2}mv^2 = \frac{2GMm}{7R} \Rightarrow v = \sqrt{\frac{4GM}{7R}} \Rightarrow a = 4$$

Sol9. $r = 0.5\,\overset{o}{A} = 0.5 \times 10^{-10}\,m$

$v = 2.2 \times 10^6\,m/s$

$I = ?$

$$I = \frac{e}{t},\ t = \frac{2\pi r}{v}$$

$$I = \frac{ev}{2\pi r} = \frac{(1.6 \times 10^{-19}) \times 2.2 \times 10^6 \times 7}{2 \times 22 \times 0.5 \times 10^{-10}}$$

$= 11.2 \times 10^{-4}$

$= 112 \times 10^{-5}\,A$

$= 112 \times 10^{-2}\,mA$

Sol10. $< t > = 30 \times 10^{-3}$ sec.

$C = 200\mu F, R$

$$\frac{Q}{A} = \text{ constant} \quad \text{[Given]}$$

$$A = A_0 e^{-\lambda t}\ \&\ Q = C\varepsilon e^{-t/RC}$$

$$\therefore \frac{d}{dt}\left[\frac{C\varepsilon e^{-t/RC}}{A_0 e^{-\lambda t}}\right] = 0$$

$$\frac{d}{dt}\left\{\frac{e^{-t/RC}}{e^{-\lambda t}}\right\}=0$$

$$\frac{e^{-\lambda t}.e^{-t/RC}\times\frac{-1}{RC}-e^{-t/RC}\times e^{-\lambda t}\times-\lambda}{\left(e^{-\lambda t}\right)^2}=0$$

$$e^{-\lambda t}e^{-t/RC}\left(-\frac{1}{RC}+\lambda\right)=0$$

$$\Rightarrow R=\frac{1}{\lambda C}=150$$

PART – B (CHEMISTRY)

SECTION - A

Sol1. Rutherford atomic model can not explain hydrogen spectrum it is explained by Bohr's atomic model and from Bohr's atomic model, uncertainity principle can't be explained.

Sol2. Rate = $K[A]^n$

$$K=\frac{Rate}{[A]^n}$$

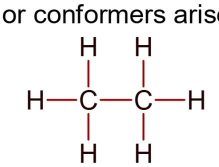
$$=\frac{moleL^{-1}s^{-1}}{\left(moleL^{-1}\right)^n}$$

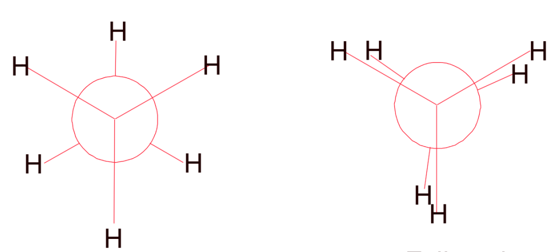
$$=mole^{1-n}L^{n-1}s^{-1}$$

Unit of K: $mole^{1-n}L^{n-1}s^{-1}$

Sol3. Barfoed test:-
In this test, copper acetate in dilute acid is reduced in 30 seconds by mono saccharides while disaccharides takes several minutes.

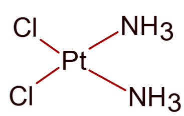
Sol4. Rotamers or conformers arises due to free rotation along σ - bond.

Ethane 



Staggered Eclipsed

Sol5. (A) $\left[PtCl_2(NH_3)_2\right]$, Number of GI = 2



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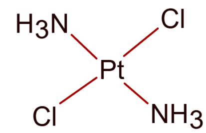
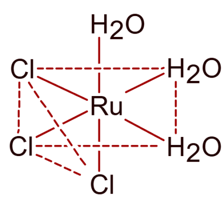


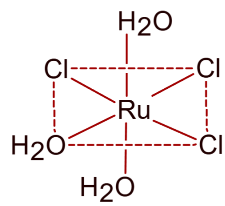
Cis

(B) $[Ni(CO)_4]$, Number of G.I = 0 as it is tetrahedral.

(C) $[Ru(H_2O)_3Cl_3]$



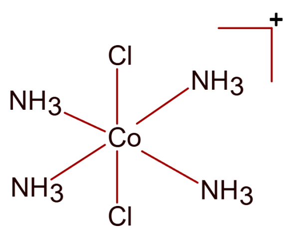
(Facial)



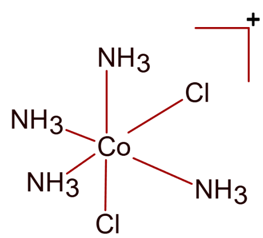
(Meridional)

Number of G.I = 2

(D) $[CoCl_2(NH_3)_4]^+$



(trans)

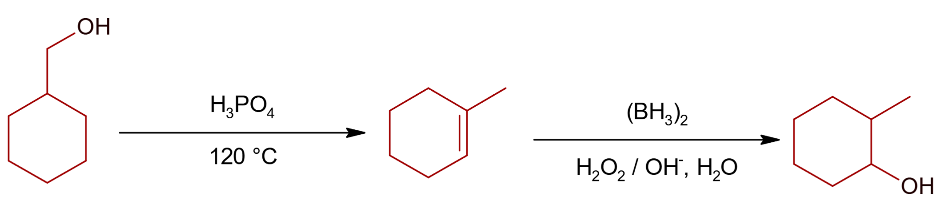


(Cis)

Number of G.I = 2

Sol6. All carbonyl compound (aldehyde and ketone) give orange precipitate with 2,4-dinitrophenyl hydrazine (Brady's reagent)

Sol7.



Anti markonikov Addition

Sol8. (a) Furacin → Antiseptic
(b) Arsphenamine → Antibiotic
(c) Dimetone → Synthetic antihistamines
(d) Valium → Tranquilizers

Sol9. In Monoclinic crystal system, $a \neq b \neq c$ and $\alpha = 90^\circ$, $\gamma = 90^\circ$ and $\beta = 120^\circ$.

Sol10.

Compound		Oxidation state of P
$H_4P_2O_7$	→	5
$H_4P_2O_5$	→	3
$H_4P_2O_6$	→	4

Sol11. $[MnCl_6]^{3-}$

$Mn(25) \rightarrow [Ar]4s^2 3d^5$

$Cl^- \rightarrow$ weak ligand

$Mn^{3+} \rightarrow 3d^4$ i.e. $t_{2g}^{1,1,1}, eg^{1,0}$

Hybridization $\rightarrow sp^3d^2$ and paramagnetic

Sol12. $CH_4 + I_2 \underset{}{\overset{hv}{\rightleftharpoons}} CH_3I + HI$

It is reversible due to reducing nature of HI but it can be irreversible by addition of oxidizing agents like, HNO_3, HIO_3, HgO, etc which will oxidize HI into I_2.

Sol13. In DNA, thymine is bind with adenine by hydrogen bonding.

Sol14. NaOH- Base

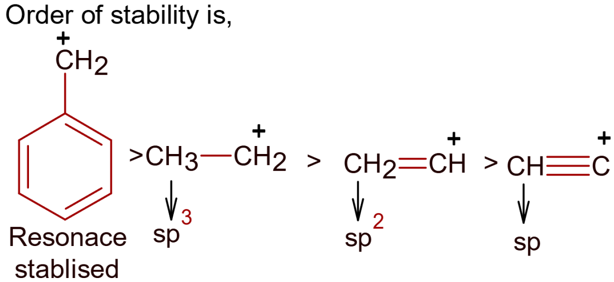
$Be(OH)_2$ – Amphoteric

$Ca(OH)_2$ – Base

$B(OH)_3$ - Acidic

$Al(OH)_3$ - Amphoteric

Sol15. Order of stability is,



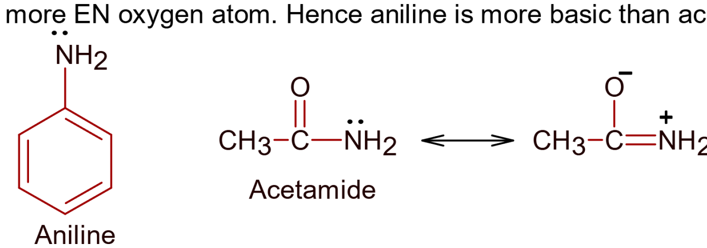
Sol16. Due to small size Li have more polarizing power so most of the compound of Li are covalent.

Sol17. It is process in which nutrient enriched water bodies support a dense plant population which kills animal life by depriving it of oxygen and result in subsequent loss of biodiversity is known as Eutrophication.

Sol18. Main product of electrolysis of conc. H_2SO_4 is $H_2S_2O_8$ i.e. $HO_3SO\text{-}OSO_3H$

Sol19. Ellingham Diagram doesn't give idea about of rate of reaction.

Sol20. In Aniline, lone pair is delocalized on less EN carbon atom while in acetamide it is delocalized on more EN oxygen atom. Hence aniline is more basic than acetamide.



SECTION – B

Sol1. $\lambda_m^c = \frac{K \times 1000}{M} = \frac{2 \times 10^{-5} \times 10^3}{10^{-3}} = 20\ Scm^2mole^{-1}$

For weak acid $(\alpha) = \frac{\lambda_m^c}{\lambda_m^\infty} = \frac{20}{190} = \frac{2}{19}$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{10^{-3}\times\left(\frac{2}{19}\right)^2}{\left(1-\frac{2}{19}\right)}$$

$= 0.01238\times10^{-3}$

$= 12.38\times10^{-6}$

So, ans is 12 (nearest integer).

Sol2. Number of GI in $\left[Co(NH_3)_3(NO_2)_3\right]$ is 2 i. e, X

Number of GI in $\left[Cr\left(C_2O_4^{2-}\right)_3\right]^{3-}$ is 0 i.e. Y

$[X+Y]=2$

Sol3. In triethyl amine, nitrogen is sp^3 hybridized, hence bond angle is approximately $109^o 23'$ but since lone pair- bond pair repulsion is greater than bp-bp repulsion. Hence, the exact angle is found to be 108^o.

Sol4. Organic compound + $Cl_2 \longrightarrow$ A (Containing Cl)

5gm

$A + AgNO_3 \longrightarrow AgCl$

0.5 gm 0.3849gm

Mole of AgCl = $\frac{0.3849}{143.87}$

Mass of Cl = $\frac{0.3849}{143.87}\times35.5 = 0.095$

% of Cl in A = $\frac{0.095}{0.5}\times100 = 19\%$

Sol5. $\Pi = CRT$

$$2.42\times10^{-3} = \left[\frac{1.46\times1000}{M_{polymer}\times100}\right]\times0.083\times300$$

$$M_{polymer} = \frac{1.46\times1000\times0.083\times300}{2.42\times10^{-3}\times100}$$

$= 14.96\times10^4 gm \cong 15\times10^4 gm$

Sol6. $\left(\frac{x}{m}\right) = k(p)^{1/n}$ ....(1)

$64\left(\frac{x}{m}\right) = k(2p)^{1/n}$ ....(2)

From equation (1) and (2)

$\frac{1}{n} = 6$

$n = \frac{1}{6}$

$\Rightarrow n = 0.167$

$= 16.7\times10^{-2}$

$\approx 17\times10^{-2}$

Sol7. $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g); K_C = 1.844$

$t=0$	3	0	0
equilibrium	$3-x$	x	x

$$K_C = \frac{x^2}{(3-x)} = 1.844$$

$$x^2 + 1.844x - 1.844\times3 = 0$$

$$x = \frac{-1.844+\sqrt{25.528}}{2} = \frac{3.208}{2} = 1.604$$

x = 1.604

At equilibrium number of moles of $PCl_5 = (3-1.604) = 1.396$ mol

$= 1396\times10^{-3}$ mol

Sol8. B.O. of CO = 3
B.O. of NO^+ = 3
Both are isoelectronic
So difference = 0
$\therefore$ x = 0

Sol9. Let volume of solution = xml
So mass of solution = 1.2x
And mass of water = x gm
Mass of solute = 0.2x

$$\text{Molality} = \frac{W\times1000}{M\times W(\text{Solvent})} = \frac{0.2x\times1000}{40\times x} = \frac{200}{40} = 5\text{ m}$$

Sol10. $H_2O(\ell) \longrightarrow H_2O(g)$

$$\Delta H^o = \Delta U^o + \Delta ngRT$$

$$\Delta H^o - \Delta U^o = \Delta ngRT$$

$= 1\times8.31\times373$

$= 3099.63$ J / mol

$= 30.9963\times10^2$ J/ mol

$\approx 31\times10^2$ J/ mol

PART – C (MATHEMATICS)

SECTION - A

Sol1. $S_1 : |z-3-2i|^2 = 8$

$$|z-3-2i| = (2\sqrt{2})$$

$$(x-3)^2 + (y-2)^2 = (2\sqrt{2})^2$$

$S_2 : Re(z) \ge 5$

$x \ge 5$

$S_3 : \left|z - \bar{z}\right| \geq 8$

$2\left|y\right| \geq 8$

$\left|y\right| \geq 4$

$y \geq 4$ or $y \leq -4$

$\therefore n\left(S_1 \cap S_2 \cap S_3\right) = 1$

Sol2. $\sin\theta + \cos\theta = \frac{1}{2}$

$16\left(\sin\left(2\theta\right) + \cos\left(4\theta\right) + \sin\left(6\theta\right)\right)$

$= 16\left[2\sin 4\theta . \cos 2\theta + \cos 4\theta\right]$

$= 16\left[4\sin 2\theta . \cos^2 2\theta + 2\cos 2\theta - 1\right]$ ....(i)

Now, $\sin\theta + \cos\theta = \frac{1}{2}$, squaring on both sides, we get

$1 + \sin 2\theta = \frac{1}{4}$

$\sin 2\theta = \frac{1}{4} - 1$

$\sin 2\theta = \frac{-3}{4}$

$\cos^2 2\theta = \frac{7}{16}$

From equation (i)

$16\left[4\sin 2\theta . \cos^2 2\theta + 2\cos^2\theta - 1\right]$

$16\left[4\left(\frac{-3}{4}\right) \times \frac{7}{16} + 2 \times \frac{7}{16} - 1\right] = -23$

Sol 3. $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a} + \vec{b} = 3\hat{j} + 5\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix} = -\hat{i} - 5\hat{j} + 3\hat{k}$

$\vec{a} . \vec{b} = -1 + 2 + 6 = 7$

$\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} \times \left(\left(\vec{a} - \vec{b}\right) \times \vec{b}\right) \times \vec{b}\right)$

$\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} \times \left(\vec{a} \times \vec{b} - \vec{b} \times \vec{b}\right) \times \vec{b}\right)$

$\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right) \times \vec{b}$

$7.\left(3\hat{j} + 5\hat{k}\right) \times \left(-\hat{i} - 5\hat{j} + k\right)$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 5 \\ -1 & -5 & 1 \end{vmatrix} = 7\left(34\hat{i} - 5\hat{j} + 3\hat{k}\right)$

Sol4. Normal on plane

$\left(2\hat{i}+\hat{j}-\hat{k}\right)\times\left(\hat{i}-\hat{j}-\hat{k}\right)=-2\hat{i}+\hat{j}-3\hat{k}$

Equation on plane

$-2\left(x+1\right)+1\left(y-0\right)-3\left(z+2\right)=0$

$-2x+y-3z-8=0$

$a+b+c=4$

Sol 5. L.H.L $=\lim_{x\to 0^-} f\left(x\right)=\lim_{x\to 0^-}\left(1+\left|\sin x\right|\right)^{\frac{3a}{\left|\sin x\right|}}=e^{\left|\sin x\right|\cdot\frac{3a}{\left|\sin x\right|}}=e^{3a}$ ....(i)

R.H.L $=\lim_{x\to 0^+} f\left(x\right)=\lim_{x\to 0^+} e^{\frac{\cot 4x}{\cot 2x}}=\lim_{x\to 0^+} e^{\frac{\tan 2x}{\tan 4x}}=\lim_{x\to 0^+} e^{\left(\frac{\frac{\tan 2x}{2x}.2x}{\frac{\tan 4x}{4x}.4x}\right)}=e^{\frac{1}{2}}$ ....(ii)

$\therefore 6a^2+b^2=\left(e+1\right)$

and $\lim_{x\to 0} f\left(x\right)=b$ ....(iii)

From (i), (ii) & (iii) we get $a=\frac{1}{6}, b=e^{\frac{1}{2}}$

$\therefore 6a^2+b^2=\left(e+1\right)$

Sol6.

P	Q	$P\vee Q$	$\sim P$	$\sim Q$	$\left(P\vee Q\right)\wedge P$	$\left(P\vee Q\right)\wedge\sim P$	$P\wedge\sim Q$	$P\Rightarrow Q$	$\sim\left(P\Rightarrow Q\right)$	$\sim\left(P\Rightarrow Q\right)\Leftrightarrow P\wedge\sim Q$
T	T	T	F	F	F	T	F	T	F	T
T	F	T	F	T	F	T	T	F	T	T
F	T	T	T	F	T	T	F	T	F	T
F	F	F	T	T	F	T	F	T	F	T

Sol7. $S_1 : x^2+y^2-x-y-\frac{1}{2}=0 \quad C_1 : \left(\frac{1}{2},\frac{1}{2}\right)$

$r_1=\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}}=1$

$S_2 : x^2+y^2-4y+\frac{7}{4}=0 \quad C_2 : \left(0,2\right)$

$r_2=\sqrt{4-\frac{7}{4}}=\frac{3}{2}$

$S_3 : x^2+y^2-4x-2y+5-r^2=0 \quad C_3 : \left(2,1\right)$

$r_3=\sqrt{4+1-5+r^2}=\left|r\right|$

$C_1C_3=\sqrt{\frac{5}{2}}, C_2C_3=\sqrt{5}$

S_1 must be inside S_3 $\therefore \sqrt{\frac{5}{2}}\le\left|r-1\right|$ and also S_2 must be inside S_3 $\therefore \sqrt{5}\le\left|r-\frac{3}{2}\right|$

Sol8. $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$

$\left|A\right|=6$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\therefore \left.\begin{matrix} \alpha + \beta = \frac{2}{3} \\ \beta = \frac{-1}{6} \end{matrix}\right\} \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$4(\alpha - \beta) = 4(1) = 4$$

Sol9. $\frac{dy}{dx} = e^{3x}.\,e^{4y}$

$= \frac{1}{e^{4y}} dy = e^{3x} dx$

$\int \frac{1}{e^{4y}}.\,dy = \int e^{3x}.\,dx$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$ ....(i)

Now $y(0) = 0$ (given)

From (i)

$\frac{-1}{4} = \frac{1}{3} + c$

$c = \frac{-7}{12}$

$\therefore \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

$\therefore \frac{e^{-4y}}{-4} = \frac{4e^{3x} - 7}{12}$

$\therefore e^{-4y} = \frac{7 - 4e^{3x}}{3}$

$4y = \ell n\left(\frac{3}{7 - 4e^{3x}}\right)$

Now, $x = \frac{-2}{3}\ln 2$

$y = \frac{1}{4}\ln\left(\frac{1}{2}\right)$

$y = \frac{-1}{4}\ln 2$

$\therefore \alpha = \frac{-1}{4}$

Sol10. $\because \frac{t-1}{2}=\frac{2}{7}$

Equation of reflected ray,

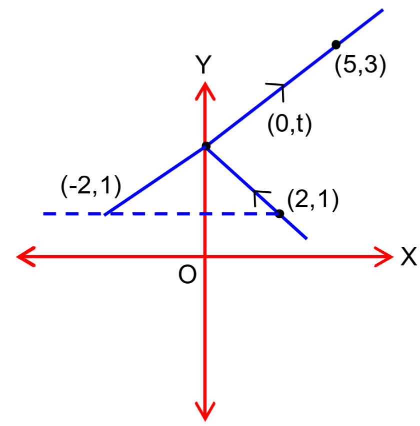
$(y-3)=\frac{2}{7}(x-5) \Rightarrow 2x-7y+11=0$

$\because e=\frac{1}{3}$

$\therefore \frac{a}{e}-ae=\frac{8}{\sqrt{53}} \Rightarrow 3a-\frac{a}{3}=\frac{8}{\sqrt{53}}$

Let equation of other directrix is $\therefore 2x-7y=\alpha$ and also distance between the directrices = distance between two parallel lines $\therefore \frac{2a}{e}=\frac{|\alpha+11|}{\sqrt{53}} \Rightarrow \alpha=7,-29$

$\therefore 2x-7y-7=0,\ 2x-7y+29=0$



Sol11. $\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$

$\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}\frac{\left(2\frac{j}{n}-\frac{1}{n}\right)+8}{\left(2\frac{j}{n}-\frac{1}{n}\right)+4}$

$=\int_0^1\frac{2x+8}{2x+4}dx$

$=\int_0^1 dx+\int_0^1\left(\frac{4}{2x+4}\right)dx$

$=1+\left[4\frac{1}{2}\ln|2x+4|\right]_0^1$

$=1+2(\ln(6)-\ln(4))$

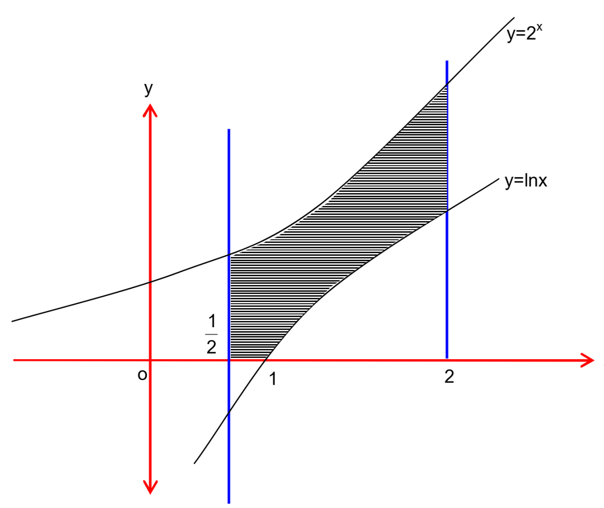
$=1+2\ln\frac{3}{2}$

Sol12. Area = $\int_{\frac{1}{2}}^{2}2^x dx-\int_1^2 \ln x dx$

$=\frac{2^2-2^{\frac{1}{2}}}{\ln 2}-(2\ln 2-1)$

$\alpha=2^2-2^{\frac{1}{2}},\beta=-2,\gamma=1$

$(\alpha+\beta-2\gamma)^2=2$



Sol13. Coefficient of x^7 is $\left(x^2+\frac{1}{bx}\right)^{11}$

$T_{r+1} = {}^{11}C_r\ x^{22-2r}, \frac{1}{b^r x^r}$

According to question, $22-3r=7$

$r=5$

Coefficient of x^{-7} is $\left(x-\frac{1}{bx^2}\right)^{11}$

Similarly r = 6

As per question

${}^{11}C_5 . \frac{1}{b^5} = {}^{11}C_6\ \frac{1}{b^6}$

b = 1

Sol14. $x^2+y^2-2x-6y+6=0$

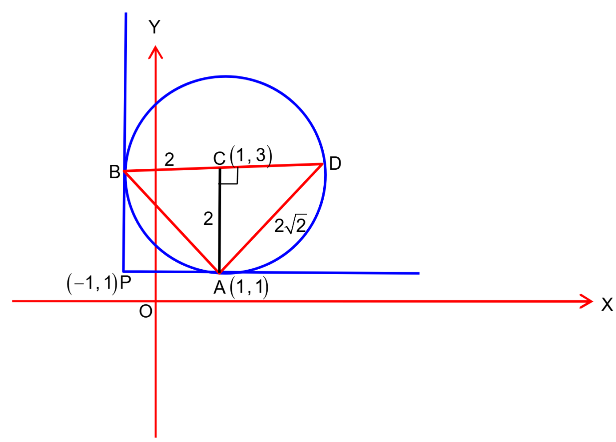
$C=(1,3), r=\sqrt{1+9-6}$

$r=2$

$\therefore$ Area of

$\Delta ABD = \frac{1}{2}\times \text{Base}\times\text{height}$

$=\frac{1}{2}\times 4\times 2 = 4$ sq. unit



Sol15. Total two digit number are 90

Possible outcome = ${}^{90}C_1 = 90$

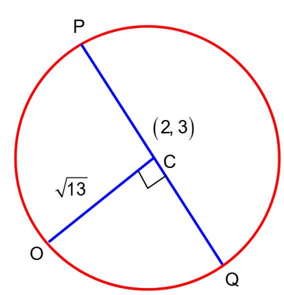
(2^n-2) is multiple of 3 when n is odd

$\therefore$ Required probability = $\frac{45}{90}=\frac{1}{2}$

Sol16. $\tan\theta = \frac{-2}{3}$

$P, Q : \left(2\pm\sqrt{13}\cos\theta, 3\pm\sqrt{13}\sin\theta\right)$

$(-1,5)$ and $(5,1)$



Sol17. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(1+e^{x.\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$ ....(i)

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(1+e^{-x.\cos x}\right)\left(\sin^4 x+\cos^4 x\right)} \quad ....(ii)$$

Adding (i) & (ii)

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(\sin^4 x+\cos^4 x\right)}$$

$$2I = 2\int_{0}^{\frac{\pi}{4}} \frac{dx}{\left(\sin^4 x+\cos^4 x\right)}$$

$$I = \int_{0}^{\frac{\pi}{4}} \left(\frac{\left(1+\tan^2 x\right)\sec^2 x}{\tan^4 x+1}\right)dx$$

$$I = \int_{0}^{\frac{\pi}{4}} \frac{\left(1+\frac{1}{\tan^2 x}\right).\sec^2 x}{\left(\tan x-\frac{1}{\tan x}\right)^2+2}dx$$

Let

$$\tan x-\frac{1}{\tan x} = t$$

$$\left(1+\frac{1}{\tan^2 x}\right)\sec^2 x\, dx = dt$$

$$\therefore I = \frac{\pi}{2\sqrt{2}}$$

Sol18. Using L'Hospital Rule

$$\lim_{x\to 2}\left(\frac{2x.f(2)-4f'(x)}{1}\right) = \frac{4(4)-4}{1} = 12$$

Sol19. $x^2+(20)^{\frac{1}{4}}x+(5)^{\frac{1}{2}} = 0$

$$\alpha+\beta = -(20)^{\frac{1}{4}}$$

$$\alpha.\beta = 5^{\frac{1}{2}}$$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 0$$

Similarly

$\alpha^4+\beta^4 = -10$ and $\alpha^8+\beta^8 = 50$

Sol20. Mean = $\frac{6+10+7+13+a+12+b+12}{8} = 9$

60 + a + b = 72

a + b = 12.........(i)

Variance = $\frac{\Sigma x_i^2}{n}-\left(\frac{\Sigma x_i}{n}\right)^2 = \frac{37}{4}$

$\Sigma x_i^2 = 6^2 + 10^2 + 7^2 + 13^2 + a^2 + 12^2 + b^2 + 12^2 = a^2 + b^2 + 642$

$\frac{a^2 + b^2 + 642}{8} - (9)^2 = \frac{37}{4}$

$\frac{a^2 + b^2}{8} + \frac{321}{4} - 81 = \frac{37}{4}$

$a^2 + b^2 = 80$

$(a+b)^2 = 144$ (from (i))

$\therefore (a-b)^2 = 16$

SECTION - B

Sol1. $\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$

$\sec y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$\sec y dy = 2\sin x \cos y dx$

$\sec^2 y dy = 2\sin x dx$

On integrating both side

$\tan y = -2\cos x + c$

y(0) = 0

$\therefore c = 2$

$\therefore \tan y = -2\cos x + 2$

at $x = \frac{\pi}{2}, \tan y = 2$

$\therefore \sec^2 y = 5$

on differentiation

$\sec^2 y . \frac{dy}{dx} = 2\sin x$

$5 . \frac{dy}{dx} = 2$

Sol2. $x + y - z = 2$

$x + 2y + \alpha z = 1$

$2x - y + z = \beta$

For infinite solution

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$\Delta = 3(2+\alpha) = 0$

$\alpha = -2$

$\Delta_2 = 0$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$\beta - 7 = 0$
$\beta = 7$
$\therefore \alpha + \beta = 5$

Sol3. Let $\overrightarrow{BA} = \hat{i} + 4\hat{j} - 5\hat{k}$ and equation of plane be $a(x-2) + b(y-3) + c(z+2) = 0$ ....(i)

$$\overrightarrow{BA} \times \vec{\ell} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix}$$

$a\hat{i} + b\hat{j} + c\hat{k} = -14\hat{i} - 14\hat{j} - 14\hat{k}$

$\therefore$ From (i) equation of plane be $(x-2) + (y-3) + (z+2) = 0$

$\Rightarrow x + y + z - 3 = 0$

$$\therefore d = \frac{|3|}{\sqrt{1+1+1}} = \sqrt{3} \Rightarrow d^2 = 3$$

Sol4. $\because F(x) = e^{-x}\int_3^x 3t^2 + 2t + 4F'(t)dt$

$\therefore F(3) = 0$

$e^x . F(x) = \int_3^x (3t^2 + 2t + 4F'(t))dt$

$e^x . F(x) + e^x . F'(x) = 3x^2 + 2x + 4F'(x)$

Let y = F(x)

$$e^x . y + e^x . \frac{dy}{dx} = 3x^2 + 2x + 4\frac{dy}{dx}$$

$$(e^x - 4)\frac{dy}{dx} = 3x^2 + 2x - e^x y$$

$$(e^x - 4)\frac{dy}{dx} + e^x y = 3x^2 + 2x$$

$$\frac{dy}{dx} + \left(\frac{e^x}{e^x - 4}\right)y = \frac{3x^2 + 2x}{e^x - 4} \text{ ....(i)}$$

I.F. $= e^{\int\left(\frac{e^x}{e^x-4}\right)dx} = e^x - 4$

From (i) $y(e^x - 4) = \int (3x^2 + 2x)dx + c$

$y(e^x - 4) = x^3 + x^2 + c$

As $F(3) = 0$

at x = 3

$C = -36$

$$F(x) = \left(\frac{x^3 + x^2 - 36}{e^x - 4}\right)$$

$$F'(x) = \frac{(3x^2 + 2x)(e^x - 4) - (x^3 + x^2 - 36)e^x}{(e^x - 4)^2}$$

$$F'(4)=\frac{56\left(e^4-4\right)-44e^4}{\left(e^4-4\right)^2}$$

$$=\frac{12e^4-224}{\left(e^4-4\right)^2}$$

$\therefore \alpha=12, \beta=4$

$\alpha+\beta=16$

Sol5. $f(x)=\begin{vmatrix}\sin^2 x & -2+\cos^2 x & \cos 2x\\ 2+\sin^2 x & \cos^2 x & \cos 2x\\ \sin^2 x & \cos^2 x & 1+\cos 2x\end{vmatrix}$

$R_1 \to R_1 - R_2 \quad R_2 \to R_2 - R_3$

$f(x)=\begin{vmatrix}-2 & -2 & 0\\ 2 & 0 & -1\\ \sin^2 x & \cos^2 x & 1+\cos 2x\end{vmatrix}$

$f(x)=-2\left(\cos^2 x\right)+2\left(2+2\cos 2x+\sin^2 x\right)$

$f(x)=-2\cos^2 x+4+4\cos 2x+2\sin^2 x$

$f(x)=4+2\cos 2x$

$\therefore f(x)_{max}=4+2=6$

Sol6. $\vec{a}\times\vec{b}=\vec{c}$

$\left(\vec{a}\times\vec{b}\right).\vec{c}=|\vec{c}|^2$

$\Rightarrow\left(\vec{a}\times\vec{b}\right).\vec{c}=2\Rightarrow\left[\vec{a}\,\vec{b}\,\vec{c}\right]=2\Rightarrow\left[\vec{b}\,\vec{a}\,\vec{c}\right]=-2\Rightarrow\vec{b}.\left(\vec{a}\times\vec{c}\right)=-2$

Projection of $\vec{b}$ on $\left(\vec{a}\times\vec{c}\right)=\ell$

$$\frac{\left|\vec{b}.\left(\vec{a}\times\vec{c}\right)\right|}{\left|\vec{a}\times\vec{c}\right|}=\ell$$

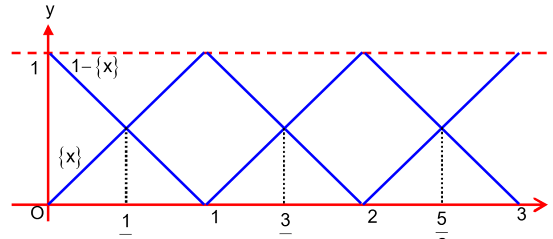
$$\ell=\frac{2}{\sqrt{6}}$$

$$\ell^2=\frac{4}{6}$$

$$3\ell^2=2$$

Sol7. $y=\{x\}$ at $y=1-\{x\}$

Not differentiable at $x=\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2}$



Sol8. $f(m.n)=f(m).f(n)$

$m = 1$

$f(n) = f(1).f(n)$

$f(1) = 1$

Put m = n = 2

$$f(4) = f(2).f(2) = \begin{cases} f(2) = 1 \Rightarrow f(4) = 1 \\ \text{or} \\ f(2) = 2 \Rightarrow f(4) = 4 \end{cases}$$

Similarly for f(5), f(6) and f(7).

Sol9. f(x) is define when

$\log_5\left(\log_3\left(18x - x^2 - 77\right)\right) > 0$

$x^2 - 18x + 80 < 0$

$x \in (8, 10)$

$a = 8, b = 10$

$$I = \int_8^{10} \frac{\sin^3 x}{\sin^3 x + \sin^3(10 + 8 - x)} dx \ldots\ldots\ldots(i)$$

Now, applying the property $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$$I = \int_8^{10} \frac{\sin^3 x}{\sin^3 x + \sin^3(18 - x)} dx \ldots\ldots\ldots(ii)$$

adding (i) & (ii), we get

$2I = (10 - 8)$

$2I = 2$

$I = 1$

Sol10. $2\log_3\left(2^x - 5\right) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$

Let $2^x = t$

$\log_3 (t - 5)^2 = \log_3 2\left(2^x - \frac{7}{2}\right)$

$(t - 5)^2 = 2t - 7$

$t^2 + 25 - 10t - 2t + 7 = 0$

$t = 4, 8$

$2^x = 4, 2^x = 8$

$x = 2 \text{ or } 3$

As $2^x - 5 > 0$

$\therefore x = 2$ is not possible

so, $x = 3$