Solutions to JEE(Main) -2021

Test Date: 27th July 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^{3} km. Find the mass of Mars.

$\left\{\text{Given } \frac{4\pi^2}{G} = 6\times10^{11}\,\text{N}^{-2}\,\text{kg}^2\right\}$

(A) 3.25×10^{21} kg
(B) 6.00×10^{23} kg
(C) 5.96×10^{19} kg
(D) 7.02×10^{25} kg

Q2. A raindrop with radius $R = 0.2$ mm falls from a cloud at a height $h = 2000$ m above the ground. Assume that the drop is spherical through its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is :

[Density of water $f_w = 1000\ \text{kg m}^{-3}$

and Density of air $f_a = 1.2\ \text{kg m}^{-3}$,

$g = 10\ \text{m/s}^2$

Coefficient of viscosity of air $= 1.8\times10^{-5}\ \text{Nsm}^{-2}$]

(A) $4.94\ \text{ms}^{-1}$
(B) $14.4\ \text{ms}^{-1}$
(C) $43.56\ \text{ms}^{-1}$
(D) $250.6\ \text{ms}^{-1}$

Q3. Figure A and B show long straight wires of circular cross – section (a and b with a < b), carrying current I which is uniformly distributed across the cross – section. The magnitude of magnetic field B varies with radius r and can be represented as :

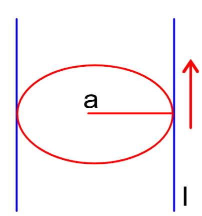


Fig. A

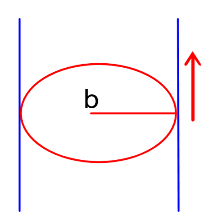
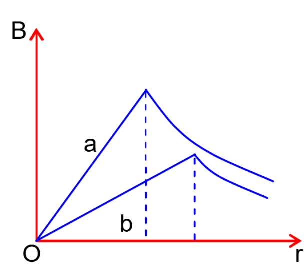
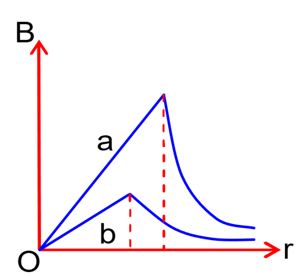


Fig. B

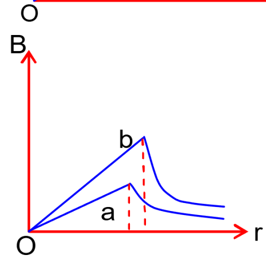
(A)



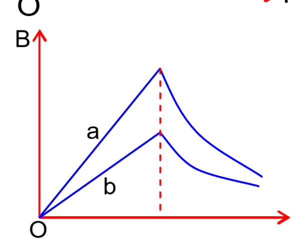
(B)



(C)



(D)



Q4. The resistance of a conductor at $15^\circ C$ is $16\,\Omega$ and at $100^\circ C$ is $20\,\Omega$. What will be the temperature coefficient of resistance of the conductor ?

(A) $0.003^\circ C^{-1}$
(B) $0.042^\circ C^{-1}$
(C) $0.033^\circ C^{-1}$
(D) $0.010^\circ C^{-1}$

Q5. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is :

(A) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$
(B) $\sqrt{\frac{G}{2R^3}}$
(C) $\frac{1}{2R}\sqrt{\frac{1}{G}}$
(D) $\sqrt{\frac{2G}{R^3}}$

Q6. Consider the following statements :
(A) Atoms of each element emit characteristics spectrum.
(B) According to Bohr's Postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.
(C) The density of nuclear matter depends on the size of the nucleus.
(D) A free neutron is stable but a free proton decay is possible.
(E) Radioactivity is an indication of the instability of nuclei.
Choose the correct answer from the options given below :

(A) B and D only
(B) A, C and E only
(C) A, B, C, D and E
(D) A, B and E only

Q7. Match **List I** with **List II.**

List I		**List II**	
(a)	Capacitance, C	(i)	$M^1L^1T^{-3}A^{-1}$
(b)	Permittivity of free space, ε_0	(ii)	$M^{-1}L^{-3}T^4A^2$
(c)	Permeability of free space, μ_0	(iii)	$M^{-1}L^{-2}T^4A^2$
(d)	Electric field, E	(iv)	$M^1L^1T^{-2}A^{-2}$

Choose the correct answer from the options given below :

(A) $(a)\to(iii),(b)\to(ii),(c)\to(iv),(d)\to(i)$
(B) $(a)\to(iii),(b)\to(iv),(c)\to(ii),(d)\to(i)$
(C) $(a)\to(iv),(b)\to(ii),(c)\to(iii),(d)\to(i)$
(D) $(a)\to(iv),(b)\to(iii),(c)\to(ii),(d)\to(i)$

Q8. One mole of an ideal gas is taken through an adiabatic process where the temperature rises from $27^\circ C$ to $37^\circ C$. If the ideal gas is composed of polyatomic molecule that has 4 vibrational modes, which of the following of the following is true ?
$[R = 8.314\ J\,mol^{-1}K^{-1}]$

(A) Work done by the gas is close to 582 J
(B) Work done on the gas in close to 582 J
(C) Work done on the gas is close to 332 J
(D) Work done by the gas is close to 332 J

Q9. Two Carnot engines A and operate in series such that engine A absorbs heat at T_1 and rejects heat to a sink at temperature T. Engine B absorbs half of the heat rejected by Engine A and rejects heat to the sink at T_3. When work done in both the cases is equal, the value of T is :

(A) $\frac{3}{2}T_1 + \frac{1}{3}T_3$ (B) $\frac{2}{3}T_1 + \frac{3}{2}T_3$

(C) $\frac{2}{3}T_1 + \frac{1}{3}T_3$ (D) $\frac{1}{3}T_1 + \frac{2}{3}T_3$

Q10. An electron and proton are separated by a large distance. The electron starts approaching the proton with energy $3\,eV$. The proton captures the electron and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength $4000\,\mathring{A}$. What is the maximum kinetic energy of the emitted photoelectron ?

(A) $1.41\,eV$ (B) $7.61\,eV$.

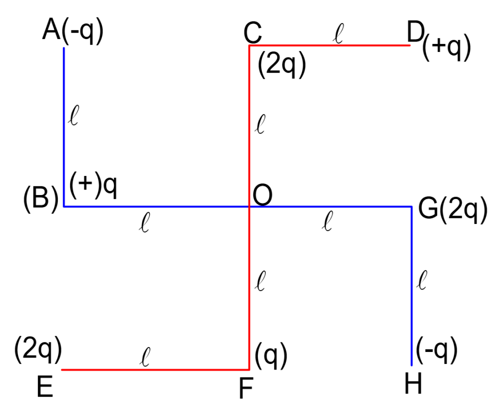
(C) 3.3 eV (D) No photoelectron would be emitted

Q11. An object of mass 0.5 kg executing simple harmonic motion. Its amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}s$ starting from mean position. Assume that the initial phase of the oscillation is zero.

(A) $6.2 \times 10^{-3}\,J$ (B) $0.62\,J$

(C) $6.2 \times 10^{3}\,J$ (D) $1.2 \times 10^{3}\,J$

Q12. What will be the magnitude of electric field at point O as shown in figure ? Each side of the figure is ℓ and perpendicular to each other ?



(A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(2\ell)}\left(2\sqrt{2}-1\right)$

(B) $\frac{q}{4\pi\varepsilon_0(2\ell)^2}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{q}{\ell^2}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2\ell^2}\left(\sqrt{2}\right)$

Q13. A $100\,\Omega$ resistance, a $0.1\,\mu F$ capacitor and an inductor are connected in series across a $250\,V$ supply at variable frequency. Calculate the value if inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz.

(A) $7.03 \times 10^{-5}\,H$ (B) 70.3 H

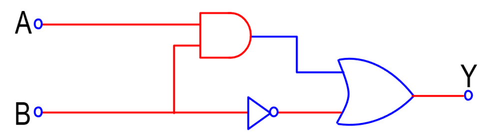
(C) 70.3 mH (D) 0.70 H

Q14. An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by :

(A) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (B) $\left(\frac{9P}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

(C) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (D) $\left(\frac{8P}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$

Q15. Find the truth table for the function Y of A and B represented in the following figure.



(A)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(B)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

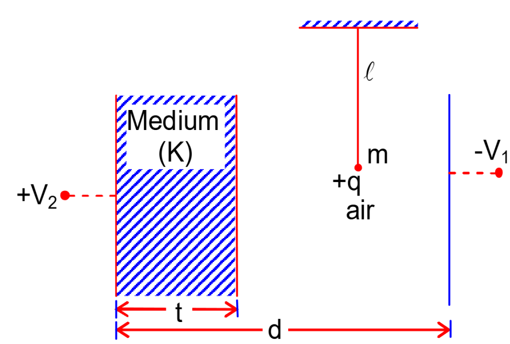
(C)

A	B	C
0	0	0
0	1	1
1	0	0
1	1	0

(D)

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Q16. A simple pendulum of mass 'm' , length 'ℓ' and charge '$+q$' suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be :



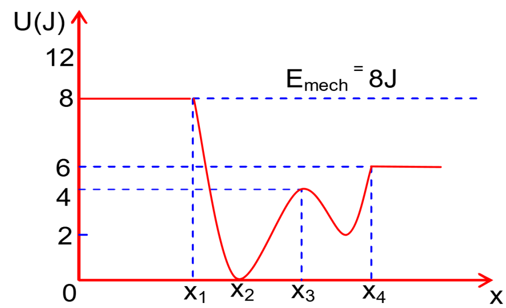
(A) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_2-V_1)}{(C_1+C_2)(d-t)}\right]$

(B) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_1(V_1+V_2)}{(C_1+C_2)(d-1)}\right]$

(C) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_1(V_2-V_1)}{(C_1+C_2)(d-t)}\right]$

(D) $\tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

Q17. Given below is the plot of a potential energy function U(x) for a system, in which a particle is in one dimensional motion, while a conservation force F(x) acts on it. Suppose that $E_{mech} = 8\,J$, the incorrect statement for this system is :



(A) at $x = x_3$, K.E. = 4 J.

(B) at $x > x_4$, K.E. is constant throughout the region .

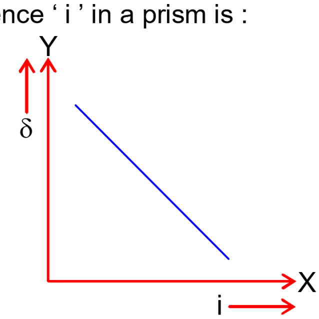
(C) at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

(D) at $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.

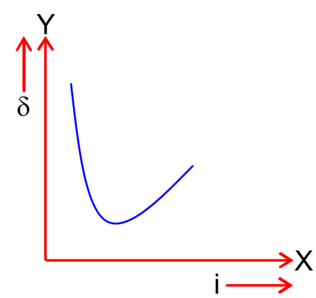
[where K.E. = kinetic energy]

Q18. The expected graphical representation of the variation of angle of deviation 'δ' with angle of incidence 'i' in a prism is :

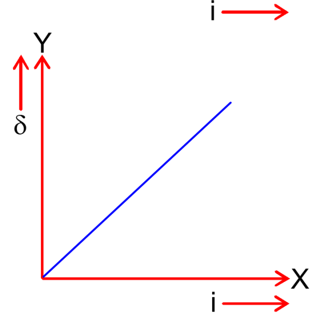
(A)



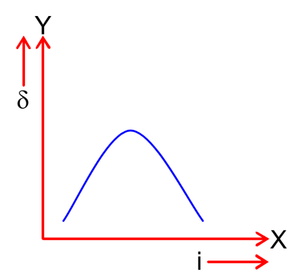
(B)



(C)



(D)



Q19. A physical 'y' is represented by the formula $y = m^2 r^{-4} g^x \ell^{-\frac{3}{2}}$ if the percentage errors found in y, m, r, ℓ and g are 18, 1, 0.5, 4 and p respectively, then find the value of x and p.

(A) 5 and $\pm$ 2

(B) $\frac{16}{3}$ and $\pm\frac{3}{2}$

(C) 8 and $\pm$ 2

(D) 4 and $\pm$ 3

Q20. A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1-\left(\frac{t-T}{T}\right)^2\right]$$

Where F_0 and T are constants. The force acts only for the time interval 2T. The velocity υ of the particle after time 2T is :

(A) $F_0T/3M$

(B) $2F_0T/M$

(C) $4F_0T/3M$

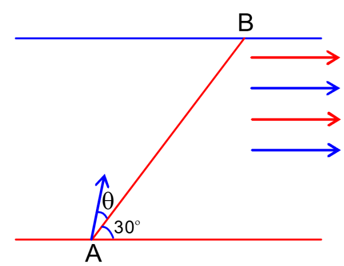
(D) $F_0T/2M$

SECTION - B

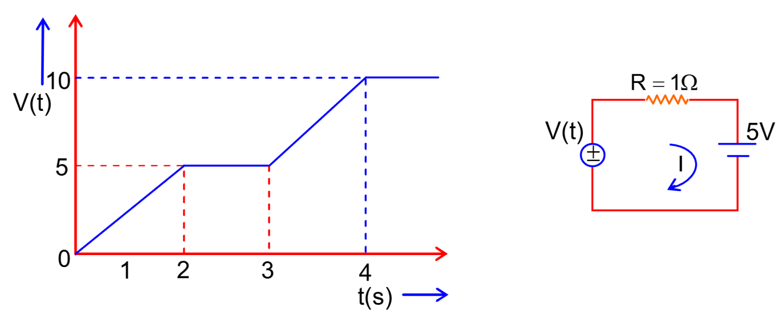
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be ________ $^{\circ}$, so that the swimmer reaches point B.



Q2. For the circuit shown, the value of current at time $t = 3.2$ s will be ________A.

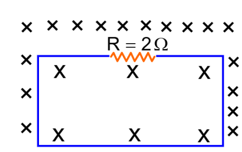


[Voltage distribution V(t) is shown by Fig.(1) and the circuit is shown in Fig.(2)]

Q3. The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is ________ mm.

[Refractive index of air $= 1.0003$, wavelength of yellow light in vacuum $= 6000\,\mathring{A}$]

Q4. In the given figure the magnetic flux through the loop increases according to the relation $\phi_{\beta}(t) 10t^2 + 20t$, where ϕ_{β} is in milliwebers and t is in seconds. The magnitude of current through $R = 2\Omega$ resistor at $t = 5\,s$ is __________mA.

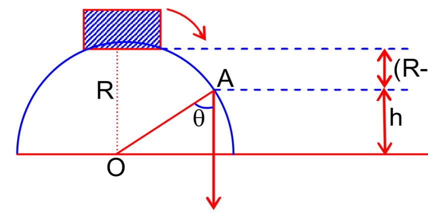


Q5. The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth 'h' below the water level. The value of 'h' for which the emerging stream of water strikes the ground at the maximum range is __________m.

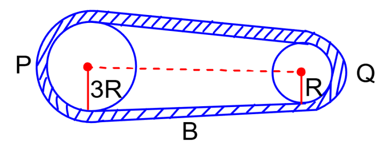
Q6. The K_{α} - X ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atom with a K electron knocked out is 27.5 ke V, the energy of this atom when an L electron is knocked out will be _______ ke V. (Round off to the nearest integer)
[$h = 4.14 \times 10^{-15}$ eVs, $c = 3 \times 10^{8}\,ms^{-1}$]

Q7. The maximum amplitude wave is found to be 12 V while the minimum amplitude is found to be 3 V. The modulation index is 0.6x where x is _________.

Q8. A small block slides down from the top of hemisphere of radius R = 3m as shown in the figure. The height ' h' at which the block will lose contact with the surface of the sphere is __________ m.
(Assume there is no friction between the block and the hemisphere)



Q9. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be __________.



Q10. A particle executes simple harmonic motion represented by displacement function as
$x(t) = A\sin(\omega t + \phi)$
If the position and velocity of the particle at $t = 0\,s$ are $2\,cm$ and 2ω cm s^{-1} respectively, then its amplitude is $x\sqrt{2}\,cm$ where the value of x is __________.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Select the correct statements.
(A) Crystalline solids have long range order.
(B) Crystalline solids are isotropic.
(C) Amorphous solids are sometimes called pseudo solids
(D) Amorphous solids soften over a range of temperatures.
(E) Amorphous solids have a definite heat of fusion
Choose the most appropriate answer from the options given below:
(A) (B), (D) only
(B) (A), (B), (E) only
(C) (A), (C), (D) only
(D) (C), (D) only

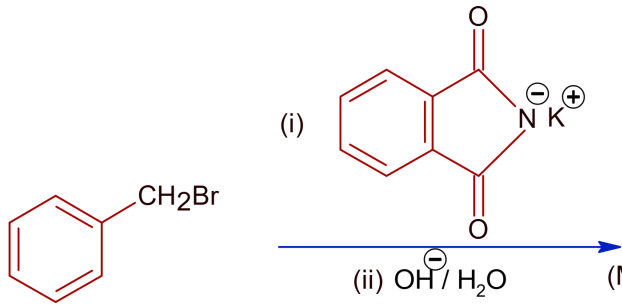
Q2. If the Thompson model of the atom was correct, then the result of Rutherford's gold foil experiment would have been:
(A) All $\alpha -$ particles get bounced back by 180^{o}.
(B) $\alpha -$ particles are deflected over a wide range of angles.
(C) All of $\alpha -$ particles pass through the gold foil without decrease in speed.
(D) $\alpha -$ particles pass through the gold foil deflected by small angles and with reduced speed.

Q3. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: SO_2(g) is adsorbed to a larger extent than H_2(g) on activated charcoal.
Reason R: SO_2(g) has a higher critical temperature than H_2(g)
In the light of the above statements, choose the most appropriate answer from the option given below.
(A) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.
(B) **A** is not correct but **R** is correct.
(C) **A** is correct but **R** is not correct.
(D) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

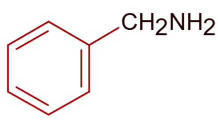
Q4. Compound A gives D-Galactose and D-Glucose on hydrolysis. The compound A is:
(A) Amylose
(B) Sucrose
(C) Maltose
(D) Lactose

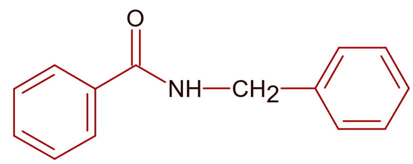
Q5. Given below are two statements:
Statement I : Hyper conjugation is permanent effect.
Statement II : Hyper conjugation in ethyl cation $\left(CH_3 - \overset{+}{C}H_2\right)$ involves the overlapping of $Csp^2 - H_{1s}$ bond with empty 2p orbital of other carbon.
Choose the correct option:
(A) **Statement I** is incorrect but **Statement II** is true.
(B) **Statement I** is correct but **Statement II** is false.
(C) Both **Statement I** and **Statement II** are false.
(D) Both **Statement I** and **Statement II** are true.

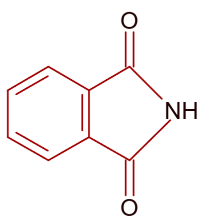
Q6. What is A in the following reaction?

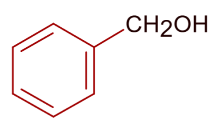


A
(Major Product)

(A) 

(B) 

(C) 

(D) 

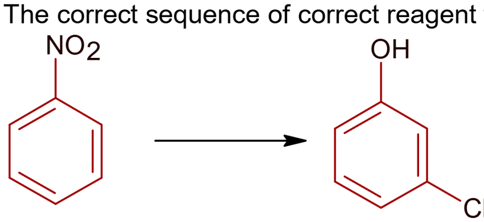
Q7. Match List – I with List – II:

List – I	**List – II**
(a) Li	(i) photoelectric cell
(b) Na	(ii) absorbent of CO_2
(c) K	(iii) coolant in fast breeder nuclear reactor
(d) Cs	(iv) treatment of cancer
	(v) bearings for motor engines

Choose the correct answer from the options given below:

(A) (a) – (v), (b) –(ii), (c) –(iv), (d) –(i)
(B) (a) –(iv), (b) – (iii), (c) –(i), (d) – (ii)
(C) (a) – (v), (b) – (i) , (c)- (ii), (d) – (iv)
(D) (a) –(v), (b) – (iii) , (c) – (ii), (d) –(i)

Q8. The correct sequence of correct reagent for the following transformation is:



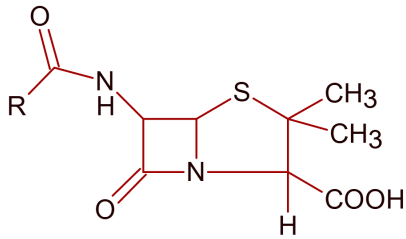
(A) (i) $Cl_2, FeCl_3$ (ii) $NaNO_2, HCl, 0^\circ C$ (iii) Fe, HCl (iv) H_2O/H^+
(B) (i) $Cl_2, FeCl_3$ (ii) Fe, HCl (iii) $NaNO_2, HCl, 0^\circ C$ (iv) H_2O/H^+
(C) (i) Fe, HCl (ii) Cl_2, HCl (iii) $NaNO_2, HCl, 0^\circ C$ (iv) H_2O/H^+
(D) (i) Fe, HCl (ii) $NaNO_2, HCl, 0^\circ C$ (iii) H_2O/H^+ (iv) $Cl_2, FeCl_3$

Q9. $R-CN \xrightarrow[(ii)H_2O]{(i)DIBAL-H} R-Y$

Consider the above reaction and identify "Y"

(A) –COOH
(B) – CH_2NH_2
(C) –CHO
(D) – $CONH_3$

Q10. Given below are two statements :
Statement I: Penicillin is a bacteriostatic type antibiotic.
Statement II: The general structure of penicillin is:

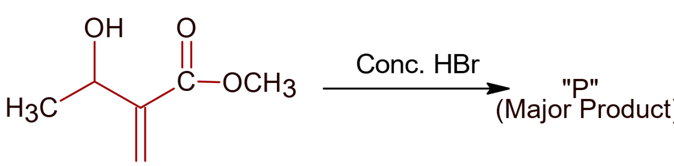


Choose the correct option:
(A) Both **statement I** and **statement II** are true
(B) **Statement I** is correct but **statement II** is false
(C) **Statement I** is incorrect but **statement II** is true
(D) Both **statement I** and **statement II** are false

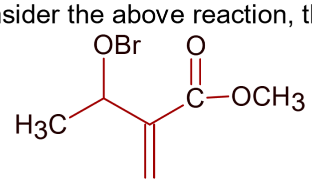
Q11. Which one of the following set of elements can be detected using sodium fusion extract?
(A) Halogens, Nitrogen, Oxygen, Sulfur
(B) Sulfur, Nitrogen, Phosphorous Halogens
(C) Phosphorous, Oxygen, Nitrogen, Halogens
(D) Nitrogen, Phosphorous, Carbon, Sulfur

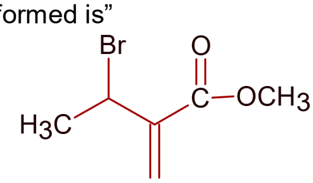
Q12. The addition of silica during the extraction of copper from its sulphide ore
(A) reduces copper sulphide into metallic copper
(B) reduces the melting point of the reaction mixture
(C) converts copper sulphide into copper silicate
(D) converts iron oxide into iron silicate

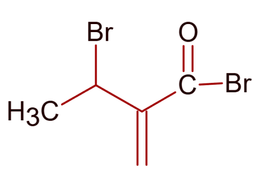
Q13.

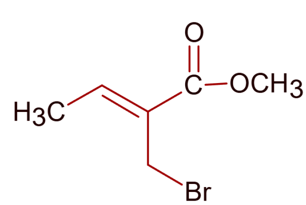


Consider the above reaction, the major product "P" formed is"

(A) 

(B) 

(C) 

(D) 

Q14. The number of neutrons and electrons respectively, present in the radioactive isotope of hydrogen is:
(A) 2 and 1
(B) 1 and 1
(C) 3 and 1
(D) 2 and 2

Q15. The CORRECT order of first ionisation enthalpy is:
(A) Mg < Al < S < P
(B) Al < Mg < S< P
(C) Mg < S < Al < P
(D) Mg < Al < P < S

Q16. Given below are two statements:
Statement I : $\left[Mn(CN)_6\right]^{3-}$, $\left[Fe(CN)_6\right]^{3-}$ and $\left[Co(C_2O_4)_3\right]^{3-}$ are d^2sp^3 hybridised
Statement II : $[MnCl_6]^{3-}$ and $[FeF_6]^{3-}$ are paramagnetic and have 4 and 5 unpaired electrons, respectively.
In the light of the above statements, choose the correct answer from the options given below:
(A) **Statement I** is correct but **Statement II** is false
(B) Both **Statement I** and **Statement II** are false
(C) **Statement I** is incorrect but **Statement II** is true
(D) Both **Statement I** and **Statement II** are true

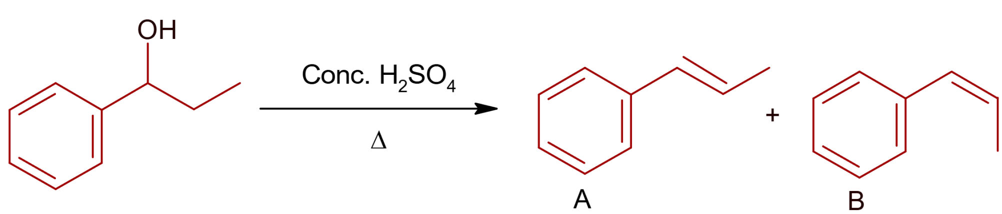
Q17. To an aqueous solution containing ions such as $Al^{3+}, Zn^{2+}, Ca^{2+}, Fe^{3+}, Ni^{2+}, Ba^{2+}$ and Cu^{2+} was added conc. HCl, followed by H_2S.
The total number of cations precipitated during this reaction is / are:
(A) 4
(B) 1
(C) 3
(D) 2

Q18. Match List – I with List – II:

List – I (compound)	**List – II (effect / affected species)**
(a) Carbon monoxide	(i) Carcinogenic
(b) Sulphur dioxide	(ii) Metabolized by pyrus plants
(c) Polychlorinated biphenyls	(iii) Haemoglobin
(d) Oxides of nitrogen	(iv) Stiffness of flower buds

Choose the correct answer from the options given below:
(A) (a) –(iv), (b) –(i), (c) – (iii), (d) – (ii)
(B) (a) – (i), (b) –(ii), (c) – (iii), (d) – (iv)
(C) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)
(D) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)

Q19.



Consider the above reaction, and choose the correct statement:
(A) Both compounds **A** and **B** are formed equally
(B) The reaction is not possible in acidic medium
(C) Compound **A** will be the major product
(D) Compound **B** will be the major product

Q20. Number of Cl=O bonds in chlorous acid, chloric acid and perchloric acid respectively are:
(A) 4,1 and 0
(B) 1,1 and 3
(C) 1,2 and 3
(D) 3,1 and 1

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. In a solvent 50% of an acid HA dimerizes and the rest dissociates. The van't Hoff factor of the acid is ________________ $\times 10^{-2}$. (Round off to the Nearest Integer).

Q2. For the first order reaction A$\rightarrow$ 2B, 1 mole of reactant A gives 0.2 moles of B after 100 minutes. The half life of the reaction is __________min. (Round off to the Nearest Integer).

[Use : ln2 = 0.69, ln 10 = 2.3 Properties of logarithms : ln x^y = y ln x; $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$]

Q3. $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$

The above reaction is carried out in a vessel starting with partial pressure $P_{SO_2} = 250$ m bar, $P_{O_2} = 750$ m bar and $P_{SO_3} = 0$ bar. When the reaction is complete, the total pressure in the reaction vessel is____________ m bar.
(Round off to the Nearest Integer).

Q4. The equilibrium constant for the reaction

$A(s) \rightleftharpoons M(s) + \frac{1}{2}O_2(g)$

In $K_p = 4$. At equilibrium, the partial pressure of O_2 is _________ atm.
(Round off to the Nearest Integer).

Q5. For the cell $Cu(s)\,|\,Cu^{2+}(aq)(0.1M)\,||\,Ag^{+}(aq)(0.01M)\,|\,Ag(s)$

The cell potential $E_1 = 0.3095V$

For the cell $Cu(s)\,|\,Cu^{2+}(aq)(0.01M)\,||\,Ag^{+}(aq)(0.001M)\,|\,Ag(s)$

The cell potential = ________$\times 10^{-2}$ V. (Round off to the Nearest Integer).

[Use : $\frac{2.303RT}{F} = 0.059$]

Q6. The dihedral angle in staggered from the Newman projection of 1,1,1,-Trichloro ethane is _____________ degree. (Round off to the Nearest Integer).

Q7. The total number of electrons in all bonding molecular orbitals of O_2^{2-} is __________.
(Round off to the Nearest Integer).

Q8. 3 moles of metal complex with formula $Co(en)_2Cl_3$, gives 3moles of silver chloride on treatment with excess of silver nitrate. The secondary valency of Co in the complex is__________. (Round off to the Nearest Integer).

Q9. When 400 mL of 0.2 M H_2SO_4 solution is mixed with 600 mL of of 0.1 M NaOH solution the increase in temperature of the final solution is__________$\times 10^{-2}$ K.
(Round off to the Nearest Integer).

[Use: $H^+(aq) + OH^-(aq) \rightarrow H_2O : \Delta_\gamma H = -57.1 kJmol^{-1}$

Specific heat of $H_2O = 4.18\,JK^{-1}g^{-1}$
Density of $H_2O = 1.0\,gcm^{-3}$
Assume no change in volume of solution on mixing]

Q10. 10.0 ml of 0.05 M $KMnO_4$ solution was consumed in a titration with 10.0 mL of given oxalic acid dehydrate solution. The strength of given oxalic acid solution is ___$\times 10^{-2}$ g/ L
(Round off to the Nearest Integer).

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to :

(A) $\sqrt{3} + 1$
(B) $\frac{\sqrt{3}+1}{\sqrt{3}}$
(C) 2
(D) 1

Q2. Let N be the set of natural numbers and a relation R on N be defined by $R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is :

(A) reflexive but neither symmetric nor transitive
(B) an equivalence relation
(C) symmetric but neither reflexive nor transitive
(D) reflexive and symmetric, but not transitive

Q3. Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx,\ x > 2$. If $y(3) = 3$ then $y(4)$ is equal to

(A) 12
(B) 4
(C) 8
(D) 16

Q4. Let $f : [0, \infty) \to [0, 3]$ be a function defined by $f(x) = \begin{cases} \max\{\sin t : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos x, & x > \pi \end{cases}$

Then which of the following is true?

(A) f is differentiable everywhere in $(0, \infty)$
(B) f is not continuous exactly at two points in $(0, \infty)$
(C) f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$
(D) f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$

Q5. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to :

(A) 1
(B) 0
(C) 4
(D) 3

Q6. The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to :

(A) $\frac{2}{3}$ (B) $\frac{9}{2}$

(C) $\frac{16}{3}$ (D) $\frac{4}{3}$

Q7. Let $f : R \to R$ be defined as

$f(x+y) + f(x-y) = 2f(x)f(y), f\left(\frac{1}{2}\right) = -1$. Then, the value of

$\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k+f(k))}$ is equal to :

(A) $\operatorname{cosec}^2(21)\cos(20)\cos(2)$ (B) $\sec^2(1)\sec(21)\cos(20)$

(C) $\sec^2(21)\sin(20)\sin(2)$ (D) $\operatorname{cosec}^2(1)\operatorname{cosec}(21)\sin(20)$

Q8. Let the mean and variance of the frequency distribution

x:	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f:	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be :

(A) 4 (B) 5

(C) $\frac{17}{3}$ (D) $\frac{16}{3}$

Q9. The point $P(a,b)$ undergoes the following three transformations successively :

(a) reflection about the line y = x

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a + b$ is equal to :

(A) 5 (B) 7

(C) 13 (D) 9

Q10. Consider a circle C which touches the y-axis at $(0, 6)$ and cuts off an intercept $6\sqrt{5}$ on the x-axis. Then the radius of the circle C is equal to :

(A) 9 (B) $\sqrt{82}$

(C) 8 (D) $\sqrt{53}$

Q11. Let A and B be two 3 x 3 real matrices such that $(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to :

(A) 2 (B) 1

(C) 0 (D) 4

Q12. Let $\alpha = \max_{x \in R}\left\{8^{2\sin 3x} . 4^{4\cos 3x}\right\}$ and $\beta = \min_{x \in R}\left\{8^{2\sin 3x} . 4^{4\cos 3x}\right\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$, then the value of c – b is equal to :

(A) 42 (B) 43
(C) 47 (D) 50

Q13. Let c be the set of all complex numbers. Let
$S_1 = \{z \in C : |z-2| \le 1\}$ and
$S_2 = \{z \in C : z(1+i) + \bar{z}(1-i) \ge 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to :

(A) $\frac{5+2\sqrt{2}}{4}$ (B) $\frac{5+2\sqrt{2}}{2}$
(C) $\frac{3+2\sqrt{2}}{4}$ (D) $\frac{3+2\sqrt{2}}{2}$

Q14. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}} + 3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is :

(A) 1 (B) 2
(C) 0 (D) –1

Q15. The value of $\lim_{x \to 0}\left(\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$ is equal to :

(A) 4 (B) 0
(C) –4 (D) –1

Q16. For real numbers α and $\beta \ne 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$, lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to :

(A) 3 (B) 9
(C) 7 (D) 5

Q17. Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point :

(A) $(2,1)$ (B) $(2,2)$
(C) $(1,3)$ (D) $(1,2)$

Q18. Which of the following is the negation of the statement "for all M > 0, there exists $x \in S$ such that $x \ge M$"?

(A) there exists M > 0, there exists $x \in S$ such that x < M
(B) there exists M > 0, such that $x \ge M$ for all $x \in S$
(C) there exists M > 0, there exists $x \in S$ such that $x \ge M$
(D) there exists M > 0, such that x < M for all $x \in S$

Q19. A student appeared in an examination consisting of 8 true – false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is :

(A) 5
(B) 6
(C) 3
(D) 4

Q20. Let $f:(a,b)\rightarrow R$ be twice differentiable such that $f(x)=\int_a^x g(t)dt$ for a differentiable function $g(x)$. If $f(x)=0$ has exactly five distinct roots in (a, b), then $g(x)g'(x)=0$ has at least :

(A) seven roots in (a, b)
(B) twelve roots in (a, b)
(C) five roots in (a, b)
(D) three roots in (a, b)

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^{x} + 1 = 0$ is equal to............

Q2. If the real part of the complex number $z = \frac{3 + 2i\cos\theta}{1 - 3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to........

Q3. Let n be a non-negative integer. Then the number of divisors of the form "$4n + 1$" of the number $(10)^{10} . (11)^{10} . (13)^{13}$ is equal to.........

Q4. The distance of the point $P(3,4,4)$ from the point of intersection of the line joining the points $Q(3,-4,-5)$ and $R(2,-3,1)$ and the plane $2x + y + z = 7$, is equal to...........

Q5. Let $A = \{n \in N | n^2 \le n + 10{,}000\}, B = \{3k + 1 | k \in N\}$ and $C = \{2k | k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to......

Q6. If $\int_0^{\pi} (\sin^3 x) e^{-\sin^2 x} dx = \alpha - \frac{\beta}{e} \int_0^1 \sqrt{t}\, e^t dt$, then $\alpha + \beta$ is equal to........

Q7. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + ... + A^{20}$, then the sum of all the elements of the matrix M is equal to..........

Q8. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at $(3,-4)$, one focus at $(4,-4)$ and one vertex at $(5,-4)$. If $mx - y = 4, m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to...........

Q9. Let $y = y(x)$ be the solution of the differential equation $dy = e^{\alpha x + y} dx; \alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to...........

Q10. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{j} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} . \vec{b} = -1$ and $\vec{b} . \vec{c} = 10$, then $(\vec{a} \times \vec{b}) . \vec{c}$ is equal to.......

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**B**	2.	**A**	3.	**A**	4.	**A**
5.	**A**	6.	**D**	7.	**A**	8.	**B**
9.	**C**	10.	**A**	11.	**B**	12.	**A**
13.	**B**	14.	**A**	15.	**B**	16.	**D**
17.	**C**	18.	**B**	19.	**B**	20.	**C**

SECTION - B

1.	**30**	2.	**1**	3.	**2**	4.	**60**
5.	**6**	6.	**10**	7.	**1**	8.	**2**
9.	**9**	10.	**2**				

PART – B (CHEMISTRY)

SECTION - A

1.	**C**	2.	**D**	3.	**D**	4.	**D**
5.	**B**	6.	**A**	7.	**D**	8.	**B**
9.	**C**	10.	**C**	11.	**B**	12.	**D**
13.	**D**	14.	**A**	15.	**B**	16.	**D**
17.	**B**	18.	**D**	19.	**C**	20.	**C**

SECTION - B

1.	**125**	2.	**654**	3.	**875**	4.	**16**
5.	**28**	6.	**60**	7.	**10**	8.	**6**
9.	**82**	10.	**1575**				

PART – C (MATHEMATICS)

SECTION - A

1.	**C**	2.	**A**	3.	**A**	4.	**A**
5.	**B**	6.	**B**	7.	**D**	8.	**C**
9.	**D**	10.	**A**	11.	**C**	12.	**A**
13.	**B**	14.	**A**	15.	**C**	16.	**C**
17.	**B**	18.	**A**	19.	**A**	20.	**A**

SECTION – B

1.	**2**	2.	**1**	3.	**792**	4.	**7**
5.	**16**	6.	**5**	7.	**2020**	8.	**9**
9.	**2**	10.	**9**				

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. $T^2 = \frac{4\pi^2 r^2}{GM}$

$\Rightarrow M = \frac{4\pi^2 r^3}{G \times T^2}$

$\Rightarrow M = \frac{6 \times 10^{11} \times \left(9 \times 10^6\right)^3}{\left(450 \times 60\right)^2}$

$= 6 \times 10^{23}\,kg$

Sol2. At terminal speed

$Mg = Fv = 6\pi\eta Rv$

$\Rightarrow V = \frac{mg}{6\pi\eta R}$

$\Rightarrow \frac{V = \delta \times \frac{4}{3}\pi R^3 g}{6\pi\eta R}$

$= \frac{2 \times \delta R^3 \times g}{9\eta}$

$= \frac{2 \times 1000 \times \left(0.2 \times 10^{-3}\right)^3 \times 10}{9 \times 1.8 \times 10^{-5}}$

$= \frac{400}{81}\ m/s$

$= 4.94\ m/s$

Sol3. As b > a

$\Rightarrow \beta_a > \beta_b$

$\beta_a = \frac{\mu_0 I}{2\pi a}$

$\beta_b = \frac{\mu_0 I}{2\pi b}$

Sol4. $R = R_0\left[1 + \alpha\,\Delta T\right]$

$\Rightarrow 16 = R_0\left[1 + \alpha\left(15 - 0\right)\right]$

$20 = R_0\left[1 + \alpha\left(100 - 0\right)\right]$

$$\Rightarrow \frac{16}{20} = \frac{1+\alpha \times 15}{1+\alpha \times 100}$$
$$\Rightarrow \alpha = 0.003^{\circ}C^{-1}$$

Sol5. $F = \frac{Gm^2}{(2R)^2} = m\omega^2 R$ Given $m = 1kg$

$$\Rightarrow \omega = \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

Sol6. Density of nucleus is constant .

Sol7. $[C] = \frac{Q}{V} = \frac{Q}{W/Q} = \frac{Q^2}{W} = \frac{A^2T^2}{M'L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

$$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$$
$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \mu. = \frac{1}{\varepsilon_0 c^2}$$
$$\Rightarrow [\mu_0] = [M^1L^1T^{-2}A^{-2}]$$
$$E = \frac{F}{q} = \frac{MLT^{-2}}{AT}$$
$$\Rightarrow [E] = MLT^{-3}A^{-1}$$

Sol8. Degrees of freedom (f) = 3 translational + 3rotational + 8vibrational = 14

$$y = 1 + \frac{2}{f}$$
$$\Rightarrow y = 1 + \frac{2}{14} = \frac{8}{7}$$
$$W = \frac{nR\Delta T}{y-1} = \frac{1\times 8.3\times 10}{8/7-1} = -582\,J$$

As $W < 0 \Rightarrow$ work is done on the gas

Sol9. $W_A = 1 - \frac{T}{T_1}$

$$W_B = 1 - \frac{T_3}{T}$$
$$W_A = W_B$$
$$\Rightarrow Q_1 - Q_2 = \frac{Q_2}{2} - Q_3$$
$$\Rightarrow 2\times\frac{Q_1}{Q_2} + \frac{2Q_3}{Q_2} = 3$$
$$\Rightarrow \frac{2T_1}{T} + \frac{T_3}{T} = 3 \Rightarrow \frac{2T_1}{3} + \frac{T_3}{3} = T$$

Sol10. Energy of electron = 3 eV
In 2nd excited state,

$$E = -\frac{(13.6)}{9} = -1.51\text{eV}$$

Photon Energy $\frac{hc}{\lambda} = 4.512\,\text{eV}$

$$kE_{max} = 4.512 - \frac{12400\text{eV}\,\mathring{A}}{4000\,\mathring{A}}$$

$= 1.41\text{eV}$

Sol11. $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow 0.2 = 2\pi\sqrt{\frac{0.5}{k}}$$

$$\Rightarrow k = 50\,\pi^2 = 500$$

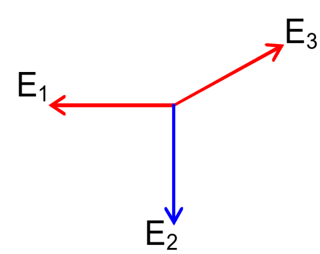
$x = A\sin\omega t$

$$\Rightarrow x = 5\sin\left(\omega\times\frac{T}{4}\right) = 5\,\text{cm}$$

$$PE = \frac{1}{2}\times k\,x^2 = \frac{1}{2}\times 500\times\left(\frac{5}{100}\right)^2$$

= 0.6255

Sol12. $E_1 = \frac{kq}{\ell^2} = E_2$



$$E_3 = \frac{kq}{\left(\sqrt{2}\ell\right)^2} = \frac{kq}{2\ell^2}$$

$$E = \frac{kq}{2\ell^2}\left(2\sqrt{2}-1\right)$$

Sol13. $\omega = 2\pi f = \frac{1}{\sqrt{LC}}$

$$\Rightarrow L = \frac{1}{4\pi^2 f^2\times C} = \frac{1}{4\pi^2\times(60)^2\times 10^{-7}}$$

$$\Rightarrow L = 70.3\text{H}$$

Sol14. P = constant

$$\Rightarrow P = FV = mv\frac{dv}{dx}V$$

$$\Rightarrow \frac{P}{m}\int dx = \int V^2\,dv$$

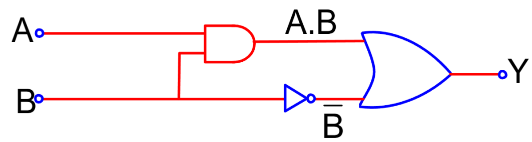
$$\Rightarrow \frac{Px}{m} = \frac{V^3}{3}$$

$$\Rightarrow \left(\frac{3P}{m}x\right)^{1/3} = v = \frac{dx}{dt}$$

$$\Rightarrow \left(\frac{3P}{m}\right)^{1/3}\int_0^t dt = \int_0^x x^{-1/3}dx \quad \Rightarrow x = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

Sol15. $Y = A.B + \overline{B}$

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1



Sol16. $T\sin\theta = qE$

$T\cos\theta = mg$

$\Rightarrow \tan\theta = \frac{qE}{mg}$

$Q = \frac{C_1C_2}{C_1+C_2} \times (V_1+V_2)$

$E = \frac{Q}{A\varepsilon_1} = \frac{C_1C_2}{C_1+C_2}\left[\frac{V_1+V_2}{A\varepsilon}\right]$

$C_1 = \frac{\varepsilon_0 A}{d-t} \Rightarrow \frac{C_2[V_1+V_2]}{(C_1+C_2)(d-t)}$

$\theta = \tan^{-1}\left[\frac{qE}{mg}\right]$

$\Rightarrow \theta = \tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$

Sol 17. $ME = PE + KE = 8\,J$

At x_3, $8 = KE + 4 \Rightarrow KE = 4\,J$

At x_2, $8 = KE + 0 \Rightarrow KE_{max} = 8\,J$

At $x < x_1$, $8 = KE + 8 \Rightarrow KE$ is O

Sol18. $\delta = i + e - A$

δ_{min} for $i = e$

Sol19. $\left(\frac{\Delta y \times 100}{y}\right) = \left[2\times\frac{\Delta m}{m} + \frac{4\times\Delta r}{r} + \frac{x\times\Delta g}{g} + \frac{3}{2}\times\frac{\Delta \ell}{\ell}\right]\times 100$

$\Rightarrow 18 = 2\times1 + 4\times0.5 + x\times p + \frac{3}{2}\times 4$

$\Rightarrow 8 = xp$

$\Rightarrow x = \frac{16}{3}$; $p = \pm\frac{3}{2}$

Sol20. $a = \frac{F_0}{M} - \frac{F_0}{MT^2}(t-T)^2 = \frac{dv}{dt}$

$\Rightarrow \int_0^V dv = \int_0^{2T}\left[\frac{F_0}{M} - \frac{F_0}{MT^2}(t-T)^2\right]dt \Rightarrow v = \frac{4F_0T}{3M}$

SECTION - B

Sol1. $\overrightarrow{V_S} = \overrightarrow{V_{SR}} + \overrightarrow{V_R}$

$\because \left|\overrightarrow{v_{SR}}\right| = \left|\overrightarrow{v_R}\right| = v$

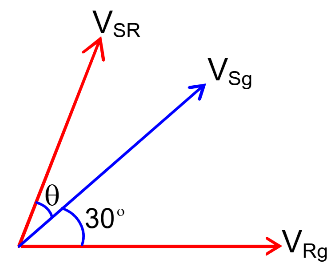
$\Rightarrow V_S = v(\cos(\theta + 30^\circ) + 1)\hat{i} + v\sin(\theta + 30^\circ)\hat{j}$

$\Rightarrow \tan 30^\circ = \dfrac{v\sin(\theta + 30^\circ)}{v\cos(\theta + 30^\circ) + v}$

$\Rightarrow \sqrt{3}\sin(\theta + 30^\circ) = \cos(\theta + 30^\circ) + 1$

$\Rightarrow \sin\theta = \dfrac{1}{2}$

$\Rightarrow \theta = 30^\circ$



Sol2. $t = 3.2s, V = 6\,\text{volts}$

$E_{net} = 6 - 5 = 1\text{volt}$

$\Rightarrow I = \dfrac{E_{net}}{R} = \dfrac{1}{1} = 1A$

Sol3. $n = \dfrac{L}{\lambda}$

$\Rightarrow \dfrac{L}{\lambda_a} - \dfrac{L}{\lambda v} = 1$

$\Rightarrow L\left[\dfrac{1}{\lambda_{v/\mu}} - \dfrac{1}{\lambda_v}\right] = 1$

$\Rightarrow \dfrac{L}{\lambda_v}[\mu - 1] = 1$

$\Rightarrow L = \dfrac{\lambda_v}{\mu - 1} = \dfrac{6000 \times 10^{-10}}{1.0003 - 1} = 2\text{mm}$

Sol4. $E = \left|\dfrac{d\phi}{dt}\right| = 20t + 20$

$I = \dfrac{E}{R} = 10t + 10$

At $t = 5s$

$I = 60\,\text{mA}$

Sol5. $R = \sqrt{2gh} \times \sqrt{\dfrac{2 \times (12 - h)}{g}}$

$\Rightarrow R = \sqrt{4h(12 - h)}$

For R_{max}

$\Rightarrow \dfrac{dR}{dh} = 0$

$\Rightarrow h = \dfrac{R}{2} = \dfrac{12}{2} = 6\,\text{m}$

Sol6. $E_{kd} = E_K - K_L$

$\Rightarrow \frac{h_c}{\lambda} = E_k - E_L$

$\Rightarrow E_L = E_k - \frac{h_c}{\lambda}$

$\Rightarrow E_L = 27.5 - \frac{12.42 \times 10^{-7}}{0.071 \times 10^{-9}}$

$\Rightarrow E_L = 27.5 - 17.5 = 10\,\text{keV}$

Sol7. Modulation Index $= \frac{Am}{Ac}$

$A_{max} = A_c + A_m = 12$

$A_{min} = A_c - A_m = 3$

$A_C = 15/2$ and $A_m = 9/2$

Modulation Index $= \frac{Am}{Ac} = \frac{9/2}{15/2} = 0.6$

$\Rightarrow 0.6x = 0.6$

$\Rightarrow x = 1$

Sol8. $mg\cos\theta = \frac{mv^2}{R}$ ...(i)

$\cos\theta = \frac{h}{R}$

Loss in potential Enooqy = Gain in kinetic Energy

$\Rightarrow mg(R-h) = \frac{1}{2}mv^2$ ...(ii)

$\Rightarrow mg\left(\frac{h}{R}\right) = \frac{2mg(R-h)}{R}$

$\Rightarrow h = \frac{2R}{3} = 2m$

Sol9. $\frac{1}{2} \times I_1\omega_1^2 = \frac{1}{2} \times I_2 \times \omega_2^2$

$\Rightarrow I_1 \times \left(\frac{v}{3R}\right)^2 = I_2 \times \left(\frac{V}{R}\right)^2$

$\Rightarrow \frac{I_1}{I_2} = \frac{9}{1} = \frac{x}{1}$

$\Rightarrow x = 9$

Sol10. $x = A\sin(\omega t + \phi)$

$V = A\omega\cos(\omega t + \phi)$

At t = 0

$\Rightarrow 2 = A\sin\phi$ ...(i)

$2\omega = A\omega\cos\phi$ ...(ii)

$\Rightarrow \tan\phi = 1$

$\Rightarrow \phi = 45^\circ$

Potting ϕ in equation (i)

$\Rightarrow 2 = A\sin 45 \Rightarrow A = 2\sqrt{2}\,cm \Rightarrow x = 2$

PART – B (CHEMISTRY)

SECTION - A

Sol1. Characteristics of crystalline solids:-
1. Crystalline solid have long range order.
2. Crystalline solid are anisotropic in nature.
3. Crystalline solid have definite heat of fusion.

Characteristics of amorphous solids:-
1. Amorphous solids have short range of order.
2. Amorphous solids are sometimes called pseudo solids.
3. Amorphous solids soften over a range of temperature.

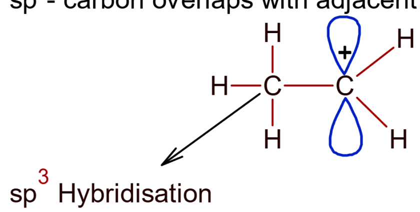
Sol2. When alpha particles pass through the gold foil, alpha particle deflected because according to Thompson model positive charge dispersed through out the atom.

Sol3. SO_2 is adsorbed more than H_2 on activated charcoal as critical temperature of SO_2 is higher than H_2. As higher the critical temperature, easier is liquifaction of gas and more is adsorption of gas on charcoal.

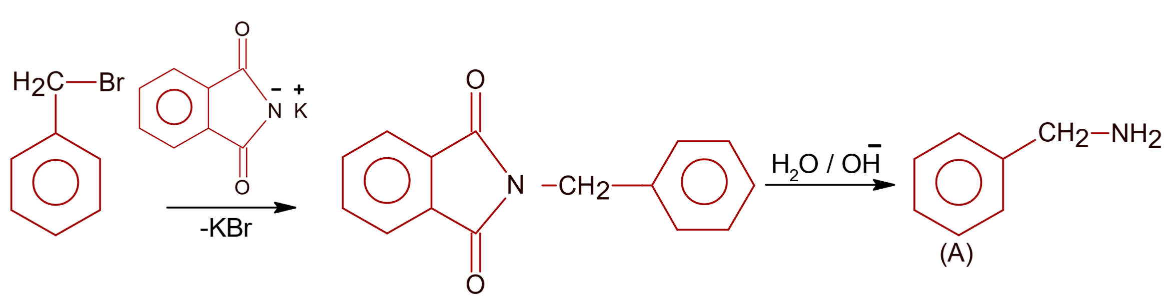
Sol4. Lactose is a disaccharide which is formed by forming C_1-C_4 glycosidic linkage between D-galactose and D- glucose.

$$\text{Lactose} \xrightarrow{\text{Hydrolysis}} \text{D - galactose + D - glucose}$$

Sol5. Hyper conjugation is a permanent electronic effect in which σ-bond electrons of hydrogen and sp^3- carbon overlaps with adjacent vacant p- orbitals

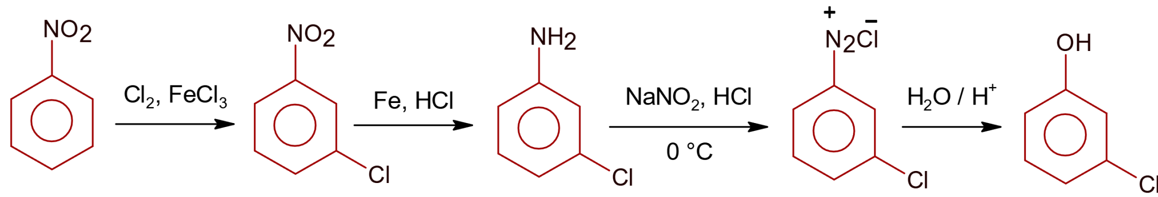


Sol6.



Sol7. (a) Li $\rightarrow$ used in bearings for motor engine.
(b) Na $\rightarrow$ used as coolant in fast breader nuclear reactor.
(c) K $\rightarrow$ used as absorbent of CO_2.
(d) Cs $\rightarrow$ used in photoelectric call.

Sol8.



Sol9. $R-CN \xrightarrow[H_2O]{DIBAL-H} R-CHO$

Sol10.

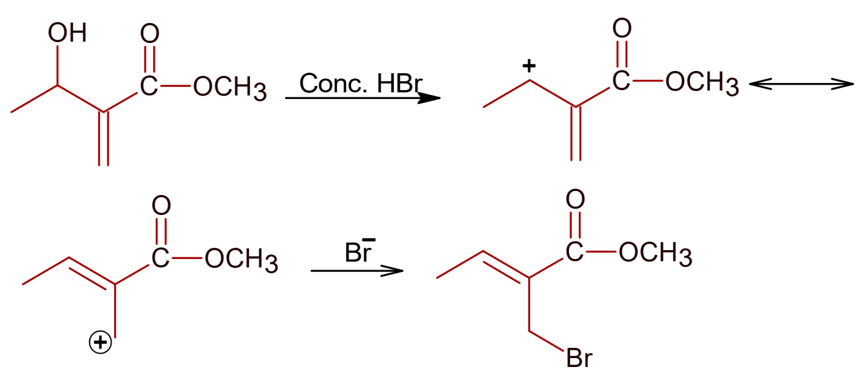
Bactericidal	Bacteriostatic
Penicillin	Erythromrycin
Aminogly cosides	Tetracycline
Ofloxacin	Chloramphenical

Structure of penicillin is correct

Sol11. Sodium fusion extract is Lassaigne method to defect sulphur, nitrogen, phosphorous and halogens.

Sol12. FeO + SiO_2 $\longrightarrow$ $FeSiO_3$ (Iron silicate)
Impurities Flux Slag

Sol13.



Sol14. Radioactive isotope of hydrogen is tritium $\left({}_1^3H\right)$

Number of neutron = 3 –1=2
Number of electron = 1

Sol15. Order of 1st I. E is Al < Mg < S < P

Sol16. $\left[Mn(CN)_6\right]^{3-} \Rightarrow Mn^{3+}$i.e $3d^4 \Rightarrow t_{2g}^{2,1,1}, e_g^{0.0}\left(d^2sp^3\right)$, CN^- is strong field ligand.

$\left[Fe(CN)_6\right]^{3-} \Rightarrow Fe^{3+}$i.e $3d^5 \Rightarrow t_{2g}^{2,2,1}, e_g^{0,0}\left(d^2sp^3\right)$, CN^- is strong field ligand.

$\left[Co(C_2O_4)_3\right]^{3-} \Rightarrow Co^{3+}$i.e, $3d^6 \Rightarrow t_{2g}^{2,2,2}, e_g^{0,0}\left(d^2sp^3\right)$, $C_2O_4^{2-}$ is strong field ligand.

$[MnCl_6]^{3-} \Rightarrow Mn^{3+} = 3d^4 \Rightarrow t_{2g}^{1,1,1} e_g^{1,0}\left(4\,\text{unpaired}\,e^-\right)$, Cl^- is weak field ligand.

$[FeF_6]^{3-} \Rightarrow Fe^{3+} = 3d^5 \Rightarrow t_{2g}^{1,1,1} e_g^{1,1}\left(5\,\text{unpaired}\,e^-\right)$ F^- is strong field ligand.

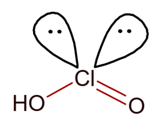
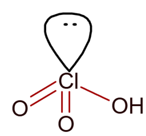
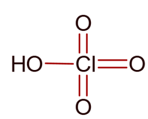
$\therefore$ S_1 and S_2 are true.

Sol17. (H_2S + dil HCl) is 2^{nd} group reagent so Cu^{2+} get precipitated as CuS.

Sol18. (A) Carbon monoxide – Haemoglobin
(B) Sulphur dioxide – Stiffness of flower buds
(C) Polychlorinated biphenyls – Carcinogenic
(D) Oxides of nitrogen – Metabolized by pyrus plants

Sol19. Trans product is more stable than Cis. (Trans product feel less repulsion due to opposite direction)

Sol20.

			(Cl=O bonds)
Chlorous Acid	($HClO_2$)		1
Chloric Acid	($HClO_3$)		2
Perchloric Acid	($HClO_4$)		3

SECTION - B

Sol1. $$i = \frac{\text{total no. of particles after dissociation / association}}{\text{total no. of particle before dissociation / association}}$$

$HA \longrightarrow H^+ + A^-$ (Dissociation)

0.5a   0.5a   0.5a

$2HA \longrightarrow (HA)_2$ (Association)

0.5a $\frac{0.5a}{2}$

$$i = \frac{a + 0.25a}{a} = 125 \times 10^{-2}$$

Ans. =125

Sol2.

	A	$\longrightarrow$	2B
t = 0	1 mole		0
after 100 min.	(1– 0.1) mol		0.2 mol

$$K = \frac{1}{t}\ell n\left(\frac{a}{a-n}\right)$$

$$K = \frac{2.303}{100}\log\left(\frac{1}{0.9}\right) \text{ min}^{-1}$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{100}[\log 10 - \log 9] = \frac{2.303}{100} \times 0.046$$

$$t_{1/2} = \frac{69.3}{2.303 \times 0.046} = 654.15\,\text{min}$$

Ans. = 654 (nearest integer)

Sol3. $2SO_2(g) \quad + \quad O_2(g) \longrightarrow 2SO_3(g)$

$t = 0 \quad 250$ m bar $\quad 750$ m bar $\quad -$

$0 \quad 625$ m bar $\quad 2 \times 250 = 500$

$P_{total} = 625 + 250\,\text{m bar}$

$= 875\,\text{m bar}$

Sol4. $K_p = (Po_2)^{1/2} = 4$

$Po_2 = 16$ atm

Sol5. $$E_{cell} = E^o_{cell} - \frac{0.059}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$$

$$E_1 = E^0_{cell} - \frac{0.059}{2}\log\frac{0.1}{(0.01)^2} = 0.3095$$

$E^0_{cell} = 0.3095 + 0.0885\,\text{V}$

$E^0_{cell} = 0.398\,\text{V}$

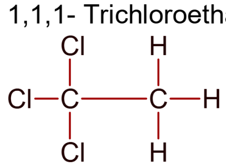
Again, $$E_2 = E^o_{cell} - \frac{0.059}{2}\log\frac{10^{-2}}{(10^{-3})^2}$$

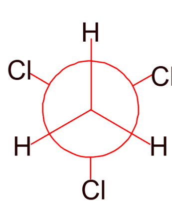
$$E_2 = 0.398 - \frac{0.059}{2} \times 4\,\text{V}$$

$E_2 = 0.28\,\text{V}$

$E_2 = 28 \times 10^{-2}\,\text{V}$

Sol6. 1,1,1- Trichloroethane





Ans. = 60

Sol7. E.C = $\sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^2\, \sigma^* 2s^2\, \sigma 2p_z^2\, \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^2 = \pi^* 2p_y^2$

Total electrons in BMO = 10

Sol8.

$3\,[Co(en)_2Cl_2]Cl + \underset{\text{excess}}{3AgNO_3} \longrightarrow \underset{\text{ppt}}{3AgCl\downarrow} + 3\,[Co(en)_2Cl_2]NO_3$

Secondary valency of complex = 6

Sol9.

	H_2SO_4	+	2NaOH $\longrightarrow$	$Na_2SO_4 + 2H_2O$
Milli mole	80		60 (limiting reagent)	
	H^+	+	OH^- $\longrightarrow$	H_2O
Milli mole	160		60	–
Milli mole	160–60		0	60

Total heat produced $= \dfrac{60}{1000}\times 57.1\times 1000$

$= 3426$ J

Total heat absorbed $= ms\Delta T$

$\Delta T = \dfrac{\text{Total heat}}{m\times s}$

$= \dfrac{3426J}{1000\times 4.18} = 0.819K$

$= 81.9\times 10^{-2}\,K$

Ans. = 82 (Nearest)

Sol10. $MnO_4^- + H_2C_2O_4.2H_2O \xrightarrow{H^+} Mn^{2+} + CO_2$

nf = (5) nf = (2)

Milli equivalent of $C_2O_4^{2-}$ = mili equivalent of MnO_4^-

$2\times M\times 10 = 5\times 0.05\times 10$

$M = 0.125\,M$

$M = \dfrac{\text{Strength}}{\text{M.M of } H_2C_2O_4\cdot 2H_2O}$

$\text{Strength} = 0.125\times 126\,g/L$

$= 15.75\,g/L$

$= 1575\times 10^{-2}\,g/L$

Ans. = 1575

PART – C (MATHEMATICS)

SECTION - A

Sol1. $\vec{a} = \vec{b}\times(\vec{b}\times\vec{c})$ ....(i)

$\Rightarrow \vec{a} = (\vec{b}.\vec{c})\vec{b} - (\vec{b}.\vec{b})\vec{c} = |\vec{b}||\vec{c}|\cos\theta\,\vec{b} - |\vec{b}|^2\vec{c} = 2\cos\theta\,\vec{b} - \vec{c}$ ....(ii)

Taking dot product with $\vec{b}$ in equation (ii)

$\vec{a}.\vec{b} = 2\cos\theta\,\vec{b}.\vec{b} - \vec{b}.\vec{c}$

$= 2\cos\theta - 2\cos\theta = 0$

Again $\vec{a}.\vec{a} = 2\cos\theta\,\vec{a}.\vec{b} - \vec{c}.\vec{a}$

$\Rightarrow \vec{c}\,.\,\vec{a} = -2$

Again, $\vec{a}\,.\,\vec{c} = 2\cos\theta\,\vec{b}\,.\,\vec{c} - 4$

$\Rightarrow -2+4 = 2\cos\theta\times 2\cos\theta$

$\Rightarrow \cos^2\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \qquad 0 < \theta < \frac{\pi}{2}$

$\theta = 45^\circ$

$\Rightarrow 1 + \tan\theta = 2$

Sol2. for reflexive (x, x)

$\Rightarrow x^3 - 3x^3 - x^3 + 3x^3 = 0$

$\therefore$ reflexive

For symmetric

$(x, y) \in R$

$x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$\Rightarrow (x - 3y)(x^2 - y^2) = 0$

For (y, x)

$(y - 3x)(y^2 - x^2) = 0$

$\Rightarrow (3x - y)(x^2 - y^2) = 0$

Not symmetric

Sol3. $xdy - ydx - x^2(xdy + ydx) + 3x^4dx = 0$

$\Rightarrow \frac{xdy - ydx}{x^2} - (xdy + ydx) + 3x^2dx = 0 \Rightarrow d\left(\frac{y}{x}\right) - d(xy) + d(x^3) = 0$

Integrate both side, we get

$\frac{y}{x} - xy + x^3 = c$

Put $x = 3, y = 3$

$\Rightarrow 1 - 9 + 27 = c$

$c = 19$

Put x = 4

$\frac{y}{4} - 4y = 19 - 64$

$\Rightarrow y = 12$

Sol4. $f(x) = \begin{cases} \sin x &, 0 \le x < \frac{\pi}{2} \\ 1 &, \frac{\pi}{2} \le x \le \pi \\ 2 + \cos x &, x > \pi \end{cases}$

$f'(x) = \begin{cases} \cos x &, 0 < x < \frac{\pi}{2} \\ 0 &, \frac{\pi}{2} < x < \pi \\ -\sin x &, x > \pi \end{cases}$

$f'\left(\frac{\pi}{2}^{+}\right) = 0$

$f'\left(\frac{\pi}{2}^{-}\right) = 0$

$f'\left(\pi^{+}\right) = 0$

$f'\left(\pi^{-}\right) = 0$

$\Rightarrow f(x)$ is differentiable in $(0, \infty)$

Sol5. $2x = \tan\frac{\pi}{9} + \tan\frac{7\pi}{18}$

$= \dfrac{\sin\left(\frac{\pi}{9} + \frac{7\pi}{18}\right)}{\cos\frac{\pi}{9}.\cos\frac{7\pi}{18}}$

$= \dfrac{1}{\cos\frac{\pi}{9}.\cos\left(\frac{7\pi}{18}\right)}$

$= \dfrac{1}{\cos\frac{\pi}{9}.\sin\left(\frac{\pi}{2} - \frac{7\pi}{18}\right)}$

$= \dfrac{1}{\cos\frac{\pi}{9}.\sin\frac{\pi}{9}}$

$\Rightarrow x = \dfrac{1}{2\cos\frac{\pi}{9}.\sin\frac{\pi}{9}}$

$= \dfrac{1}{\sin\frac{2\pi}{9}} = \operatorname{cosec}\frac{2\pi}{9}$

Again $2y = \tan\frac{\pi}{9} + \tan\frac{5\pi}{18}$

$\Rightarrow 2y = \dfrac{\sin\left(\frac{\pi}{9} + \frac{5\pi}{18}\right)}{\cos\frac{\pi}{9}.\cos\frac{5\pi}{18}}$

$= \dfrac{\sin\frac{7\pi}{18}}{\sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right).\sin\left(\frac{\pi}{2} - \frac{5\pi}{18}\right)}$

$= \dfrac{\sin\frac{7\pi}{18}}{\sin\frac{7\pi}{18} \times \sin\frac{4\pi}{18}} = \operatorname{cosec}\frac{2\pi}{9}$

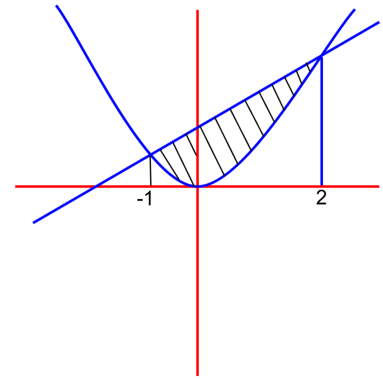
$\Rightarrow |x - 2y| = 0$

Sol6. $x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$x = -1, 2$

Required area = $\int_{-1}^{2} \left[(x+2) - x^2\right] dx = \frac{9}{2}$



Sol7. $f(x+y) + f(x-y) = 2f(x).f(y)$

$f\left(\frac{1}{2}\right) = -1$

$f(0) = 1$

Put $x = \frac{1}{2}, y = \frac{1}{2}$

$\Rightarrow f(2) = 1$

$\Rightarrow f(k) = 1$ for al integer

Now, $\sum_{k}^{20} = 1 \frac{1}{\sin k.\sin(k+1)}$

$= \frac{1}{\sin 1} \sum_{k=1}^{20} \left[\frac{\sin(k+1-k)}{\sin k.\sin(k+1)}\right]$

$= \frac{1}{\sin 1} \sum_{k=1}^{20} \cot k - \cot(k+1)]$

$= \frac{1}{\sin 1}(\cot 1 - \cot 21)$

$= \frac{1}{\sin 1} \frac{\sin 20}{\sin 1.\sin 21}$

$= \operatorname{cosec}^2 1 . \sin 20 . \operatorname{cosec} 21$

Sol8. mean = $\frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{32 + 8\alpha + 9\beta}{8 + \alpha + \beta} = 6$

$\Rightarrow 2\alpha + 3\beta = 16$ ......(i)

$d_i = x_i - \overline{x} = -4, 0, 2, 3$

$f_i d_i^2 = 64, 0, 4\alpha, 9\beta$

Variance $\sigma^2 = \frac{\Sigma f_i d_i^2}{\Sigma f_i} = 6.8$

$\Rightarrow \frac{64 + 4\alpha + 9\beta}{8 + \alpha + \beta} = 6.8$

$\Rightarrow 2.8\alpha + (-2.2\beta) = 9.6$

$\Rightarrow 28\alpha - 22\beta = 96$

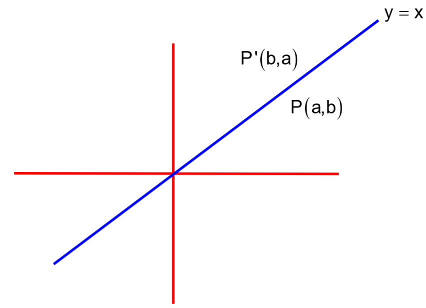
$14\alpha - 11\beta = 48$ ....(ii)

Solving (i) and (ii),

$\Rightarrow \beta = 2, \alpha = 5$

New mean = $\dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{85}{15} = \dfrac{17}{3}$

Sol9. Co-ordinate of $Q(b+2, a)$



$\Rightarrow -\dfrac{1}{2} + \dfrac{7i}{\sqrt{2}}$

$= (b+2+ai)e^{i\frac{\pi}{4}}$

$= (b+2+ai)\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$

$\Rightarrow b - a + 2 = -1$

$b + 2 - 1a = 7$

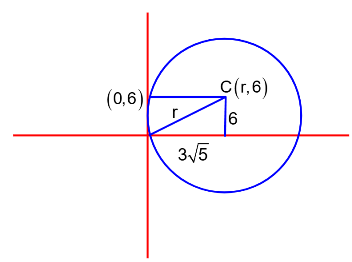
$\Rightarrow a = 4$

$b = 1$

$\Rightarrow 2a + b = 9$

Sol10. $\Rightarrow r^2 = 36 + 45$

$\Rightarrow r^2 = 81 \Rightarrow r = 9$



Sol11. $A^5 = B^5$ ....(i)

$A^3B^2 = A^2B^3$ ....(ii)

Subtract (i) & (ii)

$\Rightarrow A^3(A^2 - B^2) = B^3(B^2 - A^2)$

$\Rightarrow (A^2 - B^2)(A^3 + B^3) = 0$

$A^2 - B^2$ is invertible matrix

$\therefore |A^2 - B^2| \neq 0$

$\Rightarrow |A^2 - B^2| = 0$

$\therefore |A^3 + B^3| = 0$

Sol12. $\alpha = \max\{2^{6\sin 3x}. 2^{8\cos 3x}\}$

$= \max\{2^{6\sin 3x}. 2^{8\cos 3x}\} = 2^{10}$

$\beta = \min\{2^{6\sin 3x}. 2^{8\cos 3x}\} = 2^{-10}$

$\alpha^{\frac{1}{5}} + \beta^{\frac{1}{5}} = \dfrac{-b}{8}$

$\Rightarrow 4 + \dfrac{1}{4} = -\dfrac{b}{8}$

$\Rightarrow \frac{17}{4} = -\frac{b}{8} \Rightarrow b = -34$

Again $\alpha^{1/5}.\beta^{1/5} = \frac{e}{8}$

$\Rightarrow 4 \times \frac{1}{4} = \frac{e}{8}$

$\Rightarrow c = 8$

$\Rightarrow c - b = 8 + 34 = 42$

Sol13. $S_1 : |z-2| \le 1$ is circle with centre (2, 0) and radius less than equal to 1.

$S_2 : z(1+i) + \bar{z}(1-i) \ge 4$

Put $z = x + iy$

$y \le x - 2$

Solving with S1

$\Rightarrow x = 2 - \frac{1}{\sqrt{2}}, 2 + \frac{1}{\sqrt{2}}$

Point of intersection $P = \left(2 - \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\left|z - \frac{5}{2}\right|^2 = \left|2 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i - \frac{5}{2}\right|^2 = \frac{5\sqrt{2}+4}{4\sqrt{2}} = \frac{5+2\sqrt{2}}{4}$

Sol14. $\left[3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{\left(\frac{-1}{8}\right)\left(\log_3\left(5^{x-1}+1\right)\right)}\right]^{10}$ $\therefore T_9 = {}^{10}C_8\left(25^{x-1} \times 7\right)^{\frac{1}{2}\times 2} \times \left(5^{x-1}+1\right)^{\frac{-1}{8}\times 8} = 180$

$= \frac{45\left(5^{2x-2}+7\right)}{5^{x-1}} = 180$

$= \frac{5^{2x-2}+7}{5^{x-1}+1} = 4$

Put $5^{x-1} = t$

$\Rightarrow \frac{t^2+7}{t+1} = 4$

$\Rightarrow t^2 - 4t + 3 = 0$

$\Rightarrow t = 1, 3$

$\Rightarrow 5^{x-1} = 1$ or $5^{x-1} = 3$

$\Rightarrow x = 1$ or $x - 1 = \log_5^3$

Sol15. $\underset{x\to 0}{\text{Lt}} \frac{x}{(1-\sin x)^{\frac{1}{8}} - (1+\sin x)^{\frac{1}{8}}}$

$= \frac{2x}{(1-\sin x)^{\frac{1}{4}} - (1+\sin x)^{\frac{1}{4}}}$ Multiply by conjugate

$= \frac{4x}{(1-\sin x)^{\frac{1}{2}} - (1+\sin x)^{\frac{1}{2}}}$ Multiply by conjugate

$= \frac{8x}{1-\sin x-1-\sin x}$ Multiply by conjugate

$= -\frac{4x}{\sin x} = -4$

Sol16. Any point on line (1)

$x = \alpha + k$

$y = 1 + 2k$

$z = 1 + 3k$

Any point on line (2)

$x = 4 + K_1B$

$y = 6 + 3K_1$

$Z = 7 + 3K_1$

$\Rightarrow 1 + 2k = 6 + 3K_1$ as the intersect

$\therefore 1 + 3k = 7 + 3K_1$

$\Rightarrow K = 1, K_1 = -1$

$$\Rightarrow \begin{array}{c|c} x = \alpha + 1 & x = 4 - \beta \\ y = 3 & y = 3 \\ z = 4 & z = 4 \end{array}$$

Equation of plane

$x + 2y - z = 8$

$\Rightarrow \alpha + 1 + 6 - 4 = 8$ ....(i)

and $4 - \beta + 6 - 4 = 8$ ....(ii)

Adding (i) and (ii)

$\alpha + 5 - \beta + 12 - 8 = 16$

$\alpha - \beta + 17 = 24$

$\Rightarrow \alpha - \beta = 7$

Sol17. For co-ordinate of B

$11x + 7y = 9$

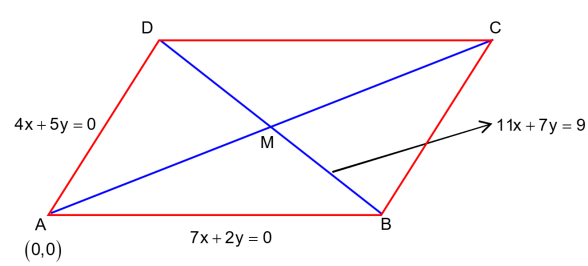
$7x + 2y = 0 \Rightarrow B\left(\frac{-2}{3}, \frac{7}{3}\right)$

For point D

$11x + 7y = 9 \Rightarrow D\left(+\frac{5}{3}, -\frac{4}{3}\right)$

$\Rightarrow$ Co-ordinate $M = \left(\frac{1}{2}, \frac{1}{2}\right)$

$\Rightarrow$ Equation of $AC \Rightarrow y = x$



Sol18. p : there exist M > 0

Such that $x \geq M$ for all $x \in S$

Obviously $\sim p : M > 0$ such that x < M for all $x \in S$

$\therefore$ Negation of 'there exists' is 'for all'.

Sol19. General form $\sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^{8-r}\left(\frac{1}{2}\right)^{r} < \frac{1}{2}$

$\sum_{r=n}^{8} {}^8C_r\left(\frac{1}{2}\right)^{8} < \frac{1}{2}$

$\Rightarrow {}^8C_n + {}^8C_{n+1} + ...{}^8C_8 < 2^7$

$\Rightarrow 256 - \left({}^8C_0 + {}^8C_1 + ...{}^8C_{n-1}\right) < 128$

$\Rightarrow {}^8C_0 + {}^8C_1 + {}^8C_{n-1} > 128$

$n \geq 5$

Sol20. $f'(x) = g(x)$

$f''(x) = g'(x)$

$\Rightarrow g(x).\, g'(x) = f'(x).\, f''(x)$

$f(x)$ has five roots

$\Rightarrow f'(x)$ has atleast 4 roots.

And $f''(x)$ has atleast of 3 roots

$\Rightarrow g(x).g'(x) = 0$ has atleast 7 roots in (a,b)

SECTION - B

Sol1. $e^{4x} - e^{3x} - 4e^{2x} - e^{x} + 1 = 0$

Divide by e^{2x}

$\Rightarrow \left(e^{2x} + e^{-2x}\right) - \left(e^{x} + e^{-x}\right) - 4 = 0$

Put $e^{x} + e^{-x} = t > 0$

$\Rightarrow e^{2x} + e^{-2x} + 2 = t^2$

$\Rightarrow t^2 - 2 - t - 4 = 0$

$\Rightarrow t^2 - t - 6 = 0$

$\Rightarrow t = 3, t = -2$ but $t \neq -2$

$\Rightarrow t = 3$

$\Rightarrow e^{x} + e^{-x} = 3$

Number of solution = 2

Sol2. $z = \frac{3 + 2i\cos\theta}{1 - 3i\cos\theta} \times \frac{1 + 7i\cos\theta}{1 + 3i\cos\theta}$

$= \frac{\left(3 - 6\cos^2\theta\right) + i\left(11\cos\theta\right)}{1 + 9\cos^2\theta}$

Real part = 0

$\Rightarrow 3 - 6\cos^2\theta = 0$

$\Rightarrow \theta = 45^\circ$

$\Rightarrow \sin^2 3\theta + \cos^2\theta = \frac{1}{2} + \frac{1}{2} = 1$

Sol3. $N = 2^{10}\ \ 5^{10}\ \ 11^{11}\ \ 13^{11}$

$5 \rightarrow 4n + 1$ type $\rightarrow$ number of choice = 11

$11 \rightarrow 4n+3$ type $\rightarrow$ number of choice = 6
$13 \rightarrow 4n+1$ type $\rightarrow$ number of choice = 12
$\therefore$ Number of divisor of $4n+1$ type = $11 \times 6 \times 12 = 792$

Sol4. Direction ratio of line $(1,-1,-6)$

Equation of line $\frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = k$

$x = k+3,\ y = -k-4,\ z = -6k-5$

Solving with plane $k = -2$

$\Rightarrow x = 1,\ y = -2,\ z = 7\ w$

$\Rightarrow$ Distance = $\sqrt{(3-1)^2 + 6^2 + 3^2} = \sqrt{49} = 7$

Sol5. $A = \{1,2,3,4,5.....100\}$

$B = \{4,7,10,13,16,19.....\}$

$C = \{2,4,6,8,10......\}$

$B - C = \{7,13,19,....97\}$

$A \cap (B - C) = \{7,13,19,.....97\}$

sum of elements = 832

Sol6. $\int_0^{\pi} \sin^2 x . \sin x\ e^{-\sin^2 x} dx$

$= \int_0^{\pi} (1-\cos^2 x) \sin x\ e^{-(1-\cos^2 x)} dx$

$= 2\int_0^{\pi/2} (1-\cos^2 x) \sin x .\ e^{-(1-\cos^2 x)} dx$

Let $\cos^2 x = t \Rightarrow \sin 2x dx = -dt$

$= -2\int_0^1 (1-t).e^{(1-t)} \frac{dt}{-2\cos x}$

$= \frac{1}{e}\int_0^1 \frac{1}{\sqrt{e}} e^t dt - \int_0^1 \sqrt{t}\ e^t dt$

$= \frac{1}{e}\left[2e - 3\int_0^1 \sqrt{t}\ e^t dt\right]$

$\Rightarrow \alpha = 2, \beta = 3$

$\alpha + \beta = 5$

Sol7. $A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = A^2 A = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow A^{20} = \begin{bmatrix} 1 & 20 & 1+2+3.....20 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 20 & 210 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 20 & 210 & 520 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$$

$$M(a_{13}) = T_n = \frac{n(n+1)}{2}$$

$$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(x+1)}{2}\right]$$

$$\Rightarrow S_{20} = 1540$$

$$\Rightarrow M = 2020$$

Sol8. Equation of the ellipse

$$\frac{(x-3)^2}{a^2} + \frac{(y+4)^2}{b^2} = 1$$

$$a = 2$$

$$ae = 1 \Rightarrow e = \frac{1}{2}$$

$$\Rightarrow b^2 = 3$$

Equation of tangent

$$y + 4 = m(x-3) \pm \sqrt{4m^2+3}$$

$$\Rightarrow mx - y = 4 + 3m \pm \sqrt{4m^2+3}$$

$$\Rightarrow 3m \pm \sqrt{4m^2+3} = 0$$

$$\Rightarrow 9m^2 = 4m^2 + 3$$

$$\Rightarrow 5m^2 = 3$$

Sol9. $e^{-y}dy = e^{\alpha x}dx$

$$\Rightarrow -e^{-y} = \frac{1}{\alpha}e^{\alpha x} + c$$

$y(\ln 2) = \ln 2$ and $y(0) = -\ln 2$

$$\Rightarrow -2 = \frac{1}{\alpha} + c$$

$$\Rightarrow c = -2 - \frac{1}{\alpha}$$

$$\Rightarrow e^{-y} = \frac{1}{\alpha}e^{\alpha x} - 2 - \frac{1}{\alpha}$$

$$\Rightarrow -e^{-\ln 2} = \frac{1}{\alpha}e^{\alpha \ln 2} - 2 - \frac{1}{\alpha}$$

$$\Rightarrow \frac{2^{\alpha} - 1}{\alpha} = \frac{3}{2}$$

$$\Rightarrow \alpha = 2$$

Sol10. $\Rightarrow 3.\alpha\beta - 2\alpha\beta = -1$

$\Rightarrow 2\alpha\beta = 4 \Rightarrow \alpha\beta = 2$ ....(i)

$\vec{b}.\vec{c} = 10$

$\Rightarrow -3\alpha - 2\beta - \alpha = 10$

$\Rightarrow 4\alpha + 2\beta = -10$

$\Rightarrow 2\alpha + \beta = 5$ ....(ii)

From (i) and (ii)

$\alpha = \frac{-1}{2}, \alpha = -2$

$\beta = -4, \beta = -1$

$\vec{a} = \hat{i} - 2\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$c = 2\hat{i} - 2\hat{j} + \hat{k}$

$\bar{a}.\left(\bar{b} \times \bar{c}\right) = 9$