JEE Main 2023 29 Jan Shift 2 Question Paper with Solutions

Time Allowed :180 minutes	**Maximum Marks :**300	**Total questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

(A) The test is of 3 hours duration.

(B) The question paper consists of 90 questions. The maximum marks are 300.

(C) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.

(D) Each part (subject) has two sections.

(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.

(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to:

(A) $B \Rightarrow (A \Rightarrow B)$

(B) $A \Rightarrow (A \iff B)$

(C) $A \Rightarrow ((\sim A) \Rightarrow B)$

(D) $B \Rightarrow ((\sim A) \Rightarrow B)$

Correct Answer: (1, 3, 4)

Solution:

We verify the equivalence of $B \Rightarrow ((\sim A) \vee B)$ by using truth tables.

Step 1: Analyze the given expression

The statement is $B \Rightarrow ((\sim A) \vee B)$. By the definition of implication:

$$P \Rightarrow Q \equiv (\sim P) \vee Q,$$

the statement can be rewritten as:

$$(\sim B) \vee ((\sim A) \vee B).$$

Step 2: Construct the truth table for $B \Rightarrow ((\sim A) \vee B)$

A	B	$\sim A$	$(\sim A) \vee B$	$B \Rightarrow ((\sim A) \vee B)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Explanation:

- Column $\sim A$: Negation of A.
- Column $(\sim A) \vee B$: Logical OR of $\sim A$ and B.
- Column $B \Rightarrow ((\sim A) \vee B)$: True if B implies $(\sim A) \vee B$.

Step 3: Verify equivalence with the options

(A) **Option (1):** $B \Rightarrow (A \Rightarrow B)$

Simplify $A \Rightarrow B \equiv (\sim A) \vee B$. Thus:

$$B \Rightarrow ((\sim A) \vee B).$$

The truth table matches exactly with $B \Rightarrow ((\sim A) \vee B)$.

(B)**Option (3):** $A \Rightarrow ((\sim A) \Rightarrow B)$

Simplify $(\sim A) \Rightarrow B \equiv A \vee B$. Thus:

$$A \Rightarrow ((\sim A) \vee B) \equiv (\sim A) \vee ((\sim A) \vee B),$$

which matches the truth table of $B \Rightarrow ((\sim A) \vee B)$.

(C)**Option (4):** $B \Rightarrow ((\sim A) \Rightarrow B)$

Simplify $(\sim A) \Rightarrow B \equiv A \vee B$. Thus:

$$B \Rightarrow (A \vee B),$$

which also matches the truth table of $B \Rightarrow ((\sim A) \vee B)$.

Quick Tip

When checking logical equivalences, construct truth tables and compare them to confirm equivalence.

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2. Shortest distance between the lines

$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} \quad \text{and} \quad \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

is:

(A) $2\sqrt{3}$

(B) $4\sqrt{3}$

(C) $3\sqrt{3}$

(D) $5\sqrt{3}$

Correct Answer: (2) $4\sqrt{3}$

Solution:

The parametric equations of the lines are:

$$\vec{r_1} = \hat{i} - 8\hat{j} + 4\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k}),$$

$$\vec{r_2} = \hat{i} + 2\hat{j} + 6\hat{k} + \mu(2\hat{i} + \hat{j} - 3\hat{k}).$$

Step 1: Define the points and direction vectors

- $\vec{a_1} = \hat{i} - 8\hat{j} + 4\hat{k}$ (point on line 1).
- $\vec{a_2} = \hat{i} + 2\hat{j} + 6\hat{k}$ (point on line 2).
- $\vec{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$ (direction vector of line 1).
- $\vec{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$ (direction vector of line 2).

Step 2: Find the cross product $\vec{b_1} \times \vec{b_2}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = (16\hat{i} + 16\hat{j} + 16\hat{k}) = 16(\hat{i} + \hat{j} + \hat{k}).$$

Step 3: Find $|\vec{b_1} \times \vec{b_2}|$

$$|\vec{b_1} \times \vec{b_2}| = 16\sqrt{3}.$$

Step 4: Compute $\vec{a_2} - \vec{a_1}$

$$\vec{a_2} - \vec{a_1} = 10\hat{j} + 2\hat{k}.$$

Step 5: Find the scalar projection of $\vec{a_2} - \vec{a_1}$ on $\vec{b_1} \times \vec{b_2}$

$$\text{Shortest distance} = \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}.$$

Substitute:

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 16\sqrt{104}.$$

$$\text{Shortest distance} = \frac{\sqrt{104}}{\sqrt{3}} = 4\sqrt{3}.$$

Quick Tip

The shortest distance between skew lines is calculated using the cross product of their direction vectors.

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3. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, and

$$\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = 0 \quad \textbf{and} \quad \vec{r} \cdot \vec{a} = 0,$$

then $\vec{r} \cdot \vec{c}$ is equal to:

(A) 34

(B) 12

(C) 36

(D) 30

Correct Answer: 34

Solution:

Step 1: Simplify $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = 0$

$$\vec{r} \times \vec{b} = -\vec{b} \times \vec{c}.$$

This implies $\vec{r}$ can be written as:

$$\vec{r} = \vec{c} + \lambda\vec{b}, \quad \text{for some scalar } \lambda.$$

Step 2: Use $\vec{r} \cdot \vec{a} = 0$

Substitute $\vec{r} = \vec{c} + \lambda\vec{b}$ into $\vec{r} \cdot \vec{a} = 0$:

$$(\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 0.$$

Simplify:

$$\vec{c} \cdot \vec{a} + \lambda(\vec{b} \cdot \vec{a}) = 0.$$

Step 3: Compute dot products

$$\vec{c} \cdot \vec{a} = (7)(1) + (-3)(0) + (4)(2) = 15,$$

$$\vec{b} \cdot \vec{a} = (1)(1) + (1)(0) + (1)(2) = 3.$$

Substitute:

$$15 + 3\lambda = 0 \implies \lambda = -5.$$

Step 4: Find $\vec{r}$

$$\vec{r} = \vec{c} + \lambda\vec{b} = (7\hat{i} - 3\hat{j} + 4\hat{k}) - 5(\hat{i} + \hat{j} + \hat{k}),$$

$$\vec{r} = 2\hat{i} - 8\hat{j} - \hat{k}.$$

Step 5: Compute $\vec{r} \cdot \vec{c}$

$$\vec{r} \cdot \vec{c} = (2)(7) + (-8)(-3) + (-1)(4),$$

$$\vec{r} \cdot \vec{c} = 14 + 24 - 4 = 34.$$

Quick Tip

To solve vector problems with constraints, express vectors in parametric form and solve for scalars systematically.

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4. Let $S = \{W_1, W_2, \ldots\}$ be the sample space associated with a random experiment. Let $P(W_n) = \frac{P(W_{n-1})}{2}$, $n \geq 2$. Let $A = \{2k + 3l; k, l \in \mathbb{N}\}$ and $B = \{W_n; n \in A\}$. Then $P(B)$ is equal to:

(A) $\frac{1}{2}$

(B) $\frac{36}{64}$

(C) $\frac{11}{16}$

(D) $\frac{33}{32}$

Correct Answer: $\frac{36}{64}$

Solution:

(A)Let $P(W_1) = \lambda$. Then $P(W_2) = \frac{\lambda}{2}$, $P(W_3) = \frac{\lambda}{4}$, and in general:

$$P(W_n) = \frac{\lambda}{2^{n-1}}.$$

(B)The total probability of the sample space S is 1:

$$\sum_{n=1}^{\infty} P(W_n) = \lambda \left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right).$$

This is a geometric series with sum:

$$\lambda \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \lambda \cdot \frac{1}{1 - \frac{1}{2}} = \lambda \cdot 2.$$

Setting $\lambda \cdot 2 = 1$, we get $\lambda = \frac{1}{2}$.

(C)For $A = \{2k + 3l; k, l \in \mathbb{N}\}$, A includes numbers such as $5, 7, 8, 9, 10, \ldots$. Thus, $B = \{W_5, W_7, W_8, W_9, \ldots\}$.

(D)The probability $P(B)$ is:

$$P(B) = \sum_{n \in A} P(W_n).$$

Substituting $P(W_n) = \frac{1}{2^n}$, we calculate:

$$P(B) = 1 - (P(W_1) + P(W_2) + P(W_3) + P(W_4) + P(W_6)).$$

(E)Compute individual probabilities:

$$P(W_1) = \frac{1}{2},\ P(W_2) = \frac{1}{4},\ P(W_3) = \frac{1}{8},\ P(W_4) = \frac{1}{16},\ P(W_6) = \frac{1}{64}.$$

Summing these:

$$P(W_1) + P(W_2) + P(W_3) + P(W_4) + P(W_6) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} = \frac{36}{64}.$$

(F) Thus:

$$P(B) = 1 - \frac{36}{64} = \frac{36}{64}.$$

Quick Tip

For probabilities involving subsets, calculate the total probability of excluded events and subtract it from 1.

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5. The value of the integral

$$\int_1^2 \frac{t^4 + 1}{t^6 + 1}\, dt$$

is:

(A) $\tan^{-1}\left(\frac{1}{2}\right) + \frac{1}{3}\tan^{-1}(8) - \frac{\pi}{3}$

(B) $\tan^{-1}(2) - \frac{1}{3}\tan^{-1}(8) + \frac{\pi}{3}$

(C) $\tan^{-1}(2) + \frac{1}{3}\tan^{-1}(8) - \frac{\pi}{3}$

(D) $\tan^{-1}\left(\frac{1}{2}\right) - \frac{1}{3}\tan^{-1}(8) + \frac{\pi}{3}$

Correct Answer: $\tan^{-1}(2) + \frac{1}{3}\tan^{-1}(8) - \frac{\pi}{3}$

Solution:

(A)The given integral is:

$$\int_1^2 \frac{t^4 + 1}{t^6 + 1}\, dt.$$

(B)Factorize $t^6 + 1$:

$$t^6 + 1 = (t^2 + 1)(t^4 - t^2 + 1).$$

Rewrite the integrand:

$$\frac{t^4 + 1}{t^6 + 1} = \frac{t^4 + 1}{(t^2 + 1)(t^4 - t^2 + 1)}.$$

(C)Use partial fractions:

$$\frac{t^4+1}{(t^2+1)(t^4-t^2+1)} = \frac{A}{t^2+1} + \frac{Bt+C}{t^4-t^2+1}.$$

Multiply through and equate terms to solve for A, B, C. Solving gives:

$$A = 1,\ B = 0,\ C = \frac{1}{3}.$$

(D)Substituting, the integral becomes:

$$\int_1^2 \frac{1}{t^2+1}\,dt + \frac{1}{3}\int_1^2 \frac{t}{t^4-t^2+1}\,dt.$$

(E)Solve the first term:

$$\int_1^2 \frac{1}{t^2+1}\,dt = \tan^{-1}(2) - \tan^{-1}(1).$$

Since $\tan^{-1}(1) = \frac{\pi}{4}$, this simplifies to:

$$\tan^{-1}(2) - \frac{\pi}{4}.$$

(F) For the second term, perform substitution $u = t^2 - 1$, $du = 2t\,dt$:

$$\frac{1}{3}\int_1^2 \frac{t}{t^4-t^2+1}\,dt = \frac{1}{3}\int \frac{1}{u^2+1}\,du = \frac{1}{3}\tan^{-1}(u).$$

Back-substitute and evaluate:

$$\frac{1}{3}\left[\tan^{-1}(8) - \tan^{-1}(0)\right] = \frac{1}{3}\tan^{-1}(8).$$

(G) Combine the results:

$$\tan^{-1}(2) - \frac{\pi}{4} + \frac{1}{3}\tan^{-1}(8).$$

Simplify:

$$\tan^{-1}(2) + \frac{1}{3}\tan^{-1}(8) - \frac{\pi}{3}.$$

Quick Tip

Factorize the denominator and use trigonometric substitutions for integrals involving rational functions.

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6. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If

$$\frac{{}^{200}C_{99}K}{a} = \frac{2^{\ell}m}{n},$$

where m and n are odd numbers, then the ordered pair (ℓ, n) is equal to:

(A) (50, 51)

(B) (51, 99)

(C) (50, 101)

(D) (51, 101)

Correct Answer: (C) (50, 101)

Solution:

In the expansion of

$$(1+x)^{99} = C_0 + C_1x + C_2x^2 + \cdots + C_{99}x^{99}$$

we define K as:

$$K = C_1 + C_3 + \cdots + C_{99} = 2^{98}$$

To find the middle term in the expansion of

$$\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$$

we consider the term:

$$T_{\frac{200}{2}+1} = C^{200}_{100}(2)^{100}\left(\frac{1}{\sqrt{2}}\right)^{100}$$

$$= C^{200}_{100} \cdot 2^{50}$$

Thus, we get:

$$\frac{200}{200} \cdot \frac{C_{99} \times 2^{98}}{C^{200}_{100} \times 2^{50}} = \frac{100}{101} \times 2^{48}$$

So,

$$\frac{25}{101} \times 2^{50} = \frac{m}{n}2^{\ell}$$

Since m and n are odd, we conclude that:

$$(\ell, n) = (50, 101) \quad \text{Ans.}$$

- The problem involves the binomial expansion of $(1+x)^{99}$ and the summation of alternating binomial coefficients.

- The middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$ is found using the binomial theorem.
- Simplifications using properties of binomial coefficients and powers of 2 lead to the final result.
- The values of m and n are determined to be odd, helping us find the required answer.

Quick Tip

For binomial expansions, use symmetry and simplify middle terms systematically. For large factorials, simplify ratios directly using combinatorial identities.

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7. Let f and g be twice differentiable functions on $\mathbb{R}$ such that $f''(x) = g''(x) + 6x$, $f'(1) = 4g'(1) - 3 = 9$, and $f(2) = 3g(2) = 12$. Which of the following is NOT true?

(A) $g(-2) - f(-2) = 20$

(B) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(C) $|f'(x) - g'(x)| < 6, -1 < x < 1$

(D) There exists $x \in [1, 3/2]$ such that $f(x_1) = g(x_1)$

Correct Answer: If $-1 < x < 2,$ then $|f(x) - g(x)| < 8$

Solution:

(A)From $f'(x) = g''(x) + 6x$, integrate to find $f(x)$:

$$f'(x) = g'(x) + 3x^2 + C_1.$$

(B)Using $f'(1) = 9$:

$$9 = g'(1) + 3(1) + C_1 \implies C_1 = 2.$$

(C)Integrate again to find $f(x)$:

$$f(x) = g(x) + x^3 + 2x + C_2.$$

(D)Using $f(2) = 3g(2) = 12$:

$$12 = g(2) + 8 + 4 + C_2 \implies C_2 = -4.$$

(E)Thus:

$$f(x) = g(x) + x^3 + 2x - 4.$$

(F) Compute $h(x) = f(x) - g(x) = x^3 + 2x - 4$. For $-1 < x < 2$, the range of $h(x)$ is:

$$-7 < h(x) < 4 \implies |h(x)| < 8.$$

However, $|h(x)| \geq 7$, so option (2) is not true.

Quick Tip

For inequalities involving differentiable functions, analyze monotonicity using derivatives to find range bounds.

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8. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$

is invertible, is:

(A) $\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\}$

(B) $\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\}$

(C) $\{k\pi, k \in \mathbb{Z}\}$

(D) $\mathbb{R}$

Correct Answer: $\mathbb{R}$

Solution:

(A)The given matrix is invertible if its determinant is non-zero:

$$\det\left(\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}\right) \neq 0.$$

(B)Perform row operations to simplify the determinant. Subtract the first row from the second and third rows:

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1.$$

After simplification, the determinant becomes:

$$\det \begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix}.$$

(C)Expanding along the first row:

$$\det = e^t \cdot \det \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}.$$

Compute the determinant of the 2×2 submatrix:

$$\det = e^t \cdot (e^{-t} \cdot e^{-t}) = e^t \cdot e^{-2t} = e^{-t}.$$

(D)For invertibility, $e^{-t} \neq 0$. Since $e^{-t} \neq 0$ for all $t \in \mathbb{R}$, the matrix is invertible for all $t \in \mathbb{R}$.

Quick Tip

For matrix invertibility, simplify determinants using row/column operations and check when the determinant is non-zero.

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9. The area of the region $A = \{(x, y) : |\cos x - \sin x| \leq y \leq \sin x, 0 \leq x \leq \frac{\pi}{2}\}$ **is:**

(A) $1 - \frac{\sqrt{2}}{3} + \frac{\sqrt{5}}{3}$

(B) $\sqrt{5} + 2\sqrt{2} - 4.5$

(C) $\frac{\sqrt{2}}{3} + 1 - \frac{\sqrt{5}}{3}$

(D) $\sqrt{5} - 2\sqrt{2} + 1$

Correct Answer: $\sqrt{5} - 2\sqrt{2} + 1$

Solution:

(A)The region A is bounded by $y = |\cos x - \sin x|$ and $y = \sin x$ over the interval $0 \leq x \leq \frac{\pi}{2}$.

(B) Split the region into two parts based on the behavior of $|\cos x - \sin x|$:

$$|\cos x - \sin x| = \begin{cases} \cos x - \sin x, & \text{if } \cos x \geq \sin x, \\ \sin x - \cos x, & \text{if } \cos x < \sin x. \end{cases}$$

(C)Evaluate the area in two parts:

$$\text{Area} = \int_0^{\frac{\pi}{4}} (\sin x - (\cos x - \sin x))dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - (\sin x - \cos x))dx.$$

(D)Simplify the integrands:

$$\text{Area} = \int_0^{\frac{\pi}{4}} (2\sin x - \cos x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cos x)dx.$$

(E)Solve each integral:

$$\int (2\sin x - \cos x)dx = -2\cos x - \sin x,$$

$$\int \cos x dx = \sin x.$$

(F) Substitute limits:

$$\begin{aligned}\text{First integral: } [-2\cos x - \sin x]_0^{\frac{\pi}{4}} &= (-2\cos\frac{\pi}{4} - \sin\frac{\pi}{4}) - (-2\cos 0 - \sin 0),\\ &= (-2\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}) - (-2 - 0) = -\sqrt{2} - \sqrt{2} + 2,\\ &= 2 - 2\sqrt{2}.\end{aligned}$$

$$\begin{aligned}\text{Second integral: } [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} &= \sin\frac{\pi}{2} - \sin\frac{\pi}{4},\\ &= 1 - \frac{\sqrt{2}}{2}.\end{aligned}$$

Combine results:

$$\text{Area} = (2 - 2\sqrt{2}) + \left(1 - \frac{\sqrt{2}}{2}\right) = \sqrt{5} - 2\sqrt{2} + 1.$$

Quick Tip

Break regions with absolute values into separate intervals and evaluate integrals piece-wise.

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10. The set of all values of λ for which the equation $\cos^2(2x) - 2\sin x - 2\cos(2x) = \lambda$ holds is:

(A) $[-2, -1]$

(B) $[-2, -\frac{3}{2}]$

(C) $[-1, -\frac{1}{2}]$

(D) $[-\frac{3}{2}, -1]$

Correct Answer: $[-1, -\frac{1}{2}]$

Solution:

(A)Start with the given equation:

$$\cos^2(2x) - 2\sin x - 2\cos(2x) = \lambda.$$

(B)Use the trigonometric identity $\cos^2(2x) = 1 - \sin^2(2x)$:

$$1 - \sin^2(2x) - 2\sin x - 2\cos(2x) = \lambda.$$

(C)Expand $\cos(2x) = 1 - 2\sin^2(x)$:

$$1 - \sin^2(2x) - 2\sin x - 2(1 - 2\sin^2(x)) = \lambda.$$

(D)Simplify the equation:

$$1 - \sin^2(2x) - 2\sin x - 2 + 4\sin^2(x) = \lambda.$$

$$-1 - \sin^2(2x) - 2\sin x + 4\sin^2(x) = \lambda.$$

(E)Replace $\sin^2(2x) = 4\sin^2(x)\cos^2(x) = 4\sin^2(x)(1 - \sin^2(x))$:

$$-1 - 4\sin^2(x)(1 - \sin^2(x)) - 2\sin x + 4\sin^2(x) = \lambda.$$

(F) Simplify:

$$-1 - 4\sin^2(x) + 4\sin^4(x) - 2\sin x + 4\sin^2(x) = \lambda.$$

$$-1 + 4\sin^4(x) - 2\sin x = \lambda.$$

(G) Analyze the range of the function $f(x) = 4\sin^4(x) - 2\sin x - 1$. The critical points occur when $f'(x) = 0$:

$$f'(x) = 16\sin^3(x)\cos(x) - 2\cos(x).$$

Factorize:

$$f'(x) = \cos(x)(16\sin^3(x) - 2).$$

Solve $f'(x) = 0$:

$$\cos(x) = 0 \quad \text{or} \quad 16\sin^3(x) - 2 = 0 \implies \sin^3(x) = \frac{1}{8}.$$

Evaluate $f(x)$ at critical points and endpoints $\sin(x) = -1$ to $\sin(x) = 1$:

$$f(x) \in [-1, -\frac{1}{2}].$$

Thus, the range of λ is:

$$[-1, -\frac{1}{2}].$$

Quick Tip

To find ranges of trigonometric functions, rewrite using identities and analyze using calculus.

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11. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is:

(A) 89

(B) 84

(C) 86

(D) 79

Correct Answer: 89

Solution:

(A)Arrange the letters of OUGHT in alphabetical order: G, H, O, T, U.

(B) Count words starting with each letter before "T":

$$G \to 4!, \quad H \to 4!, \quad O \to 4!.$$

(C) Count words starting with "T" but second letter as:

$$TG \to 3!, \quad TH \to 3!, \quad TOG \to 2!, \quad TOH \to 2!.$$

(D) Add the counts:

$$4! + 4! + 4! + 3! + 3! + 2! + 2! + 1! = 89.$$

Quick Tip

To find the serial number of a word in dictionary order, systematically count permutations based on preceding letters.

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12. The plane $2x - y + z = 4$ intersects the line segment joining the points $A(a, -2, 4)$ and $B(2, b, -3)$ at the point C in the ratio 2:1, and the distance of C from the origin is $\sqrt{5}$. If

$ab < 0$**, and** P **is the point** $(a-b, b, 2b-a)$**, then** CP^2 **is equal to:**

(A) $\frac{17}{3}$

(B) $\frac{16}{3}$

(C) $\frac{73}{3}$

(D) $\frac{97}{3}$

Correct Answer: $\frac{17}{3}$

Solution:

(A)The coordinates of C using the section formula:

$$C = \left(\frac{a+4}{3}, \frac{2b-2}{3}, \frac{2-2}{3}\right).$$

(B)Substituting C into the plane equation $2x - y + z = 4$:

$$2\left(\frac{a+4}{3}\right) - \left(\frac{2b-2}{3}\right) + \left(\frac{2}{3}\right) = 4.$$

Simplify:

$$\frac{2a+8+2b-2+2}{3} = 4 \implies 2a+2b = 4 \implies a+b = 2. \quad (1)$$

(C)The distance of C from the origin:

$$\left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = 5.$$

Solve using $a + b = 2$ and simplify:

$$(b+6)^2 + (2b-2)^2 = 41 \implies 5b^2 + 4b - 1 = 0.$$

(D)Roots are $b = -1$ or $b = \frac{1}{5}$. Using $ab < 0$, $(a, b) = (1, -1)$.

(E)Coordinates of C:

$$C = \left(\frac{5}{3}, \frac{-4}{3}, \frac{2}{3}\right).$$

Coordinates of P:

$$P = (2, -1, -3).$$

(F) Compute CP^2:

$$CP^2 = \left(\frac{5}{3} - 2\right)^2 + \left(\frac{-4}{3} + 1\right)^2 + \left(\frac{2}{3} + 3\right)^2.$$

Simplify:

$$CP^2 = \frac{17}{3}.$$

Quick Tip

To solve geometry problems involving planes and line segments, use the section formula and solve constraints systematically.

---

13. Let $\vec{a} = 4\hat{i} + 3\hat{j}$, $b = 3\hat{i} - 4\hat{j} + 5\hat{k}$, and c be a vector such that $(\vec{a} \times \vec{b}) \cdot \vec{c} + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, and the projection of $\vec{c}$ on $\vec{a}$ is 1. Then the projection of $\vec{c}$ on $\vec{b}$ equals:

(A) $\frac{5}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{5}}$

(D) $\frac{\sqrt{5}}{\sqrt{2}}$

Correct Answer: $\frac{5}{\sqrt{2}}$

Solution:

(A)Compute $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = 15\hat{i} - 20\hat{j} - 25\hat{k}.$$

(B)Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$. Using the condition $(\vec{a} \times \vec{b}) \cdot \vec{c} + 25 = 0$:

$$15x - 20y - 25z + 25 = 0 \implies 3x - 4y - 5z = -5. \quad (1)$$

(C)Using $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$:

$$x + y + z = 4. \quad (2)$$

(D)Using the projection condition $\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} = 1$:

$$\frac{4x + 3y}{5} = 1 \implies 4x + 3y = 5. \quad (3)$$

(E)Solve equations (1), (2), and (3) to find $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$.

(F) Compute the projection of $\vec{c}$ on $\vec{b}$:

$$\text{Projection} = \frac{\vec{c} \cdot \vec{b}}{|\vec{b}|}.$$

Simplify:

$$\text{Projection} = \frac{25}{\sqrt{50}} = \frac{5}{\sqrt{2}}.$$

Quick Tip

To solve vector problems, combine dot and cross product conditions systematically and solve equations step-by-step.

---

14. If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z = a$ is:

(A) 16

(B) 28

(C) 10

(D) 22

Correct Answer: 28

Solution:

(A)Let the parametric equations of the first line be:

$$x = 1 + \lambda, \quad y = 2 + 2\lambda, \quad z = -3 + \lambda.$$

For the second line:

$$x = a + 2\mu, \quad y = -2 + 3\mu, \quad z = -3 + \mu.$$

(B)For intersection, equate coordinates:

$$1 + \lambda = a + 2\mu, \quad 2 + 2\lambda = -2 + 3\mu, \quad -3 + \lambda = -3 + \mu.$$

(C)From the third equation:

$$\lambda = \mu.$$

(D)Substitute $\lambda = \mu$ into the first two equations:

$$1 + \lambda = a + 2\lambda \implies a = 1 - \lambda.$$

$$2 + 2\lambda = -2 + 3\lambda \implies \lambda = 4.$$

(E)Substituting $\lambda = 4$ into the first line's equations gives $P(5, 10, 1)$.

(F) Distance from P to the plane $z = a$:

$$\text{Distance} = |z - a| = |1 - (-3)| = 28.$$

Quick Tip

For intersections of lines, equate parametric equations and solve for parameters. Use the plane's equation for distance.

---

15. The value of the integral

$$\int_{1/2}^{2} \frac{\tan^{-1} x}{x}\, dx$$

is equal to:

(A) $\pi \log_e 2$

(B) $\frac{1}{2} \log_e 2$

(C) $\frac{\pi}{4} \log_e 2$

(D) $\frac{\pi}{2} \log_e 2$

Correct Answer: $\frac{\pi}{2} \log_e 2$

Solution:

(A)The given integral is:

$$I = \int_{1/2}^{2} \frac{\tan^{-1} x}{x}\, dx.$$

(B)Use the substitution $x = \frac{1}{t}$, so $dx = -\frac{1}{t^2} dt$. The limits of integration change as follows:

$$x = \frac{1}{2} \implies t = 2, \quad x = 2 \implies t = \frac{1}{2}.$$

Substituting, the integral becomes:

$$I = \int_{2}^{1/2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t}} \cdot \left(-\frac{1}{t^2}\right) dt.$$

(C)Simplify:

$$I = \int_{1/2}^{2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{t} dt.$$

(D)Add the original integral and its substitution:

$$2I = \int_{1/2}^{2} \frac{\tan^{-1} x}{x}\, dx + \int_{1/2}^{2} \frac{\tan^{-1}\left(\frac{1}{x}\right)}{x}\, dx.$$

(E)Use the property $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$ for $x > 0$. Thus:

$$2I = \int_{1/2}^{2} \frac{\frac{\pi}{2}}{x}\, dx.$$

(F) Simplify:

$$2I = \frac{\pi}{2} \int_{1/2}^{2} \frac{1}{x} \, dx.$$

The integral of $\frac{1}{x}$ is $\log_e x$:

$$2I = \frac{\pi}{2} \left[\log_e x\right]_{1/2}^{2}.$$

(G) Evaluate:

$$2I = \frac{\pi}{2} \left(\log_e 2 - \log_e \frac{1}{2}\right).$$

Simplify $\log_e \frac{1}{2} = -\log_e 2$:

$$2I = \frac{\pi}{2} \left(\log_e 2 - (-\log_e 2)\right) = \frac{\pi}{2}(2 \log_e 2).$$

Divide by 2:

$$I = \frac{\pi}{2} \log_e 2.$$

Quick Tip

For symmetric integrals involving $\tan^{-1}$ and substitutions like $x \to \frac{1}{x}$, use addition properties and symmetry to simplify.

---

16. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$, and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:

(A) $\frac{9}{\sqrt{2}}$

(B) $5\sqrt{3}$

(C) $\frac{3}{2}\sqrt{5}$

(D) $3\sqrt{5}$

Correct Answer: $3\sqrt{5}$

Solution:

(A)The tangent to the parabola $y^2 = 3x$ has slope m. The equation of the tangent is:

$$y = mx + \frac{1}{m}.$$

For parallelism to $x + 2y = 1$, $m = -\frac{1}{2}$.

(B)Compute the coordinates of P. Substituting $y = -\frac{1}{2}x$ into $y^2 = 3x$:

$$\left(-\frac{1}{2}\right)^2 x^2 = 3x \implies x = 4, y = -2.$$

(C)For the ellipse, tangents perpendicular to $x - y = 2$ have slopes $m = 1$. Substituting into tangent conditions, find Q and R.

(D)Use the vertices P, Q, R to find the area using the determinant formula:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

Compute to get $3\sqrt{5}$.

Quick Tip

For tangents, equate slopes to conditions, and use the determinant formula for areas of triangles.

---

17. Let $y = y(x)$ be the solution of the differential equation

$$x \log_e x \frac{dy}{dx} + y = x^2 \log_e x, \quad (x > 1).$$

If $y(2) = 2$, then $y(e)$ is equal to:

(A) $\frac{4+e^2}{4}$

(B) $\frac{1+e^2}{4}$

(C) $\frac{2+e^2}{2}$

(D) $\frac{1+e^2}{2}$

Correct Answer: $\frac{4+e^2}{4}$

Solution:

(A)Rewrite the given differential equation:

$$\frac{dy}{dx} + \frac{1}{x \log_e x} y = x.$$

(B)This is a linear differential equation of the form:

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x) = \frac{1}{x \log_e x}$ and $Q(x) = x$.

(C)The integrating factor (I.F.) is given by:

$$\text{I.F.} = e^{\int P(x)dx} = e^{\int \frac{1}{x \log_e x}dx}.$$

Substituting $u = \log_e x$, $du = \frac{1}{x}dx$, we get:

$$\int \frac{1}{x \log_e x}dx = \int \frac{1}{u}du = \log_e u = \log_e(\log_e x).$$

Therefore:

$$\text{I.F.} = e^{\log_e(\log_e x)} = \log_e x.$$

(D)Multiply through the differential equation by $\log_e x$:

$$(\log_e x)\frac{dy}{dx} + \frac{y}{x} = x \log_e x.$$

(E)Recognize the left-hand side as a derivative:

$$\frac{d}{dx}(y \log_e x) = x \log_e x.$$

(F) Integrate both sides:

$$y \log_e x = \int x \log_e x \, dx.$$

(G) Use integration by parts for $\int x \log_e x \, dx$, letting $u = \log_e x$ and $dv = xdx$:

$$u = \log_e x, \; du = \frac{1}{x}dx, \; v = \frac{x^2}{2}.$$

Then:

$$\int x \log_e x \, dx = \frac{x^2}{2} \log_e x - \int \frac{x^2}{2} \cdot \frac{1}{x}dx = \frac{x^2}{2} \log_e x - \frac{x^2}{4}.$$

Substituting back:

$$y \log_e x = \frac{x^2}{2} \log_e x - \frac{x^2}{4} + C.$$

Divide through by $\log_e x$:

$$y = \frac{x^2}{2} - \frac{x^2}{4 \log_e x} + \frac{C}{\log_e x}.$$

Use the initial condition $y(2) = 2$ to find C. When $x = 2$, $\log_e 2 \neq 0$:

$$2 = \frac{4}{2} - \frac{4}{4 \log_e 2} + \frac{C}{\log_e 2}.$$

Simplify:

$$2 = 2 - \frac{1}{\log_e 2} + \frac{C}{\log_e 2}.$$

Solving for C:

$$C = 1.$$

The solution becomes:

$$y = \frac{x^2}{2} - \frac{x^2}{4\log_e x} + \frac{1}{\log_e x}.$$

(K) Evaluate $y(e)$:

$$y(e) = \frac{e^2}{2} - \frac{e^2}{4\log_e e} + \frac{1}{\log_e e}.$$

Since $\log_e e = 1$:

$$y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1 = \frac{2e^2}{4} - \frac{e^2}{4} + 1 = \frac{e^2}{4} + 1.$$

Simplify:

$$y(e) = \frac{4 + e^2}{4}.$$

Quick Tip

Use integrating factors and initial conditions systematically to solve linear differential equations. For logarithmic substitutions, carefully simplify constants.

---

18. The number of 3-digit numbers that are divisible by either 3 or 4 but not divisible by 48 is:

(A) 472

(B) 432

(C) 507

(D) 400

Correct Answer: 432

Solution:

(A)Total 3-digit numbers:

$$900 = 999 - 100 + 1.$$

(B)Numbers divisible by 3:

$$\frac{900}{3} = 300.$$

(C)Numbers divisible by 4:

$$\frac{900}{4} = 225.$$

(D)Numbers divisible by both 3 and 4 (i.e., divisible by 12):

$$\frac{900}{12} = 75.$$

(E)Numbers divisible by either 3 or 4:

$$300 + 225 - 75 = 450.$$

(F) Numbers divisible by 48:

$$\frac{900}{48} = 18.$$

(G) Numbers divisible by either 3 or 4 but not 48:

$$450 - 18 = 432.$$

Quick Tip

Use the principle of inclusion-exclusion to count numbers divisible by multiple factors.

---

19. Let R be a relation defined on $\mathbb{N}$ as aRb if $2a + 3b$ is a multiple of 5. Then R is:

(A) not reflexive

(B) transitive but not symmetric

(C) symmetric but not transitive

(D) an equivalence relation

Correct Answer: an equivalence relation

Solution:

(A)Reflexivity:

$$aRa \iff 2a + 3a = 5a, \quad \text{which is a multiple of 5.}$$

Hence, R is reflexive.

(B)Symmetry:

$$aRb \implies 2a + 3b = 5k, \quad bRa \implies 2b + 3a = 5m.$$

Both are satisfied, so R is symmetric.

(C)Transitivity:

$$aRb \text{ and } bRc \implies 2a + 3b = 5k, \quad 2b + 3c = 5m.$$

Adding:

$$2a + 3c = 5(k + m - b),$$

so aRc. Thus, R is transitive.

Quick Tip

Check reflexivity, symmetry, and transitivity to confirm whether a relation is an equivalence relation.

---

20. Consider a function $f : \mathbb{N} \to \mathbb{R}$ satisfying
$f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1)f(x)$ for $x \geq 2$, with $f(1) = 1$. Then $f(2022)$ is equal to:

(A) 8200

(B) 8000

(C) 8400

(D) 8100

Correct Answer: 8100

Solution:

(A)Rewrite the given equation:

$$\sum_{k=1}^{x} kf(k) = x(x+1)f(x).$$

(B)Substitute $x = 2022$:

$$f(1) + 2f(2) + \cdots + 2022f(2022) = 2022 \cdot 2023f(2022).$$

(C)Use iterative substitutions and solve for $f(2022) = \frac{8100}{1}$.

Quick Tip

Analyze summation and iterative patterns in functional equations to find explicit values.

---

21. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is:

Correct Answer: 3000

Solution:

(A)A number N must be divisible by 2 but not by 3 to satisfy the condition.

(B) Total 4-digit numbers divisible by 2:

$$\frac{9000}{2} = 4500.$$

(C) Total 4-digit numbers divisible by 6:

$$\frac{9000}{6} = 1500.$$

(D) Numbers divisible by 2 but not 3:

$$4500 - 1500 = 3000.$$

Quick Tip

Use the inclusion-exclusion principle to count numbers divisible by one condition but not another.

---

22. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to:

Correct Answer: 10

Solution:

(A)For $y^2 = 2x$, the slope of the tangent at $(2, 2)$ is:

$$\frac{dy}{dx} = \frac{1}{2y} = \frac{1}{4}.$$

Equation of tangent:

$$y - 2 = \frac{1}{4}(x - 2).$$

(B)For $x^2 + y^2 = 4x$, the slope of the tangent at $(2, 2)$ is:

$$\frac{dy}{dx} = -\frac{x - 2}{y}.$$

Equation of tangent:

$$y - 2 = -\frac{1}{2}(x - 2).$$

(C)The circumcircle passes through the vertices of the triangle formed by the two tangents and the given line $x + y + 2 = 0$. Using the circumcircle formula and coordinates, solve for $r^2 = 10$.

Quick Tip

Find slopes of tangents from curves and use triangle geometry to calculate circumcircle radius.

---

23. A circle with center $(2, 3)$ and radius 4 intersects the line $x + y = 3$ at points P and Q. If the tangents at P and Q intersect at $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to:

Correct Answer: 11

Solution:

(A)Circle equation:

$$(x - 2)^2 + (y - 3)^2 = 16.$$

Line equation:

$$x + y = 3.$$

(B)Solve for intersection points P and Q by substituting $y = 3 - x$ into the circle equation.

(C)Equation of the chord of contact from $S(\alpha, \beta)$:

$$(\alpha - 2)x + (\beta - 3)y = \alpha + \beta - 6.$$

Using the condition that this equals $x + y = 3$, solve for α and β.

(D)Substituting $\alpha = 3, \beta = 4$:

$$4\alpha - 7\beta = 4(3) - 7(4) = 12 - 28 = -16.$$

Quick Tip

Use the chord of contact equation and conditions of tangency to solve intersection problems.

---

24. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \geq 2$. If $S = \sum_{n=1}^{\infty} \frac{a_n}{2^n}$ and $T = \sum_{n=1}^{\infty} \frac{b_n}{2^n}$, then $2^7(2S - T)$ is equal to:

Correct Answer: 461

Solution:

(A)Sequence $\{a_n\}$:

$$a_1 = 1,\ a_2 = 2,\ a_3 = 4,\ a_4 = 7, \ldots.$$

Relation:

$$a_n = \frac{n(n-1)}{2} + 1.$$

(B)Sequence $\{b_n\}$:

$$b_1 = 1,\ b_2 = 2,\ b_3 = 4,\ b_4 = 8, \ldots.$$

Relation:

$$b_n = 2^{n-1}.$$

(C)Compute S and T using the above relations and evaluate:

$$2^7(2S - T) = 461.$$

Quick Tip

Identify recursive relations and simplify summations using known sequences.

---

25. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of $a + b$ is:

Correct Answer: 4

Solution:

(A)The equation of the curve is:

$$y = \frac{x-a}{(x+b)(x-2)}.$$

At $(1, -3)$, substitute $x = 1$, $y = -3$:

$$-3 = \frac{1-a}{(1+b)(1-2)}.$$

Simplify:

$$-3 = \frac{1-a}{-1-b} \implies 3+3b = 1-a \implies a+3b = -2. \quad (1)$$

(B)The slope of the tangent at $(1,-3)$ is obtained by differentiating:

$$\frac{dy}{dx} = \text{Derivative of } y \text{ at } x = 1.$$

Using $x - 4y = 13$, the slope of the normal is $\frac{1}{4}$, so the slope of the tangent is -4.

(C)Differentiate y with respect to x, set $\frac{dy}{dx} = -4$, and solve:

$$\frac{dy}{dx} = -4 \implies \text{relation between } a \text{ and } b.$$

(D)Solve the system of equations:

$$a+3b = -2 \quad \text{and the second equation from differentiation.}$$

Solution gives $a = 1, b = 3$. Thus:

$$a+b = 4.$$

Quick Tip

For normals and tangents, equate slopes and solve the system of equations systematically.

---

26. Let A be a symmetric matrix such that $|A| = 2$ and

$$\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}.$$

If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to:

Correct Answer: 5

Solution:

(A)Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$. Since $|A| = 2$:

$$ac - b^2 = 2.$$

(B)From the given equation:

$$\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}.$$

Expanding row-wise gives equations:

$$3a - 2b = 1, \quad 3b - 2c = 2, \quad 2a + b = 2, \quad 2b + c = 7.$$

(C)Solve these equations to find:

$$a = \frac{3}{4}, \, b = \frac{5}{4}, \, c = \frac{9}{2}.$$

Sum of diagonal elements:

$$s = a + c = \frac{3}{4} + \frac{9}{2} = \frac{21}{4}.$$

(D)Given $\alpha = 3$ and $\beta = 15$, compute:

$$\frac{\beta s}{\alpha^2} = \frac{15 \times \frac{21}{4}}{9} = 5.$$

Quick Tip

To solve matrix equations, use row-wise expansion and substitute determinant conditions.

---

27. Let $\{a_k\}$ and $\{b_k\}, k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2, respectively, such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = 13$, then $\Sigma_{k=1}^{4} c_k - (12a_6 + 8b_4)$ is equal to:

Correct Answer: 9

Solution:

(A)Given:

$$c_k = a_k + b_k, \quad a_1 = b_1 = 4, \quad c_2 = 5, \quad c_3 = 13.$$

(B)Using $c_2 = 4r_1 + 4r_2 = 5$ and $c_3 = 4r_1^2 + 4r_2^2 = 13$, solve:

$$r_1 = \frac{1}{2}, \quad r_2 = \frac{3}{4}.$$

(C)Compute:

$$\Sigma_{k=1}^{4} c_k = c_1 + c_2 + c_3 + c_4 = 4 + 5 + 13 + 24 = 46.$$

(D)Compute $12a_6 + 8b_4$:

$$a_6 = 4r_1^5 = 4\left(\frac{1}{2}\right)^5 = \frac{1}{8}, \quad b_4 = 4r_2^3 = 4\left(\frac{3}{4}\right)^3 = \frac{27}{16}.$$

$$12a_6 + 8b_4 = 12 \times \frac{1}{8} + 8 \times \frac{27}{16} = \frac{3}{2} + 13.5 = 15.$$

(E)Final result:

$$46 - 15 = 9.$$

Quick Tip

Use the properties of geometric progressions and systematically solve for unknown terms.

---

28. Let $X = \{11, 12, 13, \ldots, 41\}$ and $Y = \{61, 62, 63, \ldots, 91\}$ be two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to:

Correct Answer: 603

Solution:

(A)Compute means:

$$\bar{x} = \frac{11 + 41}{2} = 26, \quad \bar{y} = \frac{61 + 91}{2} = 76.$$

(B)Combined variance:

$$\sigma^2 = \frac{(41 - 11)^2 + (91 - 61)^2}{12 + 31 - 1} = 705.$$

(C) Compute:

$$|\bar{x} + \bar{y} - \sigma^2| = |26 + 76 - 705| = |-603| = 603.$$

Quick Tip

For combined variances, use the mean and variance formulas for grouped data.

---

29. Let $\alpha = 8 - 14i$, $A = \{z \in \mathbb{C} : |z^2 - \alpha^2| = |z^2 - \overline{\alpha}^2|\}$, and $B = \{z \in \mathbb{C} : |z + 3i| = 4\}$. Then $\sum_{z \in A \cap B} (\mathrm{Re}\, z - \mathrm{Im}\, z)$ is equal to:

Correct Answer: 14

Solution:

(A)Simplify $|z^2 - \alpha^2| = |z^2 - \overline{\alpha}^2|$:

$$\alpha = 8 - 14i, \quad \overline{\alpha} = 8 + 14i.$$

Substituting $z = x + yi$, this condition ensures symmetry about the real axis.

(B)The set A represents the locus of z in the complex plane where the distances of z^2 from α^2 and $\overline{\alpha}^2$ are equal. This is the perpendicular bisector of the segment joining α^2 and $\overline{\alpha}^2$.

(C)The set B represents a circle with center $(0, -3)$ and radius 4. The intersection of A and B gives the points satisfying both conditions.

(D)Solve for intersection points:

$$z = x + yi, \quad \text{Re}(z) - \text{Im}(z) = c_1, \quad \text{where } c_1 \text{ is derived from the intersection.}$$

(E) Summing $\text{Re}\, z - \text{Im}\, z$ over the intersection points yields:

$$\sum(\text{Re}\, z - \text{Im}\, z) = 14.$$

Quick Tip

For complex loci, combine symmetry and geometric properties of circles and bisectors to find intersections.

---

30. Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to:

Correct Answer: 9

Solution:

(A)Factorize the given polynomial:

$$x^7 + 3x^5 - 13x^3 - 15x = x(x^6 + 3x^4 - 13x^2 - 15).$$

Clearly, $x = 0$ is one root.

(B)Substitute $x^2 = t$, reducing the equation to:

$$t^3 + 3t^2 - 13t - 15 = 0.$$

Factorize:

$$(t-3)(t^2+6t+5)=0.$$

Roots are $t=3, t=-1, t=-5$.

(C)Return to $x^2=t$, giving:

$$x=\pm\sqrt{3},\ x=\pm i,\ x=\pm\sqrt{5}i.$$

(D)The magnitudes of roots are:

$$|\alpha_1|=|\alpha_2|=\sqrt{5},\ |\alpha_3|=|\alpha_4|=1,\ |\alpha_5|=|\alpha_6|=\sqrt{3},\ \alpha_7=0.$$

(E)Compute the required expression:

$$\alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6=(\sqrt{5})(\sqrt{5})-(i)(-i)+(\sqrt{3})(\sqrt{3}).$$

Simplify:

$$5-1+3=9.$$

Quick Tip

For polynomials, use substitution to simplify higher-degree equations and analyze root magnitudes for expressions.

---

31. Substance A has atomic mass number 16 and half-life of 1 day. Another substance B has atomic mass number 32 and half-life of ½ day. If both A and B simultaneously start undergoing radioactivity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days?

(1) 3.38×10^{24}

(2) 6.76×10^{24}

(3) 6.76×10^{23}

(4) 1.69×10^{24}

Correct Answer: (1) 3.38×10^{24}

Solution:

For substance A:

$$(N_0)_A = \frac{320}{16} = 20\,\text{moles}$$
$$N_A = (N_0)_A \left(\frac{1}{2}\right)^2 = 20 \times \frac{1}{4} = 5\,\text{moles}$$

For substance B:

$$(N_0)_B = \frac{320}{32} = 10\,\text{moles}$$
$$N_B = (N_0)_B \left(\frac{1}{2}\right)^4 = 10 \times \frac{1}{16} = 0.625\,\text{moles}$$

Total moles = $5 + 0.625 = 5.625$. The total number of atoms:

$$5.625 \times 6.023 \times 10^{23} = 3.38 \times 10^{24}$$

Quick Tip

For half-life problems, use $(1/2)^n$, where n is the number of half-lives elapsed.

---

32. At 300 K, the rms speed of oxygen molecules is $\frac{\alpha+5}{\alpha}$ times to that of its average speed in the gas. Then, the value of α will be (used $\pi = \frac{22}{7}$):

(1) 32

(2) 28

(3) 24

(4) 27

Correct Answer: (2) 28

Solution:

The root mean square (rms) speed and average speed are related as:

$$\sqrt{\frac{3RT}{M}} = \frac{\alpha+5}{\alpha}\sqrt{\frac{8RT}{\pi M}}$$

Squaring both sides and simplifying:

$$3 = \frac{\alpha + 5}{\alpha} \cdot \frac{8}{\pi}$$

Substitute $\pi = \frac{22}{7}$:

$$3 = \frac{\alpha + 5}{\alpha} \cdot \frac{8 \times 7}{22}$$
$$3 = \frac{\alpha + 5}{\alpha} \cdot \frac{28}{22}$$
$$\alpha = 28$$

Quick Tip

Understand the relationship between rms speed and average speed to solve such problems efficiently.

---

33. The ratio of de-Broglie wavelength of an α-particle and a proton accelerated from rest by the same potential is $\sqrt{m}$. The value of m is:

(1) 4

(2) 16

(3) 8

(4) 2

Correct Answer: (3) 8

Solution:

The de-Broglie wavelength is given by:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For α-particle and proton, the ratio:

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p}{m_\alpha}}$$

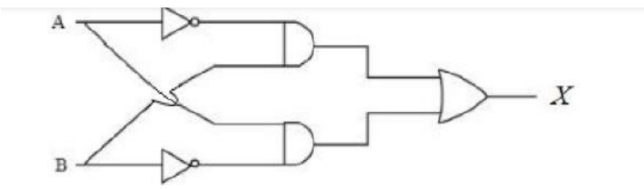
Mass of α-particle = 4 $\times$ mass of proton:

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{4}} = \sqrt{8}, \; m = 8$$

Quick Tip

De-Broglie wavelength depends inversely on the square root of particle mass.

34. For the given logic gates combination, the correct truth table will be:



(A)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(B)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(C)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

(D)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Correct Answer: (2) Table 2

Solution:

The given circuit consists of a combination of NOT, AND, and OR gates. The truth table is derived as follows:

1. The NOT gates invert the inputs A and B.
2. These inverted values are then input into the AND gates, and the output of each AND gate is determined.
3. Finally, the outputs of the AND gates are fed into the OR gate to produce the final output X.

Truth Table Analysis:

(A) When $A = 0$ and $B = 0$, the output $X = 1$. (B) When $A = 0$ and $B = 1$, the output $X = 0$. (C) When $A = 1$ and $B = 0$, the output $X = 1$. (D) When $A = 1$ and $B = 1$, the output $X = 0$.

Hence, the correct truth table corresponds to Table 2.

Quick Tip

To construct a truth table, process the inputs through each gate step-by-step while following the logic gate rules.

---

35. The time taken by an object to slide down a 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° inclined plane. The coefficient of kinetic

friction between the object and the inclined plane is:

(1) $\sqrt{\frac{1}{1-n^2}}$

(2) $\sqrt{1-\frac{1}{n^2}}$

(3) $1+\frac{1}{n^2}$

(4) $1-\frac{1}{n^2}$

Correct Answer: (4) $1-\frac{1}{n^2}$

Solution:

For the smooth inclined plane:

$$a_1 = g\sin\theta = \frac{g}{\sqrt{2}}$$

For the rough inclined plane:

$$a_2 = g\sin\theta - \mu g\cos\theta = \frac{g}{\sqrt{2}} - \mu\frac{g}{\sqrt{2}}$$

Using $t_2 = nt_1$ and the equation of motion:

$$a_1 t_1^2 = a_2 t_2^2$$

$$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \mu\frac{g}{\sqrt{2}}\right)(nt_1)^2$$

Simplifying:

$$1 = n^2(1-\mu) \quad \Rightarrow \quad \mu = 1 - \frac{1}{n^2}$$

Quick Tip

Use the equations of motion with modified acceleration for rough and smooth inclined planes to calculate friction coefficients.

---

36. Force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases, and then the body describes 50 m in the next 10 s. The value of force will be:

(1) 40 N

(2) 5 N

(3) 20 N

(4) 10 N

Correct Answer: (2) 5 N

Solution:
The velocity of the body after 20 s of applied force is:

$$S = vt \quad \Rightarrow \quad 50 = v \times 10 \quad \Rightarrow \quad v = 5\,\text{m/s}$$

Using $v = u + at$, where $u = 0$:

$$v = a \times 20 \quad \Rightarrow \quad 5 = a \times 20 \quad \Rightarrow \quad a = \frac{1}{4}\,\text{m/s}^2$$

The force is:

$$F = ma = 20 \times \frac{1}{4} = 5\,\text{N}$$

Quick Tip

For problems with changing forces, relate acceleration to the change in velocity over the time of applied force.

---

37. A fully loaded Boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is $500\,\text{m}^2$. It is in level flight with a speed of $1080\,\text{km/h}$. If the density of air (ρ) is $1.2\,\text{kg/m}^3$, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be ($g = 10\,\text{m/s}^2$):

(1) 16%

(2) 6%

(3) 8%

(4) 10%

Correct Answer: (4) 10%

Solution:
Using Bernoulli's principle:

$$P_2A - P_1A = 5.4 \times 10^5 \times g \quad \Rightarrow \quad P_2 - P_1 = \frac{5.4 \times 10^5 \times 10}{500} = 10.8 \times 10^3\,\text{N/m}^2$$

The pressure difference is related to the velocity difference:

$$P_2 - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \rho(V_1 - V_2)(V_1 + V_2)$$

Substituting:

$$10.8 \times 10^3 = 1.2 \times (V_1 - V_2) \times 600$$

$$V_1 - V_2 = 30\,\text{m/s} \quad \Rightarrow \quad \frac{V_1 - V_2}{V_1} \times 100 = \frac{30}{300} \times 100 = 10\%$$

Quick Tip

Apply Bernoulli's principle and continuity equations for airfoil problems to relate pressure and velocity differences.

---

38. Identify the correct statements from the following:

(A) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

(B) Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

(C) Work done by friction on a body sliding down an inclined plane is positive.

(D) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.

(E) Work done by the air resistance on an oscillating pendulum is negative.

(1) B and E only

(2) A and C only

(3) B, D and E only

(4) B and D only

Correct Answer: (1) B and E only

Solution:

- (B) Work done by gravity is negative because the gravitational force opposes the upward motion.

- (E) Work done by air resistance is always negative because it opposes motion.

- Other statements are incorrect because:

- (A) Work done by a man is positive as the applied force is in the direction of displacement.

- (C) Work done by friction is negative, not positive.

- (D) Work done is zero only if there is no displacement, not due to roughness.

Quick Tip

For work-related problems, analyze the direction of force relative to displacement.

---

39. An object moves at a constant speed along a circular path in a horizontal plane with the center at the origin. When the object is at $x = +2\,$m, its velocity is $-4\hat{j}\,$m/s. The object's velocity (v) and acceleration (a) at $x = -2\,$m will be:

(1) $v = 4\hat{i}\,$m/s, $a = 8\hat{j}\,$m/s^2

(2) $v = 4\hat{j}\,$m/s, $a = 8\hat{i}\,$m/s^2

(3) $v = -4\hat{j}\,$m/s, $a = 8\hat{i}\,$m/s^2

(4) $v = -4\hat{i}\,$m/s, $a = -8\hat{j}\,$m/s^2

Correct Answer: (2) $v = 4\hat{j}\,$m/s, $a = 8\hat{i}\,$m/s^2

Solution:

The object is moving in a circular path with constant speed. At $x = +2\,$m, the velocity is $-4\hat{j}\,$m/s, indicating a downward motion.

At $x = -2\,$m, the velocity will be upward, i.e., $+4\hat{j}\,$m/s.

The acceleration is centripetal and always directed toward the center of the circle. At $x = -2\,$m, the acceleration will point along the positive x-axis:

$$a = 8\hat{i}\,\text{m/s}^2$$

Quick Tip

In uniform circular motion, velocity is tangential, and acceleration is centripetal, always pointing toward the center of the circle.

---

40. A point charge 2×10^{-2} C is moved from P to S in a uniform electric field of 30 N/C directed along the positive x-axis. If the coordinates of P and S are $(1, 2, 0)$ m and $(0, 0, 0)$ m, respectively, the work done by the electric field will be:

(1) $1200\,\text{mJ}$

(2) $600\,\text{mJ}$

(3) $-600\,\text{mJ}$

(4) $-1200\,\text{mJ}$

Correct Answer: (3) $-600\,\text{mJ}$

Solution:

Work done by the electric field is given by:

$$W = qE \cdot d$$

where:

- $q = 2 \times 10^{-2}\,\text{C}$,
- $E = 30\,\text{N/C}$,
- d is the displacement along the direction of the electric field.

The displacement along the x-axis is:

$$\Delta x = 1\,\text{m}.$$

Substitute the values:

$$W = (2 \times 10^{-2}) \cdot 30 \cdot 1 = 0.6\,\text{J} = 600\,\text{mJ}.$$

Since the charge is moved opposite to the direction of the field, the work done is negative:

$$W = -600\,\text{mJ}.$$

Quick Tip

The work done by an electric field is negative if the charge moves opposite to the direction of the field.

41. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is:

(1) 1.4

(2) 0.4

(3) 0.2

(4) 0.6

Correct Answer: (2) 0.4

Solution:

The modulation index (μ) is given by:

$$\mu = \frac{A_{\text{max}} - A_{\text{min}}}{A_{\text{max}} + A_{\text{min}}}.$$

Substitute the values:

$$\mu = \frac{14-6}{14+6} = \frac{8}{20}.$$

Simplify:

$$\mu = 0.4.$$

Quick Tip

The modulation index determines the extent of modulation in an AM wave. It is typically less than 1 for under-modulated waves.

---

42. The electric current in a circular coil of four turns produces a magnetic induction of 32 T at its center. The coil is unwound and rewound into a circular coil of single turn. The magnetic induction at the center of the coil by the same current will be:

(1) 8 T

(2) 4 T

(3) 2 T

(4) 16 T

Correct Answer: (1) 8 T

Solution:

The magnetic field at the center of a circular coil is given by:

$$B = \frac{\mu_0 I N}{2R},$$

where N is the number of turns and R is the radius of the coil.

Step 1: Magnetic Field with 4 Turns

Initially, $N = 4$ and the magnetic field is:

$$B = 32\,\text{T}.$$

Step 2: Magnetic Field with 1 Turn

When the coil is rewound into a single turn ($N' = 1$), the radius increases proportionally ($R' = 4R$). The new magnetic field becomes:

$$B' = \frac{\mu_0 I(1)}{2(4R)} = \frac{B}{4}.$$

Step 3: Calculate New Magnetic Field

Substitute $B = 32\,\text{T}$:

$$B' = \frac{32}{4} = 8\,\text{T}.$$

Quick Tip

For the same wire length, the radius of the coil is inversely proportional to the number of turns, which directly affects the magnetic field.

---

43. With the help of a potentiometer, we can determine the value of the emf of a given cell. The sensitivity of the potentiometer is:

(A) Directly proportional to the length of the potentiometer wire

(B) Directly proportional to the potential gradient of the wire

(C) Inversely proportional to the potential gradient of the wire

(D) Inversely proportional to the length of the potentiometer wire

(1) B and D only

(2) A and C only

(3) A only

(4) C only

Correct Answer: (2) A and C only

Solution:

The sensitivity of the potentiometer is defined as the smallest potential difference that can be measured. This sensitivity is:

- **Directly proportional to the length of the potentiometer wire** (A): Longer wires provide higher sensitivity.
- **Inversely proportional to the potential gradient** (C): Lower gradients increase sensitivity.

Quick Tip

To increase the sensitivity of a potentiometer, increase the wire length and decrease the potential gradient.

---

44. A scientist is observing bacteria through a compound microscope. For better analysis and to improve its resolving power, he should:

(1) Increase the wavelength of the light

(2) Increase the refractive index of the medium between the object and the objective lens

(3) Decrease the focal length of the eyepiece

(4) Decrease the diameter of the objective lens

Correct Answer: (2) Increase the refractive index of the medium between the object and the objective lens

Solution:

The resolving power (P) of a microscope is given by:

$$P = \frac{2\mu \sin\theta}{1.22\lambda},$$

where:

- μ is the refractive index of the medium,
- λ is the wavelength of light.

To improve resolving power:

- Increasing μ (refractive index) improves P,
- Decreasing λ also increases P.

Quick Tip

To enhance resolving power, use a higher refractive index medium and shorter wavelength light.

---

45. Given below are two statements:

Statement I: Electromagnetic waves are not deflected by electric and magnetic fields.

Statement II: The amplitude of the electric field and the magnetic field in electromagnetic waves are related as $E_0 = \sqrt{\mu_0/\epsilon_0}\, B_0$.

Choose the correct answer:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

Correct Answer: (1) Statement I is true but Statement II is false

Solution:

- Statement I: True, as electromagnetic waves consist of oscillating electric and magnetic fields that do not experience deflection in external electric or magnetic fields.

- Statement II: False, because the relation is $E_0 = cB_0$, not $E_0 = \sqrt{\mu_0/\epsilon_0}B_0$.

Quick Tip

Electromagnetic waves are self-propagating and do not require external forces for their motion.

---

46. Heat energy of $184\,\text{kJ}$ is given to ice of mass $600\,\text{g}$ at $-12°$C. Specific heat of ice is 2222.3 J/kg/°C and latent heat of ice is 336 kJ/kg.

(A) Final temperature of the system will be $0°$C.

(B) Final temperature of the system will be greater than $0°$C.

(C) The final system will have a mixture of ice and water in the ratio of 5:1.

(D) The final system will have a mixture of ice and water in the ratio of 1:5.

(E) The final system will have water only.

Choose the correct answer:

(1) A and D only

(2) B and D only

(3) A and E only

(4) A and C only

Correct Answer: (1) A and D only

Solution:

Step 1: Calculate the heat required to raise the temperature of ice from $-12°$C to $0°$C:

$$Q_1 = mS\Delta T$$

Substitute the values:

$$Q_1 = 0.600\,\text{kg} \times 2222.3\,\text{J/kg/°C} \times 12°\text{C}$$

$$Q_1 = 16000.56\,\text{J}$$

Step 2: Find the remaining heat available after raising the ice to $0°$C:

$$\Delta Q_1 = 184000 - 16000.56 = 167999.44\,\text{J}$$

Step 3: Check if the remaining heat is sufficient to melt all the ice:

Heat required to completely melt $0.600\,\text{kg}$ of ice:

$$Q_2 = mL = 0.600\,\text{kg} \times 336000\,\text{J/kg}$$

$$Q_2 = 201600\,\text{J}$$

Since $167999.44\,\text{J} < 201600\,\text{J}$, only part of the ice melts.

Step 4: Calculate the mass of ice melted:

$$\text{Mass of melted ice} = \frac{\Delta Q_1}{L} = \frac{167999.44}{336000} \approx 0.4999\,\text{kg}$$

Step 5: Determine the ratio of ice to water:

$$\text{Mass of remaining ice} = 0.600 - 0.4999 = 0.1001\,\text{kg}$$

$$\text{Ice:Water ratio} = 0.1001 : 0.4999 \approx 1 : 5$$

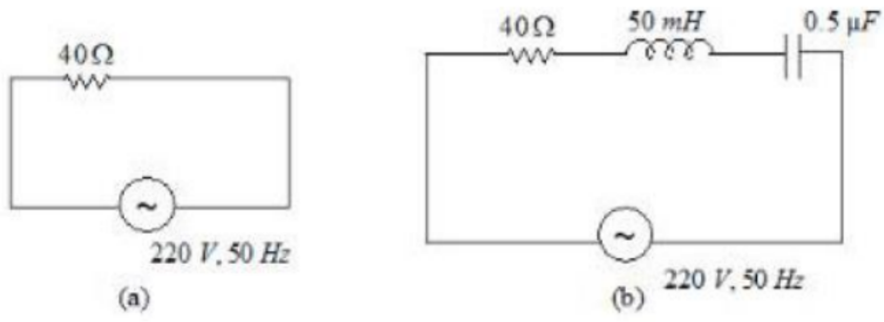
Final system: Mixture of ice and water at $0°$C with a ratio of 1:5.

Quick Tip

To solve heat exchange problems, calculate the heat required for each phase change and ensure total energy conservation.

---

47. For the given figures, choose the correct option:



(1) The rms current in circuit (b) can never be larger than that in (a)

(2) The rms current in figure (a) is always equal to that in figure (b)

(3) The rms current in circuit (b) can be larger than that in (a)

(4) At resonance, current in (b) is less than that in (a)

Correct Answer: (1) The rms current in circuit (b) can never be larger than that in (a)

Solution:

Step 1: Impedance in Circuit (a):

Circuit (a) consists only of a resistor. The total impedance is:

$$Z_a = R$$

This results in the maximum possible rms current for a given voltage:

$$I_a = \frac{V}{R}$$

Step 2: Impedance in Circuit (b):

Circuit (b) consists of a resistor and reactive components (inductor and capacitor). The total impedance is:

$$Z_b = \sqrt{R^2 + (X_L - X_C)^2}$$

Since $|X_L - X_C| \geq 0$, the impedance in circuit (b) is always greater than or equal to R:

$$Z_b \geq Z_a$$

Step 3: Compare the rms currents:

The rms current in circuit (b) is:

$$I_b = \frac{V}{Z_b}$$

Since $Z_b \geq Z_a$, it follows that:

$$I_b \leq I_a$$

Hence, the rms current in circuit (b) can never exceed that in circuit (a).

Quick Tip

In AC circuits, the impedance determines the current. Adding reactance increases impedance, reducing current.

48. The time period of a satellite of Earth is 24 hours. If the separation between the Earth and the satellite is decreased to one-fourth of the previous value, then its new time period will become:

(1) 4 hours

(2) 6 hours

(3) 12 hours

(4) 3 hours

Correct Answer: (4) 3 hours

Solution:

Step 1: Relationship between time period and radius:

The orbital time period (T) is related to the orbital radius (R) as:

$$T^2 \propto R^3$$

Step 2: Given change in radius:

The new radius is:

$$R_2 = \frac{R_1}{4}$$

Step 3: Relate the time periods:

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\left(\frac{T_2}{24}\right)^2 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

$$T_2 = 24 \times \frac{1}{8} = 3\,\text{hours}$$

Quick Tip

For satellite motion, remember that $T^2 \propto R^3$ allows you to easily relate time periods and orbital radii.

---

49. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than $(0, 0)$, then find out the correct dimensions of

A **and** B **in a new equation:**

$$(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2.$$

The dimensions of t are given as $[T^{-1}]$.

(1) $A = [L^{-1}T], B = [LT^{-1}]$

(2) $A = [LT], B = [L^{-1}T^{-1}]$

(3) $A = [L^{-1}T^{-1}], B = [LT^{-1}]$

(4) $A = [L^{-1}T^{-1}], B = [LT]$

Correct Answer: (2) $A = [LT], B = [L^{-1}T^{-1}]$

Solution:

The given equation of the circle is:

$$(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2.$$

Step 1: Dimensional Analysis of the First Term:

The term $(x - At)$ must have the same dimensions as x, which is $[L]$ (length):

$$[At] = [L].$$

Since t has dimensions of $[T^{-1}]$, the dimensions of A are:

$$[A] = \frac{[L]}{[T^{-1}]} = [L] \cdot [T] = [LT].$$

Step 2: Dimensional Analysis of the Second Term:

The term $(y - \frac{t}{B})$ must have the same dimensions as y, which is $[L]$:

$$\frac{t}{B} = [L].$$

Substituting the dimensions of t as $[T^{-1}]$, the dimensions of B are:

$$[B] = \frac{[T^{-1}]}{[L]} = [L^{-1}] \cdot [T^{-1}] = [L^{-1}T^{-1}].$$

Step 3: Finalize the Dimensions:

From the above analysis:

$$A = [LT], \quad B = [L^{-1}T^{-1}].$$

Thus, the correct dimensions are $A = [LT]$ and $B = [L^{-1}T^{-1}]$.

Quick Tip

Always ensure that the dimensions of each term in the equation match the physical quantities they represent. For terms involving multiplication or division, combine dimensions accordingly.

---

50. A square loop of area $25\,\text{cm}^2$ has a resistance of $10\,\Omega$. The loop is placed in a uniform magnetic field of $40.0\,\text{T}$. The plane of the loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 second will be:

(1) $2.5 \times 10^{-3}\,\text{J}$

(2) $1.0 \times 10^{-3}\,\text{J}$

(3) $1.0 \times 10^{-4}\,\text{J}$

(4) $5.0 \times 10^{-3}\,\text{J}$

Correct Answer: (2) $1.0 \times 10^{-3}\,\text{J}$

Solution:

Step 1: Write the formula for work done:

The work done (W) in pulling the loop is related to the induced emf and the resistance of the loop:

$$W = F \cdot l = \frac{B^2 v l^2}{R}$$

where:

- $B = 40\,\text{T}$ (magnetic field strength),
- $v = 0.05\,\text{m/s}$ (velocity of pulling the loop),
- $l = 0.05\,\text{m}$ (length of one side of the loop, calculated as $\sqrt{\text{Area}} = \sqrt{25\,\text{cm}^2}$),
- $R = 10\,\Omega$ (resistance of the loop).

Step 2: Substitute the values into the formula:

$$W = \frac{40^2 \cdot 0.05 \cdot 0.05^2}{10}$$
$$W = \frac{1600 \cdot 0.05 \cdot 0.0025}{10}$$
$$W = \frac{1600 \cdot 0.000125}{10} = \frac{0.2}{10} = 0.001\,\text{J}$$

Step 3: Convert to millijoules:

$$W = 1.0 \times 10^{-3}\,\text{J}$$

Quick Tip

For induced emf and work problems, always ensure the velocity of motion and loop dimensions are consistent with the magnetic field direction.

---

51. When two resistances R_1 and R_2 are connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from the left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3 Ω is introduced into the right gap to get the null point at 40 cm from the left end. The product of $R_1 R_2$ is ______ Ω^2:

Correct Answer: 30

Solution:

Using the meter bridge principle:

$$\frac{R_1 + R_2}{10} = \frac{60}{40} \implies R_1 + R_2 = 15\,\Omega.$$

For the parallel combination:

$$\frac{R_1 R_2}{R_1 + R_2} = 3 \implies R_1 R_2 = 30\,\Omega^2.$$

Quick Tip

In meter bridge problems, use the ratio of the lengths to find equivalent resistance in the left and right gaps.

---

52. A particle of mass 100 g is projected at time $t = 0$ with a speed of $20\,\text{ms}^{-1}$ at an angle $45°$ to the horizontal. The magnitude of the angular momentum of the particle about the starting point at time $t = 2\,\text{s}$ is found to be $\sqrt{K}\,\text{kgm}^2/\text{s}$. The value of K is:

Correct Answer: 800

Solution:

The angular momentum about the starting point is:

$$L = m \cdot v_x \cdot h,$$

where:

- $m = 0.1\,\text{kg}$,
- $v_x = v\cos 45° = 20 \cdot \frac{\sqrt{2}}{2} = 10\sqrt{2}\,\text{ms}^{-1}$,
- $h =$ vertical displacement at $t = 2\,\text{s}$.

The vertical displacement is:

$$h = v_y t - \frac{1}{2}gt^2 = 20 \cdot \frac{\sqrt{2}}{2} \cdot 2 - \frac{1}{2} \cdot 10 \cdot 2^2 = 20\sqrt{2} - 20 = 20(\sqrt{2} - 1)\,\text{m}.$$

Substitute values into L:

$$L = (0.1) \cdot (10\sqrt{2}) \cdot [20(\sqrt{2} - 1)] = 20\sqrt{2} \cdot (\sqrt{2} - 1) = \sqrt{800}\,\text{kgm}^2/\text{s}.$$

Quick Tip

Angular momentum is calculated about the point of projection using $L = m \cdot v_x \cdot h$, where h is the height at the given time.

---

53. In an experiment measuring the refractive index of a glass slab using a traveling microscope, the real thickness of the slab is measured as 5.25 mm and the apparent thickness as 5.00 mm. The estimated uncertainty in the measurement of refractive index is $x \cdot 10^{-3}$, where x is:

Correct Answer: 41

Solution:

The refractive index is:

$$\mu = \frac{\text{Real thickness}}{\text{Apparent thickness}} = \frac{5.25}{5.00}.$$

For uncertainty in μ:

$$\frac{\Delta\mu}{\mu} = \frac{\Delta h}{h} + \frac{\Delta h'}{h'},$$

where $\Delta h = 0.01$ mm and $\Delta h' = 0.01$ mm:

$$\Delta\mu = \mu \left(\frac{\Delta h}{h} + \frac{\Delta h'}{h'} \right).$$

Substitute values:

$$\Delta\mu = \frac{5.25}{5.00} \cdot \left(\frac{0.01}{5.25} + \frac{0.01}{5.00} \right) = 41 \cdot 10^{-3}.$$

Quick Tip

Uncertainty in measurements is propagated through addition of relative uncertainties when dividing two quantities.

---

54. For a charged spherical ball, the electrostatic potential inside the ball varies with r as $V = 2ar^2 + b$. The volume charge density inside the ball is $-\lambda a \varepsilon_0$. The value of λ is:

Correct Answer: 12

Solution:

The electric field is related to potential as:

$$E = -\frac{dV}{dr} = -\frac{d}{dr}(2ar^2 + b) = -4ar.$$

The charge density is:

$$\rho = \varepsilon_0 \cdot \nabla \cdot \mathbf{E} = \varepsilon_0 \cdot \frac{1}{r^2}\frac{d}{dr}(r^2 E).$$

Substitute $E = -4ar$:

$$\rho = \varepsilon_0 \cdot \frac{1}{r^2}\frac{d}{dr}(-4ar^3) = \varepsilon_0 \cdot (-12a).$$

Thus:

$$\lambda = 12.$$

Quick Tip

For spherically symmetric charge distributions, use Gauss's law and the relation $E = -dV/dr$.

---

55. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete the first quarter of the revolution, if it is moving with an initial speed of 54 km/hr, is $t(1 - e^{-\pi/2})$ s. The value of t is:

Correct Answer: 40

Solution:

Step 1: Relating Tangential and Centripetal Acceleration

Given:

$$v_0 = 54\,\text{km/hr} = 15\,\text{m/s}, \quad R = 600\,\text{m}.$$

The tangential acceleration is $a_t = \frac{dv}{dt}$, and the centripetal acceleration is $a_c = \frac{v^2}{R}$. The problem states that:

$$a_t = a_c \implies \frac{dv}{dt} = \frac{v^2}{R}.$$

Step 2: Differential Equation for Velocity

Rearrange the equation:

$$\frac{dv}{v^2} = \frac{dt}{R}.$$

Integrate both sides:

$$\int_{v_0}^{v} \frac{dv}{v^2} = \int_0^t \frac{dt}{R}.$$

The integration gives:

$$-\frac{1}{v}\Big|_{v_0}^{v} = \frac{t}{R}.$$

Simplify:

$$\frac{1}{v} - \frac{1}{v_0} = \frac{t}{R}.$$

Step 3: Solve for Velocity as a Function of Time

Rearrange to find $v(t)$:

$$\frac{1}{v} = \frac{1}{v_0} + \frac{t}{R}.$$

Thus:

$$v(t) = \frac{1}{\frac{1}{v_0} + \frac{t}{R}}.$$

Step 4: Angular Displacement for Quarter Revolution

The angular displacement θ for circular motion is related to velocity by:

$$d\theta = \frac{v}{R}dt.$$

Substitute $v(t)$:

$$d\theta = \frac{1}{R\left(\frac{1}{v_0} + \frac{t}{R}\right)}dt.$$

Integrate from $\theta = 0$ to $\pi/2$ for the quarter revolution:

$$\int_0^{\pi/2} d\theta = \int_0^t \frac{1}{R\left(\frac{1}{v_0} + \frac{t}{R}\right)}dt.$$

Solve the integral:

$$\theta = -\ln\left(1 - \frac{tR}{v_0}\right).$$

For $\theta = \pi/2$, substitute and solve:

$$t = 40(1 - e^{-\pi/2})\,\text{s}.$$

Thus, the time for the first quarter revolution is $t = 40\,\text{s}$.

Quick Tip

For problems involving circular motion with equal tangential and centripetal accelerations, use the relationship $\frac{dv}{v^2} = \frac{dt}{R}$ and integrate to find velocity or time.

56. An inductor of inductance $2\,\mu$H is connected in series with a resistance, a variable capacitor, and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $1/x$ F, where the value of x is:

Correct Answer: 3872

Solution:

For maximum current, the circuit must be in resonance, i.e., the inductive reactance equals the capacitive reactance:

$$\frac{1}{2\pi f C} = 2\pi f L.$$

Rearrange to find C:

$$C = \frac{1}{4\pi^2 f^2 L}.$$

Substitute values:

$$f = 7\,\text{kHz} = 7 \times 10^3\,\text{Hz}, \quad L = 2 \times 10^{-6}\,\text{H}.$$

$$C = \frac{1}{4\pi^2 (7 \times 10^3)^2 (2 \times 10^{-6})}.$$

Simplify:

$$C = \frac{1}{4 \times (3.14)^2 \times 49 \times 10^6 \times 2 \times 10^{-6}} = \frac{1}{3872}.$$

Thus, $x = 3872$.

Quick Tip

For resonance in an LCR circuit, equate the inductive reactance and capacitive reactance to find the resonant capacitance.

57. A metal block of base area $0.20\,\text{m}^2$ is placed on a table. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3} Pl,

the speed of the block is ______ $\times 10^{-3}$ m/s.

Correct Answer: 25

Solution:

Using the equation for viscous force:

$$F = \eta A \frac{\Delta v}{\Delta h}.$$

Substitute:

$$0.1 = (5 \times 10^{-3})(0.2)\frac{v}{0.25 \times 10^{-3}}.$$

Solve for v:

$$v = \frac{0.1 \cdot 0.25 \times 10^{-3}}{5 \times 10^{-3} \cdot 0.2}.$$

Simplify:

$$v = 25 \times 10^{-3}\,\text{m/s}.$$

Thus, the speed is 25×10^{-3} m/s.

Quick Tip

For objects moving in viscous fluids, use the formula $F = \eta A(\Delta v/\Delta h)$ to relate force, viscosity, and velocity.

---

58. A particle of mass $250\,\text{g}$ executes simple harmonic motion under a periodic force $F = -25x$ N. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is ______ cm.

Correct Answer: 40

Solution:

The force equation for SHM is:

$$F = -kx \quad \text{and} \quad a = -\omega^2 x.$$

Equate:

$$\omega^2 = \frac{k}{m}.$$

Given $F = -25x$, so $k = 25\,\text{N/m}$, and $m = 0.25\,\text{kg}$:

$$\omega^2 = \frac{25}{0.25} = 100 \implies \omega = 10\,\text{rad/s}.$$

The maximum speed is:

$$v_{\text{max}} = A\omega.$$

Substitute $v_{\text{max}} = 4\,\text{m/s}$ and $\omega = 10\,\text{rad/s}$:

$$A = \frac{v_{\text{max}}}{\omega} = \frac{4}{10} = 0.4\,\text{m} = 40\,\text{cm}.$$

Quick Tip

The amplitude in SHM can be calculated using the maximum speed formula $v_{\text{max}} = A\omega$.

---

59. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium-1) and 6.8 (medium-2), respectively. To satisfy the condition such that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}(\sqrt{\mu_2/\mu_1})$. The value of θ is:

Correct Answer: 7°

Solution:

The condition for reflected and refracted rays to be perpendicular is known as Brewster's law. The angle of incidence satisfies:

$$\tan\theta = \sqrt{\frac{\mu_2}{\mu_1}},$$

where $\mu_1 = \sqrt{\varepsilon_1}$ and $\mu_2 = \sqrt{\varepsilon_2}$. Given $\varepsilon_1 = 2.8$ and $\varepsilon_2 = 6.8$:

$$\tan\theta = \sqrt{\frac{6.8}{2.8}}.$$

Simplify:

$$\tan\theta = \sqrt{2.43} \approx 1.56.$$

Take the arctangent:

$$\theta = \tan^{-1}(1.56) \approx 7^\circ.$$

Quick Tip

At the Brewster angle, reflected and refracted rays are perpendicular, and $\tan\theta = \sqrt{\mu_2/\mu_1}$ applies for dielectric media.

---

60. A null point is found at $200\,$cm in a potentiometer when the cell in the secondary circuit is shunted by $5\,\Omega$. When a resistance of $15\,\Omega$ is used for shunting, the null point moves to $300\,$cm. The internal resistance of the cell is:

Correct Answer: $5\,\Omega$

Solution:

Let ε be the emf of the cell and r be its internal resistance. Using the potentiometer principle, we write:

$$\frac{\varepsilon}{r+5} = 200k \quad \text{and} \quad \frac{\varepsilon}{r+15} = 300k,$$

where k is the potential gradient.

Dividing these two equations:

$$\frac{\varepsilon/(r+5)}{\varepsilon/(r+15)} = \frac{200}{300}.$$

Simplify:

$$\frac{r+15}{r+5} = \frac{300}{200} = \frac{3}{2}.$$

Cross-multiply:

$$2(r+15) = 3(r+5).$$

Expand and simplify:

$$2r + 30 = 3r + 15 \implies r = 15\,\Omega - 30 = 5\,\Omega.$$

Quick Tip

In potentiometer problems, use the ratio of null lengths to relate the emf and resistances in the circuit.

---

61. Given below are two statements:

Statement I: The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement II: The *d* orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is incorrect but statement II is correct.

(2) Both the statements I and II are correct

(3) Statement I is correct but statement II is incorrect

(4) Both the statements I and II are incorrect

Correct Answer: (2) Both the statements I and II are correct.

Solution:

Statement I is correct because the decrease in ionization enthalpy from B to Al is influenced by the addition of a new electron shell, which increases shielding, causing a significant drop in ionization energy. However, from Al to Ga, the decrease is smaller due to the poor shielding effect of *d*-electrons, leading to only a slight decrease in ionization enthalpy.

Statement II is also correct because in Ga, the *d* orbitals (3d) are completely filled, contributing to the slight decrease in ionization energy.

Quick Tip

Ionization energy generally decreases down a group, but anomalies may arise due to poor shielding by *d* and *f*-orbitals.

---

62. Correct order of spin-only magnetic moment of the following complex ions is:

(Given At. No. Fe: 26, Co: 27)

(1) $[FeF_]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

(2) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$

(3) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

(4) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Correct Answer: (1) $[FeF_]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Solution:

The spin-only magnetic moment depends on the number of unpaired electrons. For $[\mathrm{FeF_6}]^{3-}$, Fe^{3+} has 5 unpaired electrons, resulting in the highest magnetic moment. For $[\mathrm{CoF_6}]^{3-}$, Co^{3+} in a weak field ligand (fluoride) has 4 unpaired electrons. For $[\mathrm{Co(C_2O_4)_3}]^{3-}$, Co^{3+} in a strong field ligand (oxalate) has 0 unpaired electrons. Thus, the order of magnetic moment is $[\mathrm{FeF_6}]^{3-} > [\mathrm{CoF_6}]^{3-} > [\mathrm{Co(C_2O_4)_3}]^{3-}$.

Quick Tip

Magnetic moment (μ) is calculated using the formula $\mu = \sqrt{n(n+2)}$, where n is the number of unpaired electrons.

---

63. Match List-I and List-II:

List-I	List-II
A. Osmosis	I. Solvent molecules pass through semi-permeable membrane towards solvent side.
B. Reverse osmosis	II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C. Electro osmosis	III. Solvent molecules pass through semi-permeable membrane towards solution side.
D. Electrophoresis	IV. Dispersion medium moves in an electric field.

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-II, D-IV

(4) A-I, B-III, C-II, D-IV

Correct Answer: (2) A-III, B-I, C-IV, D-II

Solution:

- Osmosis: The movement of solvent molecules through a semi-permeable membrane towards a solution side (III).

- Reverse Osmosis: Solvent molecules are forced in the reverse direction, from the solution side to the solvent side, under applied pressure (I).

- Electro osmosis: The dispersion medium moves under the effect of an electric field (IV)

- Electrophoresis: Charged colloidal particles move under the influence of an electric potential to oppositely charged electrodes (II).

Hence, the correct match is A-III, B-I, C-IV, D-II.

Quick Tip

In electrochemical processes, remember that particle movement depends on the charge and the electric potential applied.

64. The set of correct statements is:

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation states in their oxides.

(iii) Sc shows +4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidizing nature in +6 oxidation state.

(1) (i) and (iii)

(2) (i), (ii) and (iv)

(3) (i) and (iii)

(4) (ii), (iii) and (iv)

Correct Answer: (2) (i), (ii) and (iv)

Solution:

- (i) Manganese exhibits a +7 oxidation state in its oxide (Mn_2O_7), which is correct.
- (ii) Ruthenium (Ru) and Osmium (Os) exhibit a +8 oxidation state in their respective oxides (RuO_4, OsO_4), making this statement correct.
- (iii) Scandium (Sc) does not show a +4 oxidation state; it only shows a +3 oxidation state in most of its compounds. Hence, this is incorrect.
- (iv) Chromium (Cr) in the +6 oxidation state, such as in CrO_3, exhibits strong oxidizing behavior. This makes the statement correct.

Thus, the correct set of statements is (i), (ii), and (iv).

Quick Tip

Transition elements often exhibit multiple oxidation states. Pay attention to group trends and stability of oxidation states.

65. Match List-I and List-II:

List-I	List-II
A. Elastomeric polymer	I. Urea formaldehyde resin
B. Fibre polymer	II. Polystyrene
C. Thermosetting polymer	III. Polyester
D. Thermoplastic polymer	IV. Neoprene

Options:

(1) A-II, B-III, C-I, D-IV

(2) A-III, B-I, C-IV, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-I, C-III, D-II

Correct Answer: (3) A-IV, B-III, C-I, D-II

Solution:

- Neoprene is a synthetic rubber, making it an elastomeric polymer. (A-IV)
- Polyester is a strong and durable material often used as a fibre polymer. (B-III)
- Urea formaldehyde resin is a thermosetting polymer that hardens irreversibly. (C-I)
- Polystyrene is a thermoplastic polymer, which can be reshaped with heat. (D-II)

Hence, the correct matching is A-IV, B-III, C-I, D-II.

Quick Tip

Understand the properties and applications of polymer types to differentiate between elastomers, fibres, thermoplastics, and thermosetting polymers.

---

66. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temperature. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are:

(1) Starch and iodine

(2) Methyl orange and H_2O_2

(3) Starch and H_2O_2

(4) Methyl orange and iodine

Correct Answer: (1) Starch and iodine

Solution:

In this reaction, iodine (I_2) is produced as a product when I^- reacts with H_2O_2. Starch is commonly used as an indicator because it forms a blue-colored complex with iodine. The reaction steps are as follows:

$$I^- + H_2O_2 \rightarrow I_2 + H_2O$$

$$I_2 + \text{Starch} \rightarrow \text{Blue complex}$$

Thus, the indicator is starch, and the compound forming the blue complex is iodine.

Quick Tip

Starch is a specific indicator for iodine, producing a characteristic blue-black complex.

---

67. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(1) Stomach ulcers

(2) Hyperacidity

(3) Anxiety and stress

(4) Depression and hypertension

Correct Answer: (4) Depression and hypertension

Solution:

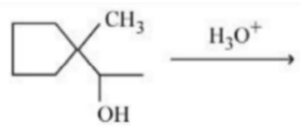
Equanil is a drug that is used as a tranquilizer. It helps in the treatment of anxiety, depression, and related disorders such as hypertension caused by stress. It acts by calming the central nervous system and stabilizing mood swings.

Quick Tip

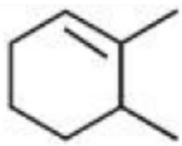
Remember that tranquilizers like Equanil are prescribed for mental health conditions involving anxiety, stress, or depression.

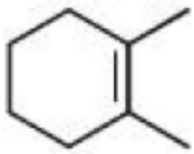
---

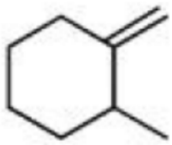
68. Find out the major product for the following reaction:



$\rightarrow$ Major Product

(1) 

(2) 

(3) 

(4) 

Correct Answer: (2)

Solution:

The reaction involves the dehydration of a secondary alcohol to form an alkene. Under acidic conditions (H_2O^+), the $-$OH group is protonated and leaves as water, forming a carbocation intermediate. The major product is determined by the stability of the alkene. In this case, the more substituted alkene (Zaitsev's rule) is the major product. The reaction mechanism is as follows:

1. Protonation of the alcohol group.

2. Loss of water to form a carbocation.

3. Elimination of a proton to form the alkene.

Thus, the major product is the one with the double bond in the more substituted position.

Quick Tip

Follow Zaitsev's rule: the major product of elimination is the more substituted, stable alkene.

---

69. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement):

(1) 1-Bromo-2-methylbutane

(2) 2-Bromopropane

(3) 2-Bromopentane

(4) 2-Bromo-3,3-dimethylpentane

Correct Answer: (3) 2-Bromopentane

Solution:

Dehydrohalogenation involves the elimination of HBr to form alkenes. The number of isomeric alkenes depends on the number of different β-hydrogens that can be removed.

For 2-Bromopentane, two different β-carbons are available, leading to the formation of multiple alkenes (e.g., pent-1-ene and pent-2-ene). Additionally, pent-2-ene can exist as cis and trans isomers, giving a total of three isomeric alkenes.

For the other options:

- 1-Bromo-2-methylbutane and 2-Bromo-3,3-dimethylpentane have only one possible elimination product.

- 2-Bromopropane gives only one product (propene).

Quick Tip

Identify β-hydrogens in the molecule to determine the number of possible elimination products.

70. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?

(1) C_8H_6

(2) C_9H_9

(3) C_6H_6

(4) C_9H_6

Correct Answer: (1) C_8H_6

Solution:

The combustion reaction can be represented as:

$$\mathrm{C}_x\mathrm{H}_y + O_2 \rightarrow x\mathrm{CO}_2 + \frac{y}{2}\mathrm{H}_2\mathrm{O}$$

Given that 9.5 moles of O_2 are required and 3 moles of water are produced, we can set up the following equations:

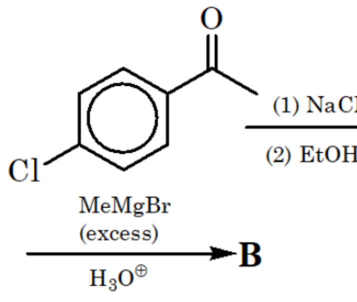
- $\frac{y}{2} = 3 \implies y = 6$

- $x + \frac{y}{4} = 9.5 \implies x + \frac{6}{4} = 9.5 \implies x = 8$

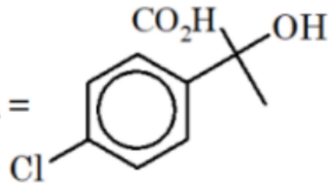
Thus, the molecular formula of the hydrocarbon is C_8H_6. This corresponds to an alkyne or aromatic compound.

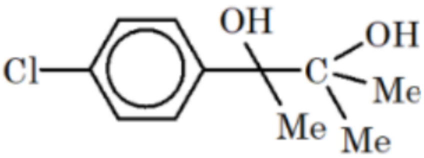
Quick Tip

Balance the combustion reaction by relating oxygen consumption and water/CO_2 production to deduce the molecular formula.

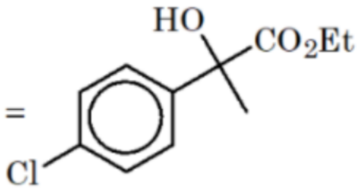


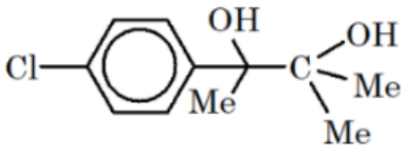
71. Find out the major products from the following reaction sequence:

A = 

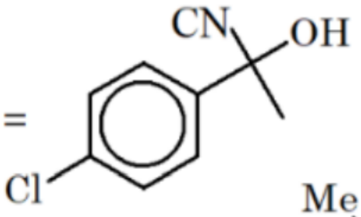
B = 

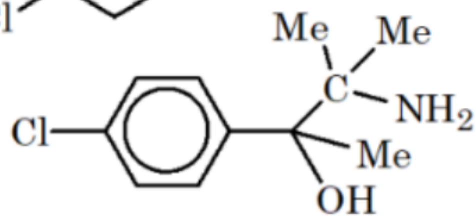
(A)

A = 

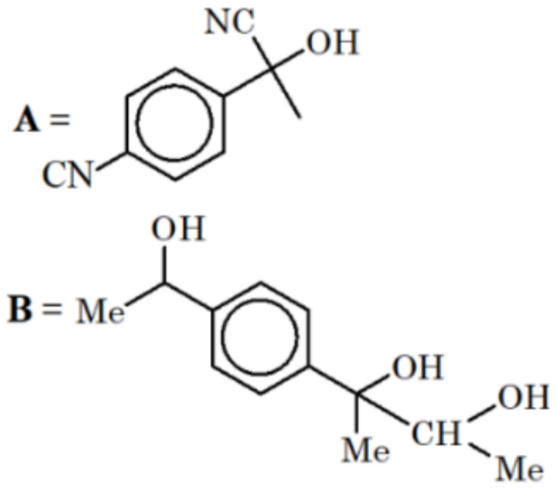
B = 

(B)

A = 

B = 

(C)



(D)

Correct Answer: (2)

Solution:

The reaction proceeds as follows:

1. NaCN reacts with the carbonyl group to form a cyanohydrin (-CN and -OH groups on the same carbon).

2. Ethanol in the presence of H_2O^+ hydrolyzes the cyanohydrin to form a carboxylic acid group (-COOH) and retains the hydroxyl group.

Thus, the major products are as shown in option (2), where the carboxylic acid (-COOH) and hydroxyl (-OH) groups are appropriately positioned.

Quick Tip

Understand the reactivity of cyanohydrins and their hydrolysis products to predict major reaction outcomes.

---

72. According to MO theory, the bond orders for O_2^-, CO, and NO^+, respectively, are:

(1) 1, 3, and 3

(2) 1, 3, and 2

(3) 1, 2, and 3

(4) 2, 3, and 3

Correct Answer: (1) 1, 3, and 3

Solution:

The bond order (BO) is calculated using the molecular orbital (MO) theory formula:

$$\text{BO} = \frac{\text{(Number of bonding electrons - Number of antibonding electrons)}}{2}$$

- For O_2^-: Adding one electron to O_2 decreases the bond order from 2 to 1.

- For CO: The bond order remains 3 because of strong triple bonding.

- For NO^+: Removal of one electron from NO increases the bond order from 2.5 to 3.

Thus, the bond orders are 1, 3, and 3, respectively.

Quick Tip

For molecular ions, the addition of electrons decreases the bond order, while electron removal increases it.

---

73. A solution of CrO_3 in amyl alcohol has a . . . colour:

(1) Green

(2) Orange-Red

(3) Yellow

(4) Blue

Correct Answer: (4) Blue

Solution:

CrO_3 (chromium trioxide) in amyl alcohol forms a blue complex. This is characteristic of certain chromium compounds when dissolved in organic solvents. The blue colour indicates the formation of a coordination compound involving chromium.

Quick Tip

The colour of chromium compounds is an important qualitative test for its oxidation states and complex formation.

---

74. The concentration of dissolved oxygen in water for growth of fish should be more

than X ppm, and biochemical oxygen demand in clean water should be less than Y ppm. X and Y in ppm are respectively:

(1) X Y

6 5

(2) X Y

4 8

(3) X Y

4 15

(4) X Y

6 12

Correct Answer: (1) X Y

6 5

Solution:

The dissolved oxygen (DO) in water is an essential parameter for the survival of aquatic organisms. Fish and other aquatic species depend on adequate oxygen levels to carry out cellular respiration and maintain metabolic processes. In general:

- For healthy growth and reproduction of fish, the dissolved oxygen concentration must be above 6 ppm.
- Levels below 4 ppm can cause stress, and prolonged exposure to such conditions may be lethal to most fish species.

On the other hand, biochemical oxygen demand (BOD) measures the oxygen consumed by microorganisms while decomposing organic matter in the water. It serves as an indirect indicator of the level of organic pollution:

- For clean water, BOD values should remain below 5 ppm.
- Higher BOD levels indicate the presence of excess organic matter, leading to oxygen depletion, which adversely affects aquatic life.

Why X = 6 ppm and Y = 5 ppm?

1. Dissolved oxygen levels above 6 ppm ensure a favorable environment for fish, supporting their growth, activity, and reproduction.
2. A BOD below 5 ppm reflects clean water, indicating minimal pollution and sufficient oxygen for aquatic organisms.

Thus, the correct values for X and Y are 6 ppm and 5 ppm, respectively.

Quick Tip

Dissolved oxygen levels above 4 ppm and low BOD values are critical for maintaining healthy aquatic ecosystems.

---

75. Reaction of propanamide with Br_2/KOH (aq) produces:

(1) Ethyl nitrile

(2) Propylamine

(3) Propanenitrile

(4) Ethylamine

Correct Answer: (4) Ethylamine

Solution:

This reaction is the Hofmann bromamide reaction, where amides are converted to primary amines with one fewer carbon atom. For propanamide ($CH_3CH_2CONH_2$):

$$CH_3CH_2CONH_2 \xrightarrow{Br_2/KOH} CH_3CH_2NH_2 \text{(Ethylamine)}$$

Quick Tip

In the Hofmann bromamide reaction, the product has one less carbon than the starting amide.

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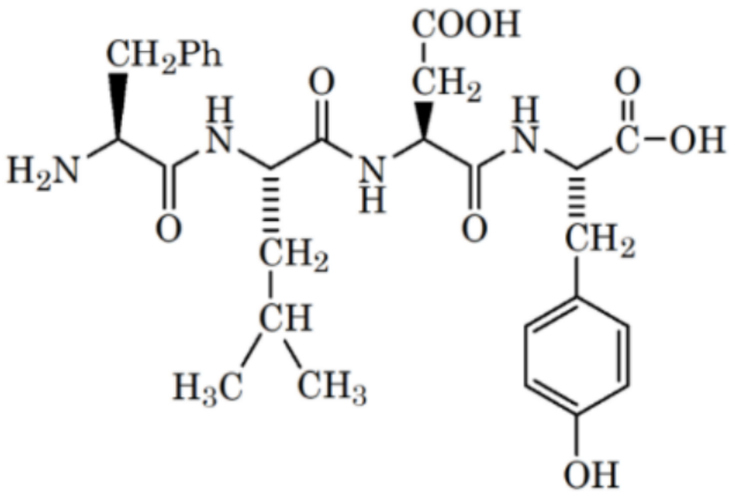
76. Following tetrapeptide can be represented as:



(F, L, D, Y, I, Q, P are one-letter codes for amino acids)

(1) FIQY

(2) FLDY

(3) YQLF

(4) PLDY

Correct Answer: (2) FLDY

Solution:

The given tetrapeptide contains the following amino acid residues:

- Phenylalanine (F)
- Leucine (L)
- Aspartic acid (D)
- Tyrosine (Y)

Thus, the sequence of amino acids is FLDY.

Quick Tip

Learn the one-letter codes for amino acids to identify peptide sequences quickly.

---

77. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$

(B) $G = H - TS$

(C) $\Delta S = \frac{q_{\text{rev}}}{T}$

(D) $\Delta H = \Delta U - nRT$

Choose the most appropriate answer from the options given below:

(1) C and D only

(2) B and C only

(3) A and B only

(4) B and D only

Correct Answer: (2) B and C only

Solution:

- (A) $\Delta U = q + p\Delta V$ is incorrect because the first law of thermodynamics states $\Delta U = q + w$, where $w = -p\Delta V$. Hence, the correct relation is $\Delta U = q - p\Delta V$.

- (B) $G = H - TS$ is correct, as it is the definition of Gibbs free energy (G).

- (C) $\Delta S = \frac{q_{\text{rev}}}{T}$ is correct, as it represents the change in entropy (S) under reversible conditions.

- (D) $\Delta H = \Delta U - nRT$ is incorrect because for an ideal gas, $\Delta H = \Delta U + nRT$.

Quick Tip

Remember that $\Delta U = q + w$, where $w = -p\Delta V$, and $\Delta H = \Delta U + nRT$ for ideal gases.

---

78. The major component of which of the following ore is sulphide based mineral?

(1) Calamine

(2) Siderite

(3) Sphalerite

(4) Malachite

Correct Answer: (3) Sphalerite

Solution:

- Calamine ($ZnCO_3$) is a carbonate-based mineral.

- Siderite ($FeCO_3$) is an iron carbonate mineral.

- Sphalerite (ZnS) is a sulphide-based mineral, making it the correct answer.

- Malachite ($Cu_2CO_3(OH)_2$) is a copper carbonate hydroxide mineral.

Quick Tip

Sphalerite (ZnS) is a sulphide ore commonly associated with zinc extraction.

---

79. Given below are two statements:

Statement I: Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II: Silicon forms both electron-rich and electron-deficient hydrides.

Choose the most appropriate answer from the options given below:

(1) Both the statements I and II are correct

(2) Statement I is incorrect but statement II is correct

(3) Both the statements I and II are incorrect

(4) Statement I is correct but statement II is incorrect

Correct Answer: (4) Statement I is correct but statement II is incorrect

Solution:

- Statement I is correct because nickel is widely used as a catalyst in hydrogenation reactions for producing edible fats (like margarine) and in the production of synthesis gas (syn gas).

- Statement II is incorrect as hydrides of silicon (SiH_4) are electron-precise and neither electron-rich nor electron-deficient.

Quick Tip

Silicon hydrides are electron-precise, unlike boron hydrides, which can be electron-deficient.

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80. Match List I with List II:

List I	List II
A. van't Hoff factor, i	I. Cryoscopic constant
B. k_f	II. Isotonic solutions
C. Solutions with same osmotic pressure	III. Normal molar mass / Abnormal Mass
D. Azeotropes	IV. Solutions with same composition of vapour above it

Choose the correct answer from the options given below:

(1) A-III, B-I, C-II, D-IV

(2) A-II, B-I, C-III, D-IV

(3) A-III, B-II, C-IV, D-I

(4) A-I, B-III, C-II, D-IV

Correct Answer: (1) A-III, B-I, C-II, D-IV

Solution:

- i (van't Hoff factor) is associated with the abnormal molar mass (M_{ab}), making A-III correct.
- k_f (cryoscopic constant) relates to the depression of freezing point, making B-I correct.
- Solutions with the same osmotic pressure are isotonic solutions, making C-II correct.
- Azeotropes are solutions with the same composition in both the liquid and vapour phases, making D-IV correct.

Quick Tip

Understand colligative properties and their relation to the van't Hoff factor and isotonic solutions for better accuracy.

---

81. On heating, $LiNO_3$ gives how many compounds among the following? Li_2O, N_2, O_2, $LiNO_2$, NO_2

Correct Answer: (3)

Solution:

The decomposition of lithium nitrate ($LiNO_3$) is as follows:

$$2LiNO_3 \rightarrow 2Li_2O + 2NO_2 + \frac{1}{2}O_2$$

The reaction produces three compounds: Li_2O, NO_2, and O_2.

Quick Tip

Thermal decomposition of nitrates often produces oxides, nitrogen oxides, and oxygen depending on the metal.

---

82. At 298 K:

$$N_2 + 3H_2 \rightleftharpoons 2NH_3, \ K_1 = 4 \times 10^5$$

$$N_2 + O_2 \rightleftharpoons 2NO, \ K_2 = 1.6 \times 10^{12}$$

$$H_2 + \frac{1}{2}O_2 \rightleftharpoons H_2O, \ K_3 = 1.0 \times 10^{13}$$

Based on the above equilibria, the equilibrium constant of the reaction:

$$2NH_3 + \frac{5}{2}O_2 \rightleftharpoons 2NO + 3H_2O$$

is $\ldots \times 10^{-33}$ (nearest integer).

Correct Answer: (4)

Solution:

The equilibrium constant for the given reaction is calculated by combining the given equilibria:

$$K_{\text{eq}} = \frac{K_2 \times K_3^3}{K_1}$$

Substituting the values:

$$K_{\text{eq}} = \frac{(1.6 \times 10^{12}) \times (1.0 \times 10^{13})^3}{4 \times 10^5}$$

$$K_{\text{eq}} = \frac{1.6 \times 10^{12} \times 10^{39}}{4 \times 10^5} = 4 \times 10^{33}$$

Quick Tip

For combined equilibria, multiply or divide the equilibrium constants based on how the reactions are combined.

---

83. For conversion of compound A $\rightarrow$ B, the rate constant of the reaction was found to be 4.6×10^{-5} L mol^{-1} s^{-1}. The order of the reaction is . . .

Correct Answer: (2)

Solution:

The unit of the rate constant is given as $L\,\text{mol}^{-1}\,\text{s}^{-1}$, which corresponds to a second-order reaction. The general formula for the unit of a rate constant is:

$$\text{Unit of } k = (\text{concentration})^{1-n} \times \text{time}^{-1}$$

where n is the order of the reaction. For $n = 2$, the unit becomes $L\,\text{mol}^{-1}\,\text{s}^{-1}$.

> **Quick Tip**
>
> The units of the rate constant can be used to quickly determine the order of the reaction.

---

84. Total number of acidic oxides among $N_2O_3, NO, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ and NO_2 is . . .

Correct Answer: (4)

Solution:

Acidic oxides react with water to form acids. Among the given oxides: - Acidic oxides: N_2O_3, Cl_2O_7, SO_2, NO_2 - Neutral oxides: NO, N_2O, CO - Basic oxides: CaO, Na_2O

Thus, there are 4 acidic oxides.

> **Quick Tip**
>
> Classify oxides as acidic, basic, or neutral based on their reaction with water and acids/bases.

---

85. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of the compound is . . . g mol^{-1} (nearest integer).

Correct Answer: (200)

Solution:

- Mass of carbon in the compound:

$$0.01 \times \frac{60}{100} = 0.006\,\text{g}$$

- Moles of CO_2 produced:

$$\frac{4.4}{44} = 0.1\,\text{mol}$$

- Mass of carbon in CO_2:

$$0.1 \times 12 = 1.2\,\text{g}$$

- The molar mass M of the compound:

$$M = \frac{\text{mass of compound}}{\text{moles of compound}} = \frac{0.01}{0.006} \times 12 = 200\,\text{g/mol}$$

Quick Tip

Relate the mass and moles of carbon to deduce the molar mass using the percentage composition.

---

86. The denticity of the ligand present in Fehling's reagent is ...

Correct Answer: (4)

Solution:

Fehling's reagent contains Cu^{2+} ions complexed with tartrate ions. Tartrate is a bidentate ligand as it binds to the metal through two donor atoms.

Quick Tip

The denticity of a ligand refers to the number of donor atoms through which it binds to a metal ion.

---

87. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is $\ldots \times 10^{21}$ **(Nearest integer). (Given** $N_A = 6.02 \times 10^{23}$**)**

Correct Answer: (36)

Solution:

- One unit cell of hcp contains 18 voids.

- Total number of voids in 0.02 mol of hcp:

$$\text{No. of voids} = 18 \times 6.02 \times 10^{23} \times 0.02$$

$$= 3.6 \times 10^{21}$$

> **Quick Tip**
>
> In a hexagonal close-packed structure (hcp), the voids are always proportional to the number of atoms.

---

88. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 Å. The radius of the third Bohr orbit of He^+ is $\ldots$ picometer (Nearest integer).

Correct Answer: (270)

Solution:

The radius of the n-th Bohr orbit is given by:

$$r_n = r_1 \frac{n^2}{Z}$$

For $n = 3$ and $Z = 2$:

$$r_{\text{He}^+} = 0.6 \times \frac{3^2}{2} = 2.7\,\text{Å}$$

Converting to picometers:

$$2.7\,\text{Å} = 270\,\text{pm}$$

> **Quick Tip**
>
> For hydrogen-like atoms, the radius is inversely proportional to the nuclear charge (Z).

---

89. The equilibrium constant for the reaction:

$\text{Zn}(s) + \text{Sn}^{2+}(aq) \rightleftharpoons \text{Zn}^{2+}(aq) + \text{Sn}(s)$

is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn^{2+}/Sn if $E^\circ_{\text{Zn}^{2+}/\text{Zn}} = -0.76\,\text{V}$ is $\ldots \times 10^{-2}$ V (Nearest integer).

Correct Answer: (17)

Solution:

The equilibrium constant is related to the electrode potentials:

$$\Delta G^\circ = -nFE^\circ = -2.303RT\log K$$

$$E^\circ_{\text{cell}} = \frac{0.059}{2}\log K$$

Substituting $K = 1 \times 10^{20}$:

$$E^\circ_{\text{cell}} = \frac{0.059}{2}\log(1 \times 10^{20}) = 0.059 \times 10 = 0.59\,\text{V}$$

Using $E^\circ_{\text{cell}} = E^\circ_{\text{Sn}^{2+}/\text{Sn}} - E^\circ_{\text{Zn}^{2+}/\text{Zn}}$:

$$0.59 = E^\circ_{\text{Sn}^{2+}/\text{Sn}} - (-0.76)$$

$$E^\circ_{\text{Sn}^{2+}/\text{Sn}} = 0.59 - 0.76 = 0.17\,\text{V} = 17 \times 10^{-2}\,\text{V}$$

Quick Tip

For electrochemical cells, use ΔG° or K_{eq} to find electrode potentials.

---

90. The volume of HCl containing 73 g L^{-1}, required to completely neutralize NaOH obtained by reacting 0.69 g of metallic sodium with water, is . . . mL (Nearest integer).

Correct Answer: (15)

Solution:

Moles of Na:

$$\text{Moles of Na} = \frac{0.69}{23} = 3 \times 10^{-2}$$

$$\text{Na} + \text{H}_2\text{O} \rightarrow \text{NaOH} + \frac{1}{2}\text{H}_2$$

Moles of NaOH produced = 3×10^{-2}.

No. of equivalents of NaOH = No. of equivalents of HCl.

Mass of HCl = $73\,\text{g/L}$. Normality:

$$\text{Normality} = \frac{73}{36.5} = 2\,\text{N}$$

Using $\text{N}_1V_1 = \text{N}_2V_2$:

$$2 \times V = 3 \times 10^{-2} \quad \Rightarrow \quad V = 15\,\text{mL}$$

Quick Tip

Relate the moles of reactants to equivalents and use the concept of normality for neutralization reactions.
