MHT CET 2023 : 10th May Morning Shift

Mathematics

Question 1

A group consists of 8 boys and 5 girls, then the number of committees of 5 persons that can be formed, if committee consists of at least 2 girls and at most 2 boys, are

Options:

A. 300

B. 320

C. 321

D. 322

Answer: C

Solution:

A committee of 5 is to be formed from 8 boys and 5 girls so that it consists of at least 2 girls and at most 2 boys.

Total number of ways

$$
\begin{aligned}
& =(\text{ Five girls })+(4 \text{ girls })(1 \text{ boy })+(3 \text{ girls })(2 \text{ boys }) \\
& ={ }^5 C_5+{ }^5 C_4 \times{ }^8 C_1+{ }^5 C_3 \times{ }^8 C_2 \\
& =1+(5 \times 8)+(10 \times 28) \\
& =1+40+280 \\
& =321
\end{aligned}
$$

Question 2

Scalar projection of the line segment joining the points $\mathrm{A}(-2,0,3), \mathrm{B}(1,4,2)$ on the line whose direction ratios are $6,-2,3$ is

Options:

A. $\frac{23}{7}$

B. 1

C. 7

D. $\frac{1}{7}$

Answer: B

Solution:

Let $\bar{a}$ be the vector joining $\mathrm{A}(-2,0,3)$ and $\mathrm{B}(1,4,2)$.

$$\therefore \quad \overline{\mathrm{a}}=(1-(-2))\hat{\mathrm{i}}+(4-0)\hat{\mathrm{j}}+(2-3)\hat{\mathrm{k}}$$
$$=3\hat{\mathrm{i}}+4\hat{\mathrm{j}}-\hat{\mathrm{k}}$$
and $\bar{b}=6\hat{i}-2\hat{j}+3\hat{k}$
$$\therefore \quad \text{Projection } =\frac{\overline{\mathrm{a}}\cdot\overline{\mathrm{b}}}{|\overline{\mathrm{b}}|}=\frac{3\times 6+4\times(-2)-1\times 3}{\sqrt{6^2+(-2)^2+3^2}}$$
$$=\frac{18-8-3}{\sqrt{49}}$$
$$=\frac{7}{7}$$
$$=1$$

Question 3

For a binomial variate X with $\mathrm{n}=6$ if $P(X=4)=\frac{135}{2^{12}}$, then its variance is

Options:

A. $\frac{8}{9}$

B. $\frac{1}{4}$

C. 4

D. $\frac{9}{8}$

Answer: D

Solution:

$$\text{Given, } P(X=4)=\frac{135}{2^{12}}$$
$$\Rightarrow {}^{6}C_{4}p^{4}q^{2}=\frac{135}{2^{12}}$$
$$\Rightarrow 15p^{4}q^{2}=\frac{135}{2^{12}}$$
$$\Rightarrow p^{4}q^{2}=\frac{3^{2}}{2^{12}}$$
$$\Rightarrow p^{2}q=\frac{3}{2^{6}}$$
$$\Rightarrow p^{2}(1-p)=\frac{3}{64}$$
$$\Rightarrow p^{2}(1-p)=\left(\frac{1}{4}\right)^{2}\cdot\left(1-\frac{1}{4}\right)$$
$$\Rightarrow p=\frac{1}{4}$$
$$\text{and } q=1-\frac{1}{4}=\frac{3}{4}$$
$$\text{Variance } = npq$$
$$=6\times\frac{1}{4}\times\frac{3}{4}$$
$$=\frac{9}{8}$$

Question 4

If $\overline{a}=2\hat{i}+3\hat{j}+2\hat{k}, \overline{b}=2\hat{i}+\hat{j}-\hat{k}$ and $\overline{c}=\hat{i}+3\hat{j}$ are such that $(\bar{a}+\lambda\bar{b})$ is perpendicular to $\bar{c}$, then the value of λ is

Options:

A. $\frac{5}{11}$

B. $\frac{11}{5}$

C. $\frac{-11}{5}$

D. $\frac{-5}{11}$

Answer: C

Solution:

Let $\overline{\mathrm{d}} = \overline{\mathrm{a}} + \lambda\overline{\mathrm{b}}$

$$\begin{aligned}\overline{\mathrm{d}} &= (2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 2\hat{\mathrm{k}}) + \lambda(2\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}) \\ &= 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 2\hat{\mathrm{k}} + 2\lambda\hat{\mathrm{i}} + \lambda\hat{\mathrm{j}} - \lambda\hat{\mathrm{k}} \\ &= (2\lambda + 2)\hat{\mathrm{i}} + (3 + \lambda)\hat{\mathrm{j}} + (2 - \lambda)\hat{\mathrm{k}}\end{aligned}$$

Now, $\overline{\mathrm{d}}$ is perpendicular to $\overline{\mathrm{c}}$.

$$\begin{aligned}\therefore \quad & \bar{c} \cdot \bar{d} = 0 \\ & \Rightarrow (\hat{i} + 3\hat{j}) \cdot [(2\lambda + 2)\hat{i} + (3 + \lambda)\hat{j} + (2 - \lambda)\hat{k}] = 0 \\ & \Rightarrow 1(2\lambda + 2) + 3(3 + \lambda) = 0 \\ & \Rightarrow 2\lambda + 2 + 9 + 3\lambda = 0 \\ & \Rightarrow 5\lambda + 11 = 0 \\ & \Rightarrow \lambda = \frac{-11}{5}\end{aligned}$$

Question 5

The number of integral values of p in the domain $[-5, 5]$, such that the equation $2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$ represents pair of lines, are

Options:

A. 3

B. 4

C. 7

D. 8

Answer: D

Solution:

Given equation of pair of lines is

$2x^2 + 4xy - py^2 + 4x + qy + 1 = 0$

Comparing with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we get $\mathrm{a} = 2,\ \mathrm{h} = 2,\ \mathrm{b} = -\mathrm{p}$

If the given equation represents a pair of straight lines, then

$\mathrm{h}^2 \geq \mathrm{ab}$
$\Rightarrow 4 \geq -2\mathrm{p}$
$\Rightarrow 2 \geq -\mathrm{p}$
$\Rightarrow \mathrm{p} \geq -2$

$\therefore$ Possible values of p from domain $[-5, 5]$ are $-2, -1, 0, 1, 2, 3, 4, 5$.

$\therefore$ Number of integral values of $p = 8$

Question 6

An open metallic tank is to be constructed, with a square base and vertical sides, having volume 500 cubic meter. Then the dimensions of the tank, for minimum area of the sheet metal used in its construction, are

Options:

A. $5\ \mathrm{m}, 5\ \mathrm{m}, 10\ \mathrm{m}$

B. $10\ \mathrm{m}, 10\ \mathrm{m}, 5\ \mathrm{m}$

C. $2\ \mathrm{m}, 2\ \mathrm{m}, 8\ \mathrm{m}$

D. $15\ \mathrm{m}, 15\ \mathrm{m}, 5\ \mathrm{m}$

Answer: B

Solution:

Let the length, breadth and depth of open tank be x, x and y respectively.

Volume $(\mathrm{V}) = x^2 y$

$\therefore$ $500 = x^2 y$ ..... (i)

Total surface area of open tank is given by

S $= x^2 + 4xy$ ..... (ii)

From (i), $y = \frac{500}{x^2}$

From (ii), S $= x^2 + 4x \times \frac{500}{x^2}$

$= x^2 + \frac{2000}{x}$

Differentiating w.r.t. x, we get

$\frac{\mathrm{dS}}{\mathrm{d}x} = 2x - \frac{2000}{x^2}$

For minimum area, $\frac{\mathrm{dS}}{\mathrm{d}x} = 0$

$$\therefore \quad 2x - \frac{2000}{x^2} = 0$$
$$\Rightarrow 2000 = 2x^3$$
$$\Rightarrow x^3 = 1000$$

$$\Rightarrow x = 10 \text{ m}$$
$$\frac{\mathrm{d}^2 \mathrm{S}}{\mathrm{d}x^2} = 2 + \frac{4000}{x^3}$$
$$\Rightarrow \left(\frac{\mathrm{d}^2 \mathrm{S}}{\mathrm{d}x^2}\right)_{x=10} > 0$$

S is minimum when $x = 10$ m and $y = 5$ m ...[From (i)]

Question 7

The p.d.f. of a discrete random variable is defined as

$$\mathrm{f}(x) = \begin{cases} \mathrm{k}x^2, 0 \leq x \leq 6 \\ 0, \text{ otherwise} \end{cases}$$

Then the value of $F(4)$ (c.d.f) is

Options:

A. $\frac{30}{91}$

B. $\frac{30}{97}$

C. $\frac{15}{47}$

D. $\frac{15}{97}$

Answer: A

Solution:

$$\begin{aligned}
&\mathrm{f}(x) = \mathrm{k}x^2, 0 \leq x \leq 6 \\
\therefore\quad & \mathrm{k}(0)^2 + \mathrm{k}(1)^2 + \mathrm{k}(2)^2 + \mathrm{k}(3)^2 + \mathrm{k}(4)^2 \\
&\quad + \mathrm{k}(5)^2 + \mathrm{k}(6)^2 = 1 \\
&\Rightarrow \mathrm{k} + 4\mathrm{k} + 9\mathrm{k} + 16\mathrm{k} + 25\mathrm{k} + 36\mathrm{k} = 1 \\
&\Rightarrow 91\mathrm{k} = 1 \\
&\Rightarrow \mathrm{k} = \frac{1}{91} \\
\mathrm{F}(4) &= \mathrm{P}(\mathrm{X} \leq 4) = \mathrm{P}(\mathrm{X}=0) + \mathrm{P}(\mathrm{X}=1) + \mathrm{P}(\mathrm{X}=2) + \mathrm{P}(\mathrm{X}=3) + \mathrm{P}(\mathrm{X}=4) \\
&= \mathrm{k}(0)^2 + \mathrm{k}(1)^2 + \mathrm{k}(2)^2 + \mathrm{k}(3)^2 + \mathrm{k}(4)^2 \\
&= \mathrm{k} + 4\mathrm{k} + 9\mathrm{k} + 16\mathrm{k} \\
&= 30\mathrm{k} \\
&= 30\left(\frac{1}{91}\right) \\
&= \frac{30}{91}
\end{aligned}$$

Question 8

The variance, for first six prime numbers greater than 5, is

Options:

A. 27

B. 28

C. 15

D. 20

Answer: B

Solution:

First six prime numbers greater than 5 are 7,11, 13, 17, 19, 23.

$$\text{Variance } \left(\sigma^2\right) = \frac{1}{\text{n}}\left(\sum_{\text{i}=1}^{\text{n}} x_{\text{i}}^2\right) - (\bar{x})^2$$

$$\therefore \quad \sigma^2 = \frac{1}{6}\left(7^2 + 11^2 + 13^2 + 17^2 + 19^2 + 23^2\right)$$

$$-\left(\frac{7 + 11 + 13 + 17 + 19 + 23}{6}\right)^2$$

$$= \frac{1518}{6} - \left(\frac{90}{6}\right)^2$$

$$= 253 - 225$$

$$= 28$$

Question 9

The value of $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$ is

Options:

A. $\frac{1}{3\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{2}{3\sqrt{3}}$

D. $\frac{4}{3\sqrt{3}}$

Answer: C

Solution:

$$\lim_{x \to a} \frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$$

$$= \lim_{x \to a} \frac{(\sqrt{a + 2x} - \sqrt{3x})(\sqrt{a + 2x} + \sqrt{3x})(\sqrt{3a + x} + 2\sqrt{x})}{(\sqrt{3a + x} - 2\sqrt{x})(\sqrt{3a + x} + 2\sqrt{x})(\sqrt{a + 2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a + 2x - 3x)(\sqrt{3a + x} + 2\sqrt{x})}{(3a + x - 4x)(\sqrt{a + 2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a - x)(\sqrt{3a + x} + 2\sqrt{x})}{3(a - x)(\sqrt{a + 2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \cdot \frac{(\sqrt{3a+a} + 2\sqrt{a})}{(\sqrt{a+2a} + \sqrt{3a})}$$
$$= \frac{4\sqrt{a}}{3 \times 2 \times \sqrt{3} \cdot \sqrt{a}}$$
$$= \frac{2}{3\sqrt{3}}$$

Question 10

The points $(1, 3)$, $(5, 1)$ are opposite vertices of a diagonal of a rectangle. If the other two vertices lie on the line $y = 2x +$ c, then one of the vertex on the other diagonal is

Options:

A. $(1, -2)$

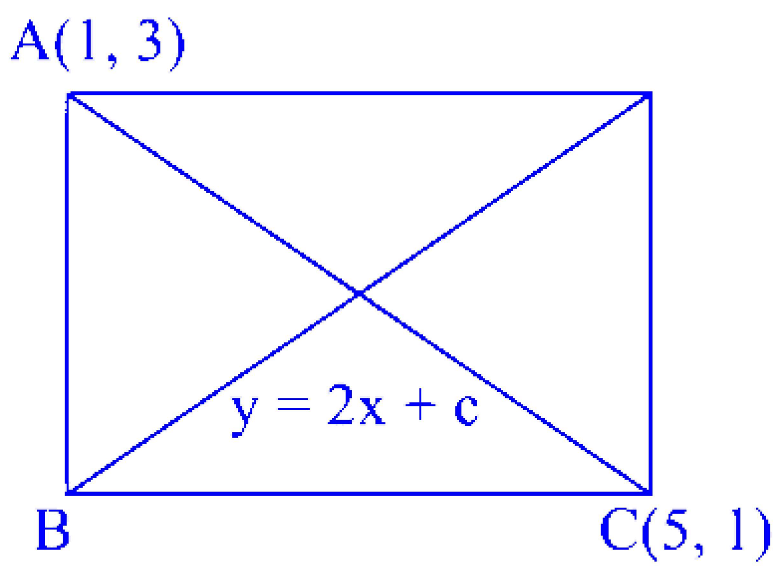
B. $(0, -4)$

C. $(2, 0)$

D. $(3, 2)$

Answer: C

Solution:



Diagonals of rectangle bisect each other.

$\therefore$ Midpoint of $(1,3)$ and $(5,1)$ is $(3,2)$.

Also, $y=2x+c$ passes through $(3,2)$.

$$\begin{aligned} \therefore \quad 2 &= 2(3)+c \\ \therefore \quad c &= -4 \end{aligned}$$

$\therefore$ Other two vertices lie on $y=2x-4$.

Let co-ordinates of B be (x,y) i.e., $(x,2x-4)$

slope of AB$\times$ slope of BC $=-1$

$$\begin{aligned} &\Rightarrow \left(\frac{2x-4-3}{x-1}\right)\left(\frac{2x-4-1}{x-5}\right)=-1 \\ &\Rightarrow \left(\frac{2x-7}{x-1}\right)\left(\frac{2x-5}{x-5}\right)=-1 \\ &\Rightarrow x^2-6x+8=0 \\ &\Rightarrow x=4,2 \end{aligned}$$

When $x=4, y=4$

When $x=2, y=0$

$\therefore$ Vertex of the other diagonal is $(2,0)$.

Question 11

Considering only the principal values of an inverse function, the set

$\mathrm{A}=\left\{x \geq 0 / \tan^{-1} x+\tan^{-1} 6x=\frac{\pi}{4}\right\}$

Options:

A. is an empty set.

B. is a singleton set.

C. contains more than two elements.

D. contains two elements.

Answer: B

Solution:

Consider, $\tan^{-1} x + \tan^{-1} 6x = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{7x}{1-6x^2}\right) = \frac{\pi}{4}$$
$$\ldots\left[\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$
$$\Rightarrow \frac{7x}{1-6x^2} = 1$$
$$\Rightarrow 7x = 1 - 6x^2$$
$$\Rightarrow 6x^2 + 7x - 1 = 0$$
$$\Rightarrow x = \frac{-7 \pm \sqrt{73}}{12}$$

Since $x \geq 0$

$$x = \frac{-7+\sqrt{73}}{12}$$

A is a singleton set.

Question 12

The line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$, then the value of $\alpha^2 + \alpha\beta + \beta^2$ is

Options:

A. 127

B. 43

C. 109

D. 61

Answer: B

Solution:

Line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$

The direction ratios of the line are $3, -5, 2$.

The direction ratios of the normal to the plane are $1, 3, -\alpha$.

The given line is perpendicular to the normal of plane.

$$\begin{aligned} \therefore \quad & 3(1)+(-5)(3)+2(-\alpha)=0 \\ & \Rightarrow 3-15-2\alpha=0 \\ & \Rightarrow -12-2\alpha=0 \\ & \Rightarrow \alpha=-6 \end{aligned}$$

Also, point $(2, 1, -2)$ lies on the plane

$$\begin{aligned} & x+3y-\alpha z+\beta=0 \\ & \Rightarrow 2+3-(-6)(-2)+\beta=0 \\ & \Rightarrow 2+3-12+\beta=0 \\ & \Rightarrow \beta=7 \\ \therefore \quad & \alpha^2+\alpha\beta+\beta^2=36-42+49=43 \end{aligned}$$

Question 13

The vector projection of $\overline{\mathrm{AB}}$ on $\overline{\mathrm{CD}}$, where $A \equiv (2, -3, 0), B \equiv (1, -4, -2), C \equiv (4, 6, 8)$ and $\mathrm{D} \equiv (7, 0, 10)$, is

Options:

A. $\frac{1}{49}(3\hat{\mathrm{i}}-6\hat{\mathrm{j}}+2\hat{\mathrm{k}})$

B. $\frac{1}{6}(-\hat{\mathrm{i}}-\hat{\mathrm{j}}-2\hat{\mathrm{k}})$

C. $-\frac{1}{49}(3\hat{\mathrm{i}}-6\hat{\mathrm{j}}+2\hat{\mathrm{k}})$

D. $-\frac{1}{6}(-\hat{\mathrm{i}}-\hat{\mathrm{j}}-2\hat{\mathrm{k}})$

Answer: C

Solution:

$\overline{\mathrm{AB}}=-\hat{\mathrm{i}}-\hat{\mathrm{j}}-2\hat{\mathrm{k}}$

$\overline{\mathrm{CD}}=3\hat{\mathrm{i}}-6\hat{\mathrm{j}}+2\hat{\mathrm{k}}$

Vector projection of $\overline{\mathrm{AB}}$ on $\overline{\mathrm{CD}}$

$$= (\overline{AB} \cdot \overline{CD}) \frac{\overline{CD}}{|\overline{CD}|^2}$$

$$= (-3 + 6 - 4) \frac{(3\hat{i} - 6\hat{j} + 2\hat{k})}{\left(\sqrt{3^2 + (-6)^2 + 2^2}\right)^2}$$

$$= \frac{-1}{49}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

Question 14

If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other internally, then the value of α^3 is

Options:

A. $\frac{27}{64}$

B. $\frac{125}{27}$

C. $\frac{27}{125}$

D. $\frac{64}{27}$

Answer: D

Solution:

$x^2 + y^2 = 9$
$C_1 = (0, 0), r_1 = 3$
$x^2 + y^2 + 2\alpha x + 2y + 1 = 0$
$C_2 = (-\alpha, -1)$
$r_2 = \sqrt{\alpha^2 + 1 - 1} = \alpha$

Since the given circles touch each other internally,

$C_1C_2 = |r_1 - r_2|$

$\Rightarrow \sqrt{a^2+1} = |3-\alpha|$

$\Rightarrow \alpha^2 + 1 = 9 + \alpha^2 - 6\alpha$

$\Rightarrow 6\alpha = 8$

$\Rightarrow \alpha = \dfrac{4}{3}$

$\Rightarrow \alpha^3 = \dfrac{64}{27}$

Question 15

If $y = \cos^{-1}\left(\frac{\mathrm{a}^2}{\sqrt{x^4+\mathrm{a}^4}}\right)$, then $\frac{\mathrm{d}y}{\mathrm{d}x}$ is

Options:

A. $\frac{2a^2x}{x^4+a^4}$

B. $\frac{2a^2x^2}{\sqrt{x^4+a^4}}$

C. $\frac{a^4x^4}{x^4+a^4}$

D. $\frac{a^4x^2}{2\sqrt{x^4+a^4}}$

Answer: A

Solution:

$$y=\cos^{-1}\left(\frac{\mathrm{a}^2}{\sqrt{x^4+a^4}}\right)$$

Put $x^2=a^2\tan\theta$

$$\therefore\quad \theta=\tan^{-1}\left(\frac{x^2}{a^2}\right)$$

$$\therefore\quad y=\cos^{-1}\left(\frac{a^2}{\sqrt{a^4\tan^2\theta+a^4}}\right)$$

$$\Rightarrow y=\cos^{-1}\left(\frac{a^2}{\sqrt{a^4\left(1+\tan^2\theta\right)}}\right)$$

$$\Rightarrow y=\cos^{-1}\left(\frac{1}{\sec\theta}\right)$$

$$\Rightarrow y=\cos^{-1}(\cos\theta)$$

$$\Rightarrow y=\theta$$

$$\Rightarrow y=\tan^{-1}\left(\frac{x^2}{a^2}\right)$$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{1+\left(\frac{x^2}{a^2}\right)^2}\cdot\frac{d}{dx}\left(\frac{x^2}{a^2}\right)$$

$$=\frac{a^4}{a^4+x^4}\cdot\frac{2x}{a^2}$$

$$=\frac{2a^2x}{x^4+a^4}$$

Question 16

The population $\mathrm{P}=\mathrm{P}(\mathrm{t})$ at time t of certain species follows the differential equation $\frac{dP}{dt}=0.5P-450$. If $P(0)=850$, then the time at which population becomes zero is

Options:

A. $2\log 18$

B. $\log 9$

C. $\frac{1}{2}\log 18$

D. $\log 18$

Answer: A

Solution:

Given differential equation is

$$\frac{dP}{dt} = 0.5P - 450$$

$$= \frac{P}{2} - \frac{900}{2}$$

$$\therefore \quad \frac{dP}{dt} = \frac{P-900}{2}$$

$$\therefore \quad \frac{2dP}{P-900} = dt$$

Integrating on both sides, we get

$$2\log|P-900| = t + c$$

$P(0) = 850$ i.e., $P = 850$ when $t = 0$

$$\therefore \quad c = 2\log 50$$

$$\therefore \quad 2\log|P-900| = t + 2\log 50$$

When $P = 0$,

$$2\log 900 = t + 2\log 50$$

$$\Rightarrow t = 2(\log 900 - \log 50)$$

$$= 2\log\frac{900}{50} = 2\log 18$$

Question 17

The vertices of the feasible region for the constraints $x+y \leq 4, x \leq 2, y \leq 1, x+y \geq 1, x, y \geq 0$ are

Options:

A. $(1,0),(2,0),(2,1),(0,4)$

B. $(0,1),(4,0),(0,4),(1,0)$

C. $(1,0),(2,0),(2,1),(0,1)$

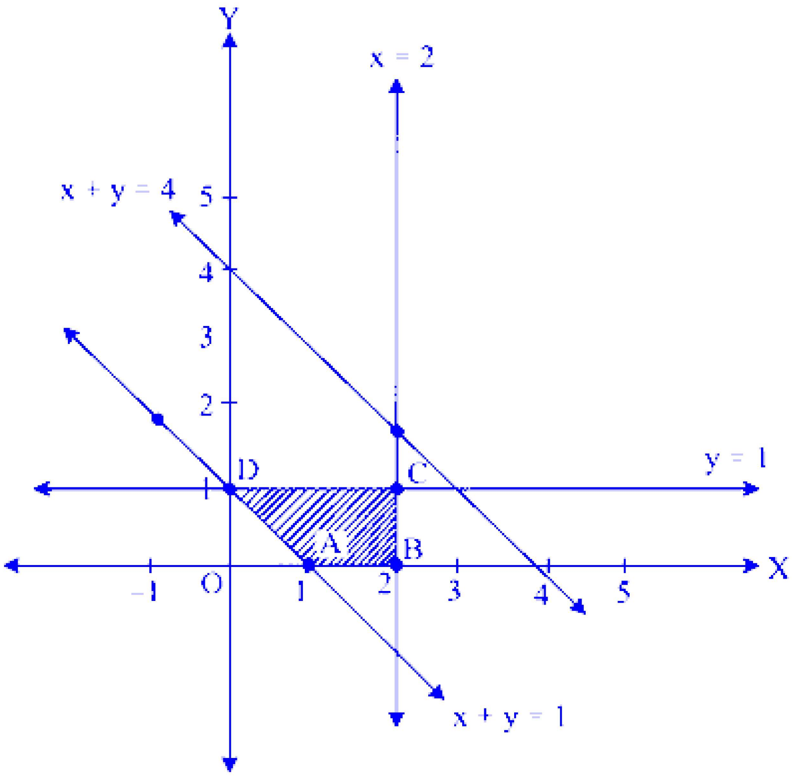
D. $(1,0),(4,0),(2,1),(0,4)$

Answer: C

Solution:

Feasible region lies on origin side of $x + y = 4, x = 2, y = 1$ and non-origin side of $x + y = 1$ and in first quadrant.

$\therefore$ Vertices of feasible region are $A(1,0), B(2,0), C(2,1), D(0,1)$



Question 18

The value of $\tan\frac{\pi}{8}$ is

Options:

A. $1 - \sqrt{2}$

B. $-1 - \sqrt{2}$

C. $\sqrt{2} - 1$

D. $\sqrt{2} + 1$

Answer: C

Solution:

Since $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\therefore \quad \tan\dfrac{\pi}{4} = \dfrac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$\Rightarrow 1 = \dfrac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

Let $y = \tan\dfrac{\pi}{8}$

$\Rightarrow 1 = \dfrac{2y}{1-y^2}$

$\Rightarrow y^2 + 2y - 1 = 0$

$\Rightarrow y = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$\Rightarrow y = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$\Rightarrow y = -1 \pm \sqrt{2}$

$\therefore \quad \tan\frac{\pi}{8} = -1 \pm \sqrt{2}$

Since $\frac{\pi}{8}$ lies in 1^{st} quadrant.

$\therefore \quad \tan\dfrac{\pi}{8} \neq -1 - \sqrt{2}$

$\therefore \quad \tan\dfrac{\pi}{8} = -1 + \sqrt{2}$

$= \sqrt{2} - 1$

Question 19

If $B = \begin{bmatrix} 1 & \alpha & 2 \\ 1 & 2 & 2 \\ 2 & 3 & 3 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 5$, then α is equal to

Options:

A. 25

B. 27

C. $3\sqrt{3}$

D. 5

Answer: B

Solution:

Using $|\text{Adj } A| = |A|^{n-1}$

But $\mathrm{B} = \operatorname{Adj}(\mathrm{A})$ .... [Given]

$\therefore \quad |\mathrm{B}| = |\mathrm{A}|^2$

$$\Rightarrow \begin{vmatrix} 1 & \alpha & 2 \\ 1 & 2 & 2 \\ 2 & 3 & 3 \end{vmatrix} = |\mathrm{A}|^2$$
$\Rightarrow \alpha - 2 = 5^2$
$\Rightarrow \alpha - 2 = 25$
$\Rightarrow \alpha = 27$

Question 20

The logical statement $[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ is equivalent to

Options:

A. $(p \wedge r) \wedge \sim q$

B. $(p \wedge \sim q) \vee r$

C. $\sim \mathrm{p} \vee \mathrm{r}$

D. $\sim p \wedge r$

Answer: A

Solution:

$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$\equiv [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$ ...[De Morgan's law]

$\equiv p \wedge (\sim q \vee r) \wedge (\sim q \wedge r)$ ...[Distributive law]

$\equiv p \wedge [(\sim q \vee r) \wedge \sim q] \wedge r$ ...[Associative law]

$\equiv p \wedge (\sim q) \wedge r$ ....[Absorption law]

$\equiv (p \wedge r) \wedge \sim q$ ...[Commutative law]

Question 21

If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1, i = \sqrt{-1}$, then z lies on

Options:

A. circle.

B. line.

C. parabola.

D. ellipse.

Answer: B

Solution:

$$w = \frac{z}{z - \frac{1}{3}i}$$
$$\Rightarrow w = \frac{3z}{3z - i}$$

Applying mod on both sides, we get

$$|w| = \frac{3|z|}{|3z - i|}$$
$$\Rightarrow 3|z| = |3z - i| \quad \ldots[|\text{w}| = 1]$$
Consider $z = a + ib$
$$\Rightarrow 3|a + ib| = |3a + 3ib - i|$$
$$\Rightarrow 3|a + ib| = |3a + (3b - 1)i|$$
$$\Rightarrow 3\left(\sqrt{a^2 + b^2}\right) = \left(\sqrt{9a^2 + (3b - 1)^2}\right)$$
$$\Rightarrow 9a^2 + 9b^2 = 9a^2 + 9b^2 - 6b + 1$$
$$\Rightarrow 6b - 1 = 0$$

$\therefore$ The above equation represents a straight line.

Question 22

If one side of a triangle is double the other and the angles opposite to these sides differ by 60°, then the triangle is

Options:

A. obtuse angled

B. right angled

C. acute angled

D. isosceles

Answer: B

Solution:

In $\triangle \mathrm{ABC}$, by sine rule,

$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

According to the given condition,

In $\triangle \mathrm{ABC}, \mathrm{a}=2 \mathrm{~b}$ and

$$\begin{aligned}
& \mathrm{A}-\mathrm{B}=60^{\circ} \Rightarrow \mathrm{A}=60^{\circ}+\mathrm{B} \\
& \Rightarrow \frac{\sin \left(60^{\circ}+\mathrm{B}\right)}{2 \mathrm{~b}}=\frac{\sin \mathrm{B}}{\mathrm{b}} \\
& \Rightarrow \frac{\sin \mathrm{B}}{\sin \left(\mathrm{B}+60^{\circ}\right)}=\frac{1}{2} \\
& \Rightarrow 2 \sin \mathrm{B}=\sin \mathrm{B} \cos 60^{\circ}+\cos \mathrm{B} \sin 60^{\circ} \\
& \Rightarrow 2 \sin \mathrm{B}=\sin \mathrm{B}\left(\frac{1}{2}\right)+\cos \mathrm{B}\left(\frac{\sqrt{3}}{2}\right) \\
& \Rightarrow \frac{3}{2} \sin \mathrm{B}=\frac{\sqrt{3}}{2} \cos \mathrm{B} \\
& \Rightarrow \tan B=\frac{1}{\sqrt{3}} \Rightarrow B=30^{\circ} \\
& \therefore \quad A=30^{\circ}+60^{\circ}=90^{\circ}
\end{aligned}$$

$\therefore \quad \triangle \mathrm{ABC}$ is right angled.

Question 23

If

$$\int \sqrt{\frac{x-7}{x-9}} dx = A\sqrt{x^2 - 16x + 63} + \log\left|(x-8) + \sqrt{x^2 - 16x + 63}\right| + c,$$

(where c is a constant of integration) then A is

Options:

A. -1

B. $\frac{1}{2}$

C. 1

D. $\frac{-1}{2}$

Answer: C

Solution:

$$\begin{aligned} \text{Let} \quad \mathrm{I} &= \int \sqrt{\frac{x-7}{x-9}} \, \mathrm{d}x \\ &= \int \sqrt{\frac{(x-7)(x-7)}{(x-9)(x-7)}} \mathrm{d}x \\ &= \int \frac{x-7}{\sqrt{x^2-16x+63}} \, \mathrm{d}x \end{aligned}$$

Let $(x-7) = A\left[\frac{d}{dx}\left(x^2 - 16x + 63\right)\right] + B$

$\therefore \quad x - 7 = A(2x - 16) + B$

$\therefore \quad x - 7 = 2Ax - 16A + B$

$\therefore \quad A = \frac{1}{2}, B = 1$

$$\therefore \quad \mathrm{I}=\int \frac{\frac{1}{2}(2x-16)+1}{\sqrt{x^2-16x+63}}\,\mathrm{d}x$$
$$=\frac{1}{2}\int \frac{2x-16}{\sqrt{x^2-16x+63}}\,\mathrm{d}x+\int \frac{1}{\sqrt{x^2-16x+63}}\,\mathrm{d}x$$
$$=\frac{1}{2}\times 2\sqrt{x^2-16x+63}+\int \frac{1}{\sqrt{(x-8)^2-(1)^2}}\,\mathrm{d}x$$
$$\ldots\left[\int \frac{\mathrm{f}'(x)}{\sqrt{\mathrm{f}(x)}}\mathrm{d}x=2\sqrt{\mathrm{f}(x)}+\mathrm{c}\right]$$
$$\therefore \quad \mathrm{I}=\sqrt{x^2-16x+63}+\log\left|x-8+\sqrt{x^2-16x+63}\right|+\mathrm{c}$$
$$\text{But, }\int \sqrt{\frac{x-7}{x-9}}\,\mathrm{d}x=\mathrm{A}\sqrt{x^2-16x+63}$$
$$+\log\left|(x-8)+\sqrt{x^2-16x+63}\right|+c$$

Comparing, we get

$\mathrm{A}=1$

Question 24

A player tosses 2 fair coins. He wins ₹5 if 2 heads appear, ₹ 2 if one head appears and ₹ 1 if no head appears. Then the variance of his winning amount in ₹ is :

Options:

A. 6

B. $\frac{5}{2}$

C. $\frac{9}{4}$

D. $\frac{17}{2}$

Answer: C

Solution:

When player tosses 2 fair coins, then

$\mathrm{S}=\{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$

Let X be a random variable that denotes the amount received by player.

Then, X can take values 5, 2 and 1

Now, $P(X=5)=\frac{1}{4}, P(X=2)=\frac{1}{2}$ and $\mathrm{P}(\mathrm{X}=1)=\frac{1}{4}$

∴ The probability distribution of X is as follows:

X	5	2	1
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$\begin{aligned}\text{Variance of } X &= \sum X^2 P(X) - \left[\sum X P(X)\right]^2 \\ &= \left[\frac{25}{4}+2+\frac{1}{4}\right]-\left[\frac{5}{4}+1+\frac{1}{4}\right]^2 \\ &= \frac{34}{4}-\left(\frac{10}{4}\right)^2 \\ &= \frac{34}{4}-\frac{25}{4} \\ &= \frac{9}{4}\end{aligned}$$

Question 25

Area of the region bounded by the curve $y=\sqrt{49-x^2}$ and X-axis is

Options:

A. 49π sq. units

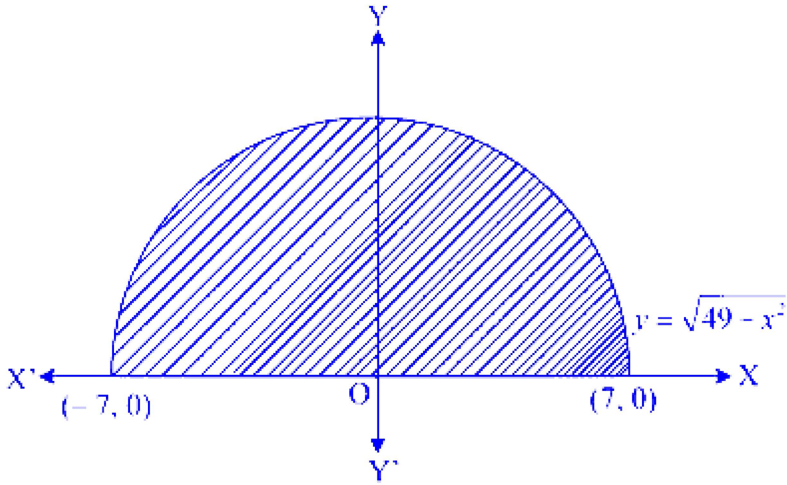
B. $\frac{49\pi}{2}$ sq. units

C. $\frac{49\pi}{4}$ sq. units

D. 98π sq. units

Answer: B

Solution:



Required area $= 2\int_0^7 \sqrt{49 - x^2}\, dx$

$$= 2\left[\frac{x}{2}\sqrt{49 - x^2} + \frac{49}{2}\sin^{-1}\left(\frac{x}{7}\right)\right]_0^7$$

$$= 2\left[\frac{7}{2}\sqrt{49 - 49} + \frac{49}{2}\sin^{-1}\left(\frac{7}{7}\right)\right] - 0$$

$$= 2 \times \frac{49}{2} \times \frac{\pi}{2}$$

$$= \frac{49\pi}{2} \text{ sq. units}$$

Question 26

The solution of the equation $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$ is

Options:

A. $x = 1$

B. $x = 0$

C. $x = -1$

D. $x = \pi$

Answer: B

Solution:

$$\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$$
$$\Rightarrow \tan^{-1}(1+x)=\frac{\pi}{2}-\tan^{-1}(1-x)$$
$$\Rightarrow \tan^{-1}(1+x)=\cot^{-1}(1-x)$$
$$\Rightarrow \tan^{-1}(1+x)=\tan^{-1}\left(\frac{1}{1-x}\right)$$
$$\Rightarrow 1+x=\frac{1}{1-x} \Rightarrow 1-x^2=1 \Rightarrow x=0$$

Question 27

The differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

Options:

A. variable radii and fixed centre at $(0,1)$.

B. variable radii and fixed centre at $(0,-1)$.

C. fixed radius of 1 unit and variable centre along the Y-axis.

D. fixed radius of 1 unit and variable centre along the X-axis.

Answer: D

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{1-y^2}}{y}$$

$\therefore \quad \int \frac{y}{\sqrt{1-y^2}}\,\mathrm{d}y=\int 1\,\mathrm{d}x$

$\therefore \quad -\sqrt{1-y^2}=x+\mathrm{c}$

$\therefore \quad (x+\mathrm{c})^2=1-y^2$

$\therefore \quad (x+c)^2+y^2=1$

$\therefore \quad$ Radius is fixed, which is 1 and the centre is $(-c,0)$ which is a variable centre on the X-axis.

Question 28

A square plate is contracting at the uniform rate $4\ \mathrm{cm}^2/\mathrm{sec}$, then the rate at which the perimeter is decreasing, when side of the square is $20\ \mathrm{cm}$, is

Options:

A. $\frac{1}{5}\ \mathrm{cm}/\mathrm{sec}.$

B. $4\ \mathrm{cm}/\mathrm{sec}.$

C. $2\ \mathrm{cm}/\mathrm{sec}.$

D. $\frac{2}{5}\ \mathrm{cm}/\mathrm{sec}.$

Answer: D

Solution:

Let A, P and X be the area, perimeter and length of side of square respectively at time 't' seconds. Then,

$$\begin{aligned} & \mathrm{A}=\mathrm{X}^2, \mathrm{P}=4\mathrm{X} \\ \therefore \quad & \mathrm{P}=4\sqrt{\mathrm{A}} \\ & \text{Differentiating w.r.t. t, we get} \\ \frac{\mathrm{dP}}{\mathrm{dt}} & =4\frac{1}{2\sqrt{\mathrm{A}}}\cdot\frac{\mathrm{dA}}{\mathrm{dt}} \\ & =\frac{2}{\mathrm{X}}\cdot\frac{\mathrm{dA}}{\mathrm{dt}} \\ & =\frac{2}{20}\times 4 \quad \ldots\left[\frac{\text{side}=20\ \mathrm{cm}}{\frac{\mathrm{dA}}{\mathrm{dt}}=4\ \mathrm{cm}^2/\mathrm{sec}}\right] \\ & =\frac{2}{5}\ \mathrm{cm}/\mathrm{sec} \end{aligned}$$

Question 29

If the function $\mathrm{f}(x)$ is continuous in $0\leq x\leq\pi$, then the value of $2a+3b$ is where

$$f(x)=\begin{cases}x+a\sqrt{2}\sin x & \text{if } 0\le x<\frac{\pi}{4}\\ 2x\cot x+b & \text{if } \frac{\pi}{4}\le x\le\frac{\pi}{2}\\ \text{a}\cos 2x-b\sin x & \text{if } \frac{\pi}{2}<x\le\pi\end{cases}$$

Options:

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{10}$

Answer: A

Solution:

$\mathrm{f}(x)$ is continuous in $0\le x\le\pi$

$\therefore\quad \mathrm{f}(x)$ is continuous at $x=\frac{\pi}{4}$.

$$\therefore\quad \lim_{x\to\frac{\pi}{4}^-}\mathrm{f}(x)=\lim_{x\to\frac{\pi}{4}^+}\mathrm{f}(x)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}^-}(x+\mathrm{a}\sqrt{2}\sin x)=\lim_{x\to\frac{\pi}{4}^+}(2x\cot x+\mathrm{b})$$

$$\Rightarrow\frac{\pi}{4}+\mathrm{a}\sqrt{2}\times\sin\frac{\pi}{4}=2\times\frac{\pi}{4}\cdot\cot\frac{\pi}{4}+\mathrm{b}$$

$$\Rightarrow\frac{\pi}{4}+\mathrm{a}=\frac{\pi}{2}+\mathrm{b}$$

$$\Rightarrow\mathrm{a}-\mathrm{b}=\frac{\pi}{4}\quad \text{.... (i)}$$

Also, $\mathrm{f}(x)$ is continuous at $x=\frac{\pi}{2}$.

$$\therefore\quad \lim_{x\to\frac{\pi}{2}^-}\mathrm{f}(x)=\lim_{x\to\frac{\pi}{2}^+}\mathrm{f}(x)$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}^-}(2x\cot x+b)=\lim_{x\to\frac{\pi}{2}^+}(a\cos 2x-b\sin x)$$

$$\Rightarrow 2\times\frac{\pi}{2}\times\cot\frac{\pi}{2}+b=a\cos\left(2\times\frac{\pi}{2}\right)-b\sin\frac{\pi}{2}$$

$$\Rightarrow b=-a-b$$

$$\Rightarrow a=-2b$$

Substituting $a=-2b$ in equation (i), we get

$$- 2b - b = \frac{\pi}{4}$$
$$\Rightarrow b = \frac{-\pi}{12}$$
$$\Rightarrow a = -2 \times \frac{-\pi}{12} = \frac{\pi}{6}$$

Now,

$$2a + 3b = 2\left(\frac{\pi}{6}\right) + 3\left(\frac{-\pi}{12}\right)$$
$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Question 30

For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $(1 + \log 2x)^2 \frac{dy}{dx}$ is equal to

Options:

A. $\frac{x \log 2x + \log 2}{x}$

B. $\frac{x \log 2x - \log 2}{x}$

C. $x \log 2x$

D. $\log 2x$

Answer: B

Solution:

$(2x)^{2y} = 4e^{2x-2y}$

Taking log on both sides, we get

$2y \log 2x = \log\left(4e^{2x-2y}\right)$
$\Rightarrow 2y \log 2x = \log 4 + \log e^{2x-2y}$
$\Rightarrow 2y \log 2x = \log 4 + 2x - 2y$
$\Rightarrow 2y \log 2x + 2y = \log 4 + 2x$
$\Rightarrow 2(y \log 2x + y) = 2 \log 2 + 2x$
$\Rightarrow y \log 2x + y = \log 2 + x$
$\Rightarrow y(1 + \log 2x) = x + \log 2$
$\Rightarrow y = \dfrac{x + \log 2}{1 + \log 2x}$

Differentiating w.r.t. x, we get

$\dfrac{dy}{dx} = \dfrac{(1 + \log 2x)(1 + 0) - (x + \log 2)\left(\frac{1}{2x}\right) \cdot 2}{(1 + \log 2x)^2}$
$\Rightarrow \dfrac{dy}{dx} = \dfrac{(1 + \log 2x) - \frac{1}{x}(x + \log 2)}{(1 + \log 2x)^2}$
$\Rightarrow (1 + \log 2x)^2 \dfrac{dy}{dx} = 1 + \log 2x - 1 - \dfrac{\log 2}{x}$
$\Rightarrow (1 + \log 2x)^2 \dfrac{dy}{dx} = \log 2x - \dfrac{\log 2}{x}$
$= \dfrac{x \log 2x - \log 2}{x}$

Question 31

The vectors are $\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \bar{b} = \hat{i} + \hat{j}$. If $\bar{c}$ is a vector such that $\bar{a} \cdot \bar{c} = |\bar{c}|$ and $|\bar{c} - \bar{a}| = 2\sqrt{2}$, angle between $\bar{a} \times \bar{b}$ and $\bar{c}$ is $\frac{\pi}{4}$, then $|(\bar{a} \times \bar{b}) \times \bar{c}|$ is

Options:

A. 3

B. $\frac{3}{\sqrt{2}}$

C. $3\sqrt{2}$

D. 1

Answer: B

Solution:

Given that angle between $\overline{\mathrm{a}} \times \overline{\mathrm{b}}$ and $\overline{\mathrm{c}}\$is\$\frac{\pi}{4}$

$\therefore \quad |(\bar{a} \times \bar{b}) \times \bar{c}| = |(\bar{a} \times \bar{b})||\bar{c}| \sin \frac{\pi}{4}$ ..... (i)

Now, $\overline{\mathrm{a}} \times \overline{\mathrm{b}} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$$\begin{aligned} &= \hat{\mathrm{i}}(0+2) - \hat{\mathrm{j}}(0+2) + \hat{\mathrm{k}}(2-1) \\ &= 2\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + \hat{\mathrm{k}} \end{aligned}$$

$\therefore \quad |\vec{\mathrm{a}} \times \overline{\mathrm{b}}| = \sqrt{2^2 + (-2)^2 + 1} = 3$

Given, $\overline{\mathrm{a}} = 2\hat{\mathrm{i}} + \hat{\mathrm{j}} - 2\hat{\mathrm{k}}$

$a = \sqrt{2^2 + 1^2 + (-2)^2} = 3$

Given, $|\bar{c} - \bar{a}| = 2\sqrt{2}$

Squaring on both sides, we get

$$\begin{aligned} &|\bar{c}|^2 + |\bar{a}|^2 - 2\bar{a} \cdot \bar{c} = 8 \\ &\Rightarrow |\bar{c}|^2 + 3^2 - 2|\bar{c}| = 8 \quad \ldots [\because \bar{a}.\bar{c} = |\bar{c}|] \end{aligned}$$

$$\begin{aligned} &\Rightarrow |\bar{c}|^2 - 2|\bar{c}| + 1 = 0 \\ &\Rightarrow (|\bar{c}| - 1)^2 = 0 \\ &\Rightarrow |\bar{c}| = 1 \end{aligned}$$

From (i),

$$\begin{aligned} |(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}| &= |(\overline{\mathrm{a}} \times \overline{\mathrm{b}})||\overline{\mathrm{c}}| \cdot \sin \frac{\pi}{4} \\ &= 3 \times 1 \times \frac{1}{\sqrt{2}} \\ &= \frac{3}{\sqrt{2}} \end{aligned}$$

Question 32

$$\int \frac{1}{7-6x-x^2} dx =$$

Options:

A. $\frac{1}{4} \log \left(\frac{7+x}{1-x}\right) + \mathrm{c}$, where c is a constant of integration.

B. $\frac{1}{8}\log\left(\frac{7+x}{1-x}\right)+$ c, where c is a constant of integration.

C. $\frac{1}{16}\log\left(\frac{7+x}{1-x}\right)+$ c, where c is a constant of integration.

D. $\frac{1}{32}\log\left(\frac{7+x}{1-x}\right)+$ c, where c is a constant of integration.

Answer: B

Solution:

$$\begin{aligned}
\text{Let I} &= \int \frac{1}{7-6x-x^2}dx \\
&= \int \frac{1}{7-6x-x^2-9+9}dx \\
&= \int \frac{1}{16-(x^2+6x+9)}dx \\
&= \int \frac{1}{(4)^2-(x+3)^2}dx \\
&= \frac{1}{8}\log\left|\frac{4+x+3}{4-(x+3)}\right|+c \\
&= \frac{1}{8}\log\left|\frac{7+x}{1-x}\right|+c
\end{aligned}$$

Question 33

$\int \frac{dx}{\sin x+\cos x} =$

Options:

A. $\sqrt{2}\log\tan\left(x+\frac{\pi}{4}\right)+$ c, where c is a constant of integration.

B. $\frac{1}{\sqrt{2}}\log\tan\left(\frac{x}{2}+\frac{\pi}{8}\right)+c$, where c is a constant of integration.

C. $\frac{1}{\sqrt{2}}\log\left(\frac{\tan\frac{x}{2}-\sqrt{2}+1}{\tan\frac{x}{2}+\sqrt{2}+1}\right)+$ c, where c is a constant of integration.

D. $-\frac{1}{\sqrt{2}}\log\left(\frac{\tan\frac{x}{2}-(\sqrt{2}+1)}{\tan\frac{x}{2}+\sqrt{2}-1}\right)+$ c, where c is a constant of integration.

Answer: D

Solution:

Let $\mathrm{I}=\int \frac{\mathrm{d}x}{\sin x+\cos x}$

Put $\tan \frac{x}{2}=\mathrm{t}$

$$\Rightarrow x=2\tan^{-1}\mathrm{t}$$
$$\Rightarrow \mathrm{d}x=\frac{2}{1+\mathrm{t}^2}\mathrm{dt}$$

and $\cos x=\frac{1-t^2}{1+t^2}, \sin x=\frac{2t}{1+t^2}$

$$\begin{aligned}\therefore \quad I &= \int \frac{\frac{2}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}dt \\ &= \int \frac{2}{2t+1-t^2}dt \\ &= -2\int \frac{1}{t^2-2t-1}dt \\ &= -2\int \frac{1}{t^2-2t+1-1-1} \\ &= -2\int \frac{1}{(t-1)^2-(\sqrt{2})^2}\end{aligned}$$

$$=-2\times\frac{1}{2\sqrt{2}}\log\left|\frac{\mathrm{t}-1-\sqrt{2}}{\mathrm{t}-1+\sqrt{2}}\right|+\mathrm{c}$$
$$=\frac{-1}{\sqrt{2}}\log\left|\frac{\tan\frac{x}{2}-1-\sqrt{2}}{\tan\frac{x}{2}-1+\sqrt{2}}\right|+\mathrm{c}$$
$$=\frac{-1}{\sqrt{2}}\log\left|\frac{\tan\frac{x}{2}-(\sqrt{2}+1)}{\tan\frac{x}{2}+\sqrt{2}-1}\right|+\mathrm{c}$$

Question 34

A ladder of length $17\ \mathrm{m}$ rests with one end against a vertical wall and the other on the level ground. If the lower end slips away at the rate of $1\ \mathrm{m/sec}$., then when it is $8\ \mathrm{m}$ away from the wall, its upper end is coming down at the rate of

Options:

A. $\frac{5}{8}\ \mathrm{m/sec}$.

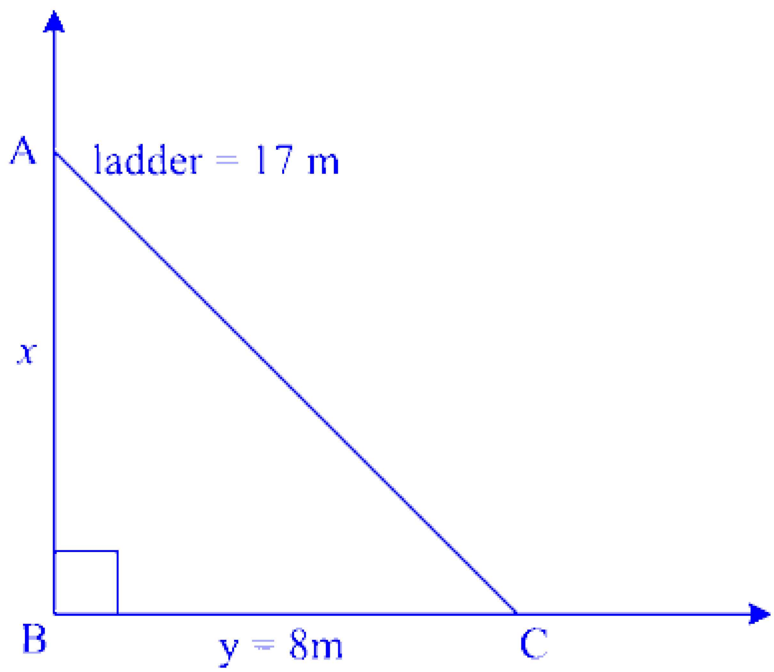
B. $\frac{8}{15}$ m/sec.

C. $\frac{-8}{15}$ m/sec.

D. $\frac{15}{8}$ m/sec.

Answer: B

Solution:



In $\triangle \mathrm{ABC}$, AC represents ladder

$\mathrm{AB} \rightarrow$ vertical wall

Let $\mathrm{AB} = x, \mathrm{BC} = y$

$\therefore \quad \angle \mathrm{ABC} = 90^\circ$

By Pythagoras theorem,

$$\mathrm{AB}^2 + \mathrm{BC}^2 = \mathrm{AC}^2$$
$$\Rightarrow x^2 + y^2 = 17^2$$
$$\Rightarrow x^2 = 289 - y^2$$
$$\Rightarrow x^2 = 289 - 64$$
$$\Rightarrow x^2 = 225$$
$$\Rightarrow x = 15 \mathrm{~m}$$

Consider equation (i),

$x^2 = 289 - y^2$

Differentiating w.r.t. t, we get

$$2x\frac{dx}{dt} = -2y\frac{dy}{dt}$$
$$\Rightarrow 15\frac{dx}{dt} = -8(1)$$
$$\Rightarrow \frac{dx}{dt} = \frac{-8}{15}\ \mathrm{m/s}$$

Negative sign shows that the ladder is moving down. i.e., vertical length is decreasing

$\therefore$ Upper end is coming down at the rate of $\frac{8}{15}$ m/s.

Question 35

Let P be a plane passing through the points $(2,1,0),(4,1,1)$ and $(5,0,1)$ and R be the point $(2,1,6)$. Then image of R in the plane P is

Options:

A. $(6,5,2)$

B. $(4,3,2)$

C. $(6,5,-2)$

D. $(3,4,-2)$

Answer: C

Solution:

Equation of the plane passing through $(2,1,0),(4,1,1)$ and $(5,0,1)$ is

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 4-2 & 1-1 & 1-0 \\ 5-2 & 0-1 & 1-0 \end{vmatrix} = 0$$
$$\Rightarrow x+y-2z=3$$

$\mathrm{R}'(x, y, \mathrm{z})$ is image of $\mathrm{R}(2,1,6)$ w.r.t. to plane

$x+y-2\mathrm{z}=3$

$\Rightarrow \quad \frac{x-2}{1}=\frac{y-1}{1}=\frac{\mathrm{z}-6}{-2}=\frac{-2[2+1-2(6)-3]}{1+1+4}$

$\Rightarrow \quad \frac{x-2}{1}=\frac{y-1}{1}=\frac{\mathrm{z}-6}{-2}=4$

$\Rightarrow \quad x=6, y=5, \mathrm{z}=-2$

$\therefore \quad \mathrm{R}'(x, y, \mathrm{z}) \equiv(6,5,-2)$

Question 36

The co-ordinates of the point, where the line through $A(3,4,1)$ and $B(5,1,6)$ crosses the XZ-plane, are

Options:

A. $\left(\frac{11}{3}, 0, \frac{21}{3}\right)$

B. $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

C. $\left(\frac{-11}{3}, 0, \frac{21}{3}\right)$

D. $\left(\frac{17}{3}, 0, \frac{-23}{3}\right)$

Answer: B

Solution:

Let A $\left(x_1, y_1, \mathrm{z}_1\right)=\mathrm{A}(3,4,1)$ and B $\left(x_2, y_2, \mathrm{z}_2\right)=\mathrm{B}(5,1,6)$

The equation of the line passing through the points $\left(x_1, y_1, z_1\right)$ and $\left(x_2, y_2, z_2\right)$ is given by

$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

$\therefore \quad \frac{x-3}{5-3}=\frac{y-4}{1-4}=\frac{z-1}{6-1}$

$\therefore \quad \frac{x-3}{2}=\frac{y-4}{-3}=\frac{z-1}{5}$

Since the line crosses the XZ plane, $y=0$

$\therefore \quad \frac{x-3}{2}=\frac{4}{3}=\frac{z-1}{5}$

$\therefore \quad \frac{x-3}{2}=\frac{4}{3}$ and $\frac{z-1}{5}=\frac{4}{3}$

$\Rightarrow x=\frac{17}{3}$ and $z=\frac{23}{3}$

$\therefore$ The required point is $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

Question 37

The number of possible solutions of $\sin\theta + \sin 4\theta + \sin 7\theta = 0, \theta \in (0, \pi)$ are

Options:

A. 3

B. 4

C. 6

D. 8

Answer: C

Solution:

$$\sin 7\theta + \sin\theta + \sin 4\theta = 0$$
$$\Rightarrow 2\sin 4\theta\cos 3\theta + \sin 4\theta = 0$$
$$\Rightarrow \sin 4\theta(2\cos 3\theta + 1) = 0$$
$$\Rightarrow \sin 4\theta = 0 \text{ or } \cos 3\theta = \frac{-1}{2}$$
$$\Rightarrow \sin 4\theta = 0 \text{ or } \cos 3\theta = \cos\left(\frac{2\pi}{3}\right)$$
$$\Rightarrow 4\theta = n\pi \quad \text{ or } 3\theta = 2n\pi \pm \frac{2\pi}{3}$$
$$\Rightarrow \theta = \frac{n\pi}{4} \quad \text{ or } \quad \theta = \frac{2n\pi}{3} \pm \frac{2\pi}{9}$$
$$\therefore \quad \theta = \frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9} \cdots [\because \theta \in (0, \pi)]$$
$$\therefore \quad \text{Number of solutions } = 6$$

Question 38

If $\mathrm{I} = \int \frac{\mathrm{d}x}{x^2(x^4+1)^{\frac{3}{4}}}$, then I is

Options:

A. $\left(\frac{x^4+1}{x}\right)^{\frac{1}{4}}+\mathrm{c}$, where c is a constant of integration.

B. $\frac{\left(x^4-1\right)^{\frac{1}{4}}}{x}+\mathrm{c}$, where c is a constant of integration.

C. $-\frac{\left(x^4+1\right)^{\frac{1}{4}}}{x}+c$, where c is a constant of integration.

D. $-\left(\frac{x^4+1}{x}\right)^{\frac{1}{4}}+\mathrm{c}$, where c is a constant of integration.

Answer: C

Solution:

Let $\mathrm{I}=\int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}} \mathrm{~d} x=\int \frac{\mathrm{d} x}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$

Put $1+\frac{1}{x^4}=t \Rightarrow \frac{-4}{x^5} d x=d t$

$$\begin{aligned} \therefore \quad \mathrm{I} & =-\frac{1}{4} \int \frac{\mathrm{dt}}{\mathrm{t}^{\frac{3}{4}}}=-\frac{1}{4} \times 4 \mathrm{t}^{\frac{1}{4}}+\mathrm{c}=-\mathrm{t}^{\frac{1}{4}}+\mathrm{c} \\ & =-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+\mathrm{c} \\ & =\frac{-\left(x^4+1\right)^{\frac{1}{4}}}{x}+\mathrm{c} \end{aligned}$$

Question 39

If $\bar{a}=\hat{i}+2 \hat{j}+\hat{k}, \bar{b}=\hat{i}-\hat{j}+\hat{k}, \bar{c}=\hat{i}+\hat{j}-\hat{k}$, then a vector in the plane of $\bar{a}$ and $\bar{b}$, whose projection on $\overline{\mathrm{c}}$ is $\frac{1}{\sqrt{3}}$, is

Options:

A. $\hat{i}+\hat{j}-2 \hat{k}$

B. $3 \hat{\mathrm{i}}+\hat{\mathrm{j}}-3 \hat{\mathrm{k}}$

C. $4\hat{i}-\hat{j}+4\hat{k}$

D. $2\hat{\mathrm{i}}+3\hat{\mathrm{j}}-\hat{\mathrm{k}}$

Answer: C

Solution:

Let $\overline{\mathrm{r}}$ be the vector coplanar to $\overline{\mathrm{a}}$ and $\overline{\mathrm{b}}$. Then,

$$\begin{aligned}\overline{\mathrm{r}} &= \overline{\mathrm{a}}+\mathrm{m}\overline{\mathrm{b}}\\ &= (\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}})+\mathrm{m}(\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}})\\ &= \hat{\mathrm{i}}(1+\mathrm{m})+\hat{\mathrm{j}}(2-\mathrm{m})+\hat{\mathrm{k}}(1+\mathrm{m}) \quad \text{.... (i)}\end{aligned}$$

Since the projection of $\overline{\mathrm{r}}$ along $\overline{\mathrm{c}}$ is $\frac{1}{\sqrt{3}}$, $\frac{\overline{\mathrm{r}}\cdot\overline{\mathrm{c}}}{|\overline{\mathrm{c}}|}=\pm\frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{(1+\mathrm{m})+(2-\mathrm{m})-(1+\mathrm{m})}{\sqrt{3}}=\pm\frac{1}{\sqrt{3}}$$

$$\Rightarrow (1+\mathrm{m})+(2-\mathrm{m})-(1+\mathrm{m})=\pm 1$$

$\therefore \quad \mathrm{m}=3$ or $\mathrm{m}=1$

Substituting $\mathrm{m}=3$ in equation (i), we get

$$\bar{r}=\hat{i}(1+3)+\hat{j}(2-3)+\hat{k}(1+3)$$

$$\Rightarrow \bar{r}=4\hat{i}-\hat{j}+4\hat{k}$$

Question 40

The number of solutions of $\tan x+\sec x=2\cos x$ in $[0,2\pi]$ are

Options:

A. 6

B. 4

C. 3

D. 2

Answer: D

Solution:

The given equation is defined for $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$.

Now, $\tan x + \sec x = 2\cos x$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$
$$\Rightarrow (\sin x + 1) = 2\cos^2 x$$
$$\Rightarrow (\sin x + 1) = 2\left(1 - \sin^2 x\right)$$
$$\Rightarrow (\sin x + 1) = 2(1 - \sin x)(1 + \sin x)$$
$$\Rightarrow (1 + \sin x)[2(1 - \sin x) - 1] = 0$$
$$\Rightarrow 2(1 - \sin x) - 1 = 0$$
$$\ldots \begin{bmatrix} \because \sin x \neq -1 \text{ otherwise } \quad \cos x = 0 \text{ and} \\ \tan x, \sec x \text{ will be undefined} \end{bmatrix}$$
$$\Rightarrow \sin x = \frac{1}{2}$$
$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ in } (0, 2\pi)$$

$\therefore$ Number of solutions = 2

Question 41

If $\int\limits_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{\frac{3}{2}}}\, \mathrm{d}x = \frac{\mathrm{k}}{6}$, then the value of k is

Options:

A. $2\sqrt{3} - \pi$

B. $2\sqrt{3} + \pi$

C. $3\sqrt{2} + \pi$

D. $3\sqrt{2} - \pi$

Answer: A

Solution:

Let $\mathrm{I}=\int\limits_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}} \mathrm{~d} x$

Put $x=\sin \theta$

$\Rightarrow d x=\cos \theta d \theta$

$$
\begin{aligned}
\left(1-x^2\right)^{\frac{3}{2}} & =\left(1-\sin ^2 \theta\right)^{\frac{3}{2}} \\
& =\left(\cos ^2 \theta\right)^{\frac{3}{2}} \\
& =\cos ^3 \theta
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad I & =\int\limits_0^{\frac{\pi}{6}} \frac{\sin ^2 \theta \cdot \cos \theta d \theta}{\cos ^3 \theta} \\
& =\int\limits_0^{\frac{\pi}{6}} \tan ^2 \theta d \theta \\
& =\int\limits_0^{\frac{\pi}{6}}\left(\sec ^2 \theta-1\right) d \theta \\
& =[\tan \theta]_0^{\frac{\pi}{6}}-[\theta]_0^{\frac{\pi}{6}} \\
& =\left(\tan \frac{\pi}{6}-\tan 0\right)-\left(\frac{\pi}{6}-0\right) \\
& =\frac{1}{\sqrt{3}}-\frac{\pi}{6}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\sqrt{3}}{3}-\frac{\pi}{6} \\
& =\frac{2 \sqrt{3}-\pi}{6}
\end{aligned}
$$

But, $\int\limits_0^{\frac{1}{2}} \frac{x^2}{\left(1-x^2\right)^{\frac{3}{2}}} \mathrm{~d} x=\frac{\mathrm{k}}{6}$ ...... [Given]

$\therefore \quad \mathrm{k}=2 \sqrt{3}-\pi$

Question 42

General solution of the differential equation $\cos x(1+\cos y) \mathrm{d} x-\sin y(1+\sin x) \mathrm{d} y=0$ is

Options:

A. $(1+\cos x)(1+\sin y)=c$

B. $1+\sin x+\cos y=c$

C. $(1+\sin x)(1+\cos y)=c$

D. $1+\sin x \cdot \cos y=c$

Answer: C

Solution:

Given differential equation is

$$\begin{aligned}
&\cos x(1+\cos y)\mathrm{d}x-\sin y(1+\sin x)\mathrm{d}y=0\\
&\Rightarrow \cos x(1+\cos y)\mathrm{d}x=\sin y(1+\sin x)\mathrm{d}y\\
&\Rightarrow \frac{\cos x}{1+\sin x}\,\mathrm{d}x=\frac{\sin y}{1+\cos y}\,\mathrm{d}y
\end{aligned}$$

Integrating on both sides, we get

$$\begin{aligned}
&\int \frac{\cos x}{1+\sin x}\,\mathrm{d}x=\frac{\sin y}{1+\cos y}\,\mathrm{d}y+\log|\mathrm{c}|\\
&\Rightarrow \log|1+\sin x|=-\log|1+\cos y|+\log|\mathrm{c}|\\
&\Rightarrow \log|1+\sin x|+\log|1+\cos y|=\log|\mathrm{c}|\\
&\Rightarrow \log|(1+\sin x)(1+\cos y)|=\log|\mathrm{c}|\\
&\Rightarrow (1+\sin x)(1+\cos y)=\mathrm{c}
\end{aligned}$$

Question 43

If the line $ax+by+c=0$ is a normal to the curve $xy=1$, then

Options:

A. $a>0, b>0$

B. $a>0, b<0$

C. $a<0, b<0$

D. $\mathrm{a}=0,\ \mathrm{b}=0$

Answer: B

Solution:

$xy = 1$

$\therefore \quad y = \frac{1}{x}$

$\therefore \quad y' = \frac{-1}{x^2}$

$\therefore \quad$ Slope of the normal $= x^2$

Slope of the line $\mathrm{a}x + \mathrm{b}y + \mathrm{c} = 0$ is $\frac{-\mathrm{a}}{\mathrm{b}}$.

Since the line $\mathrm{a}x + \mathrm{b}y + \mathrm{c} = 0$ is a normal to the curve $xy = 1$,

$x^2 = -\frac{a}{b}$

For this condition to hold true, either $\mathrm{a} < 0,\ \mathrm{b} > 0$ or $\mathrm{b} < 0, \mathrm{a} > 0$

Question 44

Let $\bar{a}, \bar{b}, \bar{c}$ be three non-zero vectors, such that no two of them are collinear and $(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$. If θ is the angle between the vectors $\bar{b}$ and $\bar{c}$, then the value of $\sin\theta$ is

Options:

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{-\sqrt{2}}{3}$

C. $\frac{\sqrt{2}}{3}$

D. $\sqrt{\frac{2}{3}}$

Answer: A

Solution:

Given: $(\bar{\mathrm{a}} \times \bar{\mathrm{b}}) \times \bar{\mathrm{c}} = \frac{1}{3}|\bar{\mathrm{b}}||\bar{\mathrm{c}}|\overrightarrow{\mathrm{a}}$

We know that,

$(\bar{\mathrm{a}} \times \bar{\mathrm{b}}) \times \bar{\mathrm{c}} = (\bar{\mathrm{a}} \cdot \bar{\mathrm{c}})\bar{\mathrm{b}} - (\bar{\mathrm{b}} \cdot \bar{\mathrm{c}})\bar{\mathrm{a}}$

On comparing, we get

$$\frac{1}{3}|\bar{b}||\overrightarrow{c}| = -\bar{b}\cdot\bar{c}$$
$$\Rightarrow\frac{1}{3}|\bar{b}||\overrightarrow{c}| = -|\bar{b}||\overrightarrow{c}|\cos\theta$$
$$\Rightarrow\cos\theta = \frac{-1}{3}$$
$$\Rightarrow\cos^2\theta = \frac{1}{9}$$
$$\sin^2\theta = 1-\cos^2\theta$$
$$= 1-\frac{1}{9}$$
$$\therefore\quad \sin^2\theta = \frac{8}{9}$$
$$\therefore\quad \sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

Question 45

If $\mathrm{f}(x)=\mathrm{e}^{x},\ \mathrm{g}(x)=\sin^{-1}x$ and $\mathrm{h}(x)=\mathrm{f}(\mathrm{g}(x))$, then $\frac{\mathrm{h}'(x)}{\mathrm{h}(x)}$ is

Options:

A. $\mathrm{e}^{\sin^{-1}x}$

B. $\frac{1}{\sqrt{1-x^2}}$

C. $\sin^{-1}x$

D. $\frac{\mathrm{e}^{\sin^{-1}x}}{\sqrt{1-x^2}}$

Answer: B

Solution:

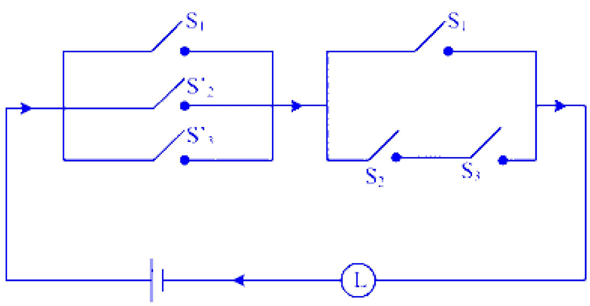
$$\mathrm{h}(x) = \mathrm{f}(\mathrm{g}(x))$$
$$= \mathrm{f}\left(\sin^{-1}x\right)$$
$$\therefore\quad \mathrm{h}(x) = \mathrm{e}^{\sin^{-1}x}$$

Differentiating w.r.t. x, we get

$$h'(x) = e^{\sin^{-1} x} \cdot \frac{d}{dx}\left(\sin^{-1} x\right)$$
$$= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1 - x^2}}$$
$$\text{Now, } \frac{h'(x)}{h(x)} = \frac{e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}}{e^{\sin^{-1} x}} = \frac{1}{\sqrt{1 - x^2}}$$

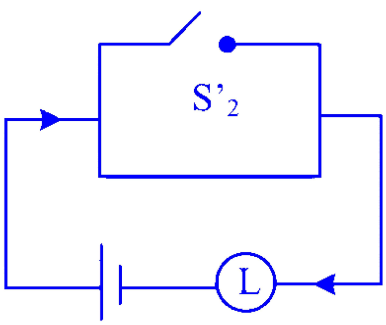
Question 46

The given circuit is equivalent to

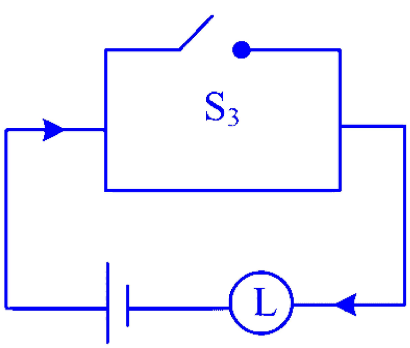


Options:

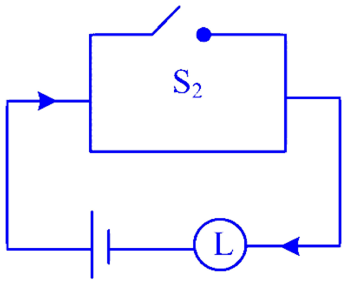
A.



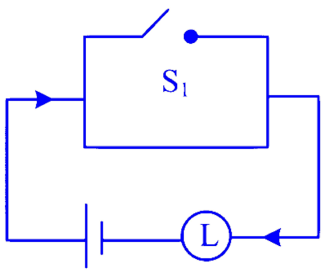
B.



C.



D.



Answer: D

Solution:

The symbolic form of the given circuit is

$(p \vee \sim q \vee \sim r) \wedge (p \vee (q \wedge r))$
$\equiv p \vee [(\sim q \vee \sim r) \wedge (q \wedge r)]$ ...[Distributive law]

$\equiv p \vee [\sim (q \wedge r) \wedge (q \wedge r)]$ ...[De Morgan's law]

$\equiv p \vee F$ ....[Complement law]

$\equiv p$ ...[Identity law]

Question 47

A kite is $120 \mathrm{~m}$ high and $130 \mathrm{~m}$ of string is out. If the kite is moving away horizontally at the rate of $39 \mathrm{~m/sec}$, then the rate at which the string is being out, is

Options:

A. $12 \mathrm{~m/sec}$.

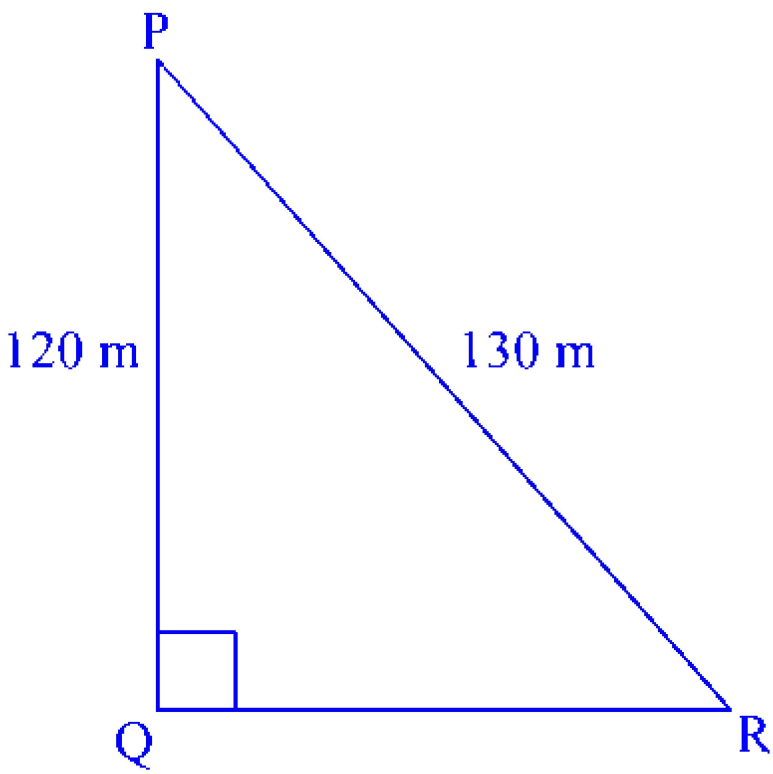
B. $15 \mathrm{~m/sec}$.

C. $18 \mathrm{~m/sec}$.

D. $20 \mathrm{~m/sec}$.

Answer: B

Solution:



Let 'P' be the position of the kite and PR be the string.

Let $\mathrm{QR}=x$ and $\mathrm{PR}=y$

By Pythagoras theorem,

$$\mathrm{PR}^2=\mathrm{PQ}^2+\mathrm{QR}^2$$
$$\Rightarrow y^2=(120)^2+x^2 \quad .... \text{(i)}$$

Differentiating w.r.t. t, we get

$$2y\frac{\mathrm{d}y}{\mathrm{dt}}=2x\frac{\mathrm{d}x}{\mathrm{dt}}$$
$$\Rightarrow y\frac{\mathrm{d}y}{\mathrm{dt}}=x\frac{\mathrm{d}x}{\mathrm{dt}} \quad .... \text{(ii)}$$

Now, kite is moving away horizontally at the rate of $39 \mathrm{~m/sec}$.

$\therefore \quad \frac{\mathrm{d}x}{\mathrm{dt}} = 39\ \mathrm{m/sec}$

From (i)

$$(130)^2 = (120)^2 + x^2$$
$$\Rightarrow x^2 = 16900 - 14400$$
$$\Rightarrow x^2 = 2500$$
$$\Rightarrow x = 50$$

From (ii),

$$130\frac{\mathrm{d}y}{\mathrm{dt}} = 50 \times 39$$
$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{dt}} = \frac{50 \times 39}{130} = 15\ \mathrm{m/sec}$$

Question 48

ABC is a triangle in a plane with vertices $\mathrm{A}(2, 3, 5), \mathrm{B}(-1, 3, 2)$ and $\mathrm{C}(\lambda, 5, \mu)$. If median through A is equally inclined to the co-ordinate axes, then value of $\lambda + \mu$ is

Options:

A. 17

B. 10

C. 7

D. 3

Answer: A

Solution:

Let AD be the median

$\therefore$ Co-ordinates of

$$D \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$
$$D \equiv \left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$$
$$\therefore \quad \overline{AD} = \left(\frac{\lambda-1}{2} - 2\right)\hat{i} + (4-3)\hat{j} + \left(\frac{\mu+2}{2} - 5\right)\hat{k}$$
$$\therefore \quad \overline{AD} = \left(\frac{\lambda-5}{2}\right)\hat{i} + \hat{j} + \left(\frac{\mu-8}{2}\right)\hat{k}$$

Since AD makes equal angle with co-ordinate axes, the direction ratios are equal.

$\therefore \quad \frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$

Consider,

$$\frac{\lambda-5}{2} = 1$$
$$\Rightarrow \lambda - 5 = 2$$
$$\Rightarrow \lambda = 7$$
and $\frac{\mu-8}{2} = 1$
$$\Rightarrow \mu - 8 = 2$$
$$\Rightarrow \mu = 10$$
$$\therefore \quad \lambda + \mu = 7 + 10 = 17$$

Question 49

Three critics review a book. For the three critics the odds in favor of the book are $2:5, 3:4$ and $4:3$ respectively. The probability that the majority is in favor of the book, is given by

Options:

A. $\frac{183}{343}$

B. $\frac{160}{343}$

C. $\frac{209}{343}$

D. $\frac{134}{343}$

Answer: D

Solution:

The probability that the first critic favors the book is $\mathrm{P(A)}=\frac{2}{2+5}=\frac{2}{7}$

$\therefore \quad \mathrm{P}\left(\mathrm{A}'\right)=1-\frac{2}{7}=\frac{5}{7}$

The probability that the second critic favors the book is $P(B)=\frac{3}{3+4}=\frac{3}{7}$

$\therefore \quad \mathrm{P}\left(\mathrm{B}'\right)=1-\frac{3}{7}=\frac{4}{7}$

The probability that the third critic favors the book is $\mathrm{P(C)}=\frac{4}{4+3}=\frac{4}{7}$

$\therefore \quad \mathrm{P}\left(\mathrm{C}'\right)=1-\frac{4}{7}=\frac{3}{7}$

$\therefore$ Majority will be in favor of the book if at least two critics favor the book.

Hence, the probability is

$$\begin{aligned}
&\mathrm{P}\left(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C}'\right)\quad+\mathrm{P}\left(\mathrm{A}\cap\mathrm{B}'\cap\mathrm{C}\right)\\
&\quad+\mathrm{P}\left(\mathrm{A}'\cap\mathrm{B}\cap\mathrm{C}\right)+\mathrm{P}(\mathrm{A}\cap\mathrm{B}\cap\mathrm{C})\\
=&\mathrm{P(A)}\cdot\mathrm{P(B)}\cdot\mathrm{P}\left(\mathrm{C}'\right)+\mathrm{P(A)}\cdot\mathrm{P}\left(\mathrm{B}'\right)\cdot\mathrm{P(C)}\\
&\quad+\mathrm{P}\left(\mathrm{A}'\right)\cdot\mathrm{P(B)}\cdot\mathrm{P(C)}+\mathrm{P(A)}\cdot\mathrm{P(B)}\cdot\mathrm{P(C)}\\
=&\frac{2}{7}\times\frac{3}{7}\times\frac{3}{7}+\frac{2}{7}\times\frac{4}{7}\times\frac{4}{7}+\frac{5}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{2}{7}\times\frac{3}{7}\times\frac{4}{7}\\
=&\frac{18}{343}+\frac{32}{343}+\frac{60}{343}+\frac{24}{343}=\frac{134}{343}
\end{aligned}$$

Question 50

$\mathrm{f}:\mathrm{R}\rightarrow\mathrm{R};\mathrm{g}:\mathrm{R}\rightarrow\mathrm{R}$ are two functions such that $\mathrm{f}(x)=2x-3,\ \mathrm{g}(x)=x^3+5$, then $(\mathrm{fog})^{-1}(-9)$ is

Options:

A. -2

B. 2

C. $-\sqrt{2}$

D. $\sqrt{2}$

Answer: A

Solution:

We have, $f(x)=2x-3,\ g(x)=x^3+5$

$$\begin{aligned} \operatorname{fog}(x) &= f(g(x)) \\ &= 2\,g(x)-3 \\ &= 2\left(x^3+5\right)-3=2x^3+7 \end{aligned}$$

Let $(\operatorname{fog})(x)=y=2x^3+7$

$$\begin{aligned} & y=2x^3+7 \\ & \Rightarrow y-7=2x^3 \\ & \Rightarrow x^3=\frac{y-7}{2} \\ & \Rightarrow x=\left(\frac{y-7}{2}\right)^{\frac{1}{3}} \end{aligned}$$

$$\therefore \quad (\operatorname{fog})^{-1}(y)=\left(\frac{y-7}{2}\right)^{\frac{1}{3}}$$

$$\therefore \quad (\operatorname{fog})^{-1}(-9)=\left(\frac{-9-7}{2}\right)^{\frac{1}{3}}=(-8)^{\frac{1}{3}}=-2$$

Chemistry

Question 51

Identify the compound with highest acidic strength from following.

Options:

A. Ethanol

B. t-Butyl alcohol

C. Phenol

D. p-Nitrophenol

Answer: D

Solution:

Phenols show weak acidic character while alcohols are neutral. Electron-withdrawing groups (like $-NO_2$) increase the acidity of substituted phenols. Hence, p-nitrophenol has the highest acidic strength among the given compound.

Question 52

Which of the following species is NOT isoelectronic with neon?

Options:

A. O^{2-}

B. Na

C. Mg^{2+}

D. Al^{3+}

Answer: B

Solution:

Atoms and ions having the same number of electrons and isoelectronic.

Species	No. of electrons
Neon	10
O^{2-}	$8+2=10$
Na	11
Mg^{2+}	$12-2=10$
Al^{3+}	$13-3=10$

Question 53

Conductivity of a solution is $1.26 \times 10^{-2} \Omega^{-1} \mathrm{~cm}^{-1}$ Calculate molar conductivity for $0.01 \mathrm{~M}$ solution.

Options:

A. $1.26 \times 10^{3} \Omega^{-1} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$

B. $2.52 \times 10^{3} \Omega^{-1} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$

C. $4.82 \times 10^{3} \Omega^{-1} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$

D. $6.30 \times 10^{3} \Omega^{-1} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$

Answer: A

Solution:

$$
\begin{aligned}
\Lambda &= \frac{1000 \mathrm{k}}{\mathrm{c}} \\
&= \frac{1000 \mathrm{~cm}^{3} \mathrm{~L}^{-1} \times 1.26 \times 10^{-2} \Omega^{-1} \mathrm{~cm}^{-1}}{0.01 \mathrm{~mol} \mathrm{~L}^{-1}} \\
&= 1.26 \times 10^{3} \Omega^{-1} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}
\end{aligned}
$$

Question 54

Identify the molecule from following that does NOT involve sp^3 hybridisation.

Options:

A. CH_4

B. C_2H_2

C. H_2O

D. NH_3

Answer: B

Solution:

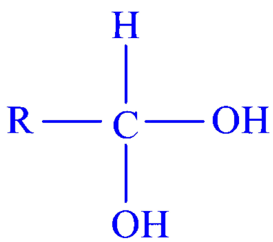
In C_2H_2, both the carbon atoms undergo sp hybridisation.

Question 55

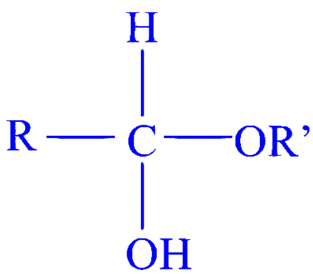
Which among the following compounds is hemiacetal?

Options:

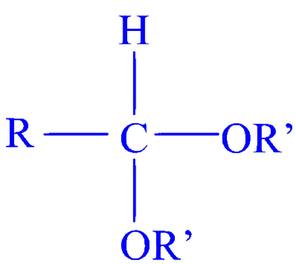
A.



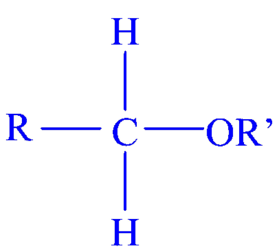
B.



C.



D.



Answer: B

Solution:

Hemiacetal is an alkoxyalcohol (has one $-\mathrm{OH}$ and one $-\mathrm{OR}'$ group bonded to the central carbon atom). Hence, compound (B) is a hemiacetal.

Question 56

Calculate the concentration of H^{+} ions in a solution if pOH is 11.

Options:

A. 10^{-11} M

B. 10^{-8} M

C. 10^{-6} M

D. 10^{-3} M

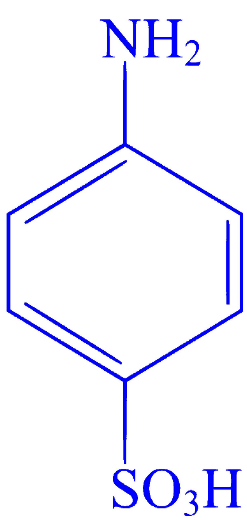
Answer: D

Solution:

$$\begin{aligned} & \mathrm{pOH}=11 \\ & \mathrm{pH}+\mathrm{pOH}=14 \\ \therefore \quad & \mathrm{pH}=14-\mathrm{pOH}=14-11=3 \\ & \mathrm{pH}=-\log_{10}[\mathrm{H}^{+}] \\ \therefore \quad & [\mathrm{H}^{+}]=10^{-\mathrm{pH}}=10^{-3}\mathrm{M} \end{aligned}$$

Question 57

Identify the name of compound



from following.

Options:

A. Sulphanilic acid

B. Sulphonic acid

C. Benzene sulphonic acid

D. Amino benzoic acid

Answer: A

Solution:

The compound shown in the image is a benzene ring with an amine $(\mathrm{NH_2})$ group and a sulfonic acid $(\mathrm{SO_3}H)$ group. The amine group is in the para position (opposite) relative to the sulfonic acid group. This compound is commonly known as sulphanilic acid.

Therefore, the correct name of the compound is Option A : Sulphanilic acid.

Question 58

Identify the physical quantity that is measured in Candela.

Options:

A. Energy

B. Work

C. Force

D. Luminous intensity

Answer: D

Solution:

The answer is Option D : Luminous intensity.

Luminous intensity is a measure of the wavelength-weighted power emitted by a light source in a particular direction per unit solid angle, based on the luminosity function, a standardized model of the sensitivity of the human eye. The SI unit of luminous intensity is the candela (cd). It is one of the seven base units in the International System of Units (SI).

Energy, work, and force are different physical quantities that are not measured in candela :

- **Energy** is the capacity to do work and is measured in joules (J) in the SI system.

- **Work** is the energy transferred to an object via the application of force along a displacement, also measured in joules.

- **Force** is an interaction that, when unopposed, will change the motion of an object. It is measured in newtons (N) in the SI system.

Therefore, the correct answer is D, as candela specifically relates to the measurement of luminous intensity, not to energy, work, or force.

Question 59

Calculate the edge length of unit cell of metal which crystallises to bcc structure.

(Radius of metal atom $= 173 \text{ pm}$)

Options:

A. 5.01×10^{-8} cm

B. 4.00×10^{-8} cm

C. 4.5×10^{-8} cm

D. 5.5×10^{-8} cm

Answer: B

Solution:

For bcc unit cell, $r = \frac{\sqrt{3}}{4}a$

$$\therefore \quad \mathrm{a} = \frac{4\mathrm{r}}{\sqrt{3}} = \frac{4 \times 173}{1.73} = 400 \text{ pm}$$
$$= 400 \times 10^{-10} \text{ cm}$$
$$= 4.00 \times 10^{-8} \text{ cm}$$

Question 60

What is new temperature of a gas when its initial volume $3\ \mathrm{dm}^3$ at $300\ \mathrm{K}$ is doubled at constant pressure?

Options:

A. 450 K

B. 600 K

C. 750 K

D. 900 K

Answer: B

Solution:

Using Charles' law,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\therefore \quad \frac{3\mathrm{dm}^3}{300\ \mathrm{K}} = \frac{2\times 3\mathrm{dm}^3}{T_2}$$

$$\therefore \quad T_2 = \frac{2\times 3\times 300}{3} = 600\ \mathrm{K}$$

At constant pressure, the volume of a given amount of gas is directly proportional to its temperature in Kelvin. Therefore, if the volume doubles, the temperature will be doubled. Hence, correct answer is option (B).

Question 61

Which among the following phenols does NOT correctly match with their IUPAC names?

Options:

A. Catechol: Benzene-1,2-diol

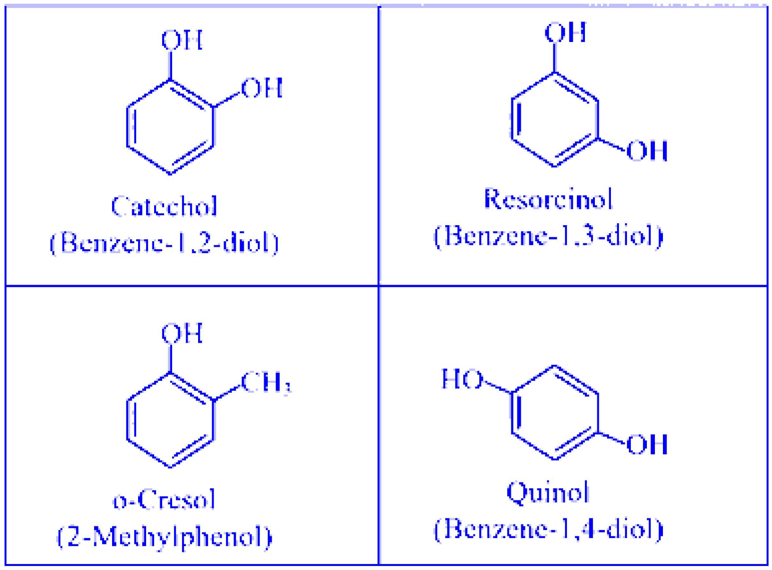
B. Resorcinol : Benzene-1,3-diol

C. o-Cresol : Benzene-1,2,3-triol

D. Quinol: Benzene-1,4-diol

Answer: C

Solution:



Question 62

Which among following statements is NOT true according to principles of green chemistry?

Options:

A. In a chemical synthesis the waste product should be zero or minimum.

B. Good atom economy means very few atoms of reactants are incorporated in the product.

C. Only small amounts of unwanted by products are formed.

D. Use of lindane is better than DDT.

Answer: B

Solution:

Good atom economy means most of the atoms of the reactants are incorporated in the desired products and only small amounts of unwanted byproducts are formed and hence lesser problems of waste disposal.

Question 63

What is the value of effective atomic number of cobalt in $[Co(NH_3)_6]^{3+}$ complex if $Co(Z = 27)$?

Options:

A. 30

B. 32

C. 35

D. 36

Answer: D

Solution:

$[Co(NH_3)_6]^{3+}$:

Oxidation state of cobalt is +3 and six ligands donate 12 electrons.

$Z = 27; X = 3; Y = 12$
EAN of $Co^{3+} = Z - X + Y$
$= 27 - 3 + 12 = 36$

Question 64

For the reaction, $3\,I + S_2O_8^{2-} \rightarrow I_3^- + 2SO_4^{2-}$, at a particular time t, $\frac{d[SO_4^{2-}]}{dt}$ is 2.2×10^{-2} mol dm^{-3} s^{-1}. What is the value of $-\frac{d[I^-]}{dt}$?

Options:

A. 1.1×10^{-2} mol dm^{-3} s^{-1}

B. 3.3×10^{-2} mol dm^{-3} s^{-1}

C. 4.4×10^{-2} mol dm^{-3} s^{-1}

D. 6.6×10^{-2} mol dm^{-3} s^{-1}

Answer: B

Solution:

$3I + S_2O_8^{2-} \rightarrow I_3^- + 2SO_4^{2-}$

$\text{Rate of reaction } = -\frac{1}{3}\frac{d\left[I^-\right]}{dt} = +\frac{1}{2}\frac{d\left[SO_4^{2-}\right]}{dt}$

$-\frac{d\left[I^-\right]}{dt} = \frac{3\, d\left[SO_4^{2-}\right]}{2} = \frac{3}{2} \times 2.2 \times 10^{-2}$

$= 3.3 \times 10^{-2} \text{ mol dm s}^{-3}$

[Note: In the question, $\frac{d[I^-]}{dt}$ is changed to $-\frac{d[I^-]}{dt}$ to apply appropriate textual concepts.]

Question 65

What is the total number of Bravais lattices present for different crystal systems?

Options:

A. 14

B. 7

C. 4

D. 3

Answer: A

Solution:

The Bravais lattices are distinct categories of crystal lattices that can be generated to fill all of space without gaps. These lattices are named after the French physicist Auguste Bravais, who described them in 1850. They represent the maximum symmetry a structure with translational symmetry can have and are used to describe the geometric structure of crystals.

For different crystal systems, there are a total of 14 Bravais lattices. These 14 lattices are divided into seven crystal systems:

1. Cubic (also known as Isometric) system: 3 Bravais lattices (Simple cubic, Body-centered cubic, Face-centered cubic)
2. Tetragonal system: 2 Bravais lattices (Simple tetragonal, Body-centered tetragonal)
3. Orthorhombic system: 4 Bravais lattices (Simple orthorhombic, Body-centered orthorhombic, Face-centered orthorhombic, Base-centered orthorhombic)
4. Hexagonal system: 1 Bravais lattice (Simple hexagonal)
5. Rhombohedral (or Trigonal) system: 1 Bravais lattice (Simple rhombohedral)
6. Monoclinic system: 2 Bravais lattices (Simple monoclinic, Base-centered monoclinic)
7. Triclinic system: 1 Bravais lattice (Simple triclinic)

Hence, the correct answer is Option A: 14 Bravais lattices.

Question 66

Identify compound Y in the following reaction.

$$\mathrm{C_2H_5Cl + Y \xrightarrow{\Delta} C_2H_5CN}$$

Options:

A. $\mathrm{NH_3}$

B. $\mathrm{HNO_3}$

C. KCN (alc)

D. AgCN (alc)

Answer: C

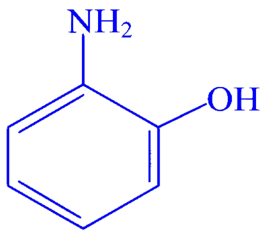
Solution:

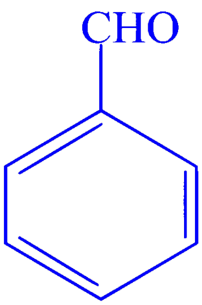
$$\underset{\text{Ethyl chloride}}{\mathrm{C_2H_5Cl}} + \mathrm{KCN_{(alc.)}} \xrightarrow{\Delta} \underset{\text{Ethyl cyanide}}{\mathrm{C_2H_5CN}} + \mathrm{KCl}$$

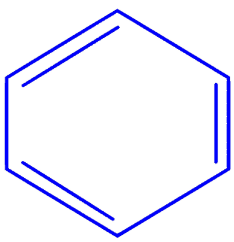
Question 67

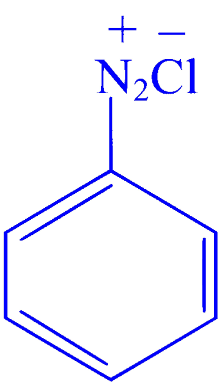
Aniline is treated with $NaNO_2 + HCl$ at low temperature to form:

Options:

A. 

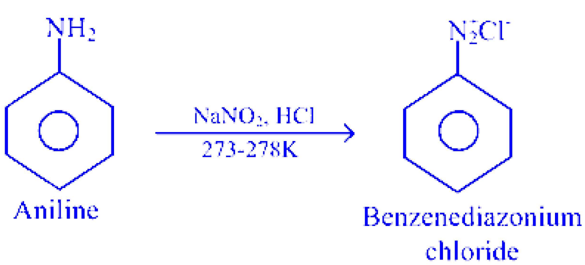
B. 

C. 

D. 

Answer: D

Solution:



Question 68

Which of the following is NOT a difficulty in setting SHE?

Options:

A. To obtain pure hydrogen gas

B. To obtain dry hydrogen gas

C. To maintain exactly 1 atm pressure

D. To bring the reaction in reverse direction

Answer: D

Solution:

In standard hydrogen electrode (SHE), a platinum plate coated with platinum black is used as electrode. The platinum black is capable of adsorbing large quantities of H_2 gas. This allows the change from gaseous to ionic form and the reverse process to occur.

Question 69

Lewis acid is a substance that :

Options:

A. gives H^+ ions aqueous solution

B. accepts a proton

C. accepts electron pair

D. donates a proton

Answer: C

Solution:

Lewis acids are defined as substances that can accept an electron pair. According to the Lewis acid-base theory, an acid is an electron pair acceptor, and a base is an electron pair donor. This definition is broader than the Bronsted-Lowry definition of acids and bases, which involves proton transfer.

Given the options :

- Option A : gives H^+ ions in aqueous solution - This describes a Brønsted-Lowry acid, not necessarily a Lewis acid.
- Option B : accepts a proton - This is the definition of a Brønsted-Lowry base.
- Option C : accepts electron pair - This accurately describes a Lewis acid.
- Option D : donates a proton - This is the definition of a Brønsted-Lowry acid.

Therefore, the correct answer is Option C : accepts electron pair.

Question 70

Which among the following α-amino acids does NOT have chiral carbon atom?

Options:

A. Histidine

B. Glutamic acid

C. Serine

D. Glycine

Answer: D

Solution:

Except glycine, all α-amino acids are chiral.

Question 71

The difference between ΔH and ΔU is usually significant for systems consisting of :

Options:

A. only solids

B. only gases

C. only liquids

D. both solids and liquids

Answer: B

Solution:

The difference between ΔH (change in enthalpy) and ΔU (change in internal energy) is particularly significant for systems consisting of gases. This is because gases are much more compressible than solids or liquids and thus their volume can change considerably with pressure, which is not the case with solids or liquids to any significant extent. Enthalpy, H, is defined as:

$H = U + PV$

where:

- H is the enthalpy
- U is the internal energy
- P is the pressure
- V is the volume

The change in enthalpy ΔH for a process can be written as:

$\Delta H = \Delta U + \Delta(PV)$

If we assume that the pressure is constant, which is a common condition for many chemical reactions that take place in open vessels at atmospheric pressure, this equation simplifies to:

$\Delta H = \Delta U + P\Delta V$

For gases, $P\Delta V$ can be significant because gases expand or contract when they are heated or cooled. In chemical reactions involving gases, when gases are produced or consumed, their volume changes can involve the absorption or release of significant amounts of energy as the work of expansion or compression against the ambient pressure.

In contrast, for solids and liquids, the volume changes (ΔV) are usually quite small because these phases are relatively incompressible compared to gases. Therefore, for processes involving only solids or liquids, $P\Delta V$ is usually negligible, and therefore ΔH approximates ΔU closely.

Hence, the correct answer is:

Option B only gases

Question 72

Which of the following elements is doped with to obtain fibre amplifiers for optical fibre communication system?

Options:

A. Zn

B. Cu

C. Er

D. La

Answer: C

Solution:

Erbium-doped fibre amplifiers are used in optical fibre communication systems.

Question 73

What is the half life of a first order reaction if rate constant is 4.2×10^{-2} per day?

Options:

A. 5.0 day

B. 16.5 day

C. 28.0 day

D. 9.0 day

Answer: B

Solution:

For a first order reaction,

$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{4.2\times10^{-2}} = 16.5$ days

---

Question 74

What type of solution is the ethyl alcohol in water?

Options:

A. Liquid in solid

B. Solid in liquid

C. Liquid in liquid

D. Gas in liquid

Answer: C

Solution:

Ethyl alcohol (ethanol) dissolved in water forms a **homogeneous mixture**, which is a type of solution where the solute and the solvent are both in the same phase. In this case, both ethanol and water are **liquid** at room temperature. Therefore, when ethanol is mixed with water, you get a liquid dissolved in another liquid. This type of solution is commonly known as a **liquid in liquid** solution.

Based on this information, the correct option is:

Option C - Liquid in liquid

Question 75

Which among the following colours is obtained in Schiff test of aldehydes?

Options:

A. Blue

B. Green

C. Magenta

D. Black

Answer: C

Solution:

When alcoholic solution of aldehyde is treated with few drops of Schiff's reagent, pink or red or magenta colour appears.

Question 76

Which from following monomers is used to prepare thermocol?

Options:

A. Bisphenol and acrylamide

B. Acrylamide

C. Butadiene

D. Styrene

Answer: D

Solution:

The correct monomer used to prepare thermocol, which is a type of expanded polystyrene (EPS) foam, is Option D, Styrene.

Thermocol is essentially polystyrene foam, which is a polymer. Polymers are long chains of repeating units called monomers. In the case of thermocol, the monomer is styrene. Through a process called polymerization, styrene molecules react to form long chains, resulting in the polymer polystyrene.

So, to summarize:

- Option A: Bisphenol and acrylamide - Bisphenol A (combined with other chemicals) is used to make polycarbonates and epoxy resins, not polystyrene.
- Option B: Acrylamide - Acrylamide is used to produce polyacrylamides, which are used as water-soluble thickeners and flocculants. It is not used to make thermocol.

- Option C: Butadiene - Butadiene is a monomer used in the production of synthetic rubbers like polybutadiene, not polystyrene.
- Option D: Styrene - Styrene is the correct monomer for the production of thermocol (polystyrene).

Therefore, the correct answer is Option D: Styrene.

Question 77

Which of the following is character of lyophilic colloid?

Options:

A. Particles of dispersed phase have no affinity for dispersion medium.

B. Particles easily detected under ultramicroscope.

C. Addition of large amount of electrolyte causes precipitation.

D. These are irreversible colloids.

Answer: C

Solution:

In lyophilic colloid, the particles of dispersed phase have a great affinity for the dispersion medium. The particles are not easily visible even under ultramicroscope. Lyophilic colloids are reversible colloids.

Question 78

Find the depression in freezing point of solution when 3.2 gram non volatile solute with molar mass $128 \text{ gram mol}^{-1}$ is dissolved in 80 gram solvent if cryoscopic constant of solvent is $4.8 \text{ K kg mol}^{-1}$.

Options:

A. 3.0 K

B. 1.5 K

C. 2.0 K

D. 2.5 K

Answer: B

Solution:

$$\begin{aligned}\Delta T_f &= \frac{1000\ K_f W_2}{M_2\ W_1} \\ &= \frac{1000\ g\ kg^{-1} \times 4.8\ K\ kg\ mol^{-1} \times 3.2\ g}{128\ g\ mol^{-1} \times 80\ g} \\ &= 1.5\ K\end{aligned}$$

Question 79

What is the value of x in order to balance following redox reaction?

$$Mn^{2+}_{(aq)} + xClO^{-}_{3(aq)} \rightarrow MnO_{2(\ s)} + xClO_{2(aq)}$$

Options:

A. $x = 1$

B. $x = 2$

C. $x = 3$

D. $x = 4$

Answer: B

Solution:

The balanced equation is:

$Mn^{2+}_{(aq)} + 2ClO_{3(sq)} \rightarrow MnO_{2(\ s)} + 2ClO_{2(aq)}$

Question 80

The reaction, $3ClO^- \rightarrow ClO_3^- + 2Cl^-$ occurs in two steps:

i. $2ClO^- \rightarrow ClO_2^- + Cl^-$

ii. $ClO_2^- + ClO^- \rightarrow ClO_3^- + Cl^-$**,**

the reaction intermediate is:

Options:

A. Cl^-

B. ClO_2^-

C. ClO_3^-

D. ClO^-

Answer: B

Solution:

ClO_2^- is produced in step (i) and consumed in step (ii). Therefore, ClO_2^- is the reaction intermediate.

Question 81

Acetic acid dissociated to 1.20% in its 0.01 M solution. What is the value of its dissociation constant?

Options:

A. 2.20×10^{-2}

B. 1.60×10^{-4}

C. 1.44×10^{-6}

D. 2.40×10^{-4}

Answer: C

Solution:

Percent dissociation $= 1.20\%$

Degree of dissociation $(\alpha) = 0.012$

For a weak monobasic acid,

$K_a = \alpha^2 c = (0.012)^2 \times 0.01 = 1.44 \times 10^{-6}$

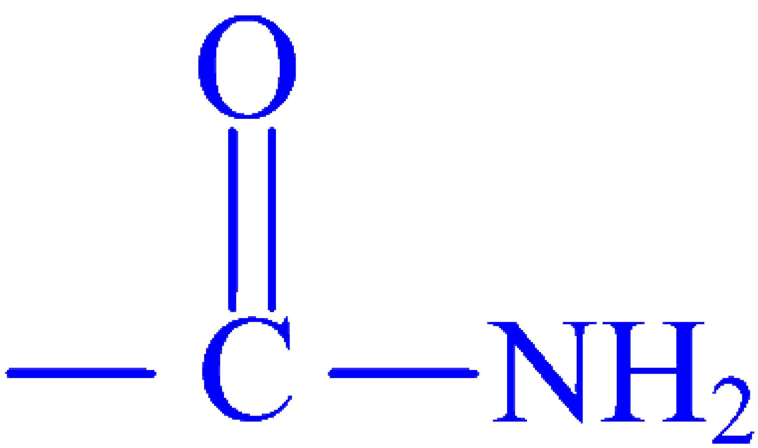
Question 82

The structure of functional group of secondary amide is :

Options:

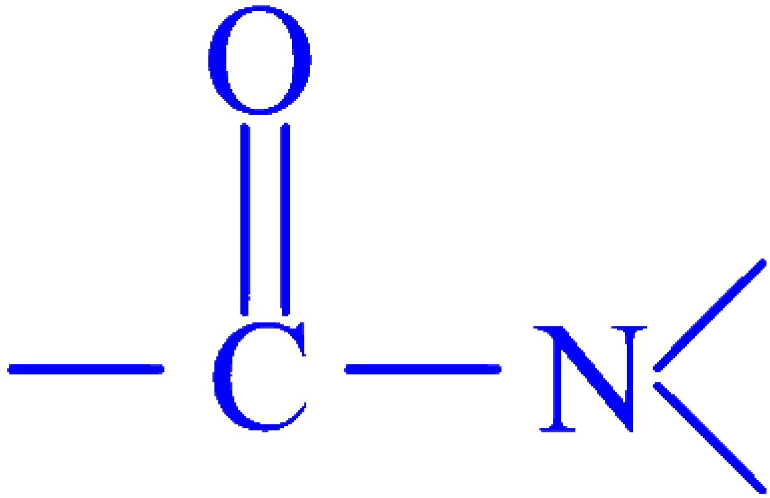
A. $-NH-$

B.



C.

D.



Answer: C

Solution:

A secondary amide functional group is characterized by the following general structure:

$R_1 - C(=O)NH - R_2$

Where R_1 and R_2 are alkyl or aryl groups and the nitrogen atom is bonded to two other carbon atoms. In other words, a secondary amide has the nitrogen atom bonded to one hydrogen and two carbon-containing groups. The structure of a secondary amide thus includes the amide bond $-C(=O)NH-$ as part of the core skeleton.

Therefore, among the options provided, the one that best represents the structure of a secondary amide is the one that shows a nitrogen atom bonded to a hydrogen and two carbon-containing groups with one of the groups being part of the carbonyl (C=O) functionality.

The correct option is C.

Question 83

Which of the following solutions exhibits lowest value of boiling point elevation assuming complete dissociation?

Options:

A. 0.1 m $AlCl_3$

B. 0.01 m $MgCl_2$

C. 1 m KCl

D. 0.5 m NaCl

Answer: B

Solution:

	Solution	Moles of particles in 1 kg solution
(A)	0.1 m $AlCl_3$	0.4
(B)	0.01 m $MgCl_2$	0.03
(C)	1 m KCl	2
(D)	0.5 m NaCl	1

0.01 m $MgCl_2$ solution has minimum number of particles in solution, so it shows the lowest value of boiling point elevation.

Question 84

Identify heteroleptic complex from following.

Options:

A. Tetraamminediaquacobalt (III) chloride

B. Hexaamminecobalt (III) bromide

C. Potassium tetrahydroxozincate (II)

D. Tetracarbonyl nickel (0)

Answer: A

Solution:

Complex	Formula
Tetraamminediaquacobalt(III) chloride	$[Co(NH_3)_4(H_2O)_2]Cl_3$

Complex	Formula
Hexaamminecobalt(III)	$[Co(NH_3)_6]Br_3$
Potassium tetrahydroxozincate(II)	$K_2[Zn(OH)_4]$
Tetracarbonyl nickel(0)	$[Ni(CO)_4]$

Question 85

Which among the following statements of group-1 elements is NOT true?

Options:

A. Unipositive ions have inert gas configuration.

B. Compounds of unipositive ions are paramagnetic.

C. These form colourless compounds in +1 state.

D. These have high negative values of standard reduction potential.

Answer: B

Solution:

The statement that is NOT true among the choices given for group-1 elements is:

Option B: Compounds of unipositive ions are paramagnetic.

Explanation:

Group 1 elements, also known as the alkali metals, include lithium, sodium, potassium, rubidium, cesium, and francium. These elements are characterized by having a single valence electron in their outer shell which they can lose easily to form cations with a +1 charge, known as unipositive ions.

Let's review each statement:

Option A: Unipositive ions have inert gas configuration.

This statement is true. When group-1 elements lose one electron, they form ions with the same electron configuration as the nearest noble gas (inert gas) in the periodic table. For example, when sodium (Na) loses an electron, it forms a Na^+ ion, which has the same electron configuration as neon (Ne), a noble gas.

Option B: Compounds of unipositive ions are paramagnetic.

This statement is not true and is the incorrect option in context. Paramagnetism arises due to the presence of unpaired electrons in the atomic or molecular electron configuration. Group 1 ions in the +1 oxidation state have full electron shells with no unpaired electrons, so they are diamagnetic rather than paramagnetic. Compounds formed by these ions, such as sodium chloride (NaCl), also do not exhibit paramagnetism because the ions involved have all paired electrons in their structure.

Option C: These form colorless compounds in +1 state.

This statement is generally true. The compounds formed by group 1 elements in the +1 oxidation state are typically colorless when dissolved in water or in the solid state. This is because the energy levels required to excite the electrons to higher energy states – and thus absorb visible light – are too high to be affected by visible light.

Option D: These have high negative values of standard reduction potential.

This statement is true. Group 1 elements have high negative standard reduction potentials, which means that they readily lose electrons to form cations. This is consistent with their position in the reactivity series of metals, where they are known to be highly reactive and easily oxidized.

Question 86

Identify glycosidic linkage present in lactose.

Options:

A. $\beta-1,2$

B. $\alpha-1,4$

C. $\beta-1,4$

D. $\beta-1,6$

Answer: C

Solution:

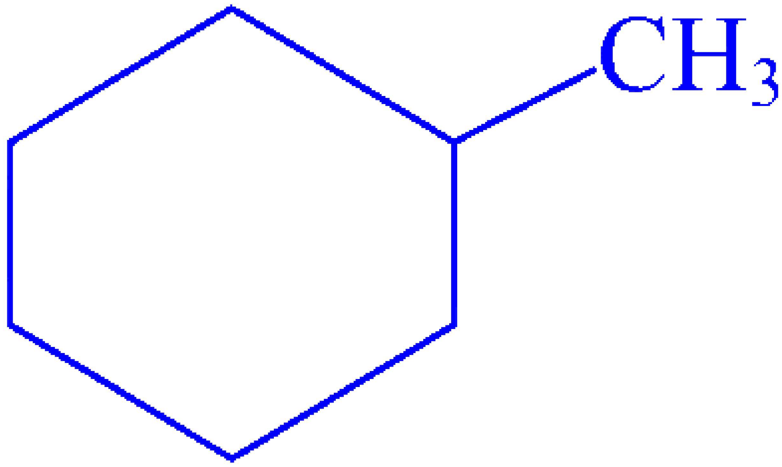
In lactose, the glycosidic linkage is formed between C-1 of β-D-galactose and C-4 of β-D-glucose.

Question 87

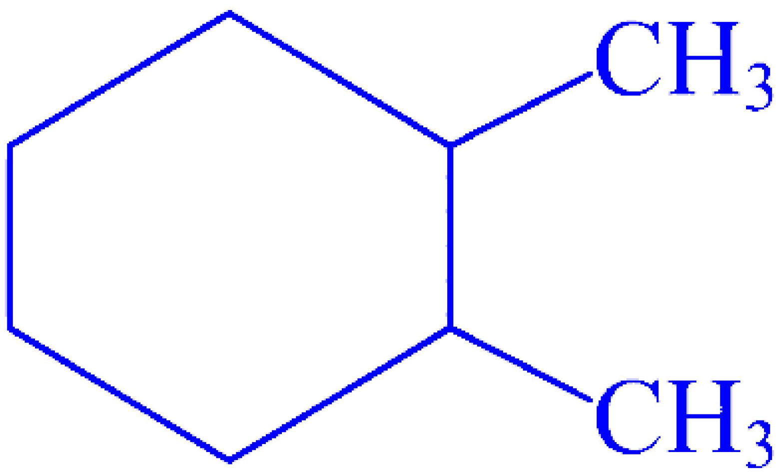
Which of the following compounds reacts with HBr to form 1-Bromo-1-methylcyclohexane?

Options:

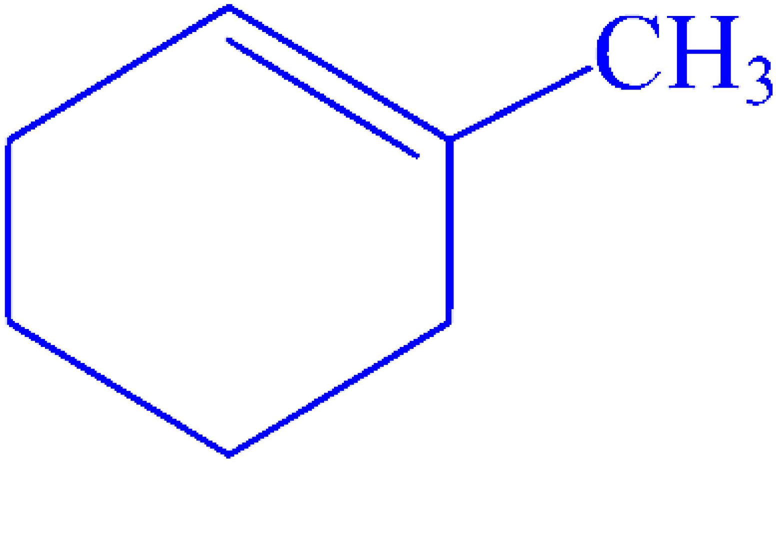
A.



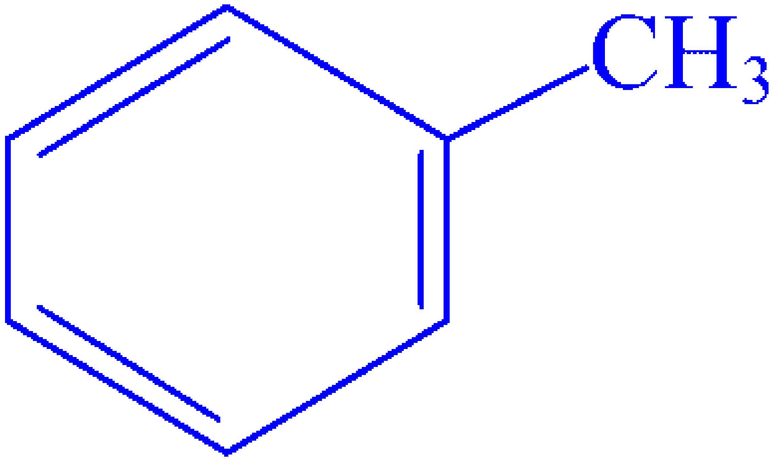
B.



C.

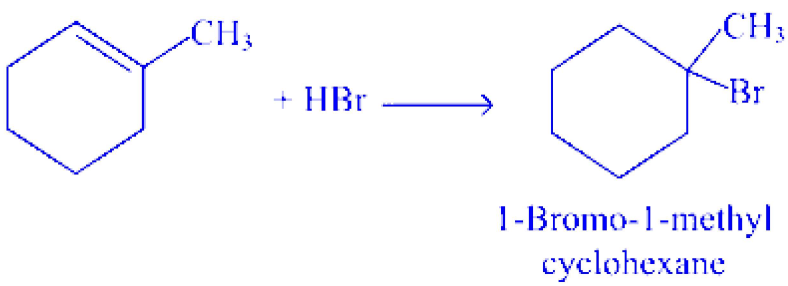


D.



Answer: C

Solution:



This reaction follows Markownikoff's rule.

Question 88

Identify degenerate orbitals from following for hydrogen atom.

Options:

A. 1s and 2s

B. 1s and 2p

C. 2s and 2p

D. 3s and 2p

Answer: C

Solution:

An increasing order of energies of orbitals in the hydrogen atom is given by:

$1\text{ s} < 2\text{ s} = 2\text{p} < 3\text{ s} = 3\text{p} = 3\text{ d} < \ldots$

Thus, in hydrogen atom $2s$ and $2p$ are degenerate orbitals.

Question 89

Identify the product P obtained in following reaction.

Benzene + ozone (excess) $\xrightarrow{\text{CCl}_4}$

Benzene triozonide $\xrightarrow{\text{Zn/H}_2\text{O}}$ $\text{P} + \text{H}_2\text{O}_2$

Options:

A. Benzoic acid

B. Benzaldehyde

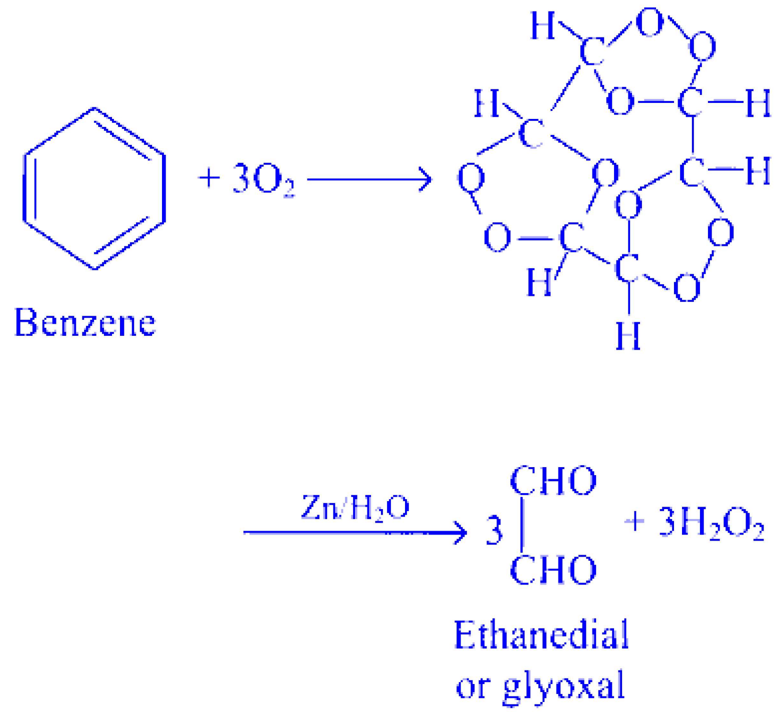
C. Phenol

D. Glyoxal

Answer: D

Solution:

When benzene is treated with ozone in presence of an inert solvent such as carbon tetrachloride, benzene triozonide is formed which is then decomposed by zinc dust and water to give glyoxal.



Question 90

Identify strongest oxoacid of halogen from following.

Options:

A. Hypochlorous acid

B. Chlorous acid

C. Chloric acid

D. Perchloric acid

Answer: D

Solution:

Increasing order of acid strength is: $HOCl < HClO_2 < HClO_3 < HClO_4$

Hence, perchloric acid ($HClO_4$) is the strongest acid, among the oxoacids of chlorine.

Question 91

Identify the reagent R used in the reaction stated below.

Benzene diazonium chloride $+\mathrm{R} \rightarrow$ Benzene

Options:

A. HCl

B. $\mathrm{H_3PO_2/H_2O}$

C. $\mathrm{H_2O/HCl}$

D. $\mathrm{CH_3-CH_2-OH}$

Answer: B

Solution:

Benzene diazonium chloride can be converted to benzene using (i) $\mathrm{H_3PO_2/H_2O}$ or (ii) $\mathrm{CH_3CH_2OH}$.

Question 92

Identify lanthanoid element from following.

Options:

A. Eu

B. Cm

C. Am

D. Np

Answer: A

Solution:

The lanthanoids, or lanthanides, are a set of 15 chemical elements in the periodic table with atomic numbers from 57 (lanthanum) to 71 (lutetium). They are named after the first element in the series, lanthanum, and they are known for their similarity to each other and for their wide range of applications in various technologies, including electronics, optics, and as catalysts.

Here's a check on each of the given options:

Option A: Eu represents Europium, which is a lanthanoid element with the atomic number 63. It falls within the range of the lanthanoids in the periodic table.

Option B: Cm stands for Curium, which is not a lanthanoid. Curium is an actinoid (or actinide) element with the atomic number 96.

Option C: Am is the symbol for Americium, another actinoid element with the atomic number 95, sitting just before Curium in the periodic table.

Option D: Np stands for Neptunium, which is also an actinoid element, bearing the atomic number 93.

Therefore, the correct answer that identifies a lanthanoid element from the provided options is:

Option A: Eu (Europium)

Question 93

What is change in internal energy when system releases $8 \mathrm{~kJ}$ of heat and performs $660 \mathrm{~J}$ of work on the surrounding?

Options:

A. $-7340 \mathrm{~J}$

B. $-5400 \mathrm{~J}$

C. $-8660 \mathrm{~J}$

D. $-1212 \mathrm{~J}$

Answer: C

Solution:

$\Delta U=Q+W=-8000 J-660 J=-8660 J$

When heat is released by the system to the surroundings, Q is negative. When work is done by the system on the surroundings, W is negative.

Question 94

Identify the method used to obtain SO_2 gas in industry.

Options:

A. By burning sulphur in air

B. By treating sodium sulphite with dil. sulphuric acid

C. By treating sodium sulphite with dil. hydrochloric acid

D. By roasting zinc sulphide and iron pyrites

Answer: D

Solution:

In industry, sulphur dioxide can be prepared by roasting zinc sulphide and iron pyrites.

$2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$

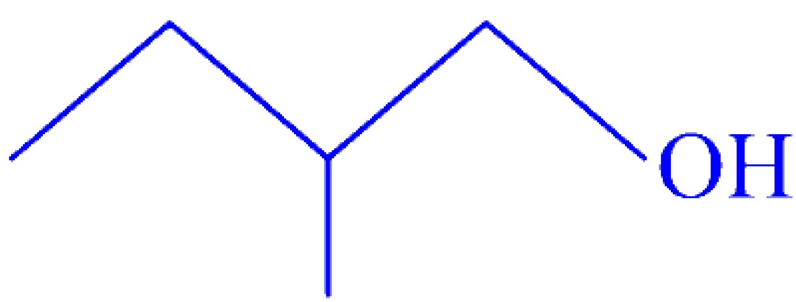
$4FeS_{2(s)} + 11O_{2(g)} \xrightarrow{\Delta} 2Fe_2O_{3(s)} + 8SO_{2(g)}$

Question 95

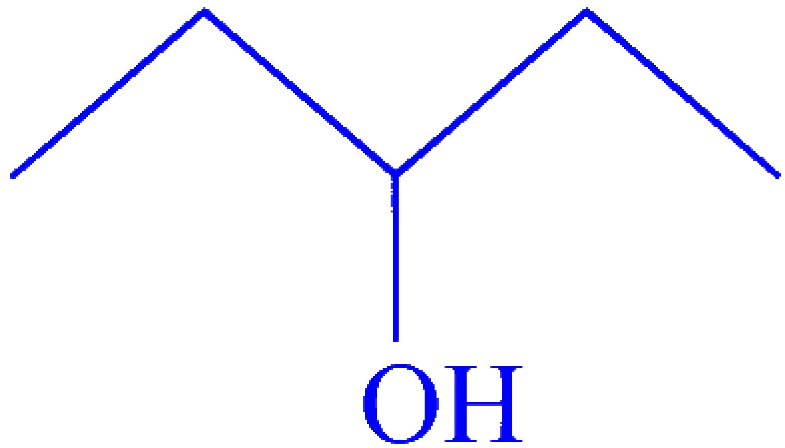
Which among the following compounds reacts fastly with HBr ?

Options:

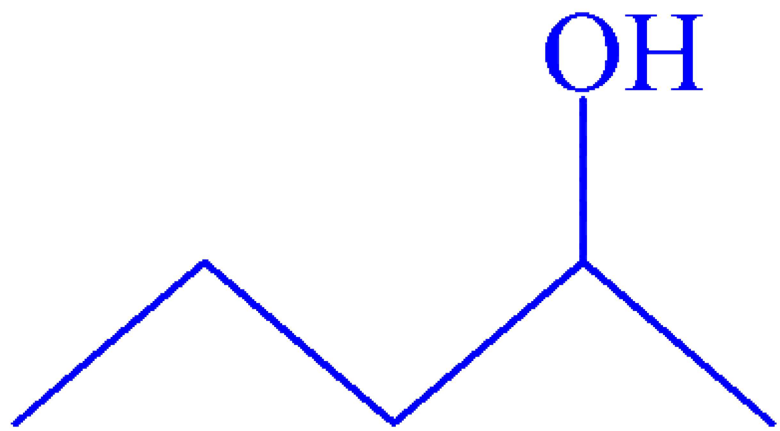
A.



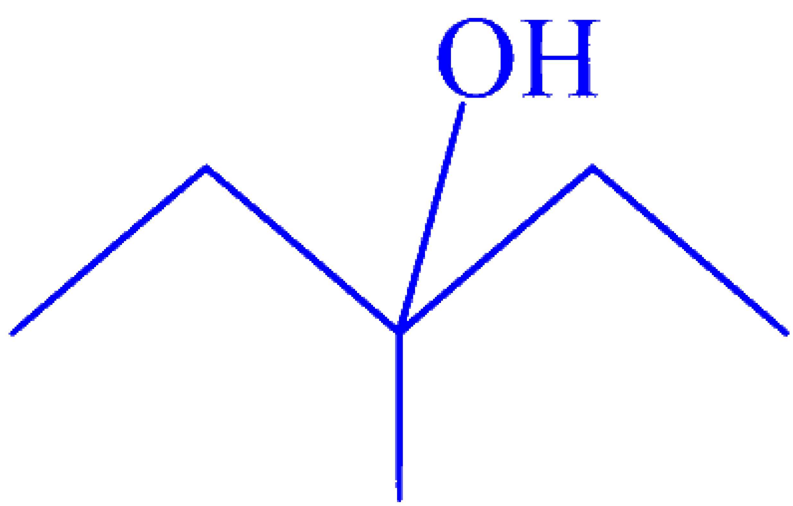
B.



C.



D.



Answer: D

Solution:

The order of reactivity of alcohols with a given haloacid is $3^{\circ} > 2^{\circ} > 1^{\circ}$.

Hence, compound (D), a tertiary alcohol, will react fastly with HBr.

Question 96

Identify biodegradable polymer from following.

Options:

A. Nylon 2-nylon 6

B. Terylene

C. Nylon 6

D. Nylon 6,6

Answer: A

Solution:

Biodegradable polymers are materials that can be broken down into their constituent parts, typically monomers, by the action of biological organisms, typically microbes. Among the options given, one stands out as a biodegradable polymer:

Option A: **Nylon 2-nylon 6**

Nylon 2-nylon 6, also known as poly(glycolide-co-caprolactam), is a copolymer consisting of two different monomers: glycolide (2) and caprolactam (6). Polymers created from monomers with a tendency to break down more easily in the presence of microorganisms can be biodegradable. Nylon 2-nylon 6 is designed to be biodegradable and is used in a variety of applications, including biomedical applications such as sutures and tissue engineering, where biodegradability is a beneficial property.

For reference, let's look at the non-biodegradable options:

Option B: **Terylene** is another name for poly(ethylene terephthalate) or PET, which is a synthetic polymer used widely in textiles and plastic bottle manufacturing. PET is known for its strength, durability, and resistance to biodegradation.

Option C: **Nylon 6** is a synthetic polymer made from caprolactam, and while it does undergo some degradation over time, it is generally not considered biodegradable in the context of being broken down by microorganisms in the environment.

Option D: **Nylon 6,6** is composed of hexamethylene diamine and adipic acid and is a strong and durable synthetic polymer that is used in various applications from fabrics to car parts. Like Nylon 6, Nylon 6,6 is not readily biodegradable.

Therefore, the biodegradable polymer in this list is **Option A: Nylon 2-nylon 6**.

Question 97

What mass of Mg is produced during electrolysis of molten $\mathrm{MgCl_2}$ by passing $2\ \mathrm{amp}$ current for 482.5 second?

(Molar mass $\mathrm{Mg} = 24\ \mathrm{g\ mol^{-1}}$)

Options:

A. 0.12 g

B. 0.24 g

C. 1.2 g

D. 0.4 g

Answer: A

Solution:

$$\mathrm{Mg^{2+}_{(s)}} + 2\mathrm{e^-} \longrightarrow \mathrm{Mg_{(s)}}$$

$$\text{Mole ratio } = \frac{1\ \mathrm{mol}}{2\ \mathrm{mole^-}}$$

$$\mathrm{W} = \frac{\mathrm{I(A)} \times \mathrm{t(s)}}{96500\,(\mathrm{C/mole^-})} \times \text{mole ratio} \times \text{ molar mass}$$

$$\mathrm{W} = \frac{2 \times 482.5}{96500\,(\mathrm{C/mole^-})} \times \frac{1\ \mathrm{mol}}{2\ \mathrm{mole^-}} \times 24\ \mathrm{g\ mol^{-1}}$$

$$\mathrm{W} = 0.12\ \mathrm{g}$$

Question 98

Calculate the final volume when 2 moles of an ideal gas expand from $3\ \mathrm{dm^3}$ at constant external pressure 1.6 bar and the work done in the process is $800\ \mathrm{J}$.

Options:

A. $2.66\ \mathrm{dm^3}$

B. 4.8 dm^3

C. 5.0 dm^3

D. 8.0 dm^3

Answer: D

Solution:

$W = -P_{ext}\ \Delta V = -P_{ext}\ (V_2 - V_1)$
$V_1 = 3\ dm^3$
$V_2 = ?$
$P_{ext} = 1.6$ bar
$W = -800\ J = 8\ dm^3$ bar $\left(\because 100\ J = 1\ dm^3 \text{ bar}\right)$
$-8 = -1.6(V_2 - 3)$
$V_2 - 3 = 5$
$V_2 = 8\ dm^3$

Question 99

Calculate the molar mass of an element with density $2.7\ g\ cm^{-3}$ that forms fcc structure. $\left[a^3 \cdot N_A = 40\ cm^3\ mol^{-1}\right]$

Options:

A. 112 $g\ mol^{-1}$

B. 54 $g\ mol^{-1}$

C. 27 $g\ mol^{-1}$

D. 78 $g\ mol^{-1}$

Answer: C

Solution:

For fcc unit cell, $n = 4$.

$$\text{Density } (\rho) = \frac{Mn}{a^3 N_A}$$

$$2.7 = \frac{M \times 4}{40}$$

$$M = \frac{2.7 \times 40}{4} = 27 \text{ g mol}^{-1}$$

Question 100

Identify ketone from the following.

Options:

A. Ethyl ethanoate

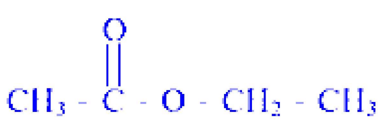
B. Acetophenone

C. N-Methylphthalimide

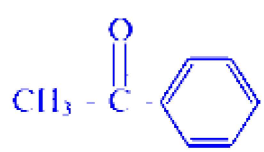
D. Methyl salicylate

Answer: B

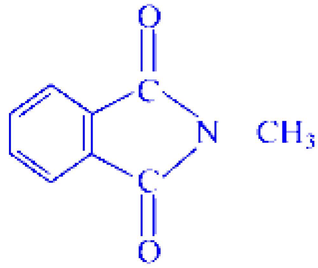
Solution:



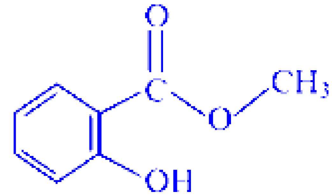
Ethyl ethanoate
(Ester)



Acetophenone
(Ketone)



N-Methylphthalimide
(Amide)



N-Methylphthalimide
(Amide)

Physics

Question 101

An open organ pipe having fundamental frequency (n) is in unison with a vibrating string. If the tube is dipped in water so that 75% of the length of the tube is inside the water then the ratio of fundamental frequency of the air column of dipped tube with that of string will be (Neglect end corrections)

Options:

A. 1 : 1

B. 2 : 1

C. 2 : 3

D. 3 : 2

Answer: B

Solution:

$n_{open} \quad = \frac{v}{2L}$ ...... (i)

When dipped in water, pipe becomes closed at one end and open at the other.

Length available for resonance is

$$\begin{aligned} l_1 &= 25\% \times L \\ &= L \times \frac{25}{100} \\ &= L/4 \end{aligned}$$

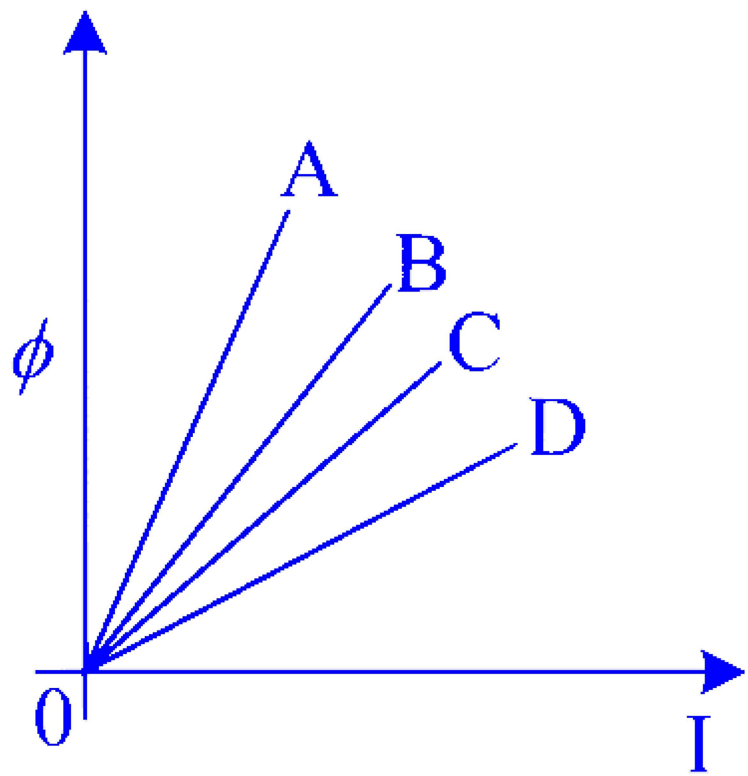
$\therefore \quad n_{closed} \quad = \frac{v}{4l_1} = \frac{v}{4 \times \frac{L}{4}} = \frac{v}{L}$ ..... (ii)

Comparing (i) and (ii),

$\therefore \quad \frac{n_{closed}}{n_{open}} = \frac{\left(\frac{v}{L}\right)}{\left(\frac{v}{2L}\right)} = \frac{2}{1}$

Question 102

A graph of magnetic flux (ϕ) versus current (I) is plotted for four inductors A, B, C, D. Larger value of self inductance is for inductor



Options:

A. A

B. B

C. C

D. D

Answer: A

Solution:

From $\phi = \mathrm{LI}$

Comparing the above equation with $\mathrm{y} = \mathrm{mx}, \mathrm{L} = \phi/\mathrm{I}$, which is the slope of the graph.

Since line A has highest slope, it indicates the largest value of self-inductance.

Question 103

An electron accelerated through a potential difference 'V_1' has a de-Broglie wavelength 'λ'. When the potential is changed to 'V_2' its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is

Options:

A. 3 : 1

B. 9 : 4

C. 3 : 2

D. 4 : 1

Answer: B

Solution:

For electron, de Broglie wavelength, $\lambda = \frac{1.228}{\sqrt{V}}$

Given: $\lambda_2 = \lambda_1 + 0.5\lambda_1 = 1.5\lambda_1$

$\therefore \quad \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{V_1}{V_2}}$

$\Rightarrow \frac{V_1}{V_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^2 = \left(\frac{1.5\lambda_1}{\lambda_1}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Question 104

In case of a stationary wave pattern which of the following statement is CORRECT?

Options:

A. The distance between the consecutive nodes is equal to the wavelength.

B. In a pipe at both ends only even harmonics are present in an air column.

C. In a pipe closed at one end, all harmonics are present in an air column.

D. In case of a stretched string when vibrated, frequency of first overtone is same as second harmonic.

Answer: D

Solution:

Frequency of first harmonic $= \mathrm{n} = \frac{\mathrm{v}}{\lambda} = \frac{1}{2l}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}}$

Frequency of second harmonic $= 2\mathrm{n} = \frac{1}{l}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}}$ ..... (i)

For the first overtone, $\lambda = l$

$\therefore$ Frequency of first overtone $\mathrm{n}_1 = \frac{1}{l}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}}$ ...... (ii)

Comparing (i) and (ii), $\mathrm{n}_1 = 2\mathrm{n}$

Question 105

If 'I' is moment of inertia of a thin circular disc about an axis passing through the tangent of the disc and in the plane of disc. The moment of inertia of same circular disc about an axis perpendicular to plane and passing through its centre is

Options:

A. $\frac{4I}{5}$

B. $\frac{2I}{5}$

C. $\frac{4I}{3}$

D. $\frac{2I}{3}$

Answer: B

Solution:

M.I. of thin circular disc through the tangent in the plane of the disc is $\mathrm{I}=\frac{5}{4} \mathrm{MR}^2$

$\Rightarrow \mathrm{MR}^2=\frac{4}{5} \mathrm{I}$

$\therefore$ M.I. of thin circular disc about an axis perpendicular to plane and passing through its

centre $=\frac{\mathrm{MR}^2}{2}=\frac{\left(\frac{4}{5} \mathrm{I}\right)}{2}=\frac{2 \mathrm{I}}{5}$

Question 106

A parallel plate capacitor has plate area 'A' and separation between plates is 'd'. It is charged to a potential difference of V_0 volt. The charging battery is then disconnected and plates are pulled apart three times the initial distance. The work done to increase the distance between the plates is ($\varepsilon_0=$ permittivity of free space)

Options:

A. $\frac{3 \varepsilon_0 \mathrm{AV}_0{ }^2}{\mathrm{~d}}$

B. $\frac{\varepsilon_0 \mathrm{AV}_0^2}{2 \mathrm{~d}}$

C. $\frac{\varepsilon_0 \mathrm{AV}_0{ }^2}{3 \mathrm{~d}}$

D. $\frac{\varepsilon_0 \mathrm{AV}_0{ }^2}{\mathrm{~d}}$

Answer: D

Solution:

Let the initial capacitance be $\mathrm{C}_0=\frac{\varepsilon_0 \mathrm{~A}}{\mathrm{~d}}$

Let the charge on the capacitor be $Q_{\text {initial }}=C_0 V_0$

Plate separation is increased by 3 times i.e., $\mathrm{d}^{\prime}=3 \mathrm{~d}$

$\mathrm{C}_{\text {final }}=\frac{\varepsilon_0 \mathrm{~A}}{3 \mathrm{~d}}=\frac{1}{3}\left(\frac{\varepsilon_0 \mathrm{~A}}{\mathrm{~d}}\right)=\frac{\mathrm{C}_0}{3}$

Let Q_{final} be the final charge on the capacitor and V_{final} be the final potential on the capacitor.

$\therefore \quad Q_{final} = C_{final} V_{final} = \frac{1}{3} C_0 V_{final}$

As the capacitor is isolated,

$$Q_{final} = Q_{initial} ,$$
$$C_0 V_0 = \frac{1}{3} C_0 V_{final}$$
$$\therefore \quad V_{final} = 3 V_0$$
$$\text{Work done} = \text{Final P.E} - \text{Initial P.E}$$
$$= \frac{1}{2} C_{final} (V_{final})^2 - \frac{1}{2} C_0 (V_0)^2$$
$$= \frac{1}{2} \frac{C_0}{3} 9 V_0^2 - \frac{1}{2} C_0 V_0^2$$
$$= C_0 V_0^2 \left(\frac{3}{2} - \frac{1}{2} \right)$$
$$= C_0 V_0^2$$
$$= \frac{\varepsilon_0 A V_0^2}{d} \quad \ldots \left(\because C_0 = \frac{\varepsilon_0 A}{d} \right)$$

Question 107

The shortest wavelength in the Balmer series of hydrogen atom is equal to the shortest wavelength in the Brackett series of a hydrogen like atom of atomic number z. The value of z is

Options:

A. 2

B. 3

C. 4

D. 6

Answer: A

Solution:

Using Rydberg's formula,

$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n^2} - \frac{1}{m^2}\right]$

where R_H is the Rydberg's constant

For calculating the shortest wavelength in the Balmer series of hydrogen atom, $\mathrm{n} = 2,\ \mathrm{m} = \infty$ and $\mathrm{Z} = 1$

$\therefore \quad \lambda_1 = \frac{4}{\mathrm{R_H}}$

For hydrogen like atom, the shortest wavelength is given by,

In Brackett series

$$\begin{aligned} &\mathrm{n} = 4,\ \mathrm{m} = \infty \\ \therefore \quad &\frac{1}{\lambda_2} = \mathrm{R_H} \cdot \mathrm{Z}^2 \left(\frac{1}{16}\right) \end{aligned}$$

$$\therefore \quad \lambda_2 = \frac{16}{\mathrm{R_H} \cdot \mathrm{Z}^2}$$

Given: $\lambda_1 = \lambda_2$,

$$\begin{aligned} \therefore \quad &\frac{4}{R_H} = \frac{16}{R_H \cdot Z^2} \Rightarrow Z^2 = 4 \\ \therefore \quad &Z = 2 \end{aligned}$$

Question 108

If the length of stretched string is reduced by 40% and tension is increased by 44% then the ratio of final to initial frequencies of stretched string is

Options:

A. 2 : 1

B. 3 : 2

C. 3 : 4

D. 1 : 3

Answer: A

Solution:

Let the initial length and tension be l and T respectively.

After shortening,

The new length $l_{\text{new}} \quad = l - \frac{40}{100} l = \frac{3}{5} l$

After increase in tension,

the new tension $\mathrm{T}_{\text{new}} \quad = \mathrm{T} + \frac{44}{100} \mathrm{~T} = \frac{144 \mathrm{~T}}{100}$

Fundamental frequency of a vibrating string is given by

$$\begin{aligned} \mathrm{n} &= \frac{1}{2l}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}} \\ \therefore \quad \mathrm{n}_1 &= \frac{1}{2l}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}} \\ \mathrm{n}_2 &= \frac{1}{2l}\sqrt{\frac{\mathrm{T}_{\text{new}}}{\mathrm{m}}} \\ \therefore \quad \frac{\mathrm{n}_1}{\mathrm{n}_2} &= \frac{l_{\text{new}}}{l} \times \frac{\sqrt{\mathrm{T}}}{\sqrt{\mathrm{T}_{\text{new}}}} \\ &= \frac{\frac{3}{5}l}{l} \times \sqrt{\frac{100 \mathrm{~T}}{144 \mathrm{~T}}} \\ &= \frac{3}{5} \times \frac{10}{12} = \frac{1}{2} \\ \therefore \quad \frac{\mathrm{n}_2}{\mathrm{n}_1} &= \frac{2}{1} \end{aligned}$$

Question 109

A square loop of area $25 \mathrm{~cm}^2$ has a resistance of 10Ω. This loop is placed in a uniform magnetic field of magnitude $40 \mathrm{~T}$. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in one second, will be

Options:

A. $1 \times 10^{-4} \mathrm{~J}$

B. $1.0 \times 10^{-3} \mathrm{~J}$

C. $5 \times 10^{-3} \mathrm{~J}$

D. $2.5 \times 10^{-3} \mathrm{~J}$

Answer: B

Solution:

Given: area of square loop $= 25$ cm

$\therefore \quad l = \sqrt{25} = 5 \text{ cm} \Rightarrow 0.05 \text{ m}$
$\text{R} = 10\Omega, \text{t} = 1\text{sec}, \text{B} = 40 \text{ T}$
$\therefore \quad \text{Velocity v} = \frac{l}{\text{t}} = \frac{0.05}{1} = 0.05 \text{ m/s}$
$\text{Motional emf } \varepsilon_{\max} = \text{B}/\text{v}$
$\therefore \quad \text{I} = \frac{\varepsilon}{\text{R}} = \frac{\text{B}/\text{v}}{\text{R}}$
$\therefore$
$\text{I} = \frac{40 \times 0.05 \times 0.05}{10} = 0.01 \text{ A}$

We know, Force acting on loop

$\therefore \quad \text{F} = \text{BI}l = 40 \times 0.01 \times 0.05$
$= 0.02 \text{ N}$
$\text{Using W} = \text{F.s},$
$\text{Work done W} = \text{BI}l \times l$
$= 0.02 \times 0.05$
$= 1 \times 10^{-3} \text{ J}$

Question 110

Two capacitors $\text{C}_1 = 3\mu\text{F}$ and $\text{C}_2 = 2\mu\text{F}$ are connected in series across d.c. source of 100 V. The ratio of the potential across C_2 to C_1 is

Options:

A. 2 : 3

B. 3 : 2

C. 6 : 5

D. 5 : 6

Answer: B

Solution:

$C_1 = 3\mu F$ and $C_2 = 2\mu F$

$$\therefore \quad C_{\text{series}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{6}{5}\mu F$$

Also, $Q = CV$

$$Q = C_{\text{series}} \times V$$

$$= \frac{6}{5} \times 100 = 120\mu C$$

Q will be the same across both the capacitors as they are in series.

$\therefore$ Potential across capacitors,

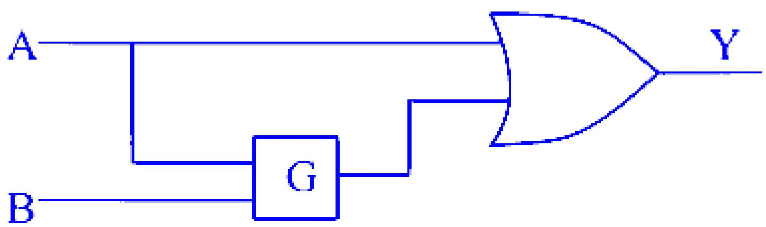
$$V_1 = \frac{Q}{C_1} = \frac{120}{3} = 40 \text{ V}$$

$$V_2 = \frac{Q}{C_2} = \frac{120}{2} = 60 \text{ V}$$

$\therefore \quad V_2 : V_1 = 60 : 40 = 3 : 2$

Question 111

To obtain the truth-table shown, from the following logic circuit, the gate G should be



A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

Options:

A. AND

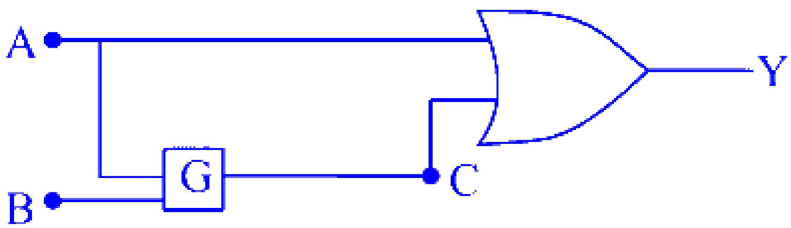
B. NAND

C. OR

D. NOR

Answer: D

Solution:



The truth table for given configuration is as shown below,

Case	A	B	C	$A + C = Y$
I	0	0	C_1	$0 + C_1 = 1$
II	0	1	C_2	$0 + C_2 = 0$
III	1	0	C_3	$1 + C_3 = 1$
IV	1	1	C_4	$1 + C_4 = 1$

Considering case (I), in order to have output (Y) as $1, C_1$ has to be 1. For input values, $A = 0$ and $B = 0$, if C_1 is to be high, the gate G could be either NAND or NOR.

Considering case (II), in order to have output (Y) as $0, C_2$ has to be 0. For input values, $A = 0$ and $B = 1$. If C_2 is to be 0 , the gate must be NOR.

Question 112

An electric dipole consisting of two opposite charges of $2 \times 10^{-6} C$ separated by a distance of 3 cm placed in an electric field of 2×10^5 N/C then the maximum torque acting on dipole is

Options:

A. 12×10^{-1} N − m

B. 24×10^{-3} N $-$ m

C. 12×10^{-3} N $-$ m

D. 24×10^{-1} N $-$ m

Answer: C

Solution:

$$\begin{aligned} &\text{Dipole moment } \mathrm{p} = \mathrm{q} \times 2\mathrm{l} = 2 \times 10^{-6} \times 3 \times 10^{-2} \\ &\qquad\qquad\qquad = 6 \times 10^{-8}\mathrm{Cm} \\ \therefore \quad &\tau_{\max} = \rho E \sin\theta = \mathrm{pE}\sin 90^{\circ} \\ &\qquad = 6 \times 10^{-8} \times 2 \times 10^{5} \times 1 \\ &\qquad = 12 \times 10^{-3}\mathrm{Nm} \end{aligned}$$

Question 113

In insulators

Options:

A. valence band is empty and conduction band is filled with electrons.

B. conduction band is empty and valence band is completely filled with electrons.

C. valence band is partially filled.

D. conduction band is partially filled with electrons.

Answer: B

Solution:

The correct answer for the description of electronic bands in insulators is:

Option B: conduction band is empty and valence band is completely filled with electrons.

In insulators, there is a large energy gap between the valence band and the conduction band. The valence band is the band of electron orbitals that electrons occupy at absolute zero temperature, and it's usually filled with electrons in an insulator. The conduction band is the band above the valence band where an electron may be excited to in order to conduct electricity. In insulators, this conduction band is empty because the energy gap (band gap) between the valence band and the conduction band is so large that electrons in the valence band do not normally have enough energy to jump into the conduction band under room temperatures or normal conditions. Hence, there is no electrical conductivity in insulators under these conditions.

Let's break down the incorrect options:

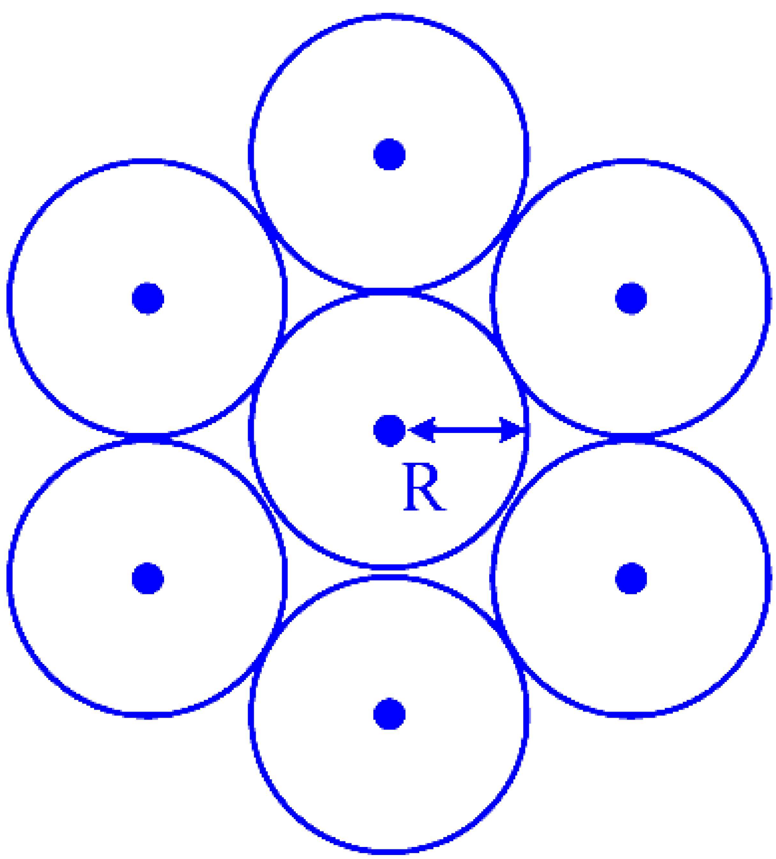
Option A: In insulators, the valence band is not empty. It is filled with electrons, which is in direct contradiction to what this option states.

Option C: In insulators, the valence band is completely filled, not partially filled. It is the complete filling of the valence band which contributes to their insulating properties, combined with the large energy gap to the conduction band.

Option D: The conduction band in insulators is not partially filled; it is empty due to the energy required to move electrons from the valence band to the conduction band being prohibitively large for thermal excitation.

Question 114

Seven identical discs each of mass M and radius R are arranged in a hexagonal plane pattern so as to touch each neighbour disc as shown in the figure. The moment of inertia of the system of seven discs about an axis passing through the centre of central disc and normal to the plane of all discs is



Options:

A. $\frac{7}{2}$ MR^2

B. $\frac{13}{2}$ MR^2

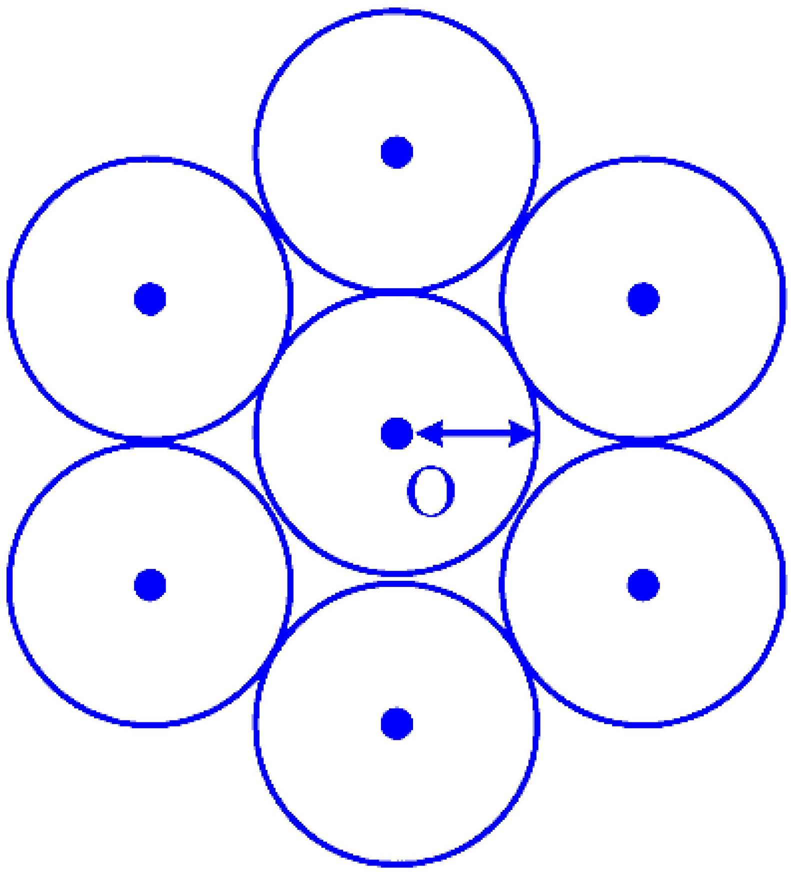
C. $\frac{29}{2}$ MR^2

D. $\frac{55}{2}$ MR^2

Answer: D

Solution:

M.I of a circular disc $= \frac{MR^2}{2}$



Using parallel axis theorem, M.I. about origin $I = I_{cm} + 6I$

where, $I_{cm} =$ M. I of the central disc

$I' =$ M. I of the each disc about the given axis.

$$\therefore \quad I = \frac{MR^2}{2} + 6\left(I_{cm} + MD^2\right)$$
$$= \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + 4MR^2\right) \quad \ldots (\because D = 2R)$$
$$= \frac{MR^2}{2} + 6\left(\frac{MR^2 + 8MR^2}{2}\right)$$
$$= \frac{55MR^2}{2}$$

Question 115

A satellite moves in a stable circular orbit round the earth if (where V_H, V_c and V_e are the horizontal velocity, critical velocity and escape velocity respectively)

Options:

A. $V_H < V_c$

B. $V_H = V_e$

C. $V_H = V_c$

D. $V_H > V_e$

Answer: C

Solution:

To understand the conditions for a stable circular orbit around Earth, we need to know the meanings of the critical velocity (V_c) and escape velocity (V_e).

Critical Velocity (V_c) is the necessary orbital velocity a body must have to be in a stable circular orbit around Earth without the need for propulsion. It depends on the gravitational force providing the necessary centripetal force to keep the satellite in orbit.

Escape Velocity (V_e) is the minimum velocity an object must have to break free from the gravitational attraction of a celestial body, like earth, without further propulsion.

For a satellite to maintain a stable orbit, its horizontal velocity (V_H) must be equal to the critical velocity (V_c). If the horizontal velocity is less than critical velocity, the gravitational pull will cause the satellite to spiral downwards towards Earth. If the horizontal velocity is greater than critical velocity but less than escape velocity, the satellite will enter an elliptical orbit. And if the horizontal velocity equals or exceeds escape velocity, the satellite will leave Earth's orbit entirely.

In simple terms, for a stable, circular orbit, the option is:

Option C : $V_H = V_c$

Options A and D would not result in a stable circular orbit. Option A would lead to a decaying orbit, and Option D would cause the satellite to leave orbit entirely. Option B implies that the satellite would be on an escape path and thus not in a stable orbit around the Earth.

Question 116

The mean electrical energy density between plates of a charged air capacitor is (where $\mathrm{q}=$ charge on capacitor, $\mathrm{A}=$ Area of capacitor plate)

Options:

A. $\frac{q^2}{2\varepsilon_0 A^2}$

B. $\frac{\mathrm{q}}{2\varepsilon_0\ \mathrm{A}^2}$

C. $\frac{\mathrm{q}^2}{2\varepsilon_0\ \mathrm{A}}$

D. $\frac{\varepsilon_0\ \mathrm{A}}{\mathrm{q}^2}$

Answer: A

Solution:

For a parallel plate capacitor, the energy density $=\frac{1}{2}E^2\varepsilon_0$

But E $=\frac{\sigma}{\varepsilon_0}$

$$\begin{aligned} \therefore \quad \text{Energy density } & =\frac{1}{2}\frac{\sigma^2}{\varepsilon_0^2}\times\varepsilon_0 \\ & =\frac{\sigma^2}{2\varepsilon_0} \\ & =\frac{\left(\frac{\mathrm{q}}{\mathrm{A}}\right)^2}{2\varepsilon_0} \quad \ldots.(\because \sigma=\mathrm{q}/\mathrm{A}) \\ & =\frac{\mathrm{q}^2}{2\ \mathrm{A}^2\cdot\varepsilon_0} \end{aligned}$$

Question 117

A person is observing a bacteria through a compound microscope. For better observation and to improve its resolving power he should

Options:

A. increase the wavelength of light.

B. increase the refractive index of the medium between the object and objective lens.

C. decrease the focal length of the eyepiece.

D. decrease the diameter of the objective lens.

Answer: B

Solution:

Resolving Power $R=\frac{\mathrm{N.A}}{0.61\lambda}$

$\mathrm{N.A}=2\mathrm{n}\sin\alpha$

$\therefore$ By increasing the refractive index of the medium between the subject and the objective lens, the resolving power can be increased.

Question 118

The inductive reactance of a coil is '$\mathrm{X_L}$'. If the inductance of a coil is tripled and frequency of a.c. supply is doubled, then the new inductive reactance will be

Options:

A. $\frac{2}{3}X_L$

B. $\frac{3}{2}X_L$

C. $\frac{1}{6}X_L$

D. $6X_L$

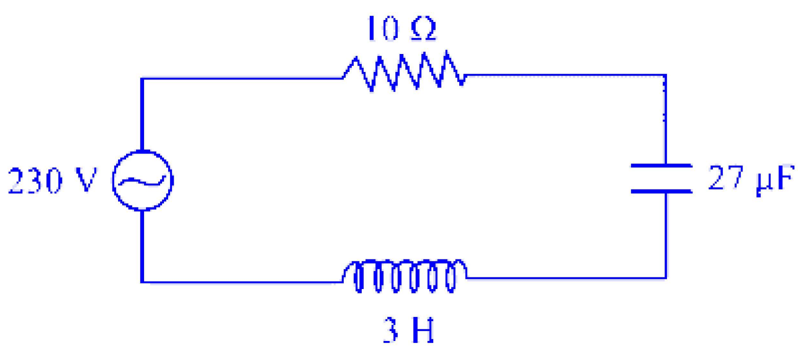
Answer: D

Solution:

$$\begin{aligned} X_L &= \omega L = 2\pi f L \\ f_{\text{new}} &= 2f \text{ and } L_{\text{new}} = 3L \\ \therefore \quad X'_L &= 2\pi(2f)3L \\ &= 6 \times 2\pi f L = 6X_L \end{aligned}$$

Question 119

In the circuit shown the ratio of quality factor and the bandwidth is



Options:

A. 10 s

B. 8 s

C. 6 s

D. 4 s

Answer: A

Solution:

$$\text{Bandwidth } = 2\Delta\omega = \frac{R}{L}$$

$$\text{Q factor } = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\frac{\text{Q factor}}{\text{Band width}} = \frac{\frac{1}{R}\sqrt{\frac{L}{C}}}{\frac{R}{L}} = \frac{1}{R^2}\frac{L^{3/2}}{\sqrt{C}}$$

$$\frac{Q_{\text{factor}}}{\text{Bandwidth}} = \frac{3\sqrt{3}}{100\sqrt{27 \times 10^{-6}}} = \frac{3\sqrt{3}}{100 \times 3\sqrt{3} \times 10^{-3}} = 10 \text{ s}$$

Question 120

Water flows through a horizontal pipe at a speed 'V'. Internal diameter of the pipe is 'd'. If the water is coming out at a speed 'V_1' then the diameter of the nozzle is

Options:

A. $d\sqrt{\frac{V_1}{V}}$

B. $d\sqrt{\frac{V}{V_1}}$

C. $\frac{dV}{V_1}$

D. $\frac{V_1}{dV}$

Answer: B

Solution:

From equation of continuity,

$A_1 V_1 = A_2 V_2$

Given: $V_1 = V$ and $V_2 = V_1$

$$\Rightarrow A_1 V = A_2 V_1$$

$$\frac{\pi d^2}{4} V = \frac{\pi d_n^2 n}{4} V_1 \quad \ldots.\left(\because A = \frac{\pi d^2}{4}\right)$$

where d_n is the diameter of the nozzle.

$$\therefore \quad d_n^2 = d^2 \frac{V}{V_1}$$

$$\therefore \quad d_n = d\sqrt{\frac{V}{V_1}}$$

Question 121

In Young's double slit experiment the separation between the slits is doubled without changing other setting of the experiment to obtain same fringe width, the distance 'D' of the screen from slit should be made

Options:

A. $\frac{\mathrm{D}}{2}$

B. $\frac{\mathrm{D}}{\sqrt{2}}$

C. 2D

D. 4D

Answer: C

Solution:

Fringe width,

$\mathrm{W} = \frac{\lambda \mathrm{D}}{\mathrm{d}}$

Given: $\mathrm{d} = 2\ \mathrm{d}$

$\Rightarrow \mathrm{W}' = \frac{\lambda \mathrm{D}'}{2\ \mathrm{d}}$

Since given,

$\mathrm{W} = \mathrm{W}'$

$$\therefore \quad \frac{\lambda \mathrm{D}}{\mathrm{d}} = \frac{\lambda \mathrm{D}'}{2\ \mathrm{d}}$$

$$\therefore \quad \mathrm{D}' = 2\mathrm{D}$$

Question 122

A body is released from the top of a tower 'H' metre high. It takes t second to reach the ground. The height of the body $\frac{t}{2}$ second after release is

Options:

A. $\frac{H}{2}$ metre from ground

B. $\frac{H}{4}$ metre from ground

C. $3\frac{H}{4}$ metre from ground

D. $\frac{H}{6}$ metre from ground

Answer: C

Solution:

Let the body be at x from the top after $\frac{t}{2}$ s.

$$\therefore \quad x = \frac{1}{2} g \frac{t^2}{4} = \frac{gt^2}{8} \quad .... (i)$$

$$H = \frac{1}{2} gt^2 \quad .... (ii)$$

Eliminating t from (i) and (ii), we get

$\frac{8x}{g} = \frac{2H}{g} \Rightarrow x = \frac{H}{4}$

$\therefore$ Height of the body from the ground

$= H - \frac{H}{4} = \frac{3H}{4}$ metres

Question 123

There is a second's pendulum on the surface of earth. It is taken to the surface of planet whose mass and radius are twice that of earth. The period of oscillation of second's pendulum on the planet will be

Options:

A. $2\sqrt{2}$ s

B. 2 s

C. $\frac{1}{\sqrt{2}}$ s

D. $\frac{1}{2}$ s

Answer: A

Solution:

$$\text{As } g=\frac{GM}{R^2}$$

$$\therefore \quad \frac{g_{\text{Earth}}}{\mathrm{g}_{\text{plunet}}}=\frac{M_{\text{Earth}}}{M_{\text{planet}}}\times\frac{R_{\text{planet}}^2}{R_{\text{Earth}}}=\frac{M}{2M}\times\frac{(2R)^2}{R}=\frac{2}{1}$$

$$\text{Also } T\propto\frac{1}{\sqrt{\mathrm{g}}}$$

$$\therefore \quad \frac{\mathrm{T}_{\text{Earth}}}{\mathrm{T}_{\text{planet}}}=\sqrt{\frac{\mathrm{g}_{\text{planet}}}{\mathrm{g}_{\text{Earrth}}}}$$

$$\frac{2}{\mathrm{T}_{\text{plunat}}}=\sqrt{\frac{1}{2}}$$

$$\therefore \quad \mathrm{T}_{\text{planet}}=2\sqrt{2}\text{ s}$$

Question 124

Two long parallel wires carrying currents 8 A and 15 A in opposite directions are placed at a distance of 7 cm from each other. A point $'\mathrm{P}'$ is at equidistant from both the wires such that the lines joining the point to the wires are perpendicular to each other. The magnitude of magnetic field at point $'\mathrm{P}'$ is $(\sqrt{2}=1.4)(\mu_0=4\pi\times10^{-7}$ SI units)

Options:

A. $68\times10^{-6}\text{ T}$

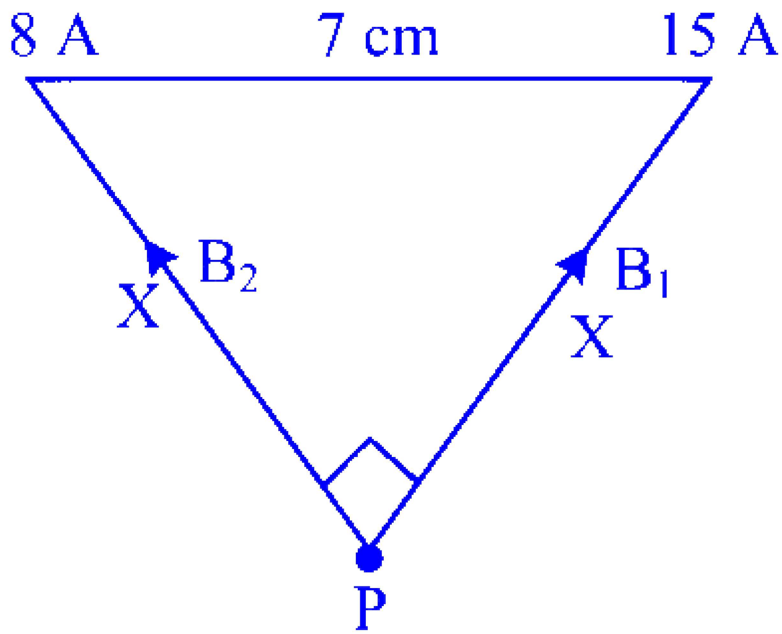
B. 48×10^{-6} T

C. 32×10^{-6} T

D. 16×10^{-6} T

Answer: A

Solution:



Magnetic field produced by two wires

$B_1 = \frac{\mu_0 I_1}{2\pi X}$ and $B_2 = \frac{\mu_0 I_2}{2\pi X}$

From Figure,

$$\begin{aligned} B_{\text{net}} &= \sqrt{B_1^2 + B_2^2} \\ &= \frac{\mu_0}{2\pi X}\sqrt{I_1^2 + I_2^2} \end{aligned}$$

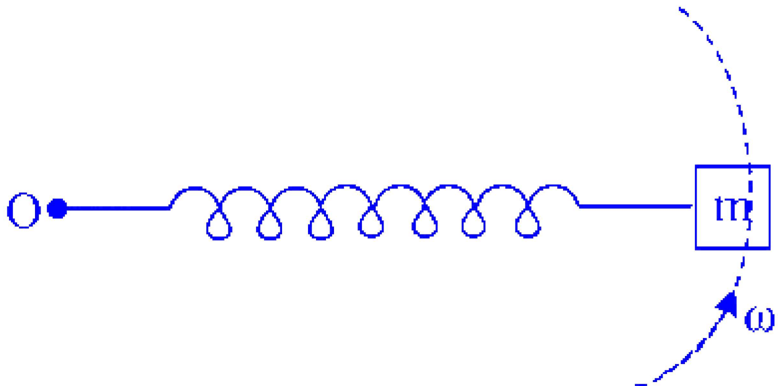
Also, using Pythagoras theorem, $2X^2 = 7 \times 7$ cm

$$\therefore \quad X = \frac{7}{\sqrt{2}} \text{ cm}$$

$$\begin{aligned} B_{\text{net}} &= \frac{4\pi \times 10^{-7}}{2\pi \times \frac{7}{\sqrt{2}} \times 10^{-2}}\sqrt{15^2 + 8^2} \\ &\approx 68 \times 10^{-6} \text{ T} \end{aligned}$$

Question 125

A body of mass 200 gram is tied to a spring of spring constant $12.5 \mathrm{~N/m}$, while other end of spring is fixed at point 'O'. If the body moves about 'O' in a circular path on a smooth horizontal surface with constant angular speed $5 \mathrm{~rad/s}$ then the ratio of extension in the spring to its natural length will be



Options:

A. 1 : 2

B. 1 : 1

C. 2 : 3

D. 2 : 5

Answer: C

Solution:

Let the normal length be L and the extension be x.

$\therefore \quad$ Restoring Force $=$ Centripetal Force

$\mathrm{kx}=\mathrm{m}(\mathrm{L}+\mathrm{x}) \omega^2$
$12.5 \mathrm{x}=0.2(\mathrm{~L}+\mathrm{x}) 25 \quad \ldots(\because \omega=5 \mathrm{~rad/s})$

$$12.5 x=5(L+x)$$
$$7.5 x=5 L$$
$$\therefore \quad \frac{x}{L}=\frac{5}{7.5}=\frac{2}{3}=2: 3$$

Question 126

Which of the following is NOT involved in the formation of secondary rainbow?

Options:

A. Refraction

B. Angular dispersion

C. Interference

D. Total internal reflection

Answer: C

Solution:

The option that is NOT involved in the formation of a secondary rainbow is Option C: Interference.

Rainbows are formed from the optical processes that occur when sunlight interacts with raindrops. A secondary rainbow, which appears outside the primary rainbow and is fainter and has its colors reversed, is created through three processes :

- **Refraction:** When sunlight enters a raindrop, it is bent (refracted) due to the change in medium from air to water. This bending of light is essential in separating the white sunlight into its component colors.
- **Angular dispersion:** This term refers to the separation of white light into its colors due to the different degrees of bending (refraction) for different wavelengths/colors. Violet light bends more than red light, resulting in dispersion.
- **Total internal reflection:** After refraction, the light rays strike the other side of the raindrop and reflect. In the secondary rainbow, light reflects twice inside the raindrop before emerging out. This double reflection causes the colors to be reversed as compared to the primary rainbow, and contributes to the formation of the secondary rainbow.

Interference, on the other hand, is the phenomenon where waves overlap and combine with each other, leading to patterns of constructive and destructive interference. While this process is crucial in some optical phenomena, like the thin film interference seen in soap bubbles, it is not a key process in the formation of the typical primary or secondary rainbow. In rainbows, each individual raindrop contributes to the rainbow separately, and interference between waves from different raindrops does not play a role in their formation.

Question 127

For a satellite orbiting around the earth in a circular orbit, the ratio of potential energy to kinetic energy at same height is

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\sqrt{2}$

D. 2

Answer: D

Solution:

$$\text{K.E} = \frac{GMm}{2r}$$

$$P.E = -\frac{GMm}{r}$$

$$\therefore \quad \frac{K.E}{|P.E|} = \frac{GMm}{2r} \times \frac{r}{GMm}$$

$$\therefore \quad \frac{P.E}{K.E} = \frac{2}{1} = 2$$

Question 128

Maximum kinetic energy of photon is 'E' when wavelength of incident radiation is 'λ'. If wavelength of incident radiations is reduced to $\frac{\lambda}{3}$ then energy of photon becomes four times. Then work function of the metal is

Options:

A. $\frac{3hc}{\lambda}$

B. $\frac{\mathrm{hc}}{3\lambda}$

C. $\frac{hc}{\lambda}$

D. $\frac{\mathrm{hc}}{2\lambda}$

Answer: B

Solution:

$\mathrm{E}=\frac{\mathrm{hc}}{\lambda}-\phi_0$ .... (i)

Given: $\lambda=\frac{\lambda}{3}$ and $\mathrm{E}=4\mathrm{E}$

$$4\mathrm{E}=\frac{\mathrm{hc}}{\lambda/3}-\phi_0 \quad \text{..... (ii)}$$
$$=\frac{3\mathrm{hc}}{\lambda}-\phi_0$$
$$4\left(\frac{\mathrm{hc}}{\lambda}-\phi_0\right)=\frac{3\mathrm{hc}}{\lambda}-\phi_0 \quad \text{.....(From (i))}$$
$$\frac{4\mathrm{hc}}{\lambda}-4\phi_0=\frac{3\mathrm{hc}}{\lambda}-\phi_0$$
$$\frac{\mathrm{hc}}{\lambda}=3\phi_0$$
$$\phi_0=\frac{\mathrm{hc}}{3\lambda}$$

Question 129

Consider the Doppler effect in two cases. In the first case, an observer moves towards a stationary source of sound with a speed of $50\ \mathrm{m/s}$. In the second case, the observer is at rest and the source moves towards the observer with the same speed of $50\ \mathrm{m/s}$. Then the frequency heard by the observer will be

[velocity of sound in air $=330\ \mathrm{m/s}$.]

Options:

A. same in both the cases.

B. more in the second case than in the first case.

C. less in the second case than in the first case.

D. less than the actual frequency in both the cases.

Answer: B

Solution:

For observer moving towards a stationary source,

$n_1 = n_0 \left[\frac{v+v_L}{v}\right]$

For source moving towards a stationary observer,

$n_2 = n_0 \left[\frac{v}{v-v_s}\right]$

Substituting the values for v, v_L and v_S in the equations above

$$n_1 = n_0 \left[\frac{330+50}{330}\right] = 1.15\, n_0$$
$$n_2 = n_0 \left[\frac{330}{330-50}\right] = 1.17\, n_0$$
$$\Rightarrow n_2 > n_1$$

$\therefore$ The frequency heard will be more in the second case than in the first case.

Question 130

According to Curie's law in magnetism, the correct relation is ($M =$ magnetization in paramagnetic sample, $B =$ applied magnetic field, $T =$ absolute temperature of the material, $C =$ curie's constant)

Options:

A. $M = \frac{T}{CB}$

B. $M = \frac{CB}{T}$

C. $C = \frac{MB}{T}$

D. $C = \frac{T^2}{MB}$

Answer: B

Solution:

The correct relation according to Curie's Law in magnetism is:

Option B $\mathrm{M} = \frac{\mathrm{CB}}{\mathrm{T}}$

Curie's Law states that the magnetization M of a paramagnetic material is directly proportional to the applied magnetic field B, and inversely proportional to the absolute temperature T of the material. The proportionality constant C is known as the Curie constant and is specific to the paramagnetic material.

To better understand this law, let's dissect Option B, which correctly reflects the Curie's Law formula. The Magnetization M, which is the extent to which the material becomes magnetized when exposed to an external magnetic field, is given by:

$\mathrm{M} = \frac{\mathrm{CB}}{\mathrm{T}}$

- C is the Curie constant,

- B is the applied magnetic field, and

- T is the absolute temperature.

So, the higher the field B, the larger the induced magnetization M, and the higher the temperature T, the smaller the magnetization due to the increased thermal motion of the magnetic moments which tend to randomize the orientation and reduce magnetization. The correct formula (Option B) reflects this inverse temperature dependence and direct magnetic field dependence of the magnetization.

Question 131

A double convex air bubble in water behaves as

Options:

A. convergent lens

B. divergent lens

C. plane slab

D. concave mirror

Answer: B

Solution:

$\mu_{\text{water}} = 1.33$ and $\mu_{\text{air}} = 1$

As air is optically rarer, $\mu_{\text{air}} < \mu_{\text{glass,}}$ converging nature of the convex lens changes to diverging. Thus it behaves as divergent lens.

Question 132

Three liquids have same surface tension and densities ρ_1, ρ_2, and $\rho_3\ (\rho_1 > \rho_2 > \rho_3)$. In three identical capillaries rise of liquid is same. The corresponding angles of contact θ_1, θ_2 and θ_3 are related as

Options:

A. $\theta_1 > \theta_2 > \theta_3$

B. $\theta_1 > \theta_3 > \theta_2$

C. $\theta_1 < \theta_2 < \theta_3$

D. $\theta_1 = \theta_2 = \theta_3$

Answer: C

Solution:

Rise in capillary tube,

$\mathrm{h} = \frac{2\ \mathrm{T} \cos \theta}{\mathrm{r} \rho \mathrm{g}}$

Given that, h, T, r and g are constant.

$$\therefore \quad \frac{\cos \theta}{\rho} = \text{constant}$$

$$\text{i.e., } \frac{\cos \theta_1}{\rho_1} = \frac{\cos \theta_2}{\rho_2} = \frac{\cos \theta_3}{\rho_3}$$

as $\rho_1 > \rho_2 > \rho_3$

$\cos \theta_1 > \cos \theta_2 > \cos \theta_3$

$\therefore \quad \theta_1 < \theta_2 < \theta_3$

Question 133

If a lighter body of mass 'M_1' and velocity 'V_1' and a heavy body (mass M_2 and velocity V_2) have the same kinetic energy then

Options:

A. $\mathrm{M}_2\mathrm{V}_2 < \mathrm{M}_1\ \mathrm{V}_1$

B. $\mathrm{M}_2\mathrm{V}_2 = \mathrm{M}_1\mathrm{V}_1$

C. $\mathrm{M}_2\mathrm{V}_1 < \mathrm{M}_1\mathrm{V}_2$

D. $\mathrm{M}_2\mathrm{V}_2 > \mathrm{M}_1\mathrm{V}_1$

Answer: D

Solution:

$\mathrm{KE}_1 = \mathrm{KE}_2$

$\mathrm{KE} = \frac{1}{2}\mathrm{mv}^2 = \frac{\mathrm{p}^2}{2\ \mathrm{m}} \quad \ldots . (\because \mathrm{p} = \mathrm{mv})$

$\therefore \quad \mathrm{p} = \sqrt{2\ \mathrm{m}(\mathrm{K} \cdot \mathrm{E})}$

$\therefore \quad \frac{\mathrm{p}_1}{\mathrm{p}_2} = \frac{\sqrt{\mathrm{M}_1}}{\sqrt{\mathrm{M}_2}}$

But $\mathrm{M}_2 > \mathrm{M}_1$

$\therefore \quad \mathrm{p}_2 > \mathrm{p}_1$

i.e., $\mathrm{M}_2\ \mathrm{V}_2 > \mathrm{M}_1\ \mathrm{V}_1$

Question 134

Electron of mass 'm' and charge 'q' is travelling with speed 'v' along a circular path of radius 'R', at right angles to a uniform magnetic field of intensity 'B'. If the speed of the electron is halved and the magnetic field is doubled, the resulting path would have radius

Options:

A. 4R

B. 2R

C. $\frac{R}{2}$

D. $\frac{R}{4}$

Answer: D

Solution:

From cyclotron motion and uniform circular motion,

i.e., $qvB \sin 90^\circ = \frac{mv^2}{R}$

$\therefore \quad R = \frac{mv}{qB}$

Given: $v' = \frac{v}{2}$ and $B' = 2B$

$$\therefore \quad R' = \frac{mv'}{qB'} = \frac{m\frac{v}{2}}{q2B} = \frac{1}{4}\frac{mv}{qB}$$

$$\Rightarrow R' = \frac{1}{4}R$$

Question 135

In a series LR circuit, $X_L = R$, power factor is P_1. If a capacitor of capacitance C with $X_C = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be

Options:

A. $1:3$

B. $1:\sqrt{2}$

C. $1:1$

D. $1:2$

Answer: B

Solution:

$$\text{Power factor } = \frac{R}{\sqrt{R^2+X_L^2}}$$

Given: $X_L = R$

$$\therefore \quad P_1 = \frac{R}{\sqrt{2R^2}} = \frac{1}{\sqrt{2}}$$

When C is connected, the circuit becomes series LCR circuit.

$$\therefore \quad \text{Power factor } = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

Given : $X_C = X_L$

$$\therefore \quad P_2 = \frac{R}{\sqrt{R^2}} = 1$$

$$\therefore \quad P_1 : P_2 = 1 : \sqrt{2}$$

Question 136

If only 1% of total current is passed through a galvanometer of resistance 'G' then the resistance of the shunt is

Options:

A. $\frac{G}{25}\Omega$

B. $\frac{G}{49}\Omega$

C. $\frac{G}{2}\Omega$

D. $\frac{G}{99}\Omega$

Answer: D

Solution:

Given: $I_g = \frac{1}{100} I$

$$\therefore \quad \frac{I}{I_g} = 100 \quad \text{..... (i)}$$

$$S = \frac{GI_g}{I - I_g} = \frac{G}{\frac{I}{I_g} - 1} \quad \text{..... (ii)}$$

Substituting, (i) into (ii),

$S = \frac{G}{100-1} = \frac{G}{99}$

Question 137

A voltmeter of resistance 150Ω connected across a cell of e.m.f. 3 V reads 2.5 V. What is the internal resistance of the cell?

Options:

A. 10Ω

B. 15Ω

C. 20Ω

D. 30Ω

Answer: D

Solution:

Internal resistance,

$$\begin{aligned} r &= \left[\frac{E}{V} - 1\right]R \\ &= \left[\frac{3}{2.5} - 1\right]150 \\ &= 30\Omega \end{aligned}$$

Question 138

Two conducting circular loops of radii 'R_1' and 'R_2' are placed in the same plane with their centres coinciding. If $R_1 > R_2$, the mutual inductance M between them will be directly proportional to

Options:

A. $\frac{R_1}{R_2}$

B. $\frac{R_2}{R_1}$

C. $\frac{R_1^2}{R_2}$

D. $\frac{R_2^2}{R_1}$

Answer: D

Solution:

Mutual inductance of two concentric coplanar circular coils,

$$M = \frac{\mu_0 \pi N_1 N_2 R_2^2}{2R_1}$$

$$\therefore \quad M \propto \frac{R_2^2}{R_1}$$

Question 139

The average force applied on the walls of a closed container depends on T^x where T is the temperature of an ideal gas. The value of 'x' is

Options:

A. 4

B. 3

C. 2

D. 1

Answer: D

Solution:

$$P = \frac{F}{A}$$

$$\therefore \quad P \propto F \quad \text{..... (i)}$$

From Ideal Gas Equation,

$P = \frac{nRT}{V}$

When V remains constant,

$P \propto T$ ..... (ii)

Comparing (i) and (ii)

$F \propto T^x$

$\therefore \quad x = 1$

Question 140

A black body radiates maximum energy at wavelength 'λ' and its emissive power is E. Now due to change in temperature of that body, it radiates maximum energy at wavelength $\frac{2\lambda}{3}$. At that temperature emissive power is

Options:

A. $\frac{51E}{8}$

B. $\frac{81E}{16}$

C. $\frac{61E}{27}$

D. $\frac{71E}{19}$

Answer: B

Solution:

From Wien's Displacement law,

$\lambda_{\max} = \frac{b}{T} \Rightarrow T = \frac{b}{\lambda_{\max}}$

From Stefan-Boltzmann law

$E = \sigma T^4 = \sigma\left(\frac{b}{\lambda_{\max}}\right)^4$

Let the new emissive power be E'.

$$\therefore \quad E' = \sigma\left(\frac{b}{\frac{2\lambda_{\max}}{3}}\right)^4$$
$$E' = \frac{81}{16}E$$

Question 141

A Carnot engine with efficiency 50% takes heat from a source at $600\ \text{K}$. To increase the efficiency to 70%, keeping the temperature of the sink same, the new temperature of the source will be

Options:

A. 360 K

B. 1000 K

C. 900 K

D. 300 K

Answer: B

Solution:

$$T_H = 600\text{k}$$
$$\eta = 1 - \frac{T_C}{T_H}$$
$$\text{but, } \eta = \frac{1}{2} \quad \ldots.(\text{Given} : \eta = 50\%)$$
$$\Rightarrow \frac{1}{2} = 1 - \frac{T_C}{600}$$
$$\therefore \quad T_C = 300K$$

With $\mathrm{T_C} = 300$ K, the efficiency is increased to 70%

$\therefore$ New temperature of the source will be $\mathrm{T_{H_{new}}}$

$$\therefore \quad \eta = 1 - \frac{300}{\mathrm{T_{H_{new}}}}$$
$$\frac{300}{\mathrm{T_{H_{new}}}} = 1 - \frac{7}{10} \quad \ldots.(\because \eta = 70\%)$$
$$\therefore \quad \mathrm{T_{H_{new}}} = \frac{3000}{3} = 1000\ \mathrm{K}$$

Question 142

The amplitude of a particle executing S.H.M. is $3 \mathrm{~cm}$. The displacement at which its kinetic energy will be 25% more than the potential energy is

Options:

A. 1 cm

B. 2 cm

C. 3 cm

D. 4 cm

Answer: B

Solution:

$$\text{Given } : \mathrm{K} \cdot \mathrm{E}=\mathrm{P} \cdot \mathrm{E}+\frac{25}{100} \cdot \mathrm{P} \cdot \mathrm{E}$$

$$\mathrm{K} \cdot \mathrm{E}=\mathrm{P} \cdot \mathrm{E}+\frac{1}{4} \mathrm{P} \cdot \mathrm{E}$$

$$\text{We know K.E }=\frac{1}{2} \mathrm{m} \omega^2\left(\mathrm{~A}^2-\mathrm{x}^2\right) \text{ and P.E }=\frac{1}{2} \mathrm{m} \omega^2 \mathrm{x}^2$$

$$\therefore \quad \mathrm{K} \cdot \mathrm{E}=\frac{5}{4} \mathrm{P} \cdot \mathrm{E}$$

$$\frac{1}{2} \mathrm{m} \omega^2\left(\mathrm{~A}^2-\mathrm{x}^2\right)=\frac{5}{4}\left(\frac{1}{2} \mathrm{m} \omega^2 \mathrm{x}^2\right)$$

$$\mathrm{A}^2-\mathrm{x}^2=\frac{5}{4} \mathrm{x}^2$$

$$\mathrm{A}^2=\frac{9}{4} \mathrm{x}^2$$

$$\therefore \quad \mathrm{A}=\frac{3}{2} \mathrm{x}$$

$$\therefore \quad \mathrm{x}=\mathrm{A} \times \frac{2}{3}=2 \mathrm{~cm}$$

Question 143

A piece of metal at $850\ \mathrm{K}$ is dropped in to $1\ \mathrm{kg}$ water at $300\ \mathrm{K}$. If the equilibrium temperature of water is $350\ \mathrm{K}$ then the heat capacity of the metal, expressed in JK^{-1} is $(1\ \mathrm{cal} = 4.2\ \mathrm{J})$

Options:

A. 420

B. 240

C. 100

D. No Solution

Answer: D

Solution:

[Note: Since mass of the metal is not mentioned in the question, it is not possible to arrive at the answer.]

Question 144

Heat energy is incident on the surface at the rate of X J/min . If 'a' and 'r' represent coefficient of absorption and reflection respectively then the heat energy transmitted by the surface in 't' minutes is

Options:

A. $(a+r)xt$

B. $\frac{(\mathrm{a}+\mathrm{r})}{\mathrm{xt}}$

C. $-(a+r)xt$

D. $\frac{\mathrm{xt}}{(\mathrm{a}+\mathrm{r})}$

Answer: A

Solution:

We know $1=a+r+t_r$

$\Rightarrow t_r=1-(a+r)$

Heat transmitted $=t_r \times$ time

$\therefore \quad Q=X \times(1-a+r) \times t \ldots$ (Substituting for t_r)

$Q=X(1-a+r) \times t$

Question 145

Identify the mismatch out of the following.

Options:

A. Zener diode - voltage regulator.

B. Photodiode-optocouplers.

C. Solar cell - Electrical energy into light.

D. Light emitting diode - optical communication.

Answer: C

Solution:

Solar cell converts light energy into electrical energy.

Question 146

Two sources of light $0.6 \mathrm{~mm}$ apart and screen is placed at a distance of $1.2 \mathrm{~m}$ from them. A light of wavelength $6000 \stackrel{o}{A}$ used. Then the phase difference between the two light waves interfering on the screen at a point at a distance $3 \mathrm{~mm}$ from central bright band is

Options:

A. 6π radian

B. 3π radian

C. 4π radian

D. 5π radian

Answer: D

Solution:

Fringe width, $W = \frac{\lambda D}{d} = \frac{6000 \times 10^{-10} \times 1.2}{0.6 \times 10^{-3}} = 1.2 \text{ mm}$

$\therefore$ Number of fringes $(n) = \frac{3}{1.2} = 2.5$

$\therefore$ Phase difference,

$\Delta\phi = 2n\pi = 2 \times 2.5\pi = 5\pi$

Question 147

The ratio of longest to shortest wavelength emitted in Paschen series of hydrogen atom is

Options:

A. $\frac{144}{63}$

B. $\frac{25}{9}$

C. $\frac{9}{25}$

D. $\frac{63}{144}$

Answer: A

Solution:

For Paschen series, Longest wavelength corresponds to

$n_1 = 3, n_2 = n_1 + 1 = 4$

$$\lambda_{max} = \frac{144}{7R}$$

Shortest wavelength corresponds to $n_1 = 3, n_2 = \infty$

$$\lambda_{\min} = \frac{9}{R}$$

$$\therefore \quad \frac{\lambda_{\max}}{\lambda_{\min}} = \frac{\left(\frac{144}{7R}\right)}{\left(\frac{9}{R}\right)} = \frac{144}{63}$$

Question 148

The height of liquid column raised in a capillary tube of certain radius when dipped in liquid 'A' vertically is 5 cm. If the tube is dipped in a similar manner in another liquid 'B' of surface tension and density double the values of liquid 'A', the height of liquid column raised in liquid 'B' would be (Assume angle of contact same)

Options:

A. 0.20 m

B. 0.5 m

C. 0.05 m

D. 0.10 m

Answer: C

Solution:

Height of liquid column, $h = \frac{2 T \cos\theta}{r\rho g}$

Given: $h_1 = 5 \text{ cm}$

also $T_2 = 2 T_1; \rho_2 = 2\rho_1$

$$\therefore \quad h_1 = \frac{2 T_1 \cos\theta}{r\rho_1 g}$$

$$h_2 = \frac{2 T_2 \cos\theta}{r\rho_2 g} = \frac{2(2 T_1)\cos\theta}{r(2\rho_1)g} = \frac{4 T_1 \cos\theta}{r(2\rho_1)g}$$

$$\therefore \quad \frac{h_1}{h_2} = \frac{5}{h_2} = \frac{2 T_1 \cos\theta}{r\rho_1 g} \times \frac{r 2\rho_1 g}{4 T_1 \cos\theta}$$

$$\frac{5}{h_2} = 1$$

$$\therefore \quad h_2 = 5 \text{ cm} = 0.05 \text{ m}$$

Question 149

A particle of mass 'm' is rotating along a circular path of radius 'r' having angular momentum 'L'. The centripetal force acting on the particle is given by

Options:

A. $\frac{L^2}{mr}$

B. $\frac{L^2}{mr^2}$

C. $\frac{mL^2}{r}$

D. $\frac{L^2}{mr^3}$

Answer: D

Solution:

$$\text{Centripetal force } = m^2 r$$
$$\therefore \quad \omega = \frac{L}{I} = \frac{L}{mr^2} \quad \ldots.(\because L = I\omega)$$
$$\therefore \quad F = m\left(\frac{L}{mr^2}\right)^2 r = \frac{mL^2 r}{m^2 r^4} = \frac{L^2}{mr^3}$$

Question 150

A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is $\left(\frac{C_P}{C_V} = \gamma = \frac{3}{2}\right) \quad (R =$ gas constant $)$

Options:

A. $TR(\sqrt{2} - 2)$

B. $\frac{T}{R}(\sqrt{2} - 2)$

C. $\frac{R}{T}(2-\sqrt{2})$

D. $RT(2-\sqrt{2})$

Answer: D

Solution:

Using the formula for adiabatic expansion,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_1 V_1^{(1/2)} = T_2 \left(2 V_1^{1/2}\right) \quad .....(\text{Given} : \gamma = \frac{3}{2})$$

$$\therefore \quad T_1 = T_2(\sqrt{2})$$

$$\therefore \quad T_2 = \frac{T}{\sqrt{2}}$$

Work done

$$W_{adi} = \frac{R(T - T_2)}{\gamma - 1} = \frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{1}{2}}$$

$$= \frac{R(\sqrt{2}\,T - T)}{\sqrt{2}} \times 2 = RT(\sqrt{2}-1)\sqrt{2}$$

$$\therefore \quad W_{adi} = RT(2-\sqrt{2})$$
