MHT CET 2023 : 11th May Evening Shift

Mathematics

Question 1

If $\mathrm{f}(x)=3^x ; \mathrm{g}(x)=4^x$, then $\frac{\mathrm{f}^{\prime}(0)-\mathrm{g}^{\prime}(0)}{1+\mathrm{f}^{\prime}(0) \mathrm{g}^{\prime}(0)}$ is

Options:

A. $\frac{\log \left(\frac{3}{4}\right)}{1+(\log 3)(\log 4)}$

B. $\frac{\log \left(\frac{3}{4}\right)}{1+\log 12}$

C. $\frac{\log 12}{1+\log 12}$

D. $\frac{\log \left(\frac{3}{4}\right)}{1-\log 12}$

Answer: A

Solution:

$$\begin{aligned} & \mathrm{f}^{\prime}(x)=3^x \log 3 \Rightarrow \mathrm{f}^{\prime}(0)=\log 3 \\ & \mathrm{g}^{\prime}(x)=4^x \log 4 \Rightarrow \mathrm{g}^{\prime}(0)=\log 4 \\ \therefore \quad & \frac{\mathrm{f}^{\prime}(0)-\mathrm{g}^{\prime}(0)}{1+\mathrm{f}^{\prime}(0) \mathrm{g}^{\prime}(0)}=\frac{\log 3-\log 4}{1+(\log 3)(\log 4)} \\ & \qquad\qquad\qquad =\frac{\log \left(\frac{3}{4}\right)}{1+(\log 3)(\log 4)}\end{aligned}$$

Question 2

If $x=\frac{5}{1-2 \mathrm{i}}, \mathrm{i}=\sqrt{-1}$, then the value of x^3+x^2-x+22 is

Options:

A. 7

B. 9

C. 17

D. 39

Answer: A

Solution:

$$
\begin{aligned}
& x=\frac{5}{1-2\mathrm{i}}=\frac{5(1+2\mathrm{i})}{1+4}=1+2\mathrm{i} \\
& \therefore \quad x^2=(1+2\mathrm{i})^2=1-4+4\mathrm{i}=-3+4\mathrm{i} \\
& \therefore \quad x^3=(-3+4\mathrm{i})(1+2\mathrm{i}) \\
& =-3-6i+4i-8 \\
& =-11-2\mathrm{i} \\
& \therefore \quad x^3+x^2-x+22 \\
& =(-11-2\mathrm{i})+(-3+4\mathrm{i})-(1+2\mathrm{i})+22 \\
& =-11-2\mathrm{i}-3+4\mathrm{i}-1-2\mathrm{i}+22 \\
& =7
\end{aligned}
$$

Question 3

Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Then mean of number of tens is

Options:

A. $\frac{1}{13}$

B. $\frac{1}{169}$

C. $\frac{2}{13}$

D. $\frac{4}{169}$

Answer: C

Solution:

Probability of getting ten $= \frac{4}{52} = \frac{1}{13}$

$\therefore$ Probability of getting a card without ten $= \frac{12}{13}$

Let random variable X denotes the number of tens.

$\therefore$ Possible values of X are $0, 1, 2$

Consider following probability distribution table.

$\mathrm{X} = x$	0	1	2
$\mathrm{P}(\mathrm{X} = x)$	$\frac{12}{13} \times \frac{12}{13}$	$\frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13}$	$\frac{1}{13} \times \frac{1}{13}$

$\therefore$ Required mean

$$= 0 + 1 \times \left(\frac{12}{13 \times 13} + \frac{12}{13 \times 13}\right) + 2 \times \left(\frac{1}{13} \times \frac{1}{13}\right)$$
$$= \frac{24}{169} + \frac{2}{169}$$
$$= \frac{26}{169}$$
$$= \frac{2}{13}$$

Question 4

$$\int x \sqrt{\frac{2\sin(x^2+1) - \sin 2(x^2+1)}{2\sin(x^2+1) + \sin 2(x^2+1)}}\, dx =$$

Options:

A. $\log\left(\sec\left(\frac{x^2+1}{2}\right)\right) +$ c, where c is a constant of integration.

B. $\log\left(\frac{x^2+1}{2}\right) + c$, where c is a constant of integration.

C. $\log\left(\sin\left(\frac{x^2+1}{2}\right)\right) +$ c, where c is a constant of integration.

D. $2\log\left(x^2+1\right) +$ c, where c is a constant of integration.

Answer: A

Solution:

Let $\mathrm{I}=\int x \sqrt{\frac{2 \sin \left(x^2+1\right)-\sin 2\left(x^2+1\right)}{2 \sin \left(x^2+1\right)+\sin 2\left(x^2+1\right)}} d x$

$=\int x \sqrt{\frac{2 \sin \left(x^2+1\right)-2 \sin \left(x^2+1\right) \cos \left(x^2+1\right)}{2 \sin x\left(x^2+1\right)+2 \sin \left(x^2+1\right) \cos \left(x^2+1\right)}} \mathrm{d} x$

$=\int x \sqrt{\frac{1-\cos \left(x^2+1\right)}{1+\cos \left(x^2+1\right)}} \mathrm{d} x$

$=\int x \sqrt{\frac{2 \sin ^2\left(\frac{x^2+1}{2}\right)}{2 \cos ^2\left(\frac{x^2+1}{2}\right)}} d x$

$=\int x \tan \left(\frac{x^2+1}{2}\right) d x$

Let $\left(\frac{x^2+1}{2}\right)=\mathrm{t} \Rightarrow x \mathrm{~d} x=\mathrm{dt}$

$$\begin{aligned} \therefore \quad \mathrm{I} & =\int \tan \mathrm{t} d \mathrm{t} \\ & =\log (\sec \mathrm{t})+\mathrm{c} \\ & =\log \left(\sec \left(\frac{x^2+1}{2}\right)\right)+\mathrm{c} \end{aligned}$$

Question 5

If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $x-3=\frac{y-\mathrm{k}}{2}=\mathrm{z}$ intersect, then the value of k is

Options:

A. $\frac{3}{2}$

B. $\frac{-2}{9}$

C. $\frac{-2}{3}$

D. $\frac{9}{2}$

Answer: D

Solution:

As the given lines are intersecting, the shortest distance between them is zero.

$$\therefore \quad \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \quad \begin{vmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore \quad \begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore \quad k = \frac{9}{2}$$

Question 6

For all real x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is

Options:

A. 0

B. 1

C. $\frac{1}{3}$

D. 3

Answer: C

Solution:

$$f(x) = \frac{1-x+x^2}{1+x+x^2}$$

$$\therefore \quad f'(x) = \frac{(1+x+x^2)(-1+2x) - (1-x+x^2)(1+2x)}{(1+x+x^2)^2}$$

$$= \frac{(-1+2x-x+2x^2-x^2+2x^3) - (-1+2x-x-2x^2+x^2+2x^3)}{(1+x+x^2)^2}$$

$$= \frac{-2+2x^2}{(1+x+x^2)^2}$$

If $f'(x) = 0$, then $\frac{-2+2x^2}{(1+x+x^2)^2} = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\therefore$ $\quad$ $f(x)$ at $x=1$ is $\frac{1}{3}$ and $f(x)$ at $x=-1$ is 1.

$\therefore$ $\quad$ Minimum value of $f(x)$ is $\frac{1}{3}$.

Question 7

If $\bar{a}=\hat{i}+4\hat{j}+2\hat{k}, \bar{b}=3\hat{i}-2\hat{j}+7\hat{k}, \bar{c}=2\hat{i}-\hat{j}+4\hat{k}$, then a vector $\overline{\mathrm{d}}$ which is parallel to vector $\overline{\mathrm{a}} \times \overline{\mathrm{b}}$ and which $\overline{\mathrm{c}} \cdot \overline{\mathrm{d}}=15$, is

Options:

A. $30\hat{i}-\hat{j}-14\hat{k}$

B. $90\hat{\mathrm{i}}-3\hat{\mathrm{j}}-42\hat{\mathrm{k}}$

C. $90\hat{\mathrm{i}}+\hat{\mathrm{j}}-7\hat{\mathrm{k}}$

D. $30\hat{\mathrm{i}}-3\hat{\mathrm{j}}+7\hat{\mathrm{k}}$

Answer: B

Solution:

Here, $\bar{c}=2\hat{i}-\hat{j}+4\hat{k}$

And given that $\bar{c}\cdot\bar{d}=15$

We verify given options one by one to satisfy the above condition.

Consider option (B),

For $\overline{\mathrm{d}}=90\hat{\mathrm{i}}-3\hat{\mathrm{j}}-42\hat{\mathrm{k}}$

$$\begin{aligned}\bar{c}\cdot\bar{d}&=(2)(90)+(-1)(-3)+(4)(-42)\\&=180+3-168=15\end{aligned}$$

$\therefore$ $\quad$ Option (B) is correct.

Question 8

If the vertices of a triangle are $(-2, 3)$, $(6, -1)$ and $(4, 3)$, then the co-ordinates of the circumcentre of the triangle are

Options:

A. $(1, 1)$

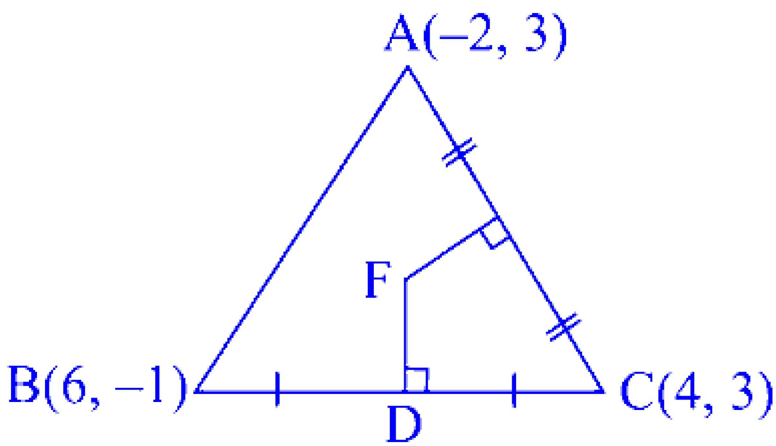
B. $(-1, -1)$

C. $(-1, 1)$

D. $(1, -1)$

Answer: D

Solution:



Here, $\mathrm{A}(-2,3), \mathrm{B}(6,-1), \mathrm{C}(4,3)$ are the vertices of $\triangle ABC$.

Let F be the circumcentre of $\triangle\mathrm{ABC}$.

Let FD and FE be the perpendicular bisectors of the sides BC and AC respectively.

$\therefore$ D and E are the midpoints of side BC and AC respectively.

$\therefore \quad D \equiv \left(\frac{6+4}{2}, \frac{-1+3}{2}\right)$
$\therefore \quad D = (5, 1)$
and $E \equiv \left(\frac{-2+4}{2}, \frac{3+3}{2}\right)$
$\therefore \quad \mathrm{E} = (1, 3)$

Now, slope of $BC = \frac{3-(-1)}{4-6} = \frac{4}{-2} = -2$

$\therefore \quad$ Slope of $\mathrm{FD} = \frac{1}{2} \quad \ldots [\because \mathrm{FD} \perp \mathrm{BC}]$

Since FD passes through $(5, 1)$ and has slope $\frac{1}{2}$, equation of FD is

$$y - 1 = \frac{1}{2}(x - 5)$$
$$\therefore \quad 2(y - 1) = x - 5$$
$$\therefore \quad 2y - 2 = x - 5$$
$$\therefore \quad x - 2y - 3 = 0 \quad \text{.... (i)}$$

Since both the points A and C have same y co-ordinates i.e. 3, the given points lie on the line $y = 3$.

Since the equation FE passes through $\mathrm{E}(1, 3)$, the equation of FE is $x = 1$. ..... (ii)

To find co-ordinates of circumcentre, we have to solve equations (i) and (ii).

Substituting the value of x in (i), we get

$$1 - 2y - 3 = 0$$
$$\therefore \quad y = -1$$

$\therefore$ Co-ordinates of circumcentre $\mathrm{F} \equiv (1, -1)$.

Question 9

The solution of $\frac{\mathrm{d}x}{\mathrm{d}y} + \frac{x}{y} = x^2$ is

Options:

A. $\frac{1}{y} = \mathrm{c}x - x \log x$, where c is a constant of integration.

B. $\frac{1}{x} = \mathrm{c}y - y \log y$, where c is a constant of integration.

C. $\frac{1}{x} = \mathrm{c}x - x \log y$, where c is a constant of integration.

D. $\frac{1}{y} = \mathrm{c}x - y \log x$, where c is a constant of integration.

Answer: B

Solution:

$$\frac{\mathrm{d}x}{\mathrm{d}y} + \frac{x}{y} = x^2$$
$$\frac{1}{x^2}\frac{\mathrm{d}x}{\mathrm{d}y} + \frac{1}{xy} = 1 \quad \text{.... (i)}$$

Let $\frac{1}{x} = \mathrm{t}$

Differentiating w.r.t. y, we get

$$\frac{-1}{x^2}\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\mathrm{dt}}{\mathrm{d}y} \Rightarrow \frac{1}{x^2}\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{-\mathrm{dt}}{\mathrm{d}y}$$

$\therefore$ (i) $\Rightarrow \frac{-\mathrm{dt}}{\mathrm{d}y} + \frac{\mathrm{t}}{y} = 1$

$\therefore$ $\frac{\mathrm{dt}}{\mathrm{d}y} - \frac{\mathrm{t}}{y} = -1$

$\therefore$ I.F. $= \mathrm{e}^{\int \frac{-1}{y}\,\mathrm{d}y} = \mathrm{e}^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}$

$\therefore$ The solution of the given equation is

$$\mathrm{t}(\text{ I.F }) = \int(-1)(\text{ I.F. })\mathrm{d}y + \mathrm{c}$$

$$\mathrm{t}\left(\frac{1}{y}\right) = \int \frac{-1}{y}\,\mathrm{d}y + \mathrm{c}$$

$\therefore$ $\frac{t}{y} = -\log y + c$

$\therefore$ $\frac{1}{xy} = -\log y + c$

$\therefore$ $\frac{1}{x} = cy - y\log y$

Question 10

If $\int \frac{\cos 8x+1}{\cot 2x-\tan 2x}\,\mathrm{d}x = \mathrm{A}\cos 8x + \mathrm{c}$, where c is an arbitrary constant, then the value of A is

Options:

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{-1}{8}$

D. $\frac{-1}{16}$

Answer: D

Solution:

$$I=\int \frac{\cos 8x+1}{\cot 2x-\tan 2x} dx$$
$$=\int \frac{2\cos^2\left(\frac{8x}{2}\right)}{\frac{\cos 2x}{\sin 2x}-\frac{\sin 2x}{\cos 2x}} dx$$
$$\ldots\left[\because 1+\cos\theta=2\cos^2\frac{\theta}{2}\right]$$

$$=\int \frac{2\cos^2(4x)\times\sin 2x\times\cos 2x}{\cos^2 2x-\sin^2 2x}\ \mathrm{d}x$$
$$=\int \frac{\cos^2(4x)\sin(4x)}{\cos(4x)} dx$$
$$\ldots\left[\begin{array}{l}\because \sin 2\theta=2\sin\theta\cos\theta \text{ and}\\ \cos 2\theta=\cos^2\theta-\sin^2\theta\end{array}\right]$$

$$=\frac{1}{2}\int 2\sin(4x)\cos(4x) dx$$
$$=\frac{1}{2}\int \sin 8x dx$$
$$=\frac{-\cos 8x}{2\times 8}+c$$
$$=\frac{-\cos 8x}{16}+c$$

Comparing with '$A\cos 8x+c$', we get $A=\frac{-1}{16}$

Question 11

The set of all points, where the derivative of the functions $\mathrm{f}(x)=\frac{x}{1+|x|}$ exists, is

Options:

A. $(-\infty,\infty)$

B. $[0,\infty)$

C. $(-\infty,0)\cup(0,\infty)$

D. $(0,\infty)$

Answer: A

Solution:

$\mathrm{f}(x)$ can be written as

$$\mathrm{f}(x)=\begin{cases}\frac{x}{1-x}, x \leq 0 \\ \frac{x}{1+x}, x>0\end{cases}$$

$$\therefore \quad \mathrm{f}^{\prime}(x)=\begin{cases}\frac{(1-x)+x}{(1+x)^2}, & x \leq 0 \\ \frac{(1+x)-x}{(1+x)^2}, & x>0\end{cases}$$

$$\therefore \quad \mathrm{f}^{\prime}(x)=\frac{1}{(1+x)^2} \forall x \in(-\infty, \infty)$$

$\therefore$ Derivative of $\mathrm{f}(x)$ exists $\forall x \in(-\infty, \infty)$

Question 12

In triangle ABC with usual notations $\mathrm{b}=\sqrt{3}, \mathrm{c}=1, \mathrm{~m} \angle \mathrm{A}=30^{\circ}$, then the largest angle of the triangle is

Options:

A. 135°

B. 90°

C. 60°

D. 120°

Answer: D

Solution:

By cosine rule, we get

$$\begin{aligned} a^2 & =b^2+c^2-2 b c \cos A \\ & =(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1) \cos \left(30^{\circ}\right) \\ & =3+1-2 \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \\ & =4-3 \\ & =1\end{aligned}$$

$\therefore \quad \mathrm{a}=1$

$\therefore$ Largest angle is angle B

$$\cos B=\frac{c^2+a^2-b^2}{2ca}=\frac{1+1-3}{2\times 1\times 1}=\frac{-1}{2}$$
$$\therefore\quad B=120^\circ$$

Question 13

A fair die is tossed twice in succession. If X denotes the number of fours in two tosses, then the probability distribution of X is given by

Options:

A.

$X=x_i$	0	1	2
P_i	$\frac{1}{36}$	$\frac{25}{36}$	$\frac{5}{18}$

B.

$X=x_i$	0	1	2
P_i	$\frac{25}{36}$	$\frac{1}{36}$	$\frac{5}{18}$

C.

$X=x_i$	0	1	2
P_i	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

D.

$X=x_i$	0	1	2
P_i	$\frac{5}{18}$	$\frac{1}{36}$	$\frac{25}{36}$

Answer: C

Solution:

A fair die is tossed twice in succession.

$\therefore$ Sample space (S)

$= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

X : Number of fours in two tosses.

$\therefore$ Possible values of X are: $0, 1, 2$.

$\therefore$ Probability distribution of X is as follows:

$\mathrm{X} = x_i$	0	1	2
$\mathrm{P_i}$	$\frac{25}{36}$	$\frac{10}{36} = \frac{5}{18}$	$\frac{1}{36}$

Question 14

If $\mathrm{f}(x) = \frac{3x+4}{5x-7}$ and $\mathrm{g}(x) = \frac{7x+4}{5x-3}$, then $\mathrm{f}(\mathrm{g}(x)) =$

Options:

A. $\frac{x^3+1}{x^2+2}$

B. $41x$

C. $\mathrm{g}(\mathrm{f}(x))$

D. $\frac{5x-7}{41}$

Answer: C

Solution:

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{7x+4}{5x-3}\right) \\
&= \frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7} \\
&= \frac{21x+12+20x-12}{35x+20-35x+21} \\
&= \frac{41x}{41} \\
&= x
\end{aligned}
$$

Now, $\mathrm{g}(\mathrm{f}(x)) = \mathrm{g}\left(\frac{3x+4}{5x-7}\right)$

$$
\begin{aligned}
&= \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3} \\
&= \frac{21x+28+20x-28}{15x+20-15x+21} \\
&= \frac{41x}{41} \\
&= x
\end{aligned}
$$

$\therefore \quad \mathrm{f}(\mathrm{g}(x)) = \mathrm{g}(\mathrm{f}(x))$

Question 15

If the function f is given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some a $\in \mathbb{R}$, is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation $\frac{\mathrm{f}(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is

Options:

A. -7

B. 6

C. 7

D. 5

Answer: C

Solution:

$f(x)=x^3-3(a-2)x^2+3ax+7$

As $\mathrm{f}(x)$ is increasing in $(0,1]$ and decreasing in $[1,5)$, we get that $\mathrm{f}(x)$ has critical point at $x=1$

$$\Rightarrow \mathrm{f}'(1)=0$$
$$\mathrm{f}'(x)=3x^2-6(\mathrm{a}-2)x+3\mathrm{a}$$
$$\therefore \quad 3(1)^2-6(\mathrm{a}-2)+3\mathrm{a}=0$$
$$\therefore \quad \mathrm{a}=5$$
$$\therefore \quad \frac{\mathrm{f}(x)-14}{(x-1)^2}=\frac{x^3-9x^2+15x-7}{(x-1)^2}$$
$$=\frac{(x-1)^2(x-7)}{(x-1)^2}$$
$$=x-7$$

$\therefore$ The required root is 7.

Question 16

The unit vector perpendicular to each of the vectors $\bar{a}+\bar{b}$ and $\bar{a}-\bar{b}$, where $\bar{a}=\hat{i}+\hat{j}+\hat{k}$ and $\overline{\mathrm{b}}=3\hat{\mathrm{i}}-2\hat{\mathrm{j}}+5\hat{\mathrm{k}}$ is

Options:

A. $\frac{-14\hat{\mathrm{i}}+4\hat{\mathrm{j}}+10\hat{\mathrm{k}}}{\sqrt{312}}$

B. $\frac{14\hat{\mathrm{i}}-4\hat{\mathrm{j}}+10\hat{\mathrm{k}}}{\sqrt{312}}$

C. $\frac{14\hat{\mathrm{i}}+4\hat{\mathrm{j}}+10\hat{\mathrm{k}}}{\sqrt{312}}$

D. $\frac{-14\hat{\mathrm{i}}-4\hat{\mathrm{j}}+10\hat{\mathrm{k}}}{\sqrt{312}}$

Answer: A

Solution:

$$\bar{a}+\bar{b}=(\hat{i}+\hat{j}+\hat{k})+(3\hat{i}-2\hat{j}+5\hat{k})$$
$$=4\hat{i}-\hat{j}+6\hat{k}$$
$$\bar{a}-\bar{b}=(\hat{i}+\hat{j}+\hat{k})-(3\hat{i}-2\hat{j}+5\hat{k})$$
$$=-2\hat{i}+3\hat{j}-4\hat{k}$$

$\therefore$ Vector perpendicular to $(\bar{a}+\bar{b})$ and $(\bar{a}-\bar{b})$ is

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 6\\ -2 & 3 & -4\end{vmatrix}=-14\hat{i}+4\hat{j}+10\hat{k}$$

$\therefore$ Required unit vector is

$$\frac{-14\hat{\mathrm{i}}+4\hat{\mathrm{j}}+10\hat{\mathrm{k}}}{\sqrt{(-14)^2+4^2+(10)^2}}=\frac{-14\hat{\mathrm{i}}+4\hat{\mathrm{j}}+10\hat{\mathrm{k}}}{\sqrt{312}}$$

Question 17

The logical statement $(\sim(\sim \mathrm{p} \vee \mathrm{q}) \vee(\mathrm{p} \wedge \mathrm{r})) \wedge(\sim \mathrm{q} \wedge \mathrm{r})$ is equivalent to

Options:

A. $\sim p \vee r$

B. $(p \wedge \sim q) \vee r$

C. $(\mathrm{p} \wedge \mathrm{r}) \wedge \sim \mathrm{q}$

D. $(\sim p \wedge \sim q) \wedge r$

Answer: C

Solution:

$[\sim(\sim p \vee q) \vee(p \wedge r)] \wedge(\sim q \wedge r)$
$\equiv[(p \wedge \sim q) \vee(p \wedge r)] \wedge(\sim q \wedge r)$ ...[De Morgan's law]

$\equiv p \wedge(\sim q \vee r) \wedge(\sim q \wedge r)$ $\ldots$[Distributive law]
$\equiv p \wedge[(\sim q \vee r) \wedge \sim q] \wedge r$ $\ldots$ A Associative law]
$\equiv p \wedge(\sim q) \wedge r$ $\ldots$[Absorption law]
$\equiv(p \wedge r) \wedge \sim q$ ...[Commutative and Associative law]

Question 18

Let $\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \bar{b} = \hat{i} + \hat{j}$ and $\bar{c}$ be a vector such that $|\bar{c} - \bar{a}| = 4, |(\bar{a} \times \bar{b}) \times \bar{c}| = 3$ and the angle between $\overline{\mathrm{c}}$ and $\overline{\mathrm{a}} \times \overline{\mathrm{b}}$ is $\frac{\pi}{6}$, then $\overline{\mathrm{a}} \cdot \overline{\mathrm{c}}$ is equal to

Options:

A. -3

B. $\frac{3}{2}$

C. 3

D. $\frac{-3}{2}$

Answer: D

Solution:

$$\begin{aligned} & \overline{\mathrm{a}} = 2\hat{\mathrm{i}} + \hat{\mathrm{j}} - 2\hat{\mathrm{k}}, \overline{\mathrm{b}} = \hat{\mathrm{i}} + \hat{\mathrm{j}} \\ \therefore \quad & |\overline{\mathrm{a}}| = \sqrt{4+1+4} = 3 \\ & \overline{\mathrm{a}} \times \overline{\mathrm{b}} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + \hat{\mathrm{k}} \\ \therefore \quad & |\overline{\mathrm{a}} \times \overline{\mathrm{b}}| = \sqrt{4+4+1} = 3 \end{aligned}$$

Angle between $\overline{\mathrm{c}}$ and $\overline{\mathrm{a}} \times \overline{\mathrm{b}}$ is $\frac{\pi}{6}$ .... [Given]

$$\begin{aligned} \therefore \quad & \sin\frac{\pi}{6} = \frac{|(\overline{\mathrm{a}} \times \overline{\mathrm{b}}) \times \overline{\mathrm{c}}|}{|\overline{\mathrm{a}} \times \overline{\mathrm{b}}||\overline{\mathrm{c}}|} \\ & \frac{1}{2} = \frac{3}{3 \times |\overline{\mathrm{c}}|} \Rightarrow |\overline{\mathrm{c}}| = 2 \end{aligned}$$

Now, $|\overline{\mathrm{c}} - \overline{\mathrm{a}}| = 4$ ..... [Given]

$$\begin{aligned} & \Rightarrow |\bar{c}|^2 + |\bar{a}|^2 - 2\bar{a} \cdot \bar{c} = 16 \\ & \Rightarrow 4 + 9 - 2a \cdot c = 16 \\ & \Rightarrow a \cdot c = \frac{-3}{2} \end{aligned}$$

Question 19

If a and b are positive number such that $a > b$, then the minimum value of $a \sec \theta - b \tan \theta \left(0 < \theta < \frac{\pi}{2}\right)$ is

Options:

A. $\frac{1}{\sqrt{a^2-b^2}}$

B. $\frac{1}{\sqrt{a^2+b^2}}$

C. $\sqrt{a^2+b^2}$

D. $\sqrt{a^2-b^2}$

Answer: D

Solution:

$$\text{let } \mathrm{f}(\theta) = \mathrm{a} \sec \theta - \mathrm{b} \tan \theta$$
$$\therefore \quad \mathrm{f}'(\theta) = \mathrm{a} \sec \theta \tan \theta - \mathrm{b} \sec^2 \theta$$
$$= \sec \theta (a \tan \theta - b \sec \theta)$$
$$\therefore \quad \mathrm{f}'(\theta) = 0 \Rightarrow \sec \theta (\mathrm{a} \tan \theta - \mathrm{b} \sec \theta) = 0$$
$$\Rightarrow \mathrm{a} \tan \theta - \mathrm{b} \sec \theta = 0$$
$$\ldots \left[\text{As } 0 < \pi < \frac{\pi}{2}, \sec \theta \neq 0\right]$$
$$\Rightarrow \mathrm{a} \sin \theta - \mathrm{b} = 0$$
$$\ldots \left[\text{As } 0 < \pi < \frac{\pi}{2}, \cos \theta \neq 0\right]$$

$$\Rightarrow \sin \theta = \frac{b}{a}$$
$$\Rightarrow \sec \theta = \frac{a}{\sqrt{a^2-b^2}} \text{ and } \tan \theta = \frac{b}{\sqrt{a^2-b^2}} \quad \text{... (i)}$$

Now,

$f''(\theta) = \sec\theta\tan\theta(a\tan\theta - b\sec\theta)$
$+\sec\theta\left(a\sec^2\theta - b\sec\theta\tan\theta\right)$
$= a\tan^2\theta\sec\theta - b\sec^2\theta\tan\theta$
$+a\sec^3\theta - b\sec^2\theta\tan\theta$
$= a\sec\theta\left(\tan^2\theta + \sec^2\theta\right)$
$= a\sec\theta\left(1 + 2\tan^2\theta\right)$
$> 0 \quad \ldots\left[\because \text{ a is positive and } 0 < \theta < \frac{\pi}{2}\right]$

$\therefore$ f(θ) is minimum when $\sin\theta = \frac{\mathrm{b}}{\mathrm{a}}$.

$\therefore$ Minimum value of $f(\theta)$

$= a\left(\frac{a}{\sqrt{a^2-b^2}}\right) - b\left(\frac{b}{\sqrt{a^2-b^2}}\right) \quad \ldots$ [From (i)]
$= \frac{a^2-b^2}{\sqrt{a^2-b^2}}$
$= \sqrt{a^2-b^2}$

Question 20

If $l = \lim_{x\to 0} \frac{x}{|x|+x^2}$, then the value of l is

Options:

A. 1

B. -1

C. 2

D. non-existant

Answer: D

Solution:

Let $f(x)=\frac{x}{|x|+x^2}$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}\frac{x}{-x+x^2}=\lim_{x\to 0}\frac{1}{-1+x}=-1$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}\frac{x}{x+x^2}=\lim_{x\to 0}\frac{1}{1+x}=1$$

Here, $\lim_{x\to 0^-} f(x)\neq\lim_{x\to 0^+} f(x)$

$\therefore$ Value of l is non-existant

Question 21

If $y=[(x+1)(2x+1)(3x+1)\ldots(nx+1)]^{\frac{3}{2}}$, then $\frac{dy}{dx}$ at $x=0$ is

Options:

A. $\frac{3n(n+1)}{4}$

B. $\frac{n(n+1)}{2}$

C. $\frac{3n(n+1)}{2}$

D. $\frac{n(n+1)}{4}$

Answer: A

Solution:

$y=[(x+1)(2x+1)(3x+1)\ldots(nx+1)]^{\frac{3}{2}}$

Taking 'log' on both sides, we get

$$\log y=\frac{3}{2}[\log(x+1)+\log(2x+1)+\log(3x+1) \\ +\ldots+\log(nx+1)]$$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{3}{2}\left[\frac{1}{x+1}+\frac{2}{2x+1}+\frac{3}{3x+1}+\ldots+\frac{n}{nx+1}\right]$$

$$\frac{dy}{dx}=\frac{3y}{2}\left[\frac{1}{x+1}+\frac{2}{2x+1}+\frac{3}{3x+1}+\ldots+\frac{n}{nx+1}\right]$$

Now at $x=0, y=[\underbrace{(1)(1)(1)\ldots(1)}_{n \text{ times}}]^{\frac{3}{2}}=1$

$$\therefore \quad \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=0}=\frac{3(1)}{2}\left[\frac{1}{0+1}+\frac{2}{0+1}+\frac{3}{0+1}+\ldots+\frac{\mathrm{n}}{0+1}\right]$$
$$=\frac{3}{2}(1+2+3+\ldots+\mathrm{n})$$
$$=\frac{3}{2} \times \frac{\mathrm{n}(\mathrm{n}+1)}{2}$$
$$=\frac{3 \mathrm{n}(\mathrm{n}+1)}{4}$$

Question 22

If $\cos x+\cos y-\cos (x+y)=\frac{3}{2}$, then

Options:

A. $x+y=0$

B. $x=2y$

C. $x=y$

D. $2x=y$

Answer: C

Solution:

$$\cos x+\cos y-\cos (x+y)=\frac{3}{2}$$
$$\therefore \quad 2 \cos \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)-\left(2 \cos ^2\left(\frac{x+y}{2}\right)-1\right)=\frac{3}{2}$$
$$\ldots\left[\begin{array}{l}\because \cos \alpha+\cos \beta=2 \cos \left(\frac{\alpha+\beta}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right) \text{ and } \\ \cos \theta=2 \cos ^2\left(\frac{\theta}{2}\right)-1\end{array}\right]$$
$$\therefore \quad 2 \cos \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)-2 \cos ^2\left(\frac{x+y}{2}\right)=\frac{3}{2}-1$$
$$\therefore \quad 2 \cos \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)-2 \cos ^2\left(\frac{x+y}{2}\right)=\frac{1}{2}$$
$$\therefore \quad 4 \cos ^2\left(\frac{x+y}{2}\right)-4 \cos \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)+1=0$$

Substituting $\cos\left(\frac{x+y}{2}\right) = \text{t}$, we get

$4t^2 - 4t\cos\left(\frac{x-y}{2}\right) + 1 = 0$

As t is real, we get $b^2 - 4ac \geq 0$

$$\Rightarrow \left[-4\cos\left(\frac{x-y}{2}\right)\right]^2 - 4 \times 4 \times 1 \geq 0$$
$$\Rightarrow 16\cos^2\left(\frac{x-y}{2}\right) - 16 \geq 0$$
$$\Rightarrow \cos^2\left(\frac{x-y}{2}\right) \geq 1$$
$$\Rightarrow \cos^2\left(\frac{x-y}{2}\right) = 1 \quad \ldots [\because -1 \leq \cos\theta \leq 1, \text{for all values of } \theta]$$
$$\Rightarrow \frac{x-y}{2} = 0$$
$$\Rightarrow x = y$$

Question 23

The joint equation of the lines pair of lines passing through the point $(3, -2)$ and perpendicular to the lines $5x^2 + 2xy - 3y^2 = 0$ is

Options:

A. $3x^2 + 2xy + 5y^2 + 14x + 26y + 5 = 0$

B. $3x^2 + 2xy - 5y^2 - 14x - 26y - 5 = 0$

C. $3x^2 - 2xy - 5y^2 - 14x - 26y + 5 = 0$

D. $3x^2 - 2xy + 5y^2 + 14x + 26y - 5 = 0$

Answer: B

Solution:

Joint equation of the liens passing through the point (x_1, y_1) and perpendicular to the lines $\text{a}x^2 + 2\,\text{h}xy + \text{b}y^2 = 0$ is:

$\text{b}(x - x_1)^2 - 2\,\text{h}\,(x - x_1)\,(y - y_1) + \text{a}(y - y_1)^2 = 0$

$\therefore$ Equation of the required line is:

$$-3(x-3)^2-2(x-3)(y+2)+5(y+2)^2=0$$

$\therefore \quad -3\left(x^2-6x+9\right)-2(xy+2x-3y-6)+5\left(y^2+4y+4\right)=0$

$\therefore \quad -3x^2+18x-27-2xy-4x+6y+12+5y^2$
$+20y+20=0$

$\therefore \quad 3x^2+2xy-5y^2-14x-26y-5=0$

Question 24

If the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane $x+2y+3z=15$ at the point P, then the distance of P from the origin is

Options:

A. $\frac{7}{2}$ units

B. $\frac{9}{2}$ units

C. $\frac{\sqrt{5}}{2}$ units

D. $2\sqrt{5}$ units

Answer: B

Solution:

Let $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}=\mathrm{k}$ (say)

Let P be the any point on the above line.

$\therefore \quad \mathrm{P} \equiv (2\mathrm{k}+1, 3\mathrm{k}-1, 4\mathrm{k}+2)$

The point P lies on the plane

$\therefore \quad 2k+1+2(3k-1)+3(4k+2)=15$
$\therefore \quad 2k+1+6k-2+12k+6=15$
$\therefore \quad 20k=10$
$\therefore \quad k=\frac{1}{2}$
$\therefore \quad P \equiv \left(2, \frac{1}{2}, 4\right)$

Distance of P from origin is

$\sqrt{2^2+\left(\frac{1}{2}\right)^2+4^2}=\sqrt{\frac{81}{4}}=\frac{9}{2}$

Question 25

If A and B are two events such that $\mathrm{P(A)}=\frac{1}{3}, \mathrm{P(B)}=\frac{1}{5}, \mathrm{P(A \cup B)}=\frac{1}{3}$, then the value of $\mathrm{P}\left(\mathrm{A}'/\mathrm{B}'\right)+\mathrm{P}\left(\mathrm{B}'/\mathrm{A}'\right)$ is

Options:

A. $\frac{5}{6}$

B. 1

C. $\frac{1}{6}$

D. $\frac{11}{6}$

Answer: D

Solution:

$$\begin{aligned}
\mathrm{P}\left(\mathrm{A}'/\mathrm{B}'\right) &= \frac{\mathrm{P}\left(\mathrm{A}' \cap \mathrm{B}'\right)}{\mathrm{P}\left(\mathrm{B}'\right)} \\
&= \frac{1-\mathrm{P(A \cup B)}}{1-\mathrm{P(B)}} \\
&= \frac{1-\frac{1}{3}}{1-\frac{1}{5}}=\frac{\frac{2}{3}}{\frac{4}{5}}=\frac{5}{6} \\
\mathrm{P}\left(\mathrm{B}'/\mathrm{A}'\right) &= \frac{\mathrm{P}\left(\mathrm{A}' \cap \mathrm{B}'\right)}{\mathrm{P}\left(\mathrm{A}'\right)} \\
&= \frac{1-\mathrm{P(A \cup B)}}{1-\mathrm{P(A)}} \\
&= \frac{\frac{2}{3}}{1-\frac{1}{3}}=\frac{\frac{2}{3}}{\frac{2}{3}}=1 \\
\therefore \quad \mathrm{P}\left(\mathrm{A}'/\mathrm{B}'\right)+\mathrm{P}\left(\mathrm{B}'/\mathrm{A}'\right) &= \frac{5}{6}+1=\frac{11}{6}
\end{aligned}$$

Question 26

If the general solution of the equation $\frac{\tan 3x-1}{\tan 3x+1}=\sqrt{3}$ is $x=\frac{\mathrm{n}\pi}{\mathrm{p}}+\frac{7\pi}{\mathrm{q}}$, n, p, q, $\in$ Z, then $\frac{p}{q}$ is

Options:

A. 12

B. $\frac{1}{12}$

C. 3

D. 36

Answer: B

Solution:

$$\frac{\tan 3x-1}{\tan 3x+1}=\sqrt{3}$$

$$\therefore \quad \frac{\tan 3x-\tan \frac{\pi}{4}}{1+(\tan 3x)\left(\tan \frac{\pi}{4}\right)}=\sqrt{3}$$

$$\therefore \quad \tan\left(3x-\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{3}\right)$$

$$\therefore \quad 3x-\frac{\pi}{4}=n\pi+\frac{\pi}{3}$$

$$\therefore \quad 3x=\mathrm{n}\pi+\frac{\pi}{3}+\frac{\pi}{4}$$

$$\therefore \quad 3x=\mathrm{n}\pi+\frac{7\pi}{12}$$

$$\therefore \quad x=\frac{\mathrm{n}\pi}{3}+\frac{7\pi}{36}$$

Comparing with $\frac{\mathrm{n}\pi}{\mathrm{p}}+\frac{7\pi}{\mathrm{q}}$, we get

$$\mathrm{p}=3, \mathrm{q}=36$$

$$\therefore \quad \frac{\mathrm{p}}{\mathrm{q}}=\frac{3}{36}=\frac{1}{12}$$

Question 27

If the area of the parallelogram with $\bar{a}$ and $\bar{b}$ as two adjacent sides is $16 \mathrm{sq}$. units, then the area of the parallelogram having $3\overline{\mathrm{a}}+2\overline{\mathrm{b}}$ and $\overline{\mathrm{a}}+3\overline{\mathrm{b}}$ as two adjacent sides (in sq. units) is

Options:

A. 96

B. 112

C. 144

D. 128

Answer: B

Solution:

Area of the parallelogram with $\bar{a}$ and $\bar{b}$ as two adjacent sides is $|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|$

$\therefore \quad |\overline{\mathrm{a}} \times \overline{\mathrm{b}}|=16$

$\therefore \quad$ Area of the required parallelogram

$$
\begin{aligned}
& =|(3\overline{\mathrm{a}}+2\overline{\mathrm{b}}) \times(\overline{\mathrm{a}}+3\overline{\mathrm{b}})| \\
& =|3(\overline{\mathrm{a}} \times \overline{\mathrm{a}})+9(\overline{\mathrm{a}} \times \overline{\mathrm{b}})+2(\overline{\mathrm{b}} \times \overline{\mathrm{a}})+(\overline{\mathrm{b}} \times \overline{\mathrm{b}})| \\
& =0+9|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|-2|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|+0 \\
& =7|\overline{\mathrm{a}} \times \overline{\mathrm{b}}| \\
& =7 \times 16 \\
& =112
\end{aligned}
$$

Question 28

The value of $\int(1-\cos x) \cdot \operatorname{cosec}^2 x d x$ is

Options:

A. $\frac{1}{2} \tan \frac{x}{2}+\mathrm{c}$, where c is a constant of integration.

B. $\tan \frac{x}{2}+\mathrm{c}$, where c is a constant of integration.

C. $2 \cot \frac{x}{2}+\mathrm{c}$, where c is a constant of integration.

D. $\cot \frac{x}{2}+\mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

Let

$$\begin{aligned}
\mathrm{I} &= \int(1-\cos x) \cdot \operatorname{cosec}^2 \mathrm{~d} x \\
&= \int \frac{2 \sin ^2 \frac{x}{2}}{[\sin x]^2} \mathrm{~d} x \quad \ldots\left[\because 1-\cos \theta=2 \sin ^2 \frac{\theta}{2}\right] \\
&= \int \frac{2 \sin ^2 \frac{x}{2}}{\left[2 \sin \frac{x}{2} \cos \frac{x}{2}\right]^2} \mathrm{~d} x \quad \ldots\left[\because \sin \theta=2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}\right] \\
&= \frac{1}{2} \int \sec ^2 \frac{x}{2} \mathrm{~d} x \\
&= \tan \frac{x}{2}+\mathrm{c}
\end{aligned}$$

Question 29

If $\cos^{-1} \sqrt{\mathrm{p}}+\cos^{-1} \sqrt{1-\mathrm{p}}+\cos^{-1} \sqrt{1-\mathrm{q}}=\frac{3 \pi}{4}$, then q is

Options:

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. $\frac{1}{3}$

Answer: A

Solution:

$\cos^{-1}\sqrt{\mathrm{p}} - \cos^{-1}\sqrt{1-\mathrm{p}} + \cos^{-1}\sqrt{1-\mathrm{q}} = \frac{3\pi}{4}$

Let $\mathrm{t} = \cos^{-1}\sqrt{\mathrm{p}}$

$\Rightarrow \mathrm{p} = \cos^2 \mathrm{t}$

$\Rightarrow \mathrm{p} = 1 - \sin^2 \mathrm{t}$

$\Rightarrow \sin \mathrm{t} = \sqrt{1-\mathrm{p}}$

$\Rightarrow \mathrm{t} = \sin^{-1}\sqrt{1-\mathrm{p}}$

$\Rightarrow \cos^{-1}\sqrt{\mathrm{p}} = \sin^{-1}\sqrt{1-\mathrm{p}}$

$\therefore$ Given equation becomes

$\therefore \quad \sin^{-1}\sqrt{1-p} - \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$

$\therefore \quad \frac{\pi}{2} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4} \quad \ldots\left[\because \cos^{-1} a + \sin^{-1} a = \frac{\pi}{2}\right]$

$\therefore \quad \cos^{-1}\sqrt{1-q} = \frac{-\pi}{4}$

$\therefore \quad \sqrt{1-q} = \cos\left(-\frac{\pi}{4}\right)$

$\therefore \quad q = 1 - \frac{1}{2}$

$\therefore \quad q = \frac{1}{2}$

Question 30

The slope of the tangent to a curve $y = \mathrm{f}(x)$ at $(x, \mathrm{f}(x))$ is $2x+1$. If the curve passes through the point $(1,2)$, then the area (in sq. units), bounded by the curve, the X-axis and the line $x=1$, is

Options:

A. $\frac{3}{2}$

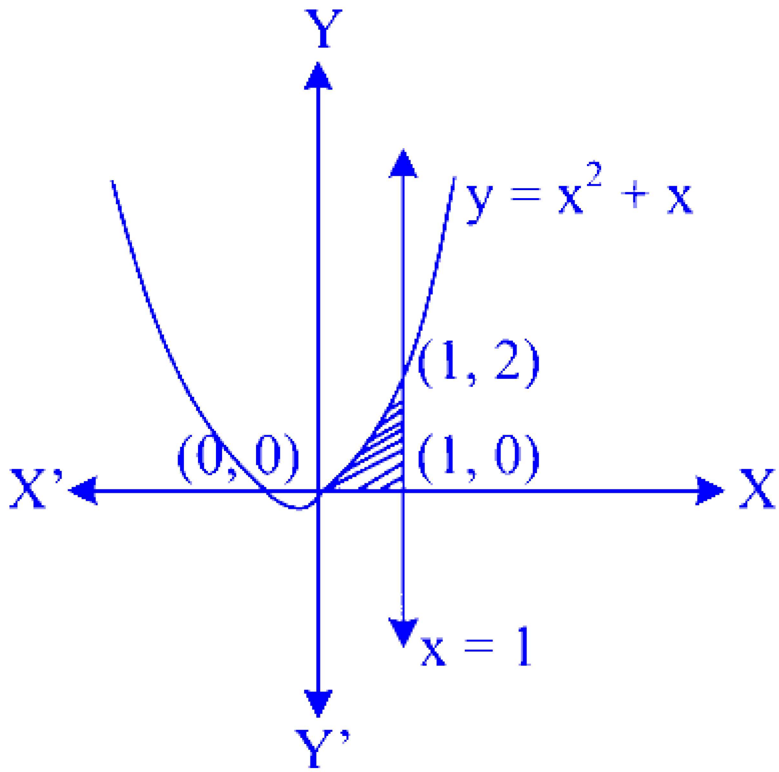
B. $\frac{4}{3}$

C. $\frac{5}{6}$

D. $\frac{1}{12}$

Answer: C

Solution:



According to the given condition, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 1$

Integrating w.r.t. x, we get $y = x^2 + x + c$

As curve passes through $(1, 2)$, we get $2 = (1)^2 + 1 + c \Rightarrow c = 0$

$\therefore$ The equation of the curve is $y = x^2 + x$.

$\therefore$ Required area

$$= \int_0^1 \left(x^2 + x\right) \mathrm{d}x$$
$$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1$$
$$= \frac{1}{3} + \frac{1}{2}$$
$$= \frac{5}{6} \text{ sq. units}$$

Question 31

A rod AB, 13 feet long moves with its ends A and B on two perpendicular lines OX and OY respectively. When A is 5 feet from O,

it is moving away at the rate of 3feet/sec. At this instant, B is moving at the rate

Options:

A. $\frac{5}{4}\text{ft/sec}$ upwards.

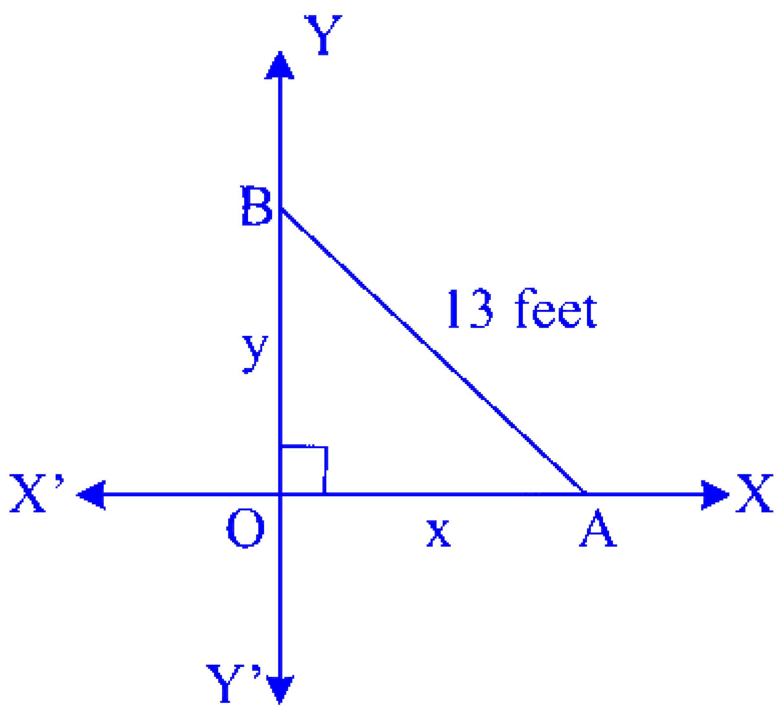
B. $\frac{4}{5}\text{ft/sec}$ upwards.

C. $\frac{5}{4}\text{ft/sec}$ downwards.

D. $\frac{4}{5}\text{ft/sec}$ downwards.

Answer: C

Solution:



Note that $\triangle\text{OAB}$ is a right angled triangle.

Let $\text{OA} = x\text{ft}$ and $\text{OB} = y\text{ft}$.

$y^2 = 169 - x^2$

Now, differentiating above function w.r.t. time 't', we get

$2y\frac{\text{d}y}{\text{dt}} = -2x\frac{\text{d}x}{\text{dt}}$ ..... (i)

At $x = 5$, $\frac{\text{d}x}{\text{dt}} = 3\text{ft/sec}$ ..... [Given]

Also, at $x=5, y=12$

$\therefore$ (i) $\Rightarrow 2(12)\frac{\mathrm{dy}}{\mathrm{dt}}=-2(5)(3)$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{d}x}=\frac{-5}{4}$

Negative sign indicates that B is moving downwards.

$\therefore$ B is moving at the rate $\frac{5}{4}\mathrm{ft}/\mathrm{sec}$ downwards.

Question 32

If $\mathrm{f}(x)=\begin{cases}\frac{x-3}{|x-3|}+\mathrm{a} & , \quad x<3 \\ \mathrm{a}+\mathrm{b} & , \quad x=3 \\ \frac{|x-3|}{x-3}+\mathrm{b}, & \quad x>3\end{cases}$

Is continuous at $x=3$, then the value of $\mathrm{a}-\mathrm{b}$ is

Options:

A. -1

B. 0

C. 1

D. 2

Answer: D

Solution:

If $x<3$, then

$\frac{x-3}{|x-3|}+\mathrm{a}=\frac{x-3}{-(x-3)}+\mathrm{a}=\mathrm{a}-1$

If $x>3$, then $\frac{|x-3|}{x-3}+b=\frac{x-3}{x-3}+b=1+b$

$\therefore$ Given function can be written as

$$f(x)=\begin{cases} a-1, x<3 \\ a+b, x=3 \\ 1+b, x>3 \end{cases}$$

As $f(x)$ is continuous at $x=3$, we get

$\lim_{x\to 3^-} f(x)=f(3)$ and $\lim_{x\to 3^+} f(x)=f(3)$

$\therefore \quad a-1=a+b$ and $1+b=a+b$
$\therefore \quad b=-1 \quad$ and $a=1$
$\therefore \quad a-b=2$

Question 33

The equation of the tangent to the curve $y=\sqrt{9-2x^2}$, at the point where the ordinate and abscissa are equal, is

Options:

A. $2x+y+\sqrt{3}=0$

B. $2x+y+3\sqrt{3}=0$

C. $2x-y-3\sqrt{3}=0$

D. $2x+y-3\sqrt{3}=0$

Answer: D

Solution:

Given curve is $y=\sqrt{9-2x^2}$

If ordinate and abscissa are equal, we get $y=x$.

$\therefore \quad$ Equation of the curve becomes $x^2=9-2x^2$

$\Rightarrow x=\pm\sqrt{3}$

If $x=-\sqrt{3}$, then $y=\sqrt{9-2(3)}=\sqrt{3}$

In this case, $x\neq y$.

Hence, $x\neq-\sqrt{3}$

$\therefore \quad x=\sqrt{3}$ and $y=\sqrt{3}$

$\therefore \quad$ Slope of the tangent to the given curve is $2y\frac{\mathrm{d}y}{\mathrm{d}x}=-4x$

$\therefore \quad$ at $(\sqrt{3},\sqrt{3})$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-2$

$\therefore \quad$ Equation of the required tangent is

$(y-\sqrt{3})=-2(x-\sqrt{3})$
i.e., $2x+y-3\sqrt{3}=0$

Question 34

If $\overline{\mathrm{a}}=2\hat{\mathrm{i}}+3\hat{\mathrm{j}}+2\hat{\mathrm{k}}, \overline{\mathrm{b}}=2\hat{\mathrm{i}}+\hat{\mathrm{j}}-\hat{\mathrm{k}}$ and $\overline{\mathrm{c}}=3\hat{\mathrm{i}}-\hat{\mathrm{j}}$ are such that $\bar{a}+\lambda\bar{b}$ is perpendicular to $\bar{c}$, then the value of λ is

Options:

A. $\frac{-1}{5}$

B. 3

C. $\frac{3}{5}$

D. $\frac{-3}{5}$

Answer: D

Solution:

According to the given condition, we get

$(\overline{\mathrm{a}}+\lambda\overline{\mathrm{b}})\cdot\overline{\mathrm{c}}=0$
$\therefore \quad [(2+2\lambda)\hat{\mathrm{i}}+(3+\lambda)\hat{\mathrm{j}}+(2-\lambda)\hat{\mathrm{k}}]\cdot(3\hat{\mathrm{i}}-\hat{\mathrm{j}})=0$
$\therefore \quad 3(2+2\lambda)-(3+\lambda)=0$
$\therefore \quad 6+6\lambda-3-\lambda=0$
$\therefore \quad 3+5\lambda=0$
$\therefore \quad \lambda=\frac{-3}{5}$

Question 35

If a circle passes through points $(4,0)$ and $(0,2)$ and its centre lies on Y-axis. If the radius of the circle is r, then the value of r^2-r+1 is

Options:

A. 25

B. 21

C. 20

D. 10

Answer: B

Solution:

Let $(0, y)$ be the centre of the circle.

$\therefore \quad \sqrt{(0-4)^2+(y-0)^2}=\sqrt{(0-0)^2+(y-2)^2}$
$\therefore \quad 16+y^2=(y-2)^2$
$\therefore \quad 16+y^2=y^2-4y+4$
$\therefore \quad y=-3$
$\therefore \quad$ centre of the circle is $(0,-3)$.
$\therefore \quad$ Radius of the circle $=\mathrm{r}=\sqrt{(0-0)^2+(-3-2)^2}$
$\quad =5$ units
$\therefore \quad \mathrm{r}^2-\mathrm{r}+1=25-5+1=21$

Question 36

If $\mathrm{A}=\begin{bmatrix}\mathrm{i} & 1 \\ 1 & 0\end{bmatrix}$ where $\mathrm{i}=\sqrt{-1}$ and $\mathrm{B}=\mathrm{A}^{2029}$, then $\mathrm{B}^{-1}=$

Options:

A. $-\mathrm{A}$

B. adj A

C. $-\mathrm{I}$

D. $-$ adj A

Answer: D

Solution:

$$A=\begin{bmatrix} i & 1 \\ 1 & 0 \end{bmatrix}$$
$$\therefore \quad A^2=\begin{bmatrix} i & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} i & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$$
$$\therefore \quad A^3=\begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} i & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$
$$\therefore \quad \mathrm{A}^6=\mathrm{A}^3 \times \mathrm{A}^3$$
$$=\begin{bmatrix} \mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix} \times \begin{bmatrix} \mathrm{i} & 0 \\ 0 & \mathrm{i} \end{bmatrix}$$
$$=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}=(-1) I_2$$
$$\text{Now, } B=A^{2029}=A^{(6 \times 338+1)}$$
$$\therefore \quad \mathrm{B}=\left(\mathrm{A}^6\right)^{338} \times \mathrm{A}$$
$$=\left((-1) I_2\right)^{338} \times A$$
$$=\mathrm{I}_2 \times \mathrm{A}$$
$$\therefore \quad \mathrm{B}=\mathrm{A}$$
$$\text{Now, } |A|=-1$$
$$\therefore \quad \mathrm{B}^{-1}=\mathrm{A}^{-1}=\frac{1}{|\mathrm{~A}|} \operatorname{adj} \mathrm{A}$$
$$\therefore \quad B^{-1}=-\operatorname{adj} \mathrm{A}$$

Question 37

The solution of the differential equation $\frac{\mathrm{d} y}{\mathrm{~d} x}+\frac{y}{x}=\sin x$ is

Options:

A. $x y+\cos x=\sin x+\mathrm{c}$, where c is a constant of integration.

B. $x(y+\cos x)=\sin x+\mathrm{c}$, where c is a constant of integration.

C. $y(x+\cos x)=\sin x+c$, where c is a constant of integration.

D. $xy+\sin x=\cos x+\mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

For given linear differential equation,

$\text{I.F. } = \mathrm{e}^{\int \frac{1}{x} \mathrm{~d} x}=\mathrm{e}^{\log x}=x$

$\therefore$ The required solution is

$yx=\int x \sin x \frac{\mathrm{d} y}{\mathrm{~d} x}$

$$\therefore \quad yx=-x \cos x+\int \cos x d x$$

$\therefore \quad yx=-x \cos x+\sin x+c$
$\therefore \quad x(y+\cos x)=\sin x+c$

Question 38

Let $f:[-1,2] \rightarrow[0, \infty)$ be a continuous function such that $\mathrm{f}(x)=\mathrm{f}(1-x), \forall x \in[-1,2]$

Let $\mathrm{R}_1=\int_{-1}^2 x \mathrm{f}(x) \mathrm{d} x$ and R_2 be the area of the region bounded by $y=\mathrm{f}(x), x=-1, x=2$ and the X-axis, then R_2 is

Options:

A. $\frac{1}{2} R_1$

B. $2 R_1$

C. $3 R_1$

D. $\frac{1}{3} R_1$

Answer: B

Solution:

Given that $\mathrm{f}(x)=\mathrm{f}(1-x)$ and $\mathrm{R}_1=\int_{-1}^{2} x\mathrm{f}(x)\mathrm{d}x$

$$\therefore \quad \mathrm{R}_1=\int_{-1}^{2}(1-x)\mathrm{f}(1-x)\mathrm{d}x \quad \ldots\left[\because \int_{\mathrm{a}}^{\mathrm{b}} \mathrm{f}(x)\mathrm{d}x=\int_{a}^{b} \mathrm{f}(\mathrm{a}+\mathrm{b}-x)\mathrm{d}x\right]$$

$$\therefore \quad \mathrm{R}_1=\int_{-1}^{2} \mathrm{f}(x)\mathrm{d}x-\int_{-1}^{2} x\mathrm{f}(x)\mathrm{d}x \quad \ldots[\because \mathrm{f}(x)=\mathrm{f}(1-x)]$$

$$\therefore \quad \mathrm{R}_1=\int_{-1}^{2} \mathrm{f}(x)\mathrm{d}x-\mathrm{R}_1$$

$$\therefore \quad 2\mathrm{R}_1=\int_{-1}^{2} \mathrm{f}(x)\mathrm{d}x$$

Note that $\mathrm{R}_2=\int_{-1}^{2} \mathrm{f}(x)\mathrm{d}x=2\mathrm{R}_1$

Question 39

The equation of line passing through the point $(1,2,3)$ and perpendicular to the lines $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+1}{-2}$ and $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ is

Options:

A. $\overline{\mathrm{r}}=(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}})+\lambda(4\hat{\mathrm{i}}+7\hat{\mathrm{j}}-13\hat{\mathrm{k}})$

B. $\overline{\mathrm{r}}=(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}})+\lambda(-4\hat{\mathrm{i}}+7\hat{\mathrm{j}}-13\hat{\mathrm{k}})$

C. $\overline{\mathrm{r}}=(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}})+\lambda(-4\hat{\mathrm{i}}-7\hat{\mathrm{j}}-13\hat{\mathrm{k}})$

D. $\overline{\mathrm{r}}=(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}})+\lambda(4\hat{\mathrm{i}}-7\hat{\mathrm{j}}-13\hat{\mathrm{k}})$

Answer: C

Solution:

Required line is perpendicular to the lines $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+1}{-2}$ and $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$

$\therefore$ Required line is parallel to vector

$$\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -2 \\ 2 & -3 & 1 \end{vmatrix} = -4\hat{i} - 7\hat{j} - 13\hat{k}$$

$\therefore$

The equation of the required line is $(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-4\hat{i} - 7\hat{j} - 13\hat{k})$

Question 40

Let a random variable X have a Binomial distribution with mean 8 and variance 4. If $\mathrm{P}(\mathrm{X} \leq 2) = \frac{\mathrm{K}}{2^{16}}$, then K is

Options:

A. 17

B. 121

C. 136

D. 137

Answer: D

Solution:

Let $\mathrm{X} \sim \mathrm{B}(\mathrm{n}, \mathrm{p})$

According to the given conditions, Mean $= np = 8$ and variance $= npq = 4$

$\Rightarrow \mathrm{p} = \mathrm{q} = \frac{1}{2}$ and $\mathrm{n} = 16$

$$P(X \leq 2) = \frac{K}{2^{16}}$$

$$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) = \frac{K}{2^{16}}$$

$$\therefore \quad {}^{16}\mathrm{C}_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{16} + {}^{16}\mathrm{C}_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{15} + {}^{16}\mathrm{C}_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14} = \frac{\mathrm{K}}{2^{16}}$$

$$\therefore \quad \frac{1 + 16 + 120}{2^{16}} = \frac{\mathrm{K}}{2^{16}}$$

$$\therefore \quad \mathrm{K} = 137$$

Question 41

$\pi + \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}\right)$ is equal to

Options:

A. $\frac{\pi}{2}$

B. $\frac{5\pi}{4}$

C. $\frac{3\pi}{2}$

D. $\frac{7\pi}{4}$

Answer: C

Solution:

$$\pi + \left[\left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13}\right) + \sin^{-1}\frac{16}{65}\right]$$
$$= \pi + \left[\left(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12}\right) + \sin^{-1}\frac{16}{65}\right]$$
$$= \pi + \left[\tan^{-1}\left(\frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}}\right) + \sin^{-1}\frac{16}{65}\right]$$
$$= \pi + \left[\tan^{-1}\left(\frac{63}{16}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$
$$= \pi + \left[\cos^{-1}\left(\frac{16}{65}\right) + \sin^{-1}\left(\frac{16}{65}\right)\right]$$
$$= \pi + \frac{\pi}{2}$$
$$= \frac{3\pi}{2}$$

Question 42

If the angles of a triangle are in the ratio $4 : 1 : 1$, then the ratio of the longest side to its perimeter is

Options:

A. $\sqrt{3} : (2 + \sqrt{3})$

B. $2 : (1 + \sqrt{3})$

C. $1 : (2 + \sqrt{3})$

D. $2 : 3$

Answer: A

Solution:

Let the angles of the triangle be $4x, x$ and x.

$$\begin{aligned} \therefore \quad & 4x + x + x = 180^\circ \Rightarrow 6x = 180^\circ \Rightarrow x = 30^\circ \\ & \frac{\sin 120^\circ}{a} = \frac{\sin 30^\circ}{b} = \frac{\sin 30^\circ}{c} \\ \therefore \quad & a : (a + b + c) \\ & = (\sin 120^\circ) : (\sin 120^\circ + \sin 30^\circ + \sin 30^\circ) \\ & = \frac{\sqrt{3}}{2} : \frac{\sqrt{3} + 2}{2} = \sqrt{3} : (\sqrt{3} + 2) \end{aligned}$$

Question 43

If truth value of logical statement $(p \leftrightarrow \sim q) \rightarrow (\sim p \wedge q)$ is false, then the truth values of p and q are respectively

Options:

A. F, T

B. T, T

C. T, F

D. F, F

Answer: C

Solution:

As $(p \leftrightarrow \sim q) \rightarrow (\sim p \wedge q)$ is false, we get

$p \leftrightarrow \sim q \equiv T$ and $\sim p \wedge q \equiv F$

$\sim p \wedge q \equiv F$

$\Rightarrow \sim p \equiv F$ and $q \equiv F$

$\Rightarrow p \equiv T$ and $q \equiv F$

Question 44

The teacher wants to arrange 5 students on the platform such that the boy B_1 occupies second position and the girls G_1 and G_2 are always adjacent to each other, then the number of such arrangements is

Options:

A. 24

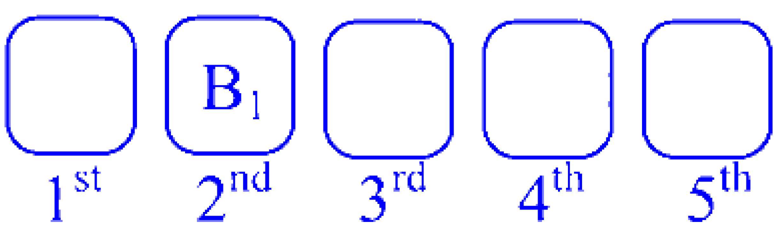
B. 12

C. 8

D. 16

Answer: C

Solution:



There are 5 positions.

Given that B_1 occupies 2^{nd} position

$\therefore B_1$ can be arranged in 1 way.

As G_1 and G_2 are always together, none of them can take 1^{st} position.

$\therefore G_1, G_2$ and one of the remaining students can be arranged on 3^{rd}, 4^{th} and 5^{th} position when G_1 and G_2 are always together in $2! \times 2!$ Ways.

And remaining 2 students can be arranged in 2 ! Ways.

$\therefore$ The required number of arrangements

$$= 2! \times 2! \times 2!$$
$$= 8$$

Question 45

If $\overline{\mathrm{a}} = \hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}, \overline{\mathrm{b}} = 4\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 4\hat{\mathrm{k}}$ and $\overline{\mathrm{c}} = \hat{\mathrm{i}} + \alpha\hat{\mathrm{j}} + \beta\hat{\mathrm{k}}$ are linearly dependent vectors and $|\bar{c}| = \sqrt{3}$, then the values of α and β are respectively.

Options:

A. $1, 1$

B. $2, 1$

C. $0, 1$

D. $1, 2$

Answer: A

Solution:

Note that only for option (A), i.e., for $\alpha = 1$ and $\beta = 1$, $|\vec{c}| = \sqrt{3}$ holds true.

$\therefore$ Option (A) is correct.

Question 46

At present a firm is manufacturing 1000 items. It is estimated that the rate of change of production P w.r.t. additional number of worker x is given by $\frac{\mathrm{dp}}{\mathrm{d}x} = 100 - 12\sqrt{x}$. If the firm employees 9 more workers, then the new level of production of items is

Options:

A. 1684

B. 1648

C. 2116

D. 1116

Answer: A

Solution:

$\frac{\mathrm{dP}}{\mathrm{d}x} = 100 - 12\sqrt{x}$

Integrating both sides, we get

$$\int \mathrm{dp} = \int (100 - 12\sqrt{x})\mathrm{d}x$$
$$\therefore \quad \mathrm{P} = 100x - 8x\sqrt{x} + \mathrm{c}$$

Given that $\mathrm{P} = 1000$, when $x = 0$

$$\Rightarrow 1000 = 100(0) - 8(0) + c$$
$$\Rightarrow c = 1000$$
$$\therefore \quad P = 100x - 8x\sqrt{x} + 1000$$

When $x = 9$, we get

$\mathrm{P} = 900 - 216 + 1000 = 1684$

$\therefore$ The new level of production of items is 1684.

Question 47

The maximum value of $z = 3x + 5y$ subject to the constraints $3x + 2y \leq 18, x \leq 4, y \leq 6, x, y \geq 0$, is

Options:

A. 27

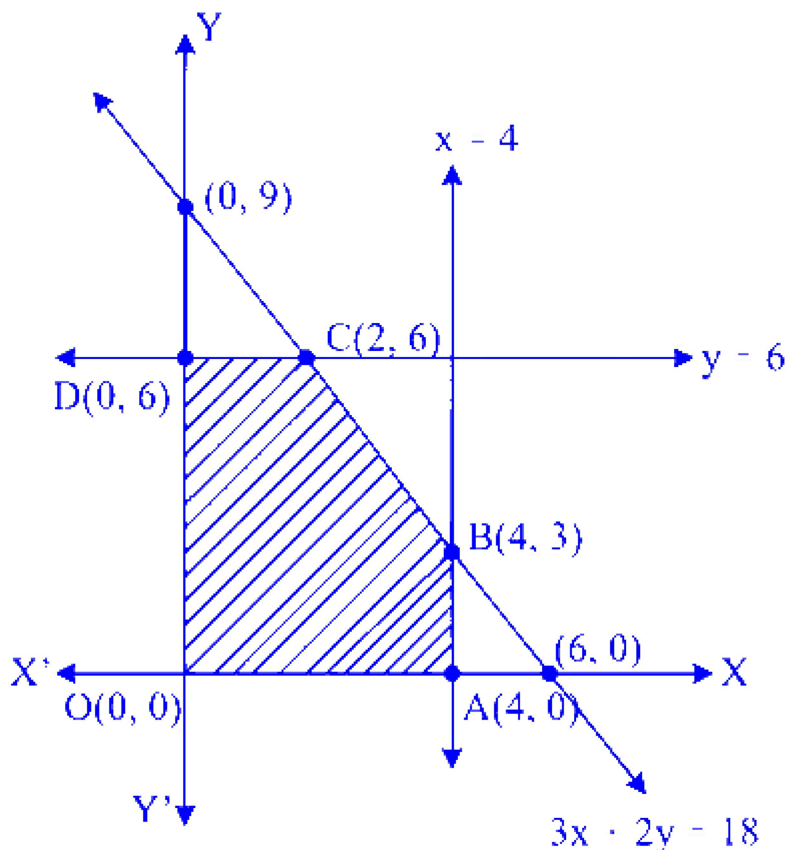
B. 36

C. 42

D. 30

Answer: B

Solution:



Objective function z $= 3x + 5y$

The corner points of the feasible region are $\mathrm{O}(0,0), \mathrm{A}(4,0), \mathrm{B}(4,3), \mathrm{C}(2,6)$ and $\mathrm{D}(0,6)$

$\therefore$ Z at $\mathrm{A}(4,0) = 12$

Z at $\mathrm{B}(4,3) = 27$

Z at $\mathrm{C}(2,6) = 36$

Z at $\mathrm{D}(0,6) = 30$

$\therefore$ Maximum value of Z is 36.

Question 48

If $\mathrm{I}=\int \sin(\log(x))\mathrm{d}x$, then I is given by

Options:

A. $-\frac{x}{2}(\sin(\log x)-\cos(\log x))+\mathrm{c}$, where c is a constant of integration.

B. $\frac{x}{2}(\sin(\log x)-\cos(\log x))+\mathrm{c}$, where c is a constant of integration.

C. $\frac{x}{2}(\sin(\log x)+\cos(\log x))+\mathrm{c}$, where c is a constant of integration.

D. $-\frac{x}{2}(\sin(\log x)+\cos(\log x))+c$, where c is a constant of integration.

Answer: B

Solution:

Let $\log x=\mathrm{t} \Rightarrow x=\mathrm{e}^{\mathrm{t}}$

Differentiating w.r.t. t, we get $\mathrm{d}x=\mathrm{e}^{\mathrm{t}}\mathrm{dt}$

$$\begin{aligned}
\therefore\quad \mathrm{I} &= \int \sin(\mathrm{t})\mathrm{e}^{\mathrm{t}}\mathrm{dt} \\
&= \sin te^{\mathrm{t}} - \int \cos \mathrm{te}^{\mathrm{t}}\mathrm{dt} \\
&= \sin \mathrm{te}^{\mathrm{t}} - \left[\cos \mathrm{te}^{\mathrm{t}} + \int \sin \mathrm{te}^{\mathrm{t}}\mathrm{dt}\right] \\
&= \sin \mathrm{te}^{\mathrm{t}} - \cos \mathrm{te}^{\mathrm{t}} - \mathrm{I} \\
\therefore\quad 2\mathrm{I} &= x\sin(\log x) - x\cos(\log x) + \mathrm{c} \\
\therefore\quad \mathrm{I} &= \frac{x}{2}(\sin(\log x) - \cos(\log x)) + \mathrm{c}
\end{aligned}$$

Question 49

If both mean and variance of 50 observations $x_1, x_2, \ldots\ldots, x_{50}$ are equal to 16 and 256 respectively, then mean of $(x_1-5)^2, (x_2-5)^2, \ldots\ldots(x_{50}-5)^2$ is

Options:

A. 357

B. 387

C. 377

D. 397

Answer: C

Solution:

Given that $\mathrm{n}=50, \bar{x}=16$ and $\sigma_x^2=256$

$\therefore \quad \sigma_x^2=\frac{1}{n}\left(\sum_{i=1}^{50} x_i^2\right)-(\bar{x})^2$

$\therefore \quad 256=\frac{1}{50}\left(\sum_{i=1}^{50} x_i^2\right)-256$

$\therefore \quad \frac{1}{50}\left(\sum_{i=1}^{50} x_i^2\right)=512$

$\therefore \quad \sum_{i=1}^{50} x_i^2=25600 \quad \text{..... (i)}$

$$\text{Now} \sum_{i=1}^{50}\left(x_i-5\right)^2=\sum_{i=1}^{50} x_i^2+25 \times 50-10 \sum_{i=1}^{50} x_i$$
$=25600+1250-8000 \quad$ ...[From (i) and (ii)]
$=18850$

$$\therefore \quad \text{Required Mean }=\frac{\sum_{\mathrm{i}=1}^{50}\left(x_{\mathrm{i}}-5\right)^2}{50}=\frac{18850}{50}=377$$

Question 50

The angle between the line $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and plane $x-2y-\lambda z=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then value of λ is

Options:

A. $\sqrt{\frac{3}{5}}$

B. $\frac{5}{\sqrt{3}}$

C. $\sqrt{\frac{5}{3}}$

D. $\frac{1}{\sqrt{3}}$

Answer: C

Solution:

The acute angle θ between line $\bar{a}+\lambda\bar{b}$ and the plane $\overline{\mathrm{r}}\cdot\overline{\mathrm{n}}=\mathrm{p}$ is given by

$\sin\theta=\left|\frac{\overline{\mathrm{b}}\cdot\overline{\mathrm{n}}}{|\overline{\mathrm{b}}|\cdot|\overline{\mathrm{n}}|}\right|$ ..... (i)

Here, $\bar{b}=2\hat{i}+\hat{j}-2\hat{k}$ and $\bar{n}=\hat{i}-2\hat{j}-\lambda\hat{k}$

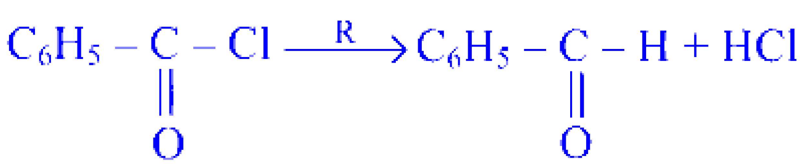
Also, $\theta=\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)=\sin^{-1}\left(\frac{1}{3}\right)$

$$\therefore\quad \text{(i)} \Rightarrow \sin\left(\sin^{-1}\left(\frac{1}{3}\right)\right)=\left|\frac{2-2+2\lambda}{\sqrt{4+1+4}\sqrt{1+4+\lambda^2}}\right|$$
$$\Rightarrow \frac{1}{3}=\left|\frac{2\lambda}{3\sqrt{5+\lambda^2}}\right|$$
$$\Rightarrow 5+\lambda^2=4\lambda^2$$
$$\Rightarrow \lambda^2=\frac{5}{3}$$
$$\Rightarrow \lambda=\sqrt{\frac{5}{3}}$$

Chemistry

Question 51

Identify the reagent 'R' used in the following reaction.



Options:

A. CO, HCl

B. H_2, Pd-$BaSO_4$

C. H_2O

D. DIBAl-H

Answer: B

Solution:

Acyl chloride is reduced to corresponding aldehyde by hydrogen using a palladium catalyst poisoned with barium sulfate. This reaction is known as Rosenmund reduction.

$$C_6H_5-\underset{\underset{O}{\|}}{C}-Cl \xrightarrow[Pd\text{-}BaSO_4]{H_2} C_6H_5-\underset{\underset{O}{\|}}{C}-H + HCl$$

Question 52

Which among the following phenols has highest melting point?

Options:

A. o-Nitrophenol

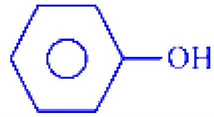
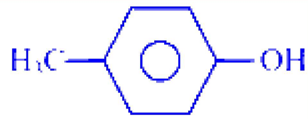
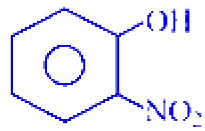
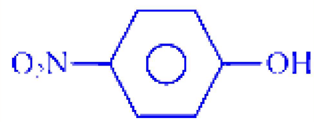
B. p-Nitrophenol

C. p-Cresol

D. Phenol

Answer: B

Solution:

Name	Formula	M.P. (°C)
Phenol		41
p-Cresol		35
o-Nitrophenol		45
p-Nitrophenol		114

p-Nitrophenol has the highest meting point because of intermolecular hydrogen bonding.

Question 53

Identify the element having general electronic configuration ns^1 from following.

Options:

A. Ca

B. Sr

C. Ba

D. Fr

Answer: D

Solution:

The general electronic configuration ns^1 is of group 1 elements. Among the given elements, francium (Fr) belongs to group 1.

Question 54

Which of the following enzyme is found in saliva?

Options:

A. Amylase

B. Lipase

C. Glucose isomerase

D. Proteoses

Answer: A

Solution:

Amylase, an enzyme present in saliva, hydrolyzes starch.

Question 55

Which from following molecules exhibits lowest thermal stability?

Options:

A. H_2O

B. H_2Te

C. H_2Se

D. H_2S

Answer: B

Solution:

The thermal stability of hydrides of group 16 elements decreases from H_2O to H_2Te.

Question 56

The common name of Benzene-1,3-diol is:

Options:

A. catechol

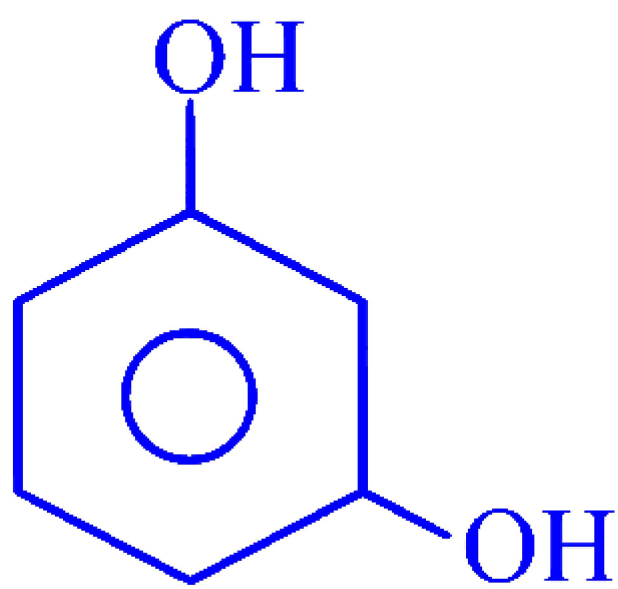
B. resorcinol

C. quinol

D. pyrogallol

Answer: B

Solution:



Benzene-1,3-diol
(Resorcinol)

Question 57

For a reaction $\mathrm{A+B \rightarrow}$ product, if $[\mathrm{A}]$ is doubled keeping $[\mathrm{B}]$ constant, the rate of reaction doubles. Calculate the order of reaction with respect to A.

Options:

A. 0

B. 1/2

C. 1

D. 2

Answer: C

Solution:

$$\begin{aligned} & \mathrm{A+B \longrightarrow \text{ Product}} \\ & \text{Rate } = \mathrm{k[A]^{x}[\,B]^{y}} \\ & \text{Rate }_1 = \mathrm{k[2\,A]^{x}[\,B]^{y}} \end{aligned}$$

$$\therefore \quad \frac{\text{Rate }_1}{\text{Rate}} = \frac{\mathrm{k2^{x}[\,A]^{x}[\,B]^{y}}}{\mathrm{k[A]^{x}[\,B]^{y}}}$$

$$\therefore \quad \frac{2\times \text{ Rate}}{\text{Rate}} = 2^{x}$$

$$\therefore \quad 2^{x} = 2$$

$$\therefore \quad x = 1$$

$\therefore$ Order of reaction with respect to A = 1

Question 58

Identify the salt that undergoes hydrolysis and forms acidic solution from following.

Options:

A. Na_2CO_3

B. NH_4NO_3

C. NH_4CN

D. KCN

Answer: B

Solution:

(A) Na_2CO_3 : Salt of weak acid and strong base: Solution is basic.

(B) NH_4NO_3 : Salt of strong acid and weak base: Solution is acidic.

(C) NH_4CN : Salt of weak acid and weak base for which $K_a < K_b$: Solution is basic.

(D) KCN : Salt of weak acid and strong base: Solution is basic.

Question 59

Which from following sentences is NOT correct?

Options:

A. ΔG° is an extensive property.

B. E°_{cell} is an intensive property.

C. Electrical work is equal to nFE_{cell} .

D. For a chemical reaction to be spontaneous E^0_{cell} must be negative.

Answer: D

Solution:

For a chemical reaction to be spontaneous E°_{cell} must be positive.

Question 60

A solution of nonvolatile solute is obtained by dissolving $1.5 \mathrm{~g}$ in $30 \mathrm{~g}$ solvent has boiling point elevation $0.65 \mathrm{~K}$. Calculate the molal elevation constant if molar mass of solute is $150 \mathrm{~g} \mathrm{~mol}^{-1}$.

Options:

A. $1.95 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

B. $2.23 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

C. $1.52 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

D. $2.72 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$

Answer: A

Solution:

$$\mathrm{M}_2=\frac{1000 \mathrm{~K}_{\mathrm{b}} \mathrm{W}_2}{\Delta \mathrm{T}_{\mathrm{b}} \mathrm{W}_1}$$

$$\mathrm{K}_{\mathrm{b}}=\frac{\mathrm{M}_2 \times \Delta \mathrm{T}_{\mathrm{b}} \times \mathrm{W}_1}{1000 \times \mathrm{W}_2}=\frac{150 \times 0.65 \times 30}{1000 \times 1.5}$$
$$=1.95 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$$

Question 61

A weak base is 1.42% dissociated in its $0.05 \mathrm{~M}$ solution. Calculate its dissociation constant.

Options:

A. 5.5×10^{-5}

B. 4.0×10^{-5}

C. 1.8×10^{-5}

D. 1.0×10^{-5}

Answer: D

Solution:

Percent dissociation $= 1.42\%$

$\therefore \quad \alpha = 0.0142$

For a weak monoacidic base,

$$\begin{aligned} K_b &= \alpha^2 C \\ &= (0.0142)^2 \times (0.05) \\ &= 1.0082 \times 10^{-5} \end{aligned}$$

Question 62

What is the value of percent atom economy when an organic compound of formula weight $75 \mathrm{~u}$ is obtained from reactants having sum formula weight $225 \mathrm{~u}$?

Options:

A. 13.5

B. 33.3

C. 40.4

D. 70.5

Answer: B

Solution:

% atom economy

$$= \frac{\text{Formula weight of the desired product}}{\text{Sum of formula weight of all the reactants used in the reaction}} \times 100$$

$$\%\text{ atom economy } = \frac{75}{225} \times 100 = 33.3$$

Question 63

Calculate ΔS_{total} for the following reaction at $300 \mathrm{~K}$.

$$NH_4NO_{3(s)} \longrightarrow NH^+_{(aq)} + NO^-_{3(aq)}$$

$$\left(\Delta H = 28.1 \text{ kJ mol}^{-1}, \Delta S_{sys} = 108.7 \text{ J K}^{-1} \text{ mol}^{-1}\right)$$

Options:

A. $15.1 \text{ J K}^{-1} \text{ mol}^{-1}$

B. $93.6 \text{ J K}^{-1} \text{ mol}^{-1}$

C. $84.3 \text{ J K}^{-1} \text{ mol}^{-1}$

D. $202.3 \text{ J K}^{-1} \text{ mol}^{-1}$

Answer: A

Solution:

$$\Delta S_{surr} = \frac{Q_{rev}}{T} = \frac{-\Delta H}{T}$$
$$= \frac{-28.1 \times 10^3 \text{ J mol}^{-1}}{300 \text{ K}}$$
$$= -93.67 \text{ J K}^{-1} \text{ mol}^{-1}$$
$$\Delta S_{total} = \Delta S_{sys} + \Delta S_{surr}$$
$$= 108.7 + (-93.67)$$
$$= 15.03 \text{ J K}^{-1} \text{ mol}^{-1}$$

Question 64

Which from following properties is NOT exhibited by LDP?

Options:

A. It is crystalline.

B. It is moisture resistant.

C. LDP films are extremely flexible.

D. It is poor conductor of electricity.

Answer: A

Solution:

HDP (High density polyethylene) is crystalline.

Question 65

Identify the FALSE statement about ideal solution from following.

Options:

A. Ideal solutions obey Raoult's law over entire range of concentration.

B. No heat is evolved or absorbed when two components forming an ideal solution are mixed.

C. Volume of ideal solution is same as sum of volumes of two components taken for mixing.

D. The vapour pressure of ideal solution is either higher or lower than vapour pressure of pure components.

Answer: D

Solution:

The vapour pressure of ideal solution always lies between vapour pressures of pure components.

Question 66

Which from following is NOT an example of amorphous solid?

Options:

A. Glass

B. Plastic

C. Rubber

D. Diamond

Answer: D

Solution:

Diamond is a crystalline solid whereas glass, plastic and rubber are amorphous solids.

Question 67

Which of the following statements is NOT true about Bohr atomic model?

Options:

A. An electron in hydrogen atom can move around the nucleus in one of the many possible orbits of fixed radius and energy.

B. The energy of an electron in the orbit does not change with time.

C. An electron can move only in those orbits for which angular momentum is integral multiple of $\frac{h}{2\pi}$.

D. This model can explain the ability of atoms to form molecules by chemical bonds.

Answer: D

Solution:

Bohr atomic model could not explain the ability of atoms to form molecules by chemical bonds.

Question 68

Calculate the rate constant of the first order reaction if 80% of the reactant reacted in 15 minute.

Options:

A. 0.11 minute^{-1}

B. 0.22 minute^{-1}

C. 0.34 minute^{-1}

D. 0.42 minute^{-1}

Answer: A

Solution:

80% of the reactant has reacted.

So, if $[A]_0 = 100$, then $[A]_\mathrm{t} = 100 - 80 = 20$

$$\begin{aligned} \mathrm{k} &= \frac{2.303}{\mathrm{t}} \log_{10} \frac{[\mathrm{A}]_0}{[\mathrm{A}]_\mathrm{t}} \\ &= \frac{2.303}{15} \log_{10} \frac{100}{20} \\ &= \frac{2.303}{15} \log_{10}(5) \\ &= \frac{2.303}{15} \times 0.699 \\ &= 0.1073 \\ &\approx 0.11 \text{ minute }^{-1} \end{aligned}$$

Question 69

Calculate the degree of dissociation of $0.01 \mathrm{~M}$ acetic acid at $25^{\circ}\mathrm{C}\left[\Lambda_\mathrm{c} = 15.0 \Omega^{-1} \mathrm{~cm}^2 \mathrm{~mol}^{-1}\right.$ and $\left.\Lambda_0 = 300 \Omega^{-1} \mathrm{~cm}^2 \mathrm{~mol}^{-1}\right]$

Options:

A. 0.042

B. 0.035

C. 0.025

D. 0.05

Answer: D

Solution:

Degree of dissociation $(\alpha) = \frac{\Lambda_c}{\Lambda_0} = \frac{15.0}{300} = 0.05$

Question 70

Which element from following does NOT exhibit spin only magnetic moment in +3 state?

Options:

A. Cr

B. V

C. Ti

D. Sc

Answer: D

Solution:

	Ions	Outer shell electronic configuration	No. of unpaired electrons
(A)	Cr^{3+}	$3d^3$	3
(B)	V^{3+}	$3d^2$	2
(C)	Ti^{3+}	$3d^1$	1
(D)	Sc^{3+}	$3d^0$	0

Sc^{3+} is does not exhibit spin-only magnetic moment because of the absence of unpaired electrons.

Question 71

Identify the final product formed on ammonolysis of benzyl chloride followed by the reaction with two moles of CH_3I.

Options:

A. $C_6H_5CH_2NHCH_3$

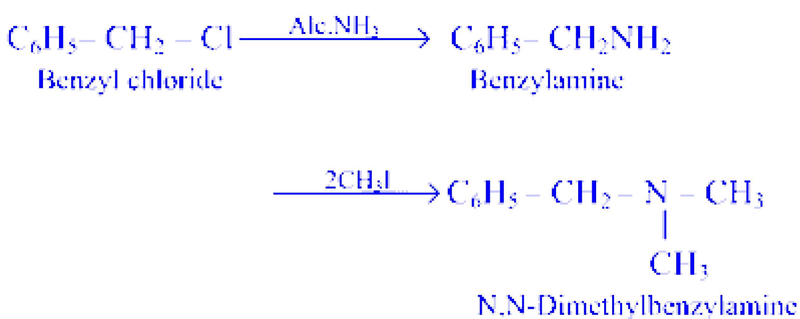
B. $C_6H_5CH_2NH_2$

C. $C_6H_5N(CH_3)_2$

D. $C_6H_5CH_2N(CH_3)_2$

Answer: D

Solution:



Question 72

Which from following elements is isoelectronic with Na^+?

Options:

A. F

B. O

C. Mg

D. Ne

Answer: D

Solution:

Atoms and ions having the same number of electrons are isoelectronic.

Species	No. of electrons
Na^+	$11 - 1 = 10$
F	9
O	8
Mg	12
Ne	10

Question 73

Which of the following is positively charged sol?

Options:

A. Haemoglobin in blood

B. Sol of starch

C. Gelatin

D. 'Ag' sol

Answer: A

Solution:

Haemoglobin in blood is a positively charged sol whereas sol of starch, gelatin sol and silver sol are negatively charged sols.

Question 74

When tert-butyl bromide is heated with silver fluoride, the major product obtained is:

Options:

A. 1-Fluoro-2-methylpropane

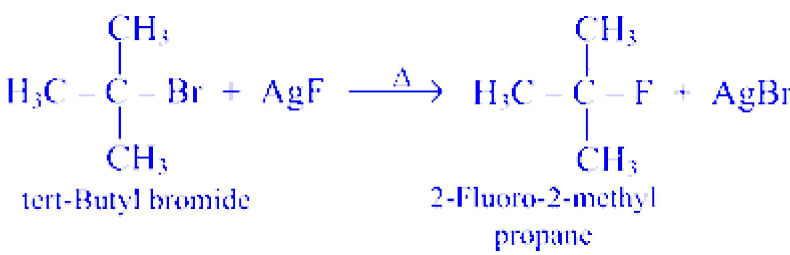
B. 2-Fluoro-2-methylpropane

C. 1-Fluorobutane

D. 2-Fluorobutane

Answer: B

Solution:



This is Swartz reaction.

Question 75

Which among the following is NOT a feature of S_N2 mechanism?

Options:

A. Single step mechanism

B. Backside attack of nucleophile

C. Formation of planar carbocation intermediate

D. Involves simultaneous bond breaking and bond forming

Answer: C

Question 76

What is the pH of 0.005 M NaOH solution?

Options:

A. 2.30

B. 12.6

C. 11.7

D. 3.2

Answer: C

Solution:

$$\underset{0.005\ \text{M}}{\text{NaOH}} \longrightarrow \underset{0.005\ \text{M}}{\text{Na}^{+}} + \underset{0.005\ \text{M}}{\text{OH}^{-}}$$

$$\begin{aligned} \text{pOH} &= -\log_{10}[\text{OH}] \\ &= -\log_{10}[0.005] \\ &= -\log_{10}\left(5 \times 10^{-3}\right) \\ &= -\left[\log 10^{-3} + \log 5\right] \\ &= -[-3 + 0.699] \\ &= -[-2.301] \\ &= 2.301 \\ \text{pOH} + \text{pH} &= 14 \\ \therefore \quad \text{pH} &= 14 - 2.301 \\ &= 11.699 \\ &\approx 11.7 \end{aligned}$$

Question 77

What is the oxidation number of sulfur in H_2SO_5 ?

Options:

A. +4

B. +6

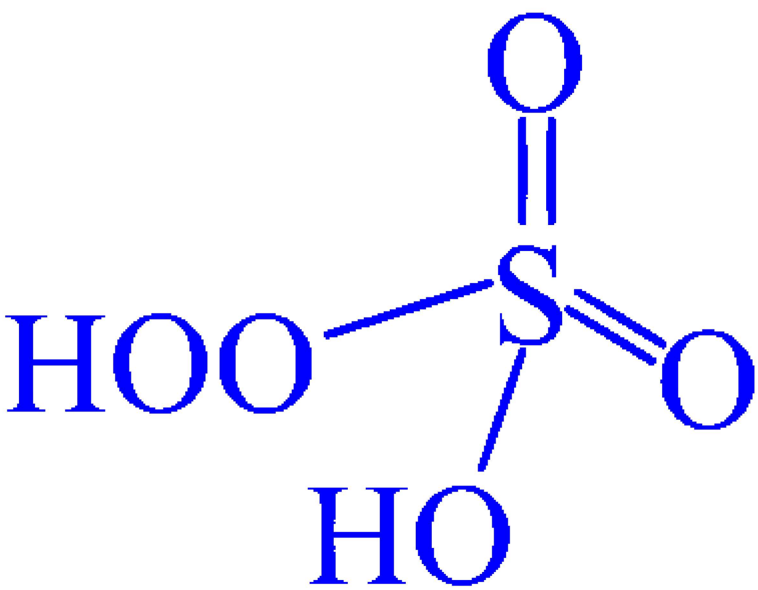
C. +8

D. +5

Answer: B

Solution:

H_2SO_5 : Peroxymonosulfuric acid



It has a peroxide linkage.

$$(2 \times 1) + x + (3 \times -2) + (2 \times -1) = 0$$
$$\therefore \quad x = +6$$

Question 78

If N_2 gas is compressed at 2 atmosphere from 9.0 L to 3.0 L at 300 K, find the final pressure at same temperature.

Options:

A. 1.66 atm

B. 3.32 atm

C. 6.0 atm

D. 9.0 atm

Answer: C

Solution:

According to Boyle's law,

$P_1 V_1 = P_2 V_2$ (at constant n and T)

$\therefore \quad P_2 = \frac{P_1 V_1}{V_2} = \frac{2 \times 9.0}{3.0} = 6.0$ bar

At constant temperature, the volume of a given amount of gas is inversely proportional to its pressure. Therefore, if the volume is reduced to one-third of its original value, the pressure will correspondingly increase by a factor of three. Hence, correct answer is option (C).

Question 79

What is the number of moles of secondary carbon atoms in n mole isopentane?

Options:

A. 4n

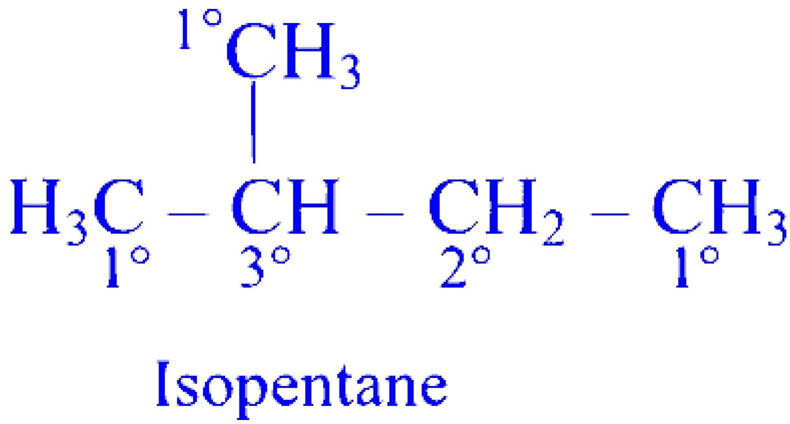
B. 3n

C. 2n

D. n

Answer: D

Solution:



1 molecule of isopentane has 1 secondary carbon atom.

$\therefore$ 1 mol isopentane $=$ 1 mol secondary carbon atoms

$\therefore$ n mol isopentane $=$ n mol secondary carbon atoms

---

Question 80

Which from following substances consists of total 1 mole atoms in it? (Molar mass of $NH_3 = 17, H_2O = 18,\ N_2 = 28, CO_2 = 44$)

Options:

A. 4.25 g NH_3

B. 1.8 g H_2O

C. 2.8 g N_2

D. 4.4 g CO_2

Answer: A

Solution:

$4.25 \text{ g } NH_3 = \frac{4.25}{17} \text{ mol } NH_3$
$= 0.25 \text{ mol } NH_3$
$1 \text{ mol } NH_3 = 1 \text{ mol N-atoms } + 3 \text{ mol H-atoms}$
$\therefore \quad 0.25 \text{ mol } NH_3 = 0.25 \times 4$
$= 1 \text{ mol atoms}$

Question 81

Identify the formula of potassium trioxalatoaluminate(III).

Options:

A. $K_3\left[Al(C_2O_4)_3\right]$

B. $Al\left[K_3(C_2O_4)_3\right]$

C. $K_3[Al(C_2O_4)_3]^{2-}$

D. $K_4[Al_3(C_2O_4)_3]^{2+}$

Answer: A

Solution:

The formula for potassium trioxalatoaluminate(III) is correctly represented by Option A :

$K_3\left[Al(C_2O_4)_3\right]$

This formula indicates that the compound is composed of a complex ion of aluminum trioxalato (with aluminum in a +3 oxidation state, as indicated by the "(III)" in the name) and three potassium ions (K^+) to balance the charge of the complex ion. The oxalate ion $C_2O_4^{2-}$ is a bidentate ligand, meaning it forms two bonds with the central aluminum ion. Since there are three oxalate ions, each carrying a -2 charge, the total charge of the complex ion is -6, which is balanced by three K^+ ions, each carrying a +1 charge, resulting in a neutral overall compound.

Question 82

If, Aniline $\xrightarrow[\text{ii) } H_2O,\Delta]{\text{i) } NaNO_2+HCl,273\text{ K}}$ Product.

Identify the product of above reaction.

Options:

A. o-Nitroaniline

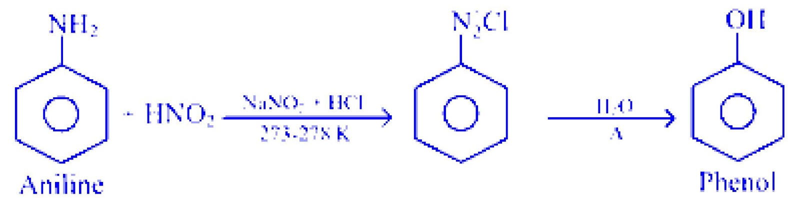
B. m-Nitroaniline

C. p-Nitroaniline

D. Phenol

Answer: D

Solution:



Question 83

Identify nonbenzenoid aromatic compound from following.

Options:

A. Aniline

B. Tropone

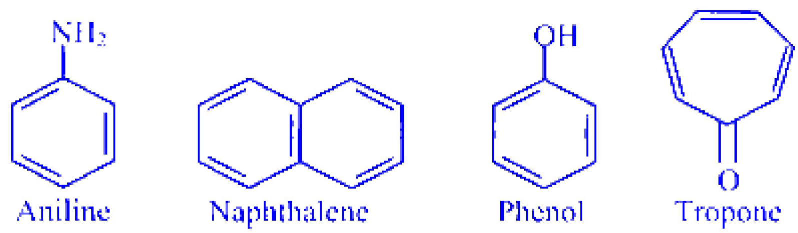
C. Naphthalene

D. Phenol

Answer: B

Solution:

Aniline, naphthalene and phenol are benzenoid aromatic compounds whereas tropone is a nonbenzenoid aromatic compound.



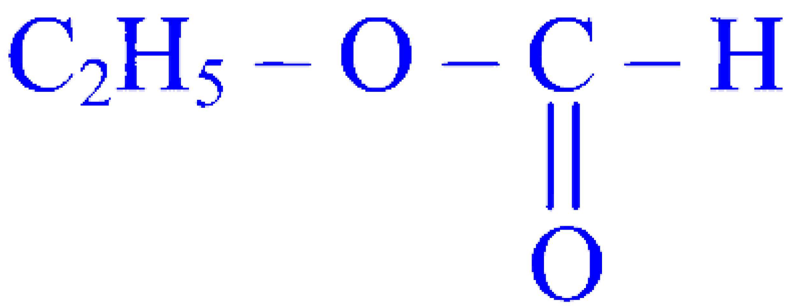
Question 84

Methyl propanoate on hydrolysis with dil NaOH forms a salt which on further acidification with conc. HCl forms _________.

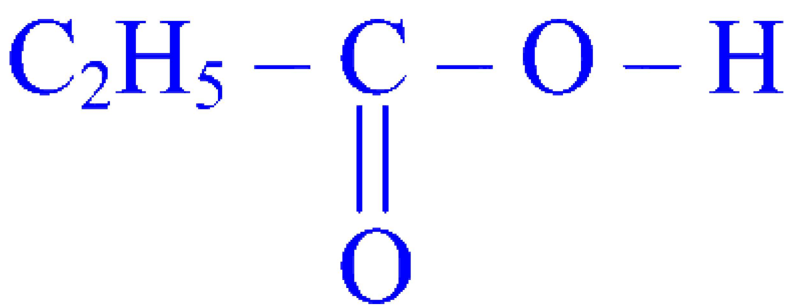
Options:

A. $CH_3 - COOH$

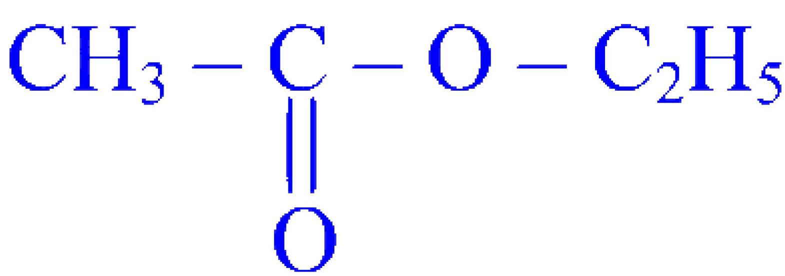
B.



C.

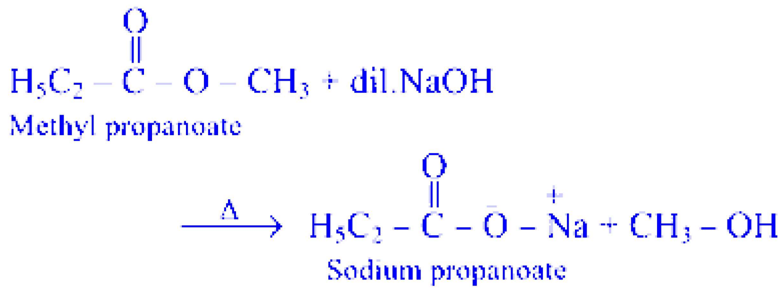


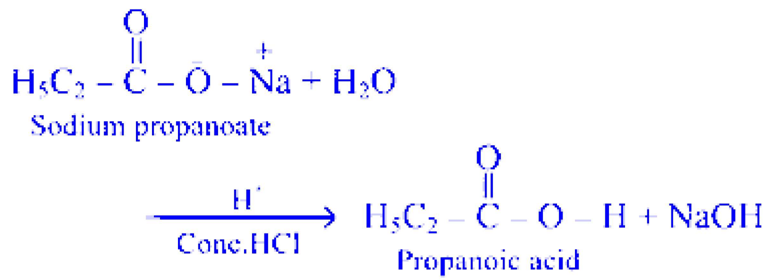
D.



Answer: C

Solution:





Question 85

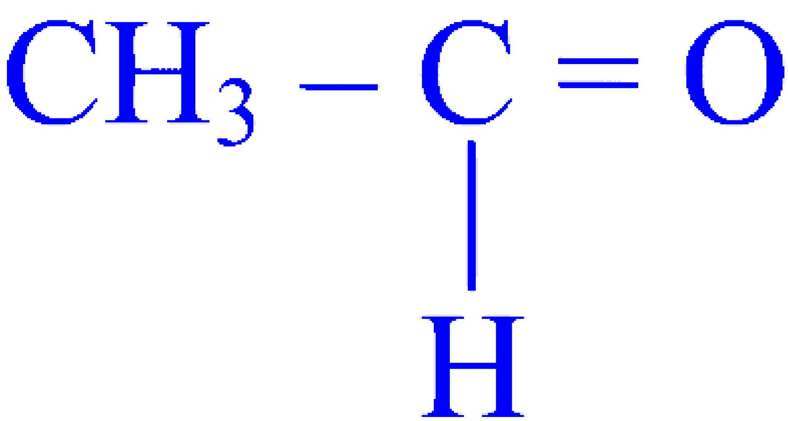
Identify the product obtained in the following reaction.

$(CH_3CO)_2O \xrightarrow{H_2O}$ Product

Options:

A. CH_3COCH_3

B.



C. $CH_3 - OH$

D. CH_3COOH

Answer: D

Solution:

Anhydrides on hydrolysis with water give carboxylic acids.

$(CH_3CO)_2O \xrightarrow{H_2O} 2CH_3COOH$

Question 86

Identify the expression for average rate for following reaction.

$$N_{2(g)} + 3H_{2(g)} \rightarrow 2NH_{3(g)}$$

Options:

A. $\frac{-\Delta[N_2]}{\Delta t} = -\frac{1}{3}\frac{\Delta[H_2]}{\Delta t} = \frac{1}{2}\frac{\Delta[NH_3]}{\Delta t}$

B. $-\frac{1}{3}\frac{\Delta[N_2]}{\Delta t} = \frac{\Delta[H_2]}{\Delta t} = \frac{1}{2}\frac{\Delta[NH_3]}{\Delta t}$

C. $\frac{-\Delta[N_2]}{\Delta t} = \frac{-\Delta[H_2]}{\Delta t} = \frac{\Delta[NH_3]}{\Delta t}$

D. $-\frac{1}{2}\frac{\Delta[N_2]}{\Delta t} = \frac{-\Delta[H_2]}{\Delta t} = \frac{1}{3}\frac{\Delta[NH_3]}{\Delta t}$

Answer: A

Solution:

In the rate expression, always make sure to express the rates with the correct sign: Negative for the reactants and positive for the products.

Question 87

The reaction of aryl halide with alkyl halide and sodium metal in dry ether to form substituted aromatic compounds is known as:

Options:

A. Wurtz reaction

B. Fittig reaction

C. Wurtz Fittig reaction

D. Friedel Craft's reaction

Answer: C

Question 88

Identify anionic complex from following.

Options:

A. Pentaammineaquacobalt(III) iodide

B. Pentamminecarbonatocabalt(III) chloride

C. Tetracyanonickelate(II) ion

D. Triamminetrinitrocobalt(III)

Answer: C

Solution:

Tetracyanonickelate(II) ion: $[Ni(CN)_4]^{2-}$

Question 89

Identify 'A' and 'B' in the following reaction.

$$CH_3Br \xrightarrow{AgNO_2} A \xrightarrow{Sn/HCl} B$$

Options:

A. CH_3NO_2 and CH_3Cl

B. CH_3NO_2 and CH_3NH_2

C. CH_3NH_2 and CH_3Cl

D. CH_3NH_2 and $CH_3CH_2NO_2$

Answer: B

Solution:

$$\underset{\text{Bromomethane}}{CH_3Br} \xrightarrow{AgNO_2} \underset{\text{Nitromethane}}{CH_3NO_2} \xrightarrow{Sn/HCl} \underset{\text{Methanamine}}{CH_3NH_2}$$

Question 90

Calculate the molar mass of metal having density $9.3\ g\ cm^{-3}$ that forms simple cubic unit cell. $\left[a^3 \cdot N_A = 22.6\ cm^3\ mol^{-1}\right]$

Options:

A. $210.2\ g\ mol^{-1}$

B. $105.3\ g\ mol^{-1}$

C. $52.6\ g\ mol^{-1}$

D. $70.2\ g\ mol^{-1}$

Answer: A

Solution:

For simple cubic unit cell, $n = 1$.

$$\text{Density } (\rho) = \frac{M\text{ n}}{a^3 N_A}$$
$$9.3 = \frac{M \times 1}{22.6}$$
$$M = \frac{9.3 \times 22.6}{1} = 210.2 \text{ g mol}^{-1}$$

Question 91

Calculate the $\mathrm{E}^{\circ}_{\text{cell}}$ **for** $\mathrm{Zn}_{(\mathrm{s})}\left|\mathrm{Zn}^{++}_{(\mathrm{IM})}\right|\left|\mathrm{Cd}^{++}_{(\mathrm{IM})}\right|\mathrm{Cd}_{(\mathrm{s})}$ **at** $25^{\circ}\mathrm{C}\left[\mathrm{E}^{0}_{\mathrm{Zn}} = -0.763\ \mathrm{V}; \mathrm{E}^{\circ}_{\mathrm{Cd}} = -0.403\ \mathrm{V}\right]$

Options:

A. 0.36 V

B. 1.17 V

C. -0.36 V

D. -1.17 V

Answer: A

Solution:

For the given cell reaction, anode is Zn and cathode is Cd.

$$\begin{aligned}\mathrm{E}^{0}_{\text{cell}} &= \mathrm{E}^{0}_{\text{cathode}} - \mathrm{E}^{0}_{\text{anode}} \\ &= -0.403 - (-0.763) \\ &= 0.36\ \mathrm{V}\end{aligned}$$

Question 92

What is the work done during oxidation of 4 moles of $\mathrm{SO}_{2(\mathrm{g})}$ **to** $\mathrm{SO}_{3(\mathrm{g})}$ **at** $27^{\circ}\mathrm{C}$**?**

$\left(\mathrm{R}=8.314 \mathrm{~J} \mathrm{~K}^{-1(\mathrm{~g})} \mathrm{mol}^{-1}\right)$

Options:

A. $4.988 \mathrm{~kJ}$

B. $-1.125 \mathrm{~kJ}$

C. $3.234 \mathrm{~kJ}$

D. $-4.988 \mathrm{~kJ}$

Answer: A

Solution:

For oxidation of 4 moles of SO_2, the reaction is given as follows:

$4 \mathrm{SO}_{2(\mathrm{~g})}+2 \mathrm{O}_{2(\mathrm{~g})} \longrightarrow 4 \mathrm{SO}_{3(\mathrm{~g})}$

$\Delta \mathrm{n}_{\mathrm{g}}=$ (moles of product gases) $-$ (moles of reactant gases)

$$
\begin{aligned}
\Delta \mathrm{n}_{\mathrm{g}} & =4-6=-2 \mathrm{~mol} \\
\mathrm{W} & =-\Delta \mathrm{n}_{\mathrm{g}} \mathrm{RT} \\
& =-(-2 \mathrm{~mol}) \times 8.314 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1} \times 300 \mathrm{~K} \\
& =+4988.4 \mathrm{~J} \\
& =+4.988 \mathrm{~kJ}
\end{aligned}
$$

Question 93

Identify the type of system if boiling water is kept in a half filled closed vessel.

Options:

A. Homogeneous closed system

B. Heterogeneous closed system

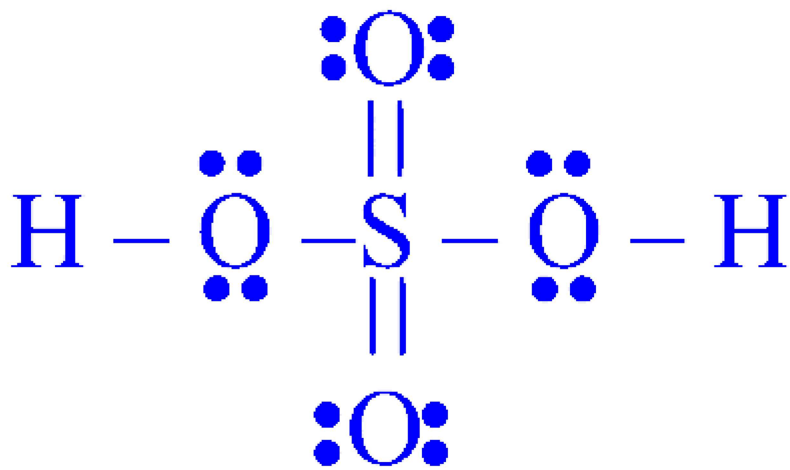
C. Homogeneous isolated system

D. Heterogeneous isolated system

Answer: B

Question 94

What is the formal charge on sulfur in following Lewis structure?



Options:

A. 2

B. -2

C. 0

D. -1

Answer: C

Solution:

Formal Charge $(FC) =$ V.E. - N.E. $-1/2$ (B.E.)

FC on S $= 6 - 0 - 1/2(12) = 0$

Question 95

Identify weakest halogen acid from following.

Options:

A. HCl

B. HI

C. HF

D. HBr

Answer: C

Solution:

Acidic strength of halogen acids increases in the order : $HF < HCl < HBr < HI$

Hence, the weakest halogen acid is HF.

Question 96

Which of the following phenomena is NOT explained by the open chain structure of glucose?

Options:

A. Formation of pentaacetate of glucose with acetic anhydride.

B. Formation of oxime with hydroxylamine.

C. Formation of silver mirror with Tollen's regent.

D. Existence of α and β forms of glucose.

Answer: D

Solution:

Glucose exists in two cyclic forms, α and β, which are formed by intramolecular hemiacetal formation between the aldehyde group and one of the alcoholic group at $C-5$ within the molecule. Thus, the open chain structure of glucose does not explain the existence of α and β forms of glucose.

Question 97

Which from following polymers is obtained from isoprene?

Options:

A. Teflon

B. Natural rubber

C. Viscose rayon

D. Cuprammonium rayon

Answer: B

Solution:

Natural rubber is a high molecular mass linear polymer of isoprene (2-methylbuta-1,3-diene).

Question 98

Find the radius of an atom in fcc unit cell having edge length $405 \mathrm{pm}$.

Options:

A. 202.5 pm

B. 175.3 pm

C. 143.2 pm

D. 181.0 pm

Answer: C

Solution:

For fcc crystal structure, $r=\frac{\sqrt{2}}{4} a$

$\therefore \quad r = \frac{1.414 \times 405}{4} = 143.2$ pm

Question 99

Which from following cations in their respective oxidation states develops colourless aqueous solution?

Options:

A. Fe^{3+}

B. Fe^{2+}

C. Cu^{2+}

D. Cu^{+}

Answer: D

Solution:

Ion	Outer electronic configuration	No. of unpaired electrons
Fe^{3+}	$3d^5$	5
Fe^{2+}	$3d^6$	4
Cu^{2+}	$3d^9$	1
Cu^{+}	$3d^{10}$	0

Cu^{+} is colourless because of the absence of unpaired electrons.

Question 100

Calculate osmotic pressure of $0.2\ \mathrm{M}$ aqueous KCl solution at $0^{\circ}\mathrm{C}$ if van't Hoff factor for KCl is 1.83. $\left[\mathrm{R} = 0.082\ \mathrm{dm}^3\ \mathrm{atm\ mol}^{-1}\ \mathrm{K}^{-1}\right]$

Options:

A. 8.2 atm

B. 9.4 atm

C. 10.6 atm

D. 6.5 atm

Answer: A

Solution:

$$\begin{aligned} \pi &= \text{iMRT} \\ &= 1.83 \times 0.2 \times 0.082 \times 273 \\ &= 8.2 \text{ atm} \end{aligned}$$

Physics

Question 101

The magnetic flux through a circuit of resistance 'R' changes by an amount $\Delta\phi$ in the time Δt. The total quantity of electric charge 'Q' which passes during this time through any point of the circuit is

Options:

A. $-\frac{\Delta\phi}{\Delta t} + R$

B. $\frac{\Delta\phi}{\text{R}}$

C. $\frac{\Delta\phi}{\Delta t}$

D. $\frac{\Delta\phi}{\Delta t} \times R$

Answer: B

Solution:

According to Faraday's law of electromagnetic induction,

$$\varepsilon=\frac{\Delta\phi}{\Delta t}$$
$$\mathrm{IR}=\frac{\Delta\phi}{\Delta \mathrm{t}}$$
$$\mathrm{I}=\frac{\Delta\phi}{\Delta \mathrm{t}\times \mathrm{R}}$$
$$\mathrm{I}\times\Delta \mathrm{t}=\frac{\Delta\phi}{\mathrm{R}}$$

$\therefore$ The total quantity of electric charge passing through the circuit is

$\mathrm{Q}=\frac{\Delta\phi}{\mathrm{R}}$

Question 102

A beam of light of wavelength $600\ \mathrm{nm}$ from a distant source falls on a single slit $1\ \mathrm{mm}$ wide and the resulting diffraction pattern is observed on a screen $2\ \mathrm{m}$ away. The distance between the first dark fringe on either side of the central bright fringe is

Options:

A. 1.2 mm

B. 2.4 mm

C. 1.2 cm

D. 2.4 cm

Answer: B

Solution:

The distance between the central bright fringe and the first dark fringe is given as:

$$y_{n_d}=\frac{n\lambda D}{d}$$
$$y_{1_d}=\frac{1\times 600\times 10^{-9}\times 2}{10^{-3}}$$
$$=1.2\times 10^{-3}=1.2\ \mathrm{mm}$$

$\therefore$ The distance between the first dark fringe on either side of the central bright fringe is $2\mathrm{y_n} = 2 \times 1.2 = 2.4\ \mathrm{mm}$

Question 103

An electron of mass 'm' and charge 'q' is accelerated from rest in a uniform electric field of strength 'E'. The velocity acquired by the electron, when it travels a distance 'L', is

Options:

A. $\sqrt{\frac{2q\ \mathrm{m}}{\mathrm{mL}}}$

B. $\sqrt{\frac{2\mathrm{qEL}}{\mathrm{m}}}$

C. $\sqrt{\frac{2\mathrm{Em}}{\mathrm{qL}}}$

D. $\sqrt{\frac{\mathrm{qE}}{\mathrm{mL}}}$

Answer: B

Solution:

We know

$F = ma$ and $F = qE$

$$\mathrm{qE} = \mathrm{ma}$$

$$\therefore \quad \frac{\mathrm{qE}}{\mathrm{m}} = \mathrm{a}$$

According to equation of motion,

$$v^2 - u^2 = 2aL$$

$$v^2 - 0^2 = 2aL$$

$$v^2 = 2aL$$

$$v = \sqrt{2aL}$$

$$\therefore \quad v = \sqrt{\frac{2qEL}{m}}$$

Question 104

Two bodies have their moments of inertia I and 2I respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

Options:

A. $1 : 2$

B. $\sqrt{2} : 1$

C. $2 : 1$

D. $1 : \sqrt{2}$

Answer: D

Solution:

The equation for angular momentum is

$$\mathrm{L} = \sqrt{2\,\mathrm{K}_{\mathrm{Rot}} \times \mathrm{I}}$$

So, $\mathrm{L} \propto \sqrt{\mathrm{I}}$

$\therefore$ The ratio of angular momentum of the two bodies is

$$\frac{\mathrm{L}_1}{\mathrm{L}_2} = \sqrt{\frac{\mathrm{I}_1}{\mathrm{I}_2}}$$

$\frac{L_1}{L_2} = \sqrt{\frac{I}{2I}} \quad \ldots \text{(given } I_2 = 2I)$

$\therefore \quad \frac{L_1}{L_2} = \frac{1}{\sqrt{2}}$

Question 105

A ball kept at 20 m height falls freely in vertically downward direction and hits the ground. The coefficient of restitution is 0.4. Velocity of the ball first rebound is $\left[g = 10 \text{ ms}^{-2}\right]$

Options:

A. 4 ms^{-1}

B. 8 ms^{-1}

C. 12 ms^{-1}

D. 16 ms^{-1}

Answer: B

Solution:

$$\begin{aligned} & v^2 = 0 + 2gh \quad \ldots \left(\because v^2 - u^2 = 2gh\right) \\ & v^2 = 2 \times 10 \times 20 = 400 \\ \therefore \quad & v = 20 \text{ m/s} \\ & e = \frac{u}{v} \end{aligned}$$

$\therefore$ The velocity of the body after the first rebound is

$u = 0.4 \times 20$
$u = 8 \text{ m/s}$

Question 106

Two long conductors separated by a distance 'd' carry currents I_1 and I_2 in the same direction. They exert a force 'F' on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance between them is also increased to 3 d. The new value of force between them is

Options:

A. $-2\ \mathrm{F}$

B. $\frac{\mathrm{F}}{3}$

C. $\frac{-2\ \mathrm{F}}{3}$

D. $\frac{-\mathrm{F}}{3}$

Answer: C

Solution:

The force per unit length of the conductors is given as:

$\mathrm{F}=\frac{\mu_0 \mathrm{I}_1 \mathrm{I}_2}{2 \pi \mathrm{d}}$

When the value and direction of current in the first conductor and the distance between the conductors are changed,

$$\therefore \quad \mathrm{F}_2=\frac{-\mu_0 2 \mathrm{I}_1 \mathrm{I}_2}{2 \pi \times 3\ \mathrm{d}}$$

$$\therefore \quad \frac{\mathrm{F}_2}{\mathrm{F}}=\frac{-2}{3}$$

$$\mathrm{F}_2=-\frac{2\ \mathrm{F}}{3}$$

Question 107

The a.c. source is connected to series LCR circuit. If voltage across R is 40 V, that across L is 80 V and that across C is 40 V, then the e.m.f. 'e' of a.c. source is

Options:

A. 40 V

B. $40\sqrt{2}$ V

C. 80 V

D. 160 V

Answer: B

Solution:

The emf across the AC source is given as:

$$\begin{aligned} & \mathrm{e}=\sqrt{\left(\mathrm{V}_{\mathrm{R}}\right)^2+\left(\mathrm{V}_{\mathrm{C}}-\mathrm{V}_{\mathrm{L}}\right)^2}=\sqrt{(40)^2+(80-40)^2} \\ & \mathrm{e}=\sqrt{3200} \\ \therefore \quad & \mathrm{e}=40 \sqrt{2} \mathrm{~V} \end{aligned}$$

Question 108

In the study of transistor as an amplifier if $\alpha=\frac{I_C}{I_E}=0.98$ and $\beta=\frac{I_C}{I_B}=49$, where I_C, I_B and I_{E} are collector, base and emitter current respectively then $\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)$ is equal to

Options:

A. zero

B. $\frac{1}{2}$

C. 2

D. 1

Answer: D

Solution:

$$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{1}{0.98}-\frac{1}{4.9}$$
$$=1.204-0.204=1$$

Question 109

A liquid drop of radius 'R' is broken into 'n' identical small droplets. The work done is [T = surface tension of the liquid]

Options:

A. $4\pi\mathrm{R}^2\left(\mathrm{n}^{\frac{2}{3}}-1\right)\mathrm{T}$

B. $4\pi R^2\left(n^{\frac{1}{3}}-1\right)T$

C. $4\pi R^2\left(1-n^{\frac{1}{3}}\right)T$

D. $4\pi\mathrm{R}^2\left(1-\mathrm{n}^{\frac{2}{3}}\right)\mathrm{T}$

Answer: B

Solution:

Volume of n smaller droplets $=$ Volume of bigger drop

$$\mathrm{n}\frac{4}{3}\pi\mathrm{r}^3=\frac{4}{3}\pi\mathrm{R}^3$$
$$\therefore \quad \mathrm{R}=\mathrm{n}^{\frac{1}{3}}\cdot\mathrm{r}$$
$$\mathrm{r}=\frac{\mathrm{R}}{\mathrm{n}^{\frac{1}{3}}}$$

Work done

$$W=\left[n\cdot 4\pi r^2-4\pi R^2\right]T$$
$$=4\pi\left[n\cdot\frac{R^2}{n^{\frac{2}{3}}}-R^2\right]T$$
$$=4\pi R^2\left[n^{\frac{1}{3}}-1\right]T$$

Question 110

For a gas, $\frac{\mathrm{R}}{\mathrm{C_v}} = 0 \cdot 4$, where R is universal gas constant and $\mathrm{C_v}$ is molar specific heat at constant volume. The gas is made up of molecules which are

Options:

A. rigid diatomic

B. monoatomic

C. non-rigid diatomic

D. polyatomic

Answer: A

Solution:

$$\text{Given: } \frac{R}{C_V} = 0.4$$

$$C_V = \frac{R}{0.4} = \frac{5R}{2}$$

$$C_P = C_V + R$$

$$\therefore \quad C_P = \frac{7R}{2}$$

$$\gamma = \frac{C_P}{C_v} = \frac{\frac{7}{5}}{\frac{5}{2}}$$

$$\gamma = \frac{7}{5}$$

$\therefore$ The gas is made up of rigid diatomic molecules.

Question 111

Two bodies A and B at temperatures '$\mathrm{T_1}$' K and '$\mathrm{T_2}$' K respectively have the same dimensions. Their emissivities are in the ratio $1 : 3$. If they radiate the same amount of heat per unit area per unit time, then the ratio of their temperatures $(\mathrm{T_1} : \mathrm{T_2})$ is

Options:

A. $1:3$

B. $3^{1/4}:1$

C. $9^{1/4}:1$

D. $81:1$

Answer: B

Solution:

From Stefan - Boltzmann's law

$\frac{dQ}{dt} = e\left(\sigma A T^4\right)$

Given A and $\frac{dQ}{dt}$ are same for both the bodies

$$\Rightarrow e_1 T_1^4 = e_2 T_2^4$$
$$\therefore \left(\frac{T_1}{T_2}\right)^4 = \frac{e_2}{e_1} = \frac{3}{1}$$
$$\Rightarrow \frac{T_1}{T_2} = \frac{\sqrt[4]{3}}{1} = \frac{3^{\frac{1}{4}}}{1}$$

Question 112

In a conical pendulum the bob of mass 'm' moves in a horizontal circle of radius 'r' with uniform speed 'V'. The string of length 'L' describes a cone of semi vertical angle 'θ'. The centripetal force acting on the bob is ($g =$ acceleration due to gravity)

Options:

A. $\frac{mgr}{\sqrt{L^2-r^2}}$

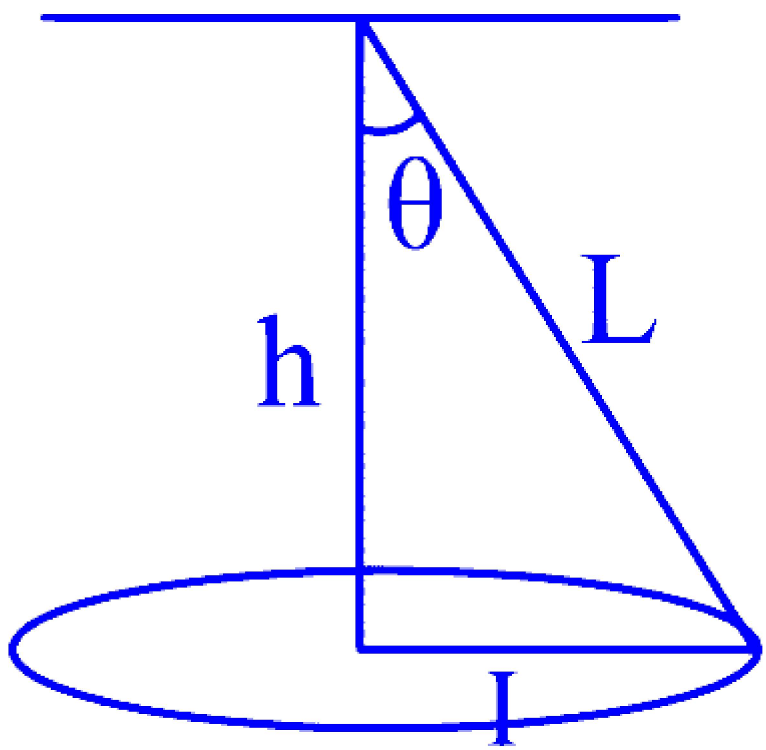
B. $\frac{mgr}{(L^2-r^2)}$

C. $\frac{\sqrt{L^2-r^2}}{mgL}$

D. $\frac{\mathrm{mgL}}{\sqrt{\mathrm{L}^2-\mathrm{r}^2}}$

Answer: A

Solution:



$$\mathrm{T}\cos\theta = \mathrm{mg}$$
$$\sin\theta = \frac{\mathrm{r}}{\mathrm{L}}$$
$$\cos\theta = \frac{\sqrt{\mathrm{L}^2-\mathrm{r}^2}}{\mathrm{L}}$$
$$\therefore \quad \mathrm{T} = \frac{\mathrm{mg}}{\cos\theta} = \frac{\mathrm{mgL}}{\sqrt{\mathrm{L}^2-\mathrm{r}^2}}$$
$$\mathrm{mr}\omega^2 = \mathrm{T}\sin\theta$$

$$\frac{\mathrm{T}\times\mathrm{r}}{\mathrm{L}} = \mathrm{m}\omega^2 \quad \ldots(\sin\theta = \frac{r}{L})$$
$$\omega^2 = \frac{\mathrm{g}}{\sqrt{\mathrm{L}^2-\mathrm{r}^2}}$$

$\therefore$ The centripetal force is

$F = \frac{mgr}{\sqrt{L^2-r^2}}$

Question 113

A fluid of density 'ρ' is flowing through a uniform tube of diameter 'd'. The coefficient of viscosity of the fluid is 'η', then critical velocity of the fluid is

Options:

A. inversely proportional to 'η'

B. directly proportional to 'η'

C. directly proportional to 'd'

D. directly proportional to 'ρ'

Answer: B

Solution:

The equation for critical velocity is given as: $v = k\frac{\eta}{\rho r}$

$\therefore$ The critical velocity is directly proportional to η.

Question 114

The self inductance 'L' of a solenoid of length 'l' and area of cross-section 'A', with a fixed number of turns 'N' increases as

Options:

A. both l and A increase

B. l decreases and A increases

C. l increases and A decreases

D. both l and A decrease

Answer: B

Solution:

The equation for self-inductance is:

$L = \frac{\mu_0 N^2 A}{l}$

$\therefore$ The self-inductance L of a solenoid of length l and area of cross section 'A', with a fixed number of turns 'N' increases as l decreases and A increases.

Question 115

A transverse wave in a medium is given by $y = A \sin 2(\omega t - kx)$. It is found that the magnitude of the maximum velocity of particles in the medium is equal to that of the wave velocity. What is the value of A ?

Options:

A. $\frac{2\lambda}{\pi}$

B. $\frac{\lambda}{\pi}$

C. $\frac{\lambda}{2\pi}$

D. $\frac{\lambda}{4\pi}$

Answer: D

Solution:

The given equation is $y = A \sin 2(\omega t - kx)$

$\therefore$ Velocity of the particle, $v = \frac{dy}{dt}$

$= 2 A\omega \cos 2(\omega t - kx)$

$\therefore$ Maximum velocity $= 2 A\omega$

Velocity of the wave $= \frac{\omega}{k}$

Given $2 A\omega = \frac{\omega}{k}$

$\therefore \quad A = \frac{1}{2k} = \frac{\lambda}{(2\pi)^2} = \frac{\lambda}{4\pi}$

Question 116

The radius of the orbit of a geostationary satellite is (mean radius of earth is 'R', angular velocity about own axis is 'ω' and acceleration due to gravity on earth's surface is 'g')

Options:

A. $\left(\frac{\mathrm{gR}^2}{\omega^2}\right)^{\frac{1}{3}}$

B. $\left(\frac{\mathrm{gR}^2}{\omega^2}\right)^{\frac{2}{3}}$

C. $\left(\frac{\mathrm{gR}^2}{\omega^2}\right)^{\frac{1}{2}}$

D. $\frac{\mathrm{gR}^2}{\omega^2}$

Answer: A

Solution:

$\mathrm{mr}\,\omega^2=\frac{GMm}{r^2}$

$\omega^2=\frac{GM}{r^3}=\frac{gR^2}{r^3} \quad \ldots.\left(\because g=\frac{GM}{R^2}\right)$

$\therefore$ Radius of the orbit of the satellite is:

$r=\left(\frac{gR^2}{\omega^2}\right)^{\frac{1}{3}}$

Question 117

According to Bohr's theory of hydrogen atom, the total energy of the electron in the n^{th} stationary orbit is

Options:

A. directly proportional to n

B. inversely proportional to n,

C. directly proportional to n^2

D. inversely proportional to n^2

Answer: D

Solution:

According to Bohr's theory of hydrogen atom, the equation for total energy of the electron in the n^{th} stationary orbit is,

$$\mathrm{E_n} = \frac{-m\mathrm{Z}^2\mathrm{e}^4}{8\varepsilon_0{}^2\,\mathrm{h}^2\mathrm{n}^2}$$

$$\therefore \quad \mathrm{E_n} \propto \frac{1}{\mathrm{n}^2}$$

Question 118

In a series LCR circuit, $\mathrm{C} = 2\mu\mathrm{F}, \mathrm{L} = 1\mathrm{mH}$ and $\mathrm{R} = 10\Omega$. The ratio of the energies stored in the inductor and the capacitor, when the maximum current flows in the circuit, is

Options:

A. 5 : 1

B. 3 : 2

C. 1 : 2

D. 1 : 5

Answer: A

Solution:

In resonance condition (current is maximum),

$\therefore \quad X_C = X_L$

$\therefore$ The ratio of energies in the inductor and capacitor is:

$$\frac{U_L}{U_C} = \frac{LI^2}{CV^2} = \frac{L}{CR^2} \quad \cdots \cdot \left(\because \frac{I}{V} = \frac{1}{R}\right)$$
$$\frac{U_L}{U_C} = \frac{10^{-3}}{2 \times 10^{-6} \times 10^2}$$
$$\frac{U_L}{U_C} = \frac{5}{1}$$

Question 119

In Young's double slit experiment, the fifth maximum with wavelength 'λ_1' is at a distance 'y_1' and the same maximum with wavelength 'λ_2' is at a distance 'y_2' measured from the central bright band. Then $\frac{y_1}{y_2}$ is equal to [D and d are constant]

Options:

A. $\frac{\lambda_1}{\lambda_2}$

B. $\frac{\lambda_2}{\lambda_1}$

C. $\frac{\lambda_1^2}{\lambda_2^2}$

D. $\frac{\lambda_2^2}{\lambda_1^2}$

Answer: A

Solution:

The equations for the position of the fringe from the central maxima are given as

$$y_1 = \frac{5\lambda_1 D}{d}$$
$$y_2 = \frac{5\lambda_2 D}{d}$$
$$\therefore \quad \frac{y_1}{y_2} = \frac{\lambda_1}{\lambda_2}$$

Question 120

Bohr model is applied to a particle of mass 'm' and charge 'q' moving in a plane under the influence of a transverse magnetic field 'B'. The energy of the charged particle in the n^{th} leve will be $[\mathrm{h} =$ Planck's constant $]$

Options:

A. $\frac{nhqB}{4\pi\mathrm{m}}$

B. $\frac{nhqB}{2\pi m}$

C. $\frac{\mathrm{nhqB}}{\pi\mathrm{m}}$

D. $\frac{2\mathrm{nhqB}}{\pi\mathrm{m}}$

Answer: A

Solution:

We know,

$$\mathrm{mvr} = \frac{\mathrm{nh}}{2\pi}$$

$$\therefore \quad \mathrm{vr} = \frac{\mathrm{nh}}{2\pi\mathrm{m}} \quad \text{.... (i)}$$

Also,

$$\mathrm{qvB} = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

$$\therefore \quad \mathrm{mv} = \mathrm{qBr} \quad \text{.... (ii)}$$

$$\mathrm{mv}^2\mathrm{r} = \mathrm{qBr} \times \frac{\mathrm{nh}}{2\pi\mathrm{m}} \quad \text{....(Multiplying (i) with (ii))}$$

$$\mathrm{E} = \frac{1}{2}\mathrm{mv}^2 = \mathrm{n}\left[\frac{\mathrm{qBh}}{4\pi\mathrm{m}}\right]$$

Question 121

A rectangular block of mass 'm' and crosssectional area A, floats on a liquid of density 'ρ'. It is given a small vertical displacement from equilibrium, it starts oscillating with frequency 'n' equal to ($g =$ acceleration due to gravity)

Options:

A. $\frac{1}{2\pi}\sqrt{\frac{\mathrm{Apg}}{\mathrm{m}}}$

B. $2\pi\sqrt{\frac{\mathrm{Apg}}{\mathrm{m}}}$

C. $\frac{1}{2\pi}\sqrt{\frac{\mathrm{m}}{\mathrm{Apg}}}$

D. $2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{Apg}}}$

Answer: A

Solution:

The formula for the time period is given as $\mathrm{T} = 2\pi\sqrt{\frac{l}{\mathrm{g}}}$

The mass of displaced fluid is

$$\text{Mass} = \text{density} \times \text{volume}$$
$$\mathrm{m} = \rho \times \mathrm{A}l$$

At equilibrium,

Weight of the block $=$ Weight of the displaced liquid

$$\therefore \quad \mathrm{mg} = \mathrm{A}l\rho\mathrm{g}$$
$$\therefore \quad l = \frac{\mathrm{m}}{\mathrm{A}\rho}$$

Substituting the values in the equation

$$\mathrm{T} = 2\pi\sqrt{\frac{l}{\mathrm{g}}}$$
$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{Apg}}}$$

The frequency $f = \frac{1}{T}$

$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{Apg}{m}}$

Question 122

Two spherical conductors of capacities $3\mu F$ and $2\mu F$ are charged to same potential having radii $3\ cm$ and $2\ cm$ respectively. If 'σ_1' and 'σ_2' represent surface density of charge on respective conductors then $\frac{\sigma_1}{\sigma_2}$ is

Options:

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: C

Solution:

We know, $C = \frac{Q}{V}$

As both the charged spheres are at the same potential, the charge on both spheres is

$Q_1 = C_1 V$
$Q_2 = C_2 V$

The charge densities of both spheres are

$\sigma_1 = \frac{Q_1}{A_1} = \frac{C_1 V}{4\pi r_1^2}$

Similiarly,

$\sigma_2 = \frac{Q_2}{A_2} = \frac{C_2 V}{4\pi r_2^2}$

Taking the ratios,

$$\frac{\sigma_2}{\sigma_1} = \frac{C_2 r_1^2}{C_1 r_2^2}$$

$$\frac{\sigma_2}{\sigma_1} = \frac{2 \times 10^{-6} \times (0.03)^2}{3 \times 10^{-6} \times (0.02)^2} = \frac{3}{2}$$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{2}{3}$$

Question 123

A circular arc of radius 'r' carrying current 'I' subtends an angle $\frac{\pi}{16}$ at its centre. The radius of a metal wire is uniform. The magnetic induction at the centre of circular arc is $[\mu_0 =$ permeability of free space]

Options:

A. $\frac{\mu_0 \mathrm{I}}{32\mathrm{r}}$

B. $\frac{\mu_0 \mathrm{I}}{16\mathrm{r}}$

C. $\frac{\mu_0 \mathrm{I}}{64\mathrm{r}}$

D. $\frac{\mu_0 \mathrm{I}}{8\mathrm{r}}$

Answer: C

Solution:

The magnetic field due to current carrying circular arc is $B = \frac{\mu_0 I}{2r}\left(\frac{\theta}{2\pi}\right)$

Here, $\theta = \frac{\pi}{16}$

$$\therefore \quad \mathrm{B} = \frac{\mu_0 \mathrm{I}}{2\mathrm{r}}\left(\frac{1}{2\pi} \times \frac{\pi}{16}\right)$$

$$\mathrm{B} = \frac{\mu_0 \mathrm{I}}{2\mathrm{r}}\left(\frac{1}{32}\right)$$

$$\mathrm{B} = \frac{\mu_0 \mathrm{I}}{64\mathrm{r}}$$

Question 124

A sound of frequency $480 \mathrm{~Hz}$ is emitted from the stringed instrument. The velocity of sound in air is $320 \mathrm{~m/s}$. After completing 180 vibrations, the distance covered by a wave is

Options:

A. 60 m

B. 90 m

C. 120 m

D. 180 m

Answer: C

Solution:

Given: $\mathrm{v}=320 \mathrm{~m/s}, \mathrm{f}=480 \mathrm{~Hz}, \mathrm{~N}=180$

$$\begin{aligned} & \mathrm{v}=\mathrm{f} \lambda \\ \therefore \quad & \lambda=\frac{\mathrm{v}}{\mathrm{f}} \end{aligned}$$

Substituting the values, we get

$\lambda=\frac{320}{480}=\frac{2}{3}$

$\therefore$ The total distance covered after 180 vibrations is

$$\begin{aligned} & D=N \times \lambda \\ & D=180 \times \frac{2}{3} \\ & D=120 \mathrm{~m} \end{aligned}$$

Question 125

A sonometer wire 'A' of diameter 'd' under tension 'T' having density 'ρ_1' vibrates with fundamental frequency 'n'. If we use another wire 'B

' which vibrates with same frequency under tension '$2 \mathrm{~T}$' and diameter '$2 \mathrm{D}$' then density 'ρ_2' of wire 'B' will be

Options:

A. $\rho_2=2 \rho_1$

B. $\rho_2=\rho_1$

C. $\rho_2=\frac{\rho_1}{2}$

D. $\rho_2=\frac{\rho_1}{4}$

Answer: C

Solution:

The formula for frequency of a sonometer is $\mathrm{f}=\frac{1}{2 l} \sqrt{\frac{\mathrm{T}}{\pi \rho \mathrm{D}^2}}$

Here l is length, T is tension, D is diameter and ρ is density.

The frequency of both the wires is same.

The frequency of the wire A is $f_A=\frac{1}{2 l} \sqrt{\frac{T}{\pi \rho_1 D^2}}$

The frequency of the wire B is $f_B=\frac{1}{2 l} \sqrt{\frac{2 T}{\pi \rho_2(2 D)^2}}$

Equating both the frequencies

$$
\begin{aligned}
& \frac{1}{2 l} \sqrt{\frac{\mathrm{T}}{\pi \rho \mathrm{D}^2}}=\frac{1}{2 l} \sqrt{\frac{2 \mathrm{~T}}{\pi \rho_2(2 \mathrm{D})^2}} \\
& \sqrt{\frac{1}{\rho_1}}=\sqrt{\frac{1}{2 \rho_2}} \\
& \frac{1}{\rho_1}=\frac{1}{2 \rho_2} \\
\therefore \quad & \rho_2=\frac{\rho_1}{2}
\end{aligned}
$$

Question 126

The upper end of the spring is fixed and a mass 'm' is attached to its lower end. When mass is slightly pulled down and released, it oscillates with time period 3 second. If mass 'm' is increased by $1 \mathrm{~kg}$, the time period becomes 5 second. The value of 'm' is (mass of spring is negligible)

Options:

A. $\frac{3}{8} \mathrm{~kg}$

B. $\frac{5}{9} \mathrm{~kg}$

C. $\frac{8}{13} \mathrm{~kg}$

D. $\frac{9}{16} \mathrm{~kg}$

Answer: D

Solution:

The formula for the time period of a spring mass system is $T=2 \pi \sqrt{\frac{m}{k}}$

For mass $\mathrm{m}+1, \mathrm{~T}^{\prime}=2 \pi \sqrt{\frac{\mathrm{m}+1}{\mathrm{k}}}$

Taking the ratio,

$$\begin{aligned} \frac{\mathrm{T}}{\mathrm{T}^{\prime}} & =\frac{2 \pi \sqrt{\frac{\mathrm{m}}{\mathrm{k}}}}{2 \pi \sqrt{\frac{\mathrm{m}+1}{\mathrm{k}}}} \\ \frac{\mathrm{T}}{\mathrm{T}^{\prime}} & =\sqrt{\frac{\mathrm{m}}{\mathrm{m}+1}} \\ \frac{3}{5} & =\sqrt{\frac{\mathrm{m}}{\mathrm{m}+1}} \\ \frac{9}{25} & =\frac{\mathrm{m}}{\mathrm{m}+1} \\ \therefore \quad 9 \mathrm{~m}+9 & =25 \mathrm{~m} \\ \therefore \quad 16 \mathrm{~m} & =9 \\ \therefore \quad \mathrm{m} & =\frac{9}{16} \mathrm{~kg} \end{aligned}$$

Question 127

What should be the diameter of a soap bubble, in order that the excess pressure inside it is $25.6\ \mathrm{Nm}^{-2}$? [surface tension of soap solution $= 3 \cdot 2 \times 10^{-2}\ \mathrm{Nm}^{-2}$]

Options:

A. 2 cm

B. 1.5 cm

C. 1 cm

D. 0.5 cm

Answer: C

Solution:

The formula for excess pressure inside the bubble is given as $P_0 = \frac{4T}{R}$

Where, T is surface tension and R is the radius of the sphere

Rearranging the formula for R and substituting the values,

$$R = \frac{4T}{P_0}$$
$$R = \frac{4 \times 3.2 \times 10^{-2}}{25.6}$$
$$R = 0.5 \times 10^{-2}\ \mathrm{m} = 0.5\ \mathrm{cm}$$

The diameter then becomes, $2\mathrm{R} = 1\ \mathrm{cm}$

Question 128

If temperature of gas molecules is raised from $127^\circ\mathrm{C}$ to $527^\circ\mathrm{C}$, the ratio of r.m.s. speed of the molecules is respectively

Options:

A. $1 : 2$

B. 2 : 1

C. $1 : \sqrt{2}$

D. $2 : \sqrt{2}$

Answer: C

Solution:

We know

$V_{rms} = \sqrt{\frac{3RT}{m}}$

The temperature of the same gas molecule is raised.

$V_{ms} \propto \sqrt{T}$

$\therefore$ The ratio of the velocities is

$$\frac{V_1}{V_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$
$$\frac{V_1}{V_2} = \frac{\sqrt{400}}{\sqrt{800}}$$
$$\frac{V_1}{V_2} = \frac{1}{\sqrt{2}}$$

Convert temperatures given in centigrade to Kelvin before calculation.

Question 129

The ratio of energy required to raise a satellite to a height 'h' above the earth's surface to that required to put it into the orbit at the same height is ($R =$ radius of earth)

Options:

A. $\frac{2h}{R}$

B. $\frac{h}{R}$

C. $\frac{R}{h}$

D. $\frac{R}{2h}$

Answer: A

Solution:

The formula for the energy required to raise a satellite to height h is

$E_1 = \Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mghR}{R+h}$

The formula for the energy required to set the satellite in orbit is

$$
\begin{aligned}
E_2 &= \frac{-GMm}{2(R+h)} + \frac{GMm}{R} \\
&= mgR\left[1 - \frac{1}{2\left(1+\frac{h}{R}\right)}\right] \left(\because GM = gR^2\right) \\
\therefore \quad E_2 &= \frac{mgR\left(\frac{2h}{R}+1\right)}{2\left(1+\frac{h}{R}\right)} \\
\therefore \quad \frac{E_1}{E_2} &= \frac{mgh}{1+\frac{h}{R}} \times \frac{2\left(1+\frac{h}{R}\right)}{mgR}
\end{aligned}
$$

$= \frac{2h}{R} \quad \left(\because h < R \Rightarrow 1 + \frac{2h}{R} \approx 0\right)$

Question 130

According to Boyle's law, the product PV remains constant. The unit of PV is same as that of

Options:

A. energy

B. force

C. impulse

D. momentum

Answer: A

Solution:

The units of PV can be calculated as follows:

The unit of pressure is $\mathrm{Kgm^{-1}\ s^{-2}}$

The unit of volume is $\mathrm{m^3}$

$\therefore$ The unit of PV is

$\mathrm{Unit = Kgm^{-1}\ s^{-2}\ m^3 = Kgm^2\ s^{-2}}$

This is a unit of energy.

Question 131

When a metallic surface is illuminated with radiation of wavelength 'λ', the stopping potential is 'V'. If the same surface is illuminated with radiation of wavelength '2λ', the stopping potential is '$\left(\frac{\mathrm{V}}{4}\right)$'. The threshold wavelength for the metallic surface is

Options:

A. $\frac{5}{2}\lambda$

B. 3λ

C. 4λ

D. 5λ

Answer: B

Solution:

For stopping potential V, $\mathrm{eV} = \frac{\mathrm{hc}}{\lambda} - \frac{\mathrm{hc}}{\lambda_0}$

For stopping potential $\frac{\mathrm{V}}{4}$, $\frac{\mathrm{eV}}{4} = \frac{\mathrm{hc}}{2\lambda} - \frac{\mathrm{hc}}{\lambda_0}$

Taking the ratio, we get

$$4=\frac{\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)}{\frac{1}{2\lambda}-\frac{1}{\lambda_0}}$$
$$4\left(\frac{1}{2\lambda}-\frac{1}{\lambda_0}\right)=\frac{1}{\lambda}-\frac{1}{\lambda_0}$$
$$\left(\frac{2}{\lambda}-\frac{4}{\lambda_0}\right)=\frac{1}{\lambda}-\frac{1}{\lambda_0}$$
$$\left(\frac{2}{\lambda}-\frac{1}{\lambda}\right)=-\frac{1}{\lambda_0}+\frac{4}{\lambda_0}$$
$$\left(\frac{1}{\lambda}\right)=\frac{3}{\lambda_0}$$
$$\therefore \quad \lambda_0=3\lambda$$

Question 132

Three identical capacitors of capacitance 'C' each are connected in series and this connection is connected in parallel with one more such identical capacitor. Then the capacitance of whole combination is

Options:

A. 3C

B. 2C

C. $\frac{4}{3}$C

D. $\frac{3}{4}$C

Answer: C

Solution:

The equivalent capacitance of three capacitances connected in series is

$$\frac{1}{C_{eq}}=\frac{1}{C}+\frac{1}{C}+\frac{1}{C}$$
$$\frac{1}{C_{eq}}=\frac{3}{C}$$
$$C_{eq}=\frac{C}{3}$$

This capacitance is connected in parallel with another capacitance.

$$\Rightarrow C_{\text{total}} \quad = \frac{C}{3} + C$$
$$C_{\text{total}} \quad = \frac{4C}{3}$$

Question 133

In Young's double slit experiment, green light is incident on two slits. The interference pattern is observed on a screen. Which one of the following changes would cause the observed fringes to be more closely spaced?

Options:

A. Reducing the separation between the slits

B. Using blue light instead of green light

C. Using red light instead of green light

D. Moving the screen away from the slits

Answer: B

Solution:

The formula for fringe width is $W = \frac{\lambda D}{d}$

$\therefore \quad$ W $\propto \lambda$, W $\propto$ D and W $\propto \frac{1}{\lambda}$

If the distance from the screen is increased, the width will increase.

If the distance between the slit is decreased, the width will increase.

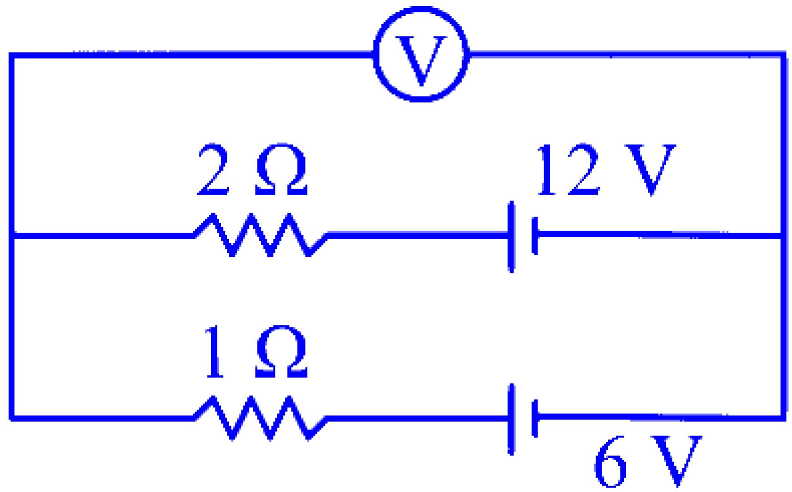
As the wavelength will decrease the distance between the fringes will decrease.

$\lambda_{\text{red}} \quad > \lambda_{\text{green}} \quad > \lambda_{\text{blue}}$

$\therefore \quad$ Blue light should be used.

Question 134

Two batteries, one of e.m.f. $12 \mathrm{~V}$ and internal resistance 2Ω and other of e.m.f. $6 \mathrm{~V}$ and internal resistance 1Ω, are connected as shown in the figure. What will be the reading of the voltmeter 'V'?



Options:

A. 12 V

B. 8 V

C. 6 V

D. 4 V

Answer: B

Solution:

The formula for the equivalent emf of the parallel combination of batteries is

$\varepsilon_{\mathrm{e}}=r_{\mathrm{eq}}\left(\frac{e_1}{r_1}+\frac{e_2}{r_2}\right)$

Here, r_{eq} is the equivalent resistance

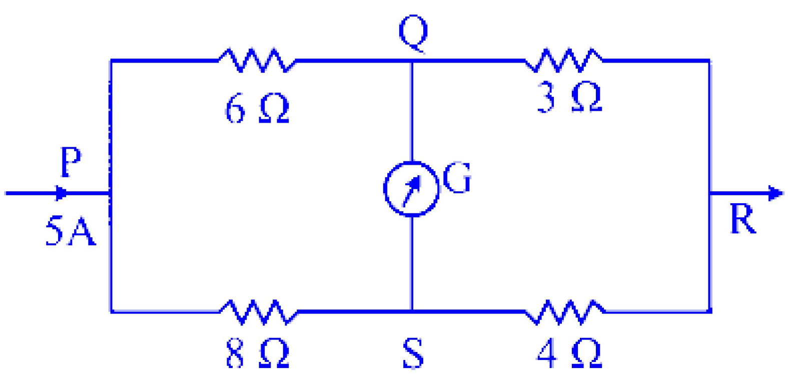
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$
$$\frac{1}{r_{eq}} = \frac{1}{2} + \frac{1}{1}$$
$$\frac{1}{r_{eq}} = \frac{3}{2}$$

Substituting the values

$$\varepsilon_e = r_{eq}\left(\frac{e_1}{r_1} + \frac{e_2}{r_2}\right)$$
$$\varepsilon_c = \frac{2}{3}\left(\frac{12}{2} + \frac{6}{1}\right)$$
$$\varepsilon_e = \frac{2}{3} \times 12$$
$$\therefore \quad \varepsilon_e = 8 \text{ V}$$

Question 135

Potential difference between the points P and Q is nearly



Options:

A. 17 V

B. 14 V

C. 12 V

D. 8 V

Answer: A

Solution:

Let total current be denoted as I. The given circuit is a Wheatstone bridge.

$$\Rightarrow R_1 = 6 + 3 = 9\Omega$$
$$\Rightarrow R_2 = 8 + 4 = 12\Omega$$

According to KCL, the current will get divided into two parts I_1 and I_2

$\therefore \quad I_1 = \frac{R_2 I}{(R_1 + R_2)}$

Substituting the values,

$$I_1 = \frac{12}{9 + 12} \times 5$$
$$I_1 = 2.85 \text{ A}$$

Potential difference between P and Q is

$$V = I_1 R$$
$$V = 2.85 \times 6$$
$$V = 17V$$

Question 136

A coil having effective area 'A' is held with its plane normal to a magnitude field of induction 'B'. The magnetic induction is quickly reduced to 25% of its initial value in 1 second. The e.m.f. induced in the coil (in volt) will be

Options:

A. $\frac{BA}{4}$

B. $\frac{BA}{2}$

C. $\frac{3BA}{8}$

D. $\frac{3BA}{4}$

Answer: D

Solution:

The formula for induced emf is $e = \frac{\Delta\phi}{\Delta t}$, where $\phi = BA$

Here, the area is constant and the magnetic field is changing.

$$\therefore \quad \Delta\phi = \Delta BA$$
$$\therefore \quad \Delta\phi = A \cdot \Delta B$$
$$\therefore \quad \Delta B = B_1 - B_2$$

$B_1 = B$ and $B_2 = \frac{25}{200} B = \frac{1}{4} B$

$$\therefore \quad B = B - \frac{1}{4} B$$
$$\therefore \quad B = \frac{3}{4} B$$

Substituting the values,

$$e = \frac{\Delta\phi}{\Delta t}$$
$$e = \frac{A \times \frac{3}{4} B}{1}$$
$$\therefore \quad e = \frac{3}{4} AB$$

Question 137

The path difference between two waves, represented by $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and $y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$ is

Options:

A. $\frac{\lambda}{2\pi}(\phi)$

B. $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

C. $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$

D. $\frac{2\pi}{\lambda}(\phi)$

Answer: B

Solution:

$$y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$$
$$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$$

y_2 can also be written as

$$y_2 = a_2 \sin\left[\frac{\pi}{2} + \left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)\right]$$
$$= a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$$

The phase difference between the two waves is

Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference

Path difference $= \frac{\lambda}{2\pi} \times \left(\phi + \frac{\pi}{2}\right)$

Question 138

An electromagnetic wave, whose wave normal makes an angle of 45° with the vertical, travelling in air strikes a horizontal liquid surface. While travelling through the liquid it gets deviated through 15°. What is the speed of the electromagnetic wave in the liquid, if the speed of electromagnetic wave in air is 3×10^8 m/s ?
$\left(\sin 30^\circ = 0.5, \sin 45^\circ = \frac{1}{\sqrt{2}}\right)$

Options:

A. $\frac{\sqrt{2}}{3} \times 10^8$ m/s

B. 1.5×10^8 m/s

C. 2.1×10^8 m/s

D. 2.5×10^8 m/s

Answer: C

Solution:

The angle of incidence is $\mathrm{i}=45^{\circ}$

The angle of deviation is $\delta=15^{\circ}$

The angle of refraction is

$$\begin{aligned}
&\delta=i-r\\
&r=i-\delta\\
&r=45^{\circ}-15^{\circ}\\
&r=30^{\circ}
\end{aligned}$$

The relation between the speed of the wave in the medium, the angle of incidence and the angle of refraction is given by the formula:

$$\begin{aligned}
&\frac{v_2}{v_1}=\frac{\sin r}{\sin i}\\
&\frac{v_2}{v_1}=\frac{\sin 30^{\circ}}{\sin 45^{\circ}}\\
&\frac{v_2}{v_1}=\frac{1}{\sqrt{2}}\\
&\mathrm{v}_2=\frac{\mathrm{v}_1}{\sqrt{2}}\\
&\mathrm{v}_2=\frac{3\times 10^8}{\sqrt{2}}\\
&\mathrm{v}_2=2.1\times 10^8\ \mathrm{m/s}
\end{aligned}$$

Question 139

The difference in length between two rods A and B is $60\ \mathrm{cm}$ at all temperatures. If $\alpha_{\mathrm{A}}=18\times 10^{-6}/^{\circ}\mathrm{C}$ and $\beta_{\mathrm{B}}=27\times 10^{-6}/^{\circ}\mathrm{C}$, the lengths of the two rods are

Options:

A. $l_{\mathrm{A}}=200\ \mathrm{cm}, l_{\mathrm{B}}=140\ \mathrm{cm}$

B. $l_{\mathrm{A}}=180\ \mathrm{cm}, l_{\mathrm{B}}=120\ \mathrm{cm}$

C. $l_{\mathrm{A}}=160\ \mathrm{cm}, l_{\mathrm{B}}=100\ \mathrm{cm}$

D. $l_{\mathrm{A}}=120\ \mathrm{cm}, l_{\mathrm{B}}=60\ \mathrm{cm}$

Answer: B

Solution:

Given: $\Delta l = 60 \text{ cm}, \alpha_{\text{A}} = 18 \times 10^{-6}/^{\circ}\text{C}$,

$\alpha_B = 27 \times 10^{-6}/^{\circ}\text{C}$

Δl is constant at all temperatures.

We know $\Delta l = l\alpha\Delta t$

Let the length of the rods at a temperature 0°C be l_{A} and l_{B}

$\therefore$ At temperature t°C

$$
\begin{aligned}
& l_{\text{A}}\alpha_{\text{A}}\text{t}_{\text{A}} = l_{\text{B}}\alpha_{\text{B}}\text{t}_{\text{B}} \\
& l_{\text{A}}(18) \times 10^{-6} = l_{\text{B}}(27) \times 10^{-6} \quad \text{.... (i)} \\
& \Delta l = l_{\text{A}} - l_{\text{B}} \\
& \Delta l = \tfrac{3}{2} l_{\text{B}} - l_{\text{B}} \quad \text{.... from (i)} \\
& \Delta l = \tfrac{1}{2} l_{\text{B}} \\
\therefore \quad & l_{\text{B}} = 2\Delta l \\
\therefore \quad & l_{\text{B}} = 2 \times 60 = 120 \text{ cm} \\
\therefore \quad & l_{\text{A}} = \tfrac{3}{2} \times 120 = 180 \text{ cm}
\end{aligned}
$$

Question 140

A parallel plate capacitor is charged by a battery and battery remains connected. The dielectric slab of constant 'K' is inserted between the plates and then taken out. Then electric field between the plates

Options:

A. remains the same

B. increases

C. decreases

D. becomes zero

Answer: A

Question 141

From a disc of mass 'M' and radius 'R', a circular hole of diameter 'R' is cut whose rim passes through the centre. The moment of inertia of the remaining part of the disc about perpendicular axis passing through the centre is

Options:

A. $\frac{13MR^2}{32}$

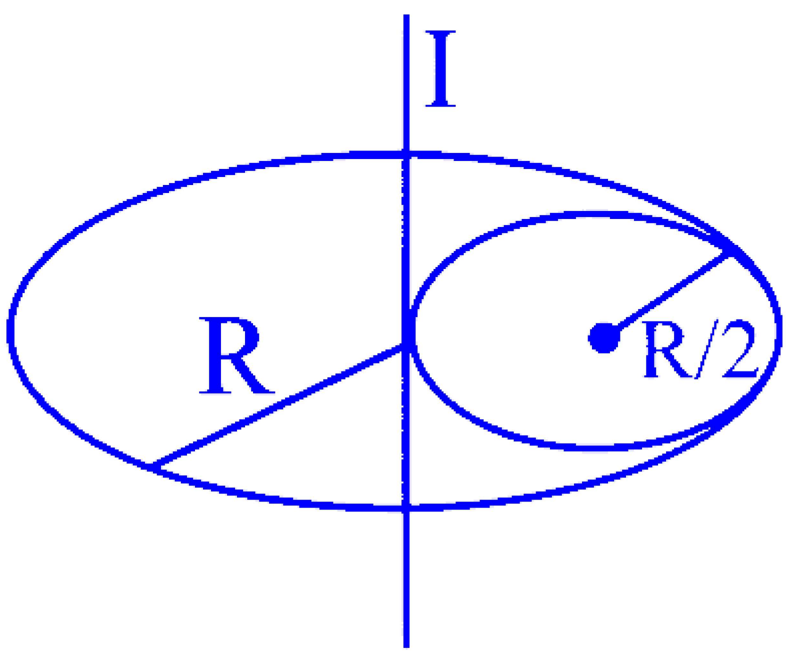
B. $\frac{11MR^2}{32}$

C. $\frac{9MR^2}{32}$

D. $\frac{7MR^2}{32}$

Answer: A

Solution:



Moment of inertia of disc is given by

$I_{disc} = I_r + I_{hole} \quad \ldots . \{I_r = \text{M.I. of remaining part}\}$

$\therefore \quad I_r = I_{disc} - I_{hole} \quad \ldots\ldots \text{(i)}$

$I_{disc} = \frac{MR^2}{2}$ ..... (ii)

By parallel axes theorem we get,

$$I_{hole} = \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$$

$$\ldots \left\{\begin{array}{l} \because M_{hole} = \frac{M_{disc}}{4} \\ \because \text{ the surface density is same} \end{array}\right\}$$

$$\therefore \quad I_{hole} = \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right] \text{ ..... (iii)}$$

Substituting eq (iii) and eq (ii) in eq (i) we get,

$$\begin{aligned} I_r &= \frac{MR^2}{2} - \frac{MR^2}{32} - \frac{MR^2}{16} \\ &= MR^2\left[\frac{1}{2} - \frac{1}{32} - \frac{1}{16}\right] \\ &= \frac{13}{32}MR^2 \end{aligned}$$

Question 142

If p-n junction diode is in forward bias then

Options:

A. width of depletion layer increases

B. electric conduction is not possible at all

C. barrier voltage increases

D. width of depletion layer decreases

Answer: D

Question 143

The orbital magnetic moment associated with orbiting electron of charge 'e' is

Options:

A. inversely proportional to angular momentum

B. directly proportional to mass of electron

C. directly proportional to angular momentum

D. inversely proportional to charge on electron

Answer: C

Solution:

$\therefore$ Orbital magnetic moment:

$M_0 = \frac{-e}{2m_e} L$

$\therefore$ The orbital magnetic moment is directly proportional to angular momentum L of the electron

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Question 144

An ideal gas expands adiabatically. $(\gamma = 1 \cdot 5)$ To reduce the r.m.s. velocity of the molecules 3 times, the gas has to be expanded

Options:

A. 81 times

B. 27 times

C. 9 times

D. 3 times

Answer: A

Solution:

We know,

$V_{\text{rms}} = \sqrt{\frac{3RT}{M_0}}$

$\Rightarrow T \propto V^2$

$\frac{T_2}{T_1} = \frac{V_2^2}{V_1^2} = \frac{\left(\frac{V_1}{3}\right)^2}{V_1^2} = \frac{1}{9}$ .... (i)

Also, $TV^{\gamma-1} =$ Constant

$\Rightarrow \frac{V_2}{V_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$

$\frac{V_2}{V_1} = (9)^2 = 81$

Question 145

A metal surface of work function $1 \cdot 13 \text{ eV}$ is irradiated with light of wavelength 310 nm. The retarding potential required to stop the escape of photoelectrons is [Take $\frac{\text{hc}}{\text{e}} = 1240 \times 10^{-9}$SI units]

Options:

A. 1.13 V

B. 2.87 V

C. 3.97 V

D. 4.23 V

Answer: B

Solution:

Energy of incident light:

$$E = \frac{hc}{e\lambda} = \frac{1240 \times 10^{-9}}{310 \times 10^{-9}} = 4 \text{ ev}$$

Stopping potential $\text{V}_0 = \frac{\text{hc}}{\text{e}\lambda} - \frac{\phi_0}{\text{e}}$

$V_0 = 4 - 1.13 = 2.87 \text{ V}$

Question 146

Two cars A and B start from a point at the same time in a straight line and their positions are represented by $R_A(t) =$ at $+ bt^2$ and $R_B(t) = xt - t^2$. At what time do the cars have same velocity?

Options:

A. $\frac{x-a}{2(b+1)}$

B. $\frac{x+a}{2(b-1)}$

C. $\frac{x-a}{(b+1)}$

D. $\frac{x+a}{(b-1)}$

Answer: A

Solution:

$\therefore$ Velocity of car A and B:

$$V_A = \frac{d(R_A)}{dt} = a + 2bt$$

$$V_B = \frac{d(R_B)}{dt} = x - 2t$$

$\therefore$ So, time at which cars have same velocity is

$$V_A = V_B$$

$$a + 2bt = x - 2t$$

$\therefore$ $t = \frac{x-a}{2(b+1)}$

Question 147

The a.c. source of e.m.f. with instantaneous value 'e' is given by $e = 200\sin(50t)$ volt. The r.m.s. value of current in a circuit of resistance 50Ω is

Options:

A. 0.2828 A

B. 2.828 A

C. 28.28 A

D. 282.8 A

Answer: B

Solution:

$\therefore$ e.m.f $\mathrm{e} = 200\sin(50\mathrm{t})$ V .... (i)

$\therefore$ Current is given by:

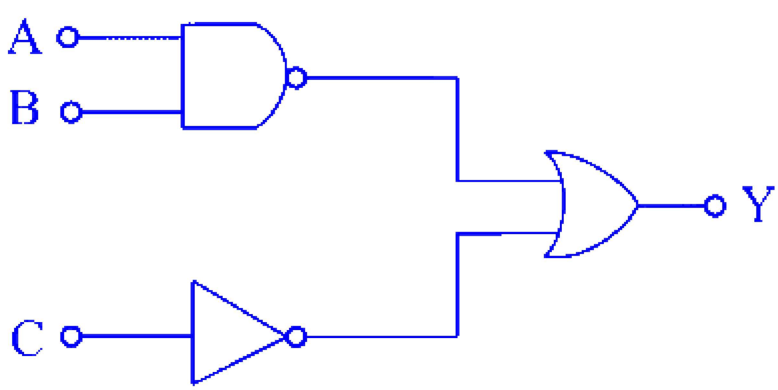
$\mathrm{I_o} = \frac{\mathrm{e_o}}{\mathrm{R}} = \frac{200}{50} = 4$ A .... (from (i))

$\therefore$ R.M.S value of current:

$\mathrm{I_{rms}} = \frac{\mathrm{I_o}}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828$ A

Question 148

In the digital circuit the inputs are as shown in figure. The Boolean expression for output Y is



Options:

A. $\overline{A+B}+\overline{C}$

B. $\overline{A\cdot B}\cdot\overline{C}$

C. $\overline{A\cdot B}+\overline{C}$

D. $\overline{A+B}\cdot\overline{C}$

Answer: C

Solution:

In the given circuit, there is AND, NOR and a NOT gate.

Output of AND gate: $\overline{A\cdot B}$

Output of NOT gate: $\overline{C}$

$\therefore$ The boolean expression will be:

$\overline{A\cdot B}+\overline{C}$

Question 149

A double convex lens of focal length 'F' is cut into two equal parts along the vertical axis. The focal length of each part will be

Options:

A. 2 F

B. F

C. $\frac{F}{2}$

D. 4 F

Answer: A

Solution:

For bifocal convex lens

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$
$$= \frac{(\mu - 1) \times 2}{R} \quad \ldots .(R_1 = R_2 = R)$$

For plane surface $R_2 = \infty$

For half plane-convex lens

$$\frac{1}{\mathrm{f}'} = (\mu - 1)\frac{1}{\mathrm{R}}$$
$$\frac{1/\mathrm{f}}{1/\mathrm{f}'} = \frac{(\mu - 1)}{\mathrm{R}} \times 2 \times \frac{\mathrm{R}}{\mu - 1} = 2$$
$$\frac{\mathrm{f}}{\mathrm{f}} = 2$$
$$\mathrm{f}' = 2\mathrm{f}$$

As focal length is $\mathrm{F}, \mathrm{f}' = 2\ \mathrm{F}$.

Question 150

Two progressive waves are travelling towards each other with velocity $50\ \mathrm{m/s}$ and frequency $200\ \mathrm{Hz}$. The distance between the two consecutive antinodes is

Options:

A. $0.125\ \mathrm{m}$

B. $0.150\ \mathrm{m}$

C. $0.175\ \mathrm{m}$

D. $0.200\ \mathrm{m}$

Answer: A

Solution:

Velocity of wave, $v = f\lambda$

$\therefore \quad \lambda = \frac{\mathrm{v}}{\mathrm{f}} = \frac{50}{200} = 0.25\ \mathrm{m}$

$\therefore$ Dividing the wavelength for antinodes:

$$\lambda = \frac{0.25}{2}$$
$$= 0.125 \text{ m}$$
