MHT CET 2023 : 11th May Morning Shift

Mathematics

Question 1

$$\int_0^\pi \frac{dx}{4+3\cos x}=$$

Options:

A. $\frac{2\pi}{7}$

B. $\frac{\pi}{\sqrt{7}}$

C. $\frac{\pi}{2\sqrt{7}}$

D. $\frac{\pi}{7}$

Answer: B

Solution:

Let $I=\int_0^\pi \frac{dx}{4+3\cos x}$

Put $\tan\frac{x}{2}=\mathrm{t}$

$\therefore \quad \mathrm{d}x=\frac{2\mathrm{dt}}{1+\mathrm{t}^2}$ and $\cos x=\frac{1-\mathrm{t}^2}{1+\mathrm{t}^2}$

$$\therefore \quad \mathrm{I}=\int_0^{\pi} \frac{\mathrm{d}x}{4+3\cos x}=\int_0^{\infty} \frac{2\mathrm{dt}}{7+\mathrm{t}^2}$$

$$=\left[\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{\mathrm{t}}{\sqrt{7}}\right)\right]_0^{\infty}$$

$$=\frac{2}{\sqrt{7}}\left[\tan^{-1}\infty-0\right]$$

$$=\frac{2}{\sqrt{7}} \cdot \frac{\pi}{2}=\frac{\pi}{\sqrt{7}}$$

Question 2

If $y=\log_{\sin x} \tan x$, then $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=\frac{\pi}{4}}$ has the value

Options:

A. $\frac{4}{\log 2}$

B. $-3\log 2$

C. $\frac{-4}{\log 2}$

D. $3\log 2$

Answer: C

Solution:

$$y = \frac{\log \tan x}{\log \sin x}$$

$$\frac{dy}{dx} = \frac{(\log \sin x)\left(\frac{1}{\tan x}\right) \cdot \sec^2 x - (\log \tan x)\left(\frac{1}{\sin x}\right)(\cos x)}{(\log \sin x)^2}$$

At $x = \frac{\pi}{4}$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{\log\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{1}\right)(\sqrt{2})^2 - (\log 1)\left(\frac{\sqrt{2}}{1}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left[\log\left(\frac{1}{\sqrt{2}}\right)\right]^2}$$

$$= \frac{-2 \times \frac{1}{2}(\log 2) - 0}{\frac{1}{4}(\log 2)^2} \quad \ldots [\because \log 1 = 0]$$

$$= \frac{-4}{\log 2}$$

Question 3

$\lim_{x \to 2} \left[\frac{1}{x-2} - \frac{2}{x^3 - 3x^2 + 2x}\right]$ **is equal to**

Options:

A. $\frac{2}{3}$

B. $\frac{-2}{3}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer: C

Solution:

$$\lim_{x \to 2}\left[\frac{1}{x-2}-\frac{2}{x\left(x^2-3x+2\right)}\right]$$
$$=\lim_{x \to 2}\left[\frac{1}{(x-2)}-\frac{2}{x(x-2)(x-1)}\right]$$
$$=\lim_{x \to 2}\left[\frac{x^2-x-2}{x(x-1)(x-2)}\right]$$
$$=\lim_{x \to 2}\frac{(x-2)(x+1)}{x(x-1)(x-2)}$$
$$=\lim_{x \to 2}\frac{x+1}{x(x-1)}$$
$$=\frac{3}{2}$$

Question 4

From a lot of 20 baskets, which includes 6 defective baskets, a sample of 2 baskets is drawn at random one by one without replacement. The expected value of number of defective basket is

Options:

A. 0.6

B. 0.06

C. 0.006

D. 1.07

Answer: A

Solution:

Let X denotes the number of defective baskets.

$\therefore$ Possible values of X are $0, 1, 2$.

Now, selection of baskets is done without replacement, we get

$P(X=0)=\frac{14}{20}\times\frac{13}{19}=\frac{182}{380}$,

$$P(X=1)=\frac{14}{20}\times\frac{6}{19}+\frac{6}{20}\times\frac{14}{19}=\frac{168}{380},$$
$$P(X=2)=\frac{6}{20}\times\frac{5}{19}=\frac{30}{380}$$

$\therefore$ Required Expected Value

$$=0\times \mathrm{P}(\mathrm{X}=0)+1\times \mathrm{P}(\mathrm{X}=1)+2\times \mathrm{P}(\mathrm{X}=2)$$
$$=\frac{168+60}{380}=\frac{228}{380}=0.6$$

Question 5

If the angle between the lines represented by the equation $x^2+\lambda xy-y^2\tan^2\theta=0$ is 2θ, then the value of λ is

Options:

A. 0

B. 1

C. $\tan\theta$

D. 2

Answer: A

Solution:

Given equation of pair of lines is

$$x^2+\lambda xy-y^2\tan^2\theta=0$$
$$\therefore \quad a=1, h=\frac{\lambda}{2},\ \mathrm{b}=-\tan^2\theta$$
$$\therefore \quad \tan 2\theta=\left|\frac{2\sqrt{\mathrm{h}^2-\mathrm{ab}}}{\mathrm{a}+\mathrm{b}}\right|$$
$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta}=\left|\frac{2\sqrt{\frac{\lambda^2}{4}+\tan^2\theta}}{1-\tan^2\theta}\right|$$
$$\Rightarrow \frac{\lambda^2}{4}+\tan^2\theta=\tan^2\theta$$
$$\Rightarrow \lambda=0$$

Question 6

Number of common tangents to the circles $x^2+y^2-6x-14y+48=0$ and $x^2+y^2-6x=0$ are

Options:

A. 0

B. 1

C. 4

D. 2

Answer: C

Solution:

$$x^2+y^2-6x-14y+48=0$$

$\therefore \quad C_1(3,7), r_1=\sqrt{10}$

Again $x^2+y^2-6x=0$

$\therefore \quad C_2(3,0), r_2=3$

Now $l(C_1C_2)=$ distance between centres

$\therefore \quad l(C_1C_2)=\sqrt{0^2+7^2}=7$ and

$r_1+r_2=\sqrt{10}+3<l(C_1C_2)$

$\Rightarrow$ The given circles are disjoint.

$\Rightarrow$ Number of common tangents is 4 .

Question 7

The left-hand derivative of $f(x)=[x]\sin(\pi x)$, at $x=k$, k is an integer and [.] is the greatest integer function, is

Options:

A. $(-1)^{\mathrm{k}}(\mathrm{k}-1)\pi$

B. $(-1)^{\mathrm{k}-1}(\mathrm{k}-1)\pi$

C. $(-1)^{\mathrm{k}}\mathrm{k}\pi$

D. $(-1)^{\mathrm{k}-1}\mathrm{k}\pi$

Answer: A

Solution:

$$
\begin{aligned}
& f(x)=[x]\sin(\pi x) \\
& \text{LHD } =\lim_{h\to 0}\frac{f(k-h)-f(k)}{-h} \\
& \quad =\lim_{h\to 0}\frac{[k-h]\sin\pi(k-h)-[k]\sin k\pi}{-h} \\
& \quad =\lim_{h\to 0}\frac{(k-1)\sin(k\pi-\pi h)-k\sin k\pi}{-h} \qquad \ldots[\because k\in I] \\
& \quad =\lim_{h\to 0}\frac{(-1)^{k+1}(k-1)\sinh\pi-0}{-h} \qquad \ldots[\because \sin k\pi=0, k\in I] \\
& \quad =(-1)^k(k-1)\pi
\end{aligned}
$$

Question 8

Let $\mathrm{f}(x)=\int\frac{\sqrt{x}}{(1+x)^2}\,\mathrm{d}x, x\geq 0$, then $\mathrm{f}(3)-\mathrm{f}(1)$ is equal to

Options:

A. $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$

B. $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$

C. $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

D. $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Answer: D

Solution:

$f(x)=\int \frac{\sqrt{x}}{(1+x)^2}\,\mathrm{d}x, x\geq 0$

$\mathrm{f}(3)-\mathrm{f}(1)=\int_1^3 \frac{\sqrt{x}}{(1+x)^2}\,\mathrm{d}x=\mathrm{I}\text{ (say)}$

Put $\sqrt{x}=\tan\theta \Rightarrow \mathrm{d}x=2\tan\theta\sec^2\theta\mathrm{d}\theta$

When $x=1, \theta=\frac{\pi}{4}$ and when $x=3, \theta=\frac{\pi}{3}$

$$\therefore \quad I=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\tan^2\theta\sec^2\theta}{\left(1+\tan^2\theta\right)^2} d\theta$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\tan^2\theta}{1+\tan^2\theta} d\theta$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\sin^2\theta}{\sin^2\theta+\cos^2\theta} d\theta$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1-\cos 2\theta)\mathrm{d}\theta$$

$$=\left[\theta-\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$=\left(\frac{\pi}{3}-\frac{\pi}{4}\right)-\frac{1}{2}\left(\frac{\sqrt{3}}{2}-1\right)$$

$$=\frac{\pi}{12}-\frac{\sqrt{3}}{4}+\frac{1}{2}$$

Question 9

Value of c satisfying the conditions and conclusions of Rolle's theorem for the function $\mathrm{f}(x)=x\sqrt{x+6}, x\in[-6,0]$ is

Options:

A. -4

B. 4

C. 3

D. -3

Answer: A

Solution:

$$\begin{aligned} & \mathrm{f}(x)=x\sqrt{x+6} \\ \therefore \quad & \mathrm{f}'(x)=x\left(\frac{1}{2\sqrt{x+6}}\right)+\sqrt{x+6}(1) \\ & \qquad =\frac{x}{2\sqrt{x+6}}+\sqrt{x+6} \end{aligned}$$

Since $\mathrm{f}(x)$ satisfies all the conditions of Rolle's Theorem,

There exists $c \in (-6, 0)$ such that

$$\begin{aligned} & f'(c)=0 \\ & \Rightarrow \frac{c}{2\sqrt{c+6}}+\sqrt{c+6}=0 \\ & \Rightarrow c+2c+12=0 \\ & \Rightarrow c=-4 \end{aligned}$$

Question 10

Three of six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral, equals ____________.

Options:

A. $\frac{1}{2}$

B. $\frac{1}{5}$

C. $\frac{1}{10}$

D. $\frac{1}{20}$

Answer: C

Solution:

The number of triangles that can be drawn using 6 vertices is given by

$n(S) = {}^6C_3 = 20$

A : Event of selecting equilateral triangle. The equilateral triangle can be drawn if selected three vertices are alternate.

$\therefore \quad n(A) = 2$
$\therefore \quad P(A) = \frac{2}{20} = \frac{1}{10}$

Question 11

If the direction cosines $l,\ m, n$ of two lines are connected by relations $l - 5\ m + 3n = 0$ and $7l^2 + 5\ m^2 - 3n^2 = 0$, then value of $l + m + n$ is

Options:

A. $\frac{2}{\sqrt{6}}$ or $\frac{6}{\sqrt{14}}$

B. $\frac{1}{\sqrt{6}}$ or $\frac{5}{\sqrt{14}}$

C. $\frac{2}{\sqrt{6}}$ or $\frac{5}{\sqrt{14}}$

D. $\frac{1}{\sqrt{6}}$ or $\frac{6}{\sqrt{14}}$

Answer: A

Solution:

$l - 5\ m + 3n = 0$ and $7l^2 + 5\ m^2 - 3n^2 = 0$
$\Rightarrow l = 5\ m - 3n$ and $7l^2 = 3n^2 - 5\ m^2$

$\Rightarrow l = 5m - 3n$ and $7l^2 = 3n^2 - 5m^2$
Putting $l = (5m - 3n)$ in $7l^2 = 3n^2 - 5m^2$, we
$7(5m - 3n)^2 = 3n^2 - 5m^2$
$\Rightarrow 7\left(25m^2 - 30mn + 9n^2\right) = 3n^2 - 5m^2$
$\Rightarrow 180m^2 - 210mn + 60n^2 = 0$
$\Rightarrow 6m^2 - 7mn + 2n^2 = 0$
$\Rightarrow (3m - 2n)(2m - n) = 0$

$\Rightarrow 3\ m = 2n$ or $2\ m = n$ ..... (i)

If 3 m = 2n, then $l = \frac{n}{3}$

$$\therefore \quad \frac{m}{2} = \frac{n}{3} = \frac{l}{1} = \frac{1}{\sqrt{14}}$$

$$\therefore \quad l + m + n = \frac{6}{\sqrt{14}}$$

If 2 m = n, then $l = \frac{-n}{2}$

$\therefore \quad \frac{m}{1} = \frac{n}{2} = \frac{l}{-1} = \frac{1}{\sqrt{6}}$

$\therefore \quad l + m + n = \frac{2}{\sqrt{6}}$

$\therefore$ The possible values of $l + m + n$ is $\frac{2}{\sqrt{6}}$ or $\frac{6}{\sqrt{14}}$

Question 12

Let $f(x) = \log(\sin x), 0 < x < \pi$ and $g(x) = \sin^{-1}(e^{-x}), x \geq 0$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then

Options:

A. $a\alpha^2 - b\alpha - a = 0$

B. $a\alpha^2 - b\alpha - a = 1$

C. $a\alpha^2 + b\alpha - a = -2\alpha^2$

D. $a\alpha^2 + b\alpha + a = 0$

Answer: B

Solution:

$f(x) = \log(\sin x), 0 < x < \pi$ and

$g(x) = \sin^{-1}(e^{-x}), x \geq 0$

$\therefore \quad (fog)(x) = \log\left[\sin\left(\sin^{-1} e^{-x}\right)\right] = \log\left(e^{-x}\right) = -x$

$\therefore \quad (fog)'(x) = -1$

$\therefore \quad a = (fog)'(\alpha) = -1$ and $b = (fog)(\alpha) = -\alpha$

These values satisfy only option (B).

$\therefore$ Option (B) is correct.

Question 13

Five students are to be arranged on a platform such that the boy B_1 occupies the second position and such that the girl G_1 is always adjacent to the girl G_2. Then, the number of such possible arrangements is

Options:

A. 4

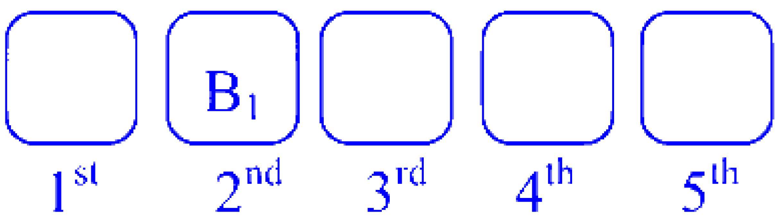
B. 7

C. 8

D. 6

Answer: C

Solution:



There are 5 positions.

Given that B_1 occupies 2^{nd} position

$\therefore$ B_1 can be arranged in 1 way. As G_1 and G_2 are always together, none of them can take 1^{st} position.

$\therefore$ G_1, G_2 and one of the remaining students can be arranged on 3^{rd}, 4^{th} and 5^{th} position when G_1 and G_2 are always together in $2! \times 2!$ Ways. And remaining 2 students can be arranged in $2!$ Ways.

$\therefore$ The required number of arrangements $= 2! \times 2! \times 2! = 8$

Question 14

If the volume of the parallelopiped is 158 cu. units whose coterminus edges are given by the vectors $\bar{a}=(\hat{i}+\hat{j}+n\hat{k}), \bar{b}=2\hat{i}+4\hat{j}-n\hat{k}$ and $\bar{c}=\hat{i}+n\hat{j}+3\hat{k}$, where $n \geq 0$, then the value of n is

Options:

A. 8

B. $\frac{19}{3}$

C. 7

D. 19

Answer: A

Solution:

$$\text{Volume of parallelopiped } = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$$\Rightarrow 1\left(12+n^2\right)-1(6+n)+n(2n-4)=158$$

$$\Rightarrow 3n^2-5n-152=0$$

$$\Rightarrow (3n+19)(n-8)=0$$

$$\Rightarrow n=8 \quad \ldots[\because n \geq 0]$$

Question 15

Let $\mathrm{f}(x)$ be positive for all real x. If $\mathrm{I}_1=\int\limits_{1-\mathrm{h}}^{\mathrm{h}} x\mathrm{f}(x(1-x))\mathrm{d}x$ and $\mathrm{I}_2=\int\limits_{1-\mathrm{h}}^{\mathrm{h}} \mathrm{f}(x(1-x))\mathrm{d}x$, where $(2h-1)>0$, then $\frac{I_1}{I_2}$ is

Options:

A. 2

B. h

C. $\frac{1}{2}$

D. 1

Answer: C

Solution:

$$I_1 = \int_{1-h}^{h} x f(x(1-x)) dx \text{ and } \quad \ldots (i)$$

$$I_2 = \int_{1-h}^{h} f(x(1-x)) dx \quad \ldots (ii)$$

$$I_1 = \int_{1-h}^{h} (1-x) f[(1-x)(1-1+x)] dx$$

$$\ldots \left[\because \int_{a}^{b} f(x) dx = \int_{a}^{b} f(a+b-x) dx \right]$$

$$\therefore \quad I_1 = \int_{1-h}^{h} (1-x) f(x(1-x)) dx$$

$$= \int_{1-h}^{h} f(x(1-x)) dx - \int_{1-h}^{h} x f(x(1-x)) dx$$

$\Rightarrow I_1 = I_2 - I_1$ ....[From (i) and (ii)]

$\Rightarrow 2I_1 = I_2$

$\Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$

Question 16

The mirror image of the point $(1, 2, 3)$ in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Thus, the point __________ lies on this plane.

Options:

A. $(1, -1, 1)$

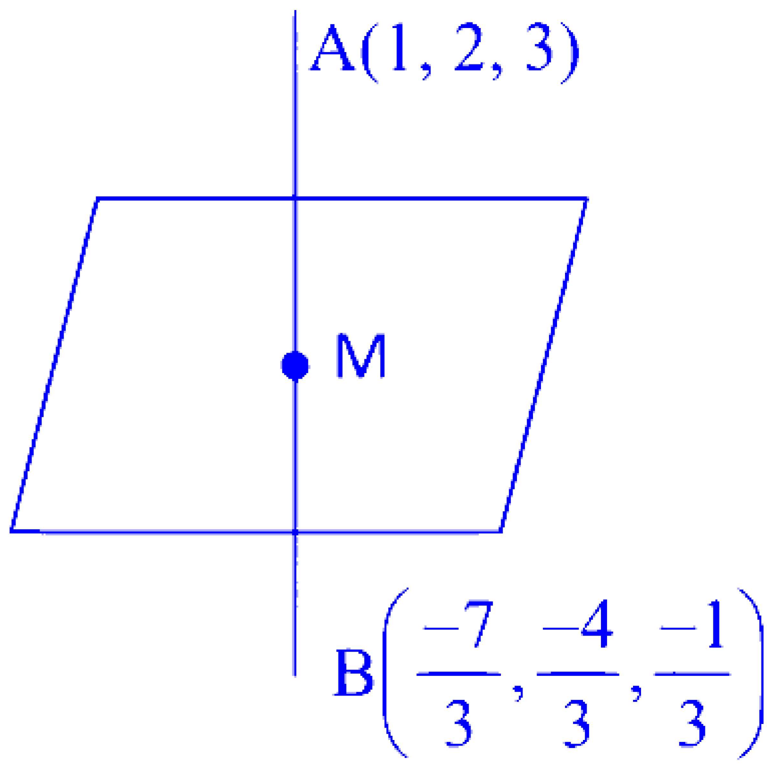
B. $(-1, -1, 1)$

C. $(1, 1, 1)$

D. $(-1, -1, -1)$

Answer: A

Solution:



M is the midpoint.

$\therefore \quad \mathrm{M} \equiv \left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

D.r.s of AB are $\frac{-10}{3}, \frac{-10}{3}, \frac{-10}{3}$ i.e., $1, 1, 1$

Equation of plane is

$$1\left(x+\frac{2}{3}\right)+1\left(y-\frac{1}{3}\right)+1\left(z-\frac{4}{3}\right)=0$$
$$\Rightarrow x+y+z=1$$

Option (A) satisfies this equation of the plane.

Question 17

If $\alpha = 3\sin^{-1}\frac{6}{11}$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option is

Options:

A. $\cos\beta > 0$

B. $\sin\beta < 0$

C. $\cos(\alpha+\beta) > 0$

D. $\cos\alpha < 0$

Answer: A

Solution:

$\alpha = 3\sin^{-1}\frac{6}{11}$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$

Since $\frac{6}{11} > \frac{6}{12}$

Taking $\sin^{-1}$ on both sides, we get

$\sin^{-1}\left(\frac{6}{11}\right) > \sin^{-1}\left(\frac{6}{12}\right)$ ..... $[\because \sin^{-1}x$ is an increasing function $]$

$$\Rightarrow 3\sin^{-1}\left(\frac{6}{11}\right) > 3\sin^{-1}\left(\frac{1}{2}\right)$$
$$\Rightarrow \alpha > 3\left(\frac{\pi}{6}\right)$$
$$\Rightarrow \alpha > \frac{\pi}{2} \quad \text{.... (i)}$$

Now, $\frac{4}{9} < \frac{4}{8}$

Taking $\cos^{-1}$ on both sides, we get

$\cos^{-1}\left(\frac{4}{9}\right) > \cos^{-1}\left(\frac{4}{8}\right)$

$\ldots [\because \cos^{-1} x$ is a decreasing function $]$

$\Rightarrow 3\cos^{-1}\left(\frac{4}{9}\right) > 3\cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \beta > 3\left(\frac{\pi}{3}\right)$

$\Rightarrow \beta > \pi$ .... (ii)

From (i) and (ii), we have α lies in II$^{\text{nd}}$ quadrant and β lies in III$^{\text{rd}}$ quadrant.

$\therefore$ $\cos\alpha < 0, \cos\beta < 0$ and $\sin\beta < 0$

Also, $\alpha + \beta > \frac{\pi}{2} + \pi$ $\ldots$ [From (i) and (ii)]

$\therefore$ $\alpha + \beta > \frac{3\pi}{2}$

Thus, $\alpha + \beta$ lies in IV$^{\text{th}}$ quadrant.

So, $\cos(\alpha + \beta) > 0$

[Note: Options (B), (C) and (D) are correct.]

Question 18

The value of $\sec^2\left(\tan^{-1} 2\right) + \operatorname{cosec}^2\left(\cot^{-1} 3\right)$ is

Options:

A. 4

B. 9

C. 2

D. 15

Answer: D

Solution:

$$\text{Let } \tan^{-1} 2 = \alpha \Rightarrow \tan\alpha = 2$$
$$\text{And } \cot^{-1} 3 = \beta \Rightarrow \cot\beta = 3$$
$$\begin{aligned} \therefore \quad & \sec^2\left(\tan^{-1} 2\right) + \operatorname{cosec}^2\left(\cot^{-1} 3\right) \\ & = \sec^2\alpha + \operatorname{cosec}^2\beta \\ & = 1 + \tan^2\alpha + 1 + \cot^2\beta \\ & = 2 + (2)^2 + (3)^2 \\ & = 15 \end{aligned}$$

Question 19

The value of $\tan\left(\frac{\pi}{8}\right)$ is __________.

Options:

A. $\sqrt{2} - 1$

B. $1 - \sqrt{2}$

C. $\sqrt{2}$

D. $\sqrt{2} + 1$

Answer: A

Solution:

Since, $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$

Putting $\frac{A}{2} = \frac{\pi}{8}$, we get

$$\begin{aligned} \tan\left(\frac{\pi}{8}\right) &= \frac{1 - \cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} \\ &= \frac{1 - \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)} \end{aligned}$$
$$\therefore \quad \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$$

Question 20

A plane is parallel to two lines, whose direction ratios are $1, 0, -1$ and $-1, 1, 0$ and it contains the point $(1, 1, 1)$. If it cuts co-ordinate axes $(\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ - axes resp.) at $\mathrm{A}, \mathrm{B}, \mathrm{C}$, then the volume of the tetrahedron OABC is _________ cu. units.

Options:

A. 9

B. $\frac{9}{4}$

C. $\frac{9}{2}$

D. 27

Answer: C

Solution:

Equation of the plane passing through $(1, 1, 1)$ is given as

$\mathrm{a}(x-1)+\mathrm{b}(y-1)+\mathrm{c}(\mathrm{z}-1)=0$ .... (i)

As the plane is parallel to the lines having direction ratios $1, 0, -1$ and $-1, 1, 0$, we get

$\mathrm{a}-\mathrm{c}=0$ and $-\mathrm{a}+\mathrm{b}=0$

$\Rightarrow \mathrm{a}=\mathrm{b}=\mathrm{c}$ .... (ii)

$\therefore$ From (i) and (ii), we get

$x \quad x-1+y-1+z-1=0$

$\therefore \quad x+y+z=3 \quad \Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{3}=1$

$\therefore$ Co-ordinates of A, B, C are $(3, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 3)$ respectively.

$$\therefore \quad \text{Volume of tetrahedron OABC} = \frac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix}$$

$$= \frac{1}{6} \times 27$$

$$= \frac{9}{2} \text{ cu. units}$$

Question 21

The domain of the function given by $2^x + 2^y = 2$ is

Options:

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

Solution:

$2^x + 2^y = 2$
$\Rightarrow 2^y = 2 - 2^x$
$\Rightarrow y = \log_2(2 - 2^x)$ is defined, if $2 - 2^x > 0$
$\Rightarrow 2^x < 2$
$\Rightarrow 2^{x-1} < 1$
$\Rightarrow x - 1 < 0$
$\Rightarrow -\infty < x < 1$

Question 22

If $\mathrm{f}(x) = x\mathrm{e}^{x(1-x)}, x \in \mathrm{R}$, then $\mathrm{f}(x)$ is

Options:

A. increasing in $\left[-\frac{1}{2}, 1\right]$

B. decreasing R

C. increasing in R

D. decreasing in $\left[-\frac{1}{2}, 1\right]$

Answer: A

Solution:

$$\begin{aligned} & \mathrm{f}(x)=x\mathrm{e}^{x(1-x)} \\ \therefore \quad & \mathrm{f}'(x)=x\mathrm{e}^{x(1-x)}[x(-1)+(1-x)]+\mathrm{e}^{x(1-x)} \\ & \quad =\mathrm{e}^{x(1-x)}\left(x-2x^2+1\right) \end{aligned}$$

For $\mathrm{f}(x)$ to be increasing, $\mathrm{f}'(x)\geq 0$

$$\begin{aligned} & \Rightarrow \mathrm{e}^{x(1-x)}\left(x-2x^2+1\right)\geq 0 \\ & \Rightarrow x-2x^2+1\geq 0 \\ & \Rightarrow 2x^2-x-1\leq 0 \\ & \Rightarrow (2x+1)(x-1)\leq 0 \\ & \Rightarrow x\in\left[-\frac{1}{2},1\right] \end{aligned}$$

For $\mathrm{f}(x)$ to be decreasing, $\mathrm{f}'(x)\leq 0$

$$\begin{aligned} & \Rightarrow (2x+1)(x-1)\geq 0 \\ & \Rightarrow x\in\left(-\infty,-\frac{1}{2}\right]\cup[1,\infty) \end{aligned}$$

Question 23

The solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1+y^2}{1+x^2}$ is

Options:

A. $x+y=\mathrm{c}$, where c is a constant of integration.

B. $x-y=\mathrm{c}(xy)$, where c is a constant of integration.

C. $x+y=\mathrm{c}(1+xy)$, where c is a constant of integration.

D. $y-x=\mathrm{c}(1+xy)$, where c is a constant of integration.

Answer: D

Solution:

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$
$$\Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

Integrating on both sides, we get

$$\int \frac{1}{1+y^2} dy - \int \frac{1}{1+x^2} dx = \tan^{-1} c$$
$$\Rightarrow \tan^{-1} y - \tan^{-1} x = \tan^{-1} c$$
$$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = \tan^{-1} c$$
$$\Rightarrow \frac{y-x}{1+xy} = c$$
$$\Rightarrow y - x = c(1+xy)$$

Question 24

If $\bar{a}, \bar{b}, \bar{c}$ are three vectors such that $\overline{\mathrm{a}} \cdot (\overline{\mathrm{b}} + \overline{\mathrm{c}}) + \overline{\mathrm{b}} \cdot (\overline{\mathrm{c}} + \overline{\mathrm{a}}) + \overline{\mathrm{c}} \cdot (\overline{\mathrm{a}} + \overline{\mathrm{b}}) = 0$ and $|\overline{\mathrm{a}}| = 1, |\bar{b}| = 8$ and $|\bar{c}| = 4$, then $|\bar{a} + \bar{b} + \bar{c}|$ has the value __________.

Options:

A. 81

B. 9

C. 5

D. 4

Answer: B

Solution:

$|\bar{a}| = 1, |\bar{b}| = 8, |\bar{c}| = 4$, and

$$\overline{\mathrm{a}} \cdot (\overline{\mathrm{b}} + \overline{\mathrm{c}}) + \overline{\mathrm{b}} \cdot (\overline{\mathrm{c}} + \overline{\mathrm{a}}) + \overline{\mathrm{c}} \cdot (\overline{\mathrm{a}} + \overline{\mathrm{b}}) = 0$$
$$\Rightarrow 2(\overline{\mathrm{a}} \cdot \overline{\mathrm{b}} + \overline{\mathrm{b}} \cdot \overline{\mathrm{c}} + \overline{\mathrm{c}} \cdot \overline{\mathrm{a}}) = 0 \quad \text{.... (i)}$$

Now,

$$|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})$$
$$\Rightarrow |\bar{a}+\bar{b}+\bar{c}|^2 = 1+64+16+0 \quad \ldots [\text{ From (i) }]$$
$$\Rightarrow |\bar{a}+\bar{b}+\bar{c}|^2 = 81$$
$$\Rightarrow |\bar{a}+\bar{b}+\bar{c}| = 9$$

Question 25

Let $\bar{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\bar{b}=\hat{i}+\hat{j}$. If $\bar{c}$ is a vector such that $\overline{\mathrm{a}}\cdot\overline{\mathrm{c}}=|\overline{\mathrm{c}}|, |\overline{\mathrm{c}}-\overline{\mathrm{a}}|=2\sqrt{2}$ and the angle between $(\bar{a}\times\bar{b})$ and $\bar{c}$ is $\frac{\pi}{6}$, then $|(\bar{a}\times\bar{b})\times\bar{c}|$ is

Options:

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. 1

D. $\frac{3}{4}$

Answer: A

Solution:

$$|\bar{c}-\bar{a}| = 2\sqrt{2},$$
$$\Rightarrow |\bar{c}|^2+|\bar{a}|^2-2(\bar{a}\cdot\bar{c}) = 8$$
$$\Rightarrow |\bar{c}|^2+9-2|\bar{c}| = 8 \quad \ldots [\because \overline{\mathrm{a}}\cdot\overline{\mathrm{c}}=|\overline{\mathrm{c}}|]$$
$$\Rightarrow (|\bar{c}|-1)^2 = 0$$
$$\Rightarrow |\bar{c}| = 1 \quad \text{.... (i)}$$

Now,

$$|(\overline{\mathrm{a}}\times\overline{\mathrm{b}})\times\overline{\mathrm{c}}| = |(\overline{\mathrm{a}}\times\overline{\mathrm{b}})||\overline{\mathrm{c}}|\sin\frac{\pi}{6}$$
$$= |\overline{\mathrm{a}}\times\overline{\mathrm{b}}|(1)\left(\frac{1}{2}\right) \quad \text{... [From (i)]}$$
$$= \frac{3}{2} \quad \ldots [\because \overline{\mathrm{a}}\times\overline{\mathrm{b}} = 2\hat{\mathrm{i}}-2\hat{\mathrm{j}}+\hat{\mathrm{k}}]$$

Question 26

$$\int_{-1}^{3}\left(\cot^{-1}\left(\frac{x}{x^2+1}\right)+\cot^{-1}\left(\frac{x^2+1}{x}\right)\right)\mathrm{d}x =$$

Options:

A. $\left(\frac{\pi}{4}\right)$

B. π

C. $\left(\frac{\pi}{2}\right)$

D. (2π)

Answer: D

Solution:

$$\text{Let} \quad \mathrm{I} = \int_{-1}^{3}\left(\cot^{-1}\left(\frac{x}{x^2+1}\right)+\cot^{-1}\left(\frac{x^2+1}{x}\right)\right)\mathrm{d}x$$

$$= \int_{-1}^{3}\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)+\cot^{-1}\left(\frac{x^2+1}{x}\right)\right)\mathrm{d}x$$

$$\ldots\left[\because \cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)\right]$$

$$= \int_{-1}^{3}\frac{\pi}{2}\,\mathrm{d}x \quad \ldots\left[\tan^{-1}x+\cot^{-1}x = \frac{\pi}{2}\right]$$

$$= \frac{\pi}{2}[x]_{-1}^{3}$$

$$= \frac{\pi}{2}(4)$$

$$\therefore \quad \mathrm{I} = 2\pi$$

Question 27

If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then value of α is

Options:

A. 4

B. 11

C. 5

D. 0

Answer: B

Solution:

$$\begin{aligned} & P = \text{adj } A \quad \text{.... [Given]} \\ \therefore \quad & |P| = |A|^2 \quad \ldots . [\because |\text{adjA}| = |\text{A}|^{\text{n}-1}] \\ & \Rightarrow \begin{vmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{vmatrix} = 4^2 \\ & \Rightarrow 2\alpha - 6 = 16 \\ & \Rightarrow \alpha = 11 \end{aligned}$$

Question 28

If $\mathrm{f}(x) = \begin{cases} \frac{\sqrt{1+\mathrm{m}x} - \sqrt{1-\mathrm{m}x}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2} & , 0 \leq x \leq 1 \end{cases}$ is continuous in the interval $[-1, 1]$, then m is equal to

Options:

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. -1

D. $-\frac{1}{4}$

Answer: B

Solution:

Since $\mathrm{f}(x)$ is continuous in $[-1, 1]$, it is continuous at $x = 0$.

$$\therefore \quad \lim_{x \to 0^-} \mathrm{f}(x) = \lim_{x \to 0^+} \mathrm{f}(x)$$
$$\Rightarrow \lim_{x \to 0} \frac{\sqrt{1 + \mathrm{m}x} - \sqrt{1 - \mathrm{m}x}}{x} = \lim_{x \to 0} \frac{2x + 1}{x - 2}$$
$$\Rightarrow \lim_{x \to 0} \frac{(1 + \mathrm{m}x - 1 + \mathrm{m}x)}{x(\sqrt{1 + \mathrm{m}x} + \sqrt{1 - \mathrm{m}x})} = \frac{2(0) + 1}{0 - 2}$$
$$\Rightarrow \frac{2\,\mathrm{m}}{1 + 1} = \frac{1}{-2}$$
$$\Rightarrow \mathrm{m} = \frac{-1}{2}$$

Question 29

The area of the region bounded by the parabola $y = x^2$ and the curve $y = |x|$ is

Options:

A. $\frac{1}{2}$ sq. units

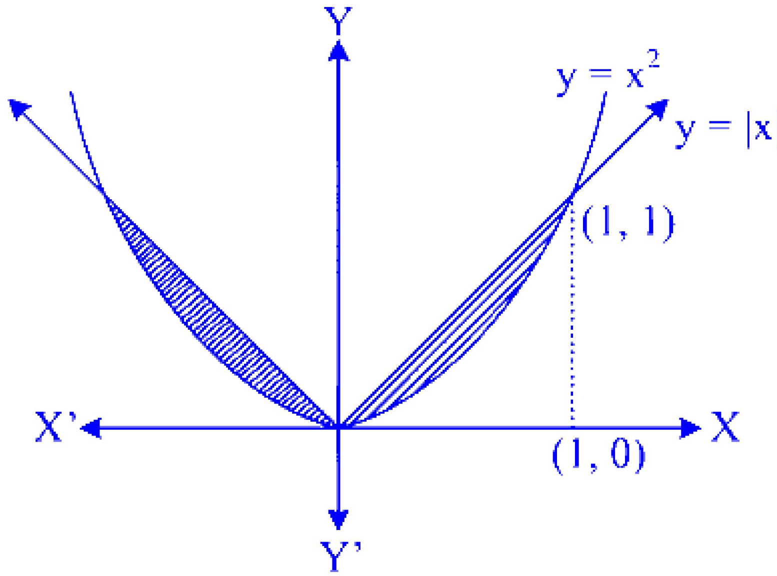
B. $\frac{1}{3}$ sq. units

C. $\frac{1}{4}$ sq. units

D. $\frac{1}{6}$ sq. units

Answer: B

Solution:



Required area $= 2\int_0^1 \left(x - x^2\right) \mathrm{d}x$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3} \text{ sq.units}$$

Question 30

Derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ w.r.t. $\cos^{-1} x^2$ is

Options:

A. $-\frac{1}{2}$

B. -1

C. $\frac{1}{2}$

D. 1

Answer: A

Solution:

Let $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ and $z=\cos^{-1}\left(x^2\right)$

Put $x^2=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1} x^2$

$$\therefore \quad y=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$
$$\Rightarrow y=\tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$
$$\Rightarrow y=\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$
$$\Rightarrow y=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$
$$\Rightarrow y=\frac{\pi}{4}-\frac{1}{2}\cos^{-1} x^2$$
$$\Rightarrow y=\frac{\pi}{4}-\frac{1}{2}z$$
$$\therefore \quad \frac{\mathrm{dy}}{\mathrm{dz}}=-\frac{1}{2}$$

Question 31

A binomial random variable X satisfies $9.p(X=4)=p(X=2)$ when $n=6$. Then p is equal to

Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{8}$

D. $\frac{1}{5}$

Answer: A

Solution:

9. $p(X=4)=p(X=2)$ and $n=6$
$\Rightarrow 9\times {}^6C_4p^4q^2={}^6C_2p^2q^4$
$\Rightarrow 9p^2=q^2$
$\Rightarrow 3p=q \quad \ldots[\because 0<p,q<1]$
$\Rightarrow 3p=1-p$
$\Rightarrow p=\frac{1}{4}$

Question 32

If in $\triangle$ABC, with usual notations, $a\cdot\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$, then

Options:

A. a, b, c are in G.P.

B. a, b, c are in H.P.

C. a, b, c are in A.P.

D. a, b, c are in Arithmetico Geometric Progression

Answer: C

Solution:

$$a\cdot\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$$
$$\Rightarrow a\left(\frac{1+\cos C}{2}\right)+c\left(\frac{1+\cos A}{2}\right)=\frac{3b}{2}$$
$$\Rightarrow a+a\cos C+c+c\cos A=3b$$
$$\Rightarrow a+b+c=3b \quad \ldots[\because b=c\cos A+a\cos C]$$
$$\Rightarrow a+c=2b$$

$\therefore \quad a,b,c$ are in A.P.

Question 33

The lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{2}$

Options:

A. intersect each other and point of intersection is $(2, 1, 3)$

B. intersect each other and point of intersection is $(3, 2, 4)$

C. intersect each other and point of intersection is $(-2, 3, 3)$

D. do not intersect.

Answer: D

Solution:

The given lines are $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{2}$

Here,

$$(x_1, y_1, z_1) \equiv (1, -1, 1)$$
$$(x_2, y_2, z_2) \equiv (-2, 1, -1)$$
$$(a_1, b_1, c_1) \equiv (3, 2, 5)$$
$$(a_2, b_2, c_2) \equiv (4, 3, 2)$$

$$\text{Consider } \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$
$$= \begin{vmatrix} -3 & 2 & -2 \\ 3 & 2 & 5 \\ 4 & 3 & 2 \end{vmatrix}$$
$$= -3(-11) - 2(-14) - 2(1)$$
$$= 33 + 28 - 2$$
$$= 59 \neq 0$$

$\therefore$ The lines are not intersecting.

Question 34

A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$, then, the equation of the curve is

Options:

A. $\sin\left(\frac{y}{x}\right)=\log(x)+\frac{1}{2}$

B. $\operatorname{cosec}\left(\frac{y}{x}\right)=\log(x)+2$

C. $\sec\left(\frac{2y}{x}\right)=\log(x)+2$

D. $\cos\left(\frac{2y}{x}\right)=\log(x)+\frac{1}{2}$

Answer: A

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{x}+\sec\left(\frac{y}{x}\right) \quad \text{.... (i)}$$

Put $y=\mathrm{v}x$

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{v}+x\frac{\mathrm{dv}}{\mathrm{d}x}$$

$$\therefore \quad \mathrm{v}+x\frac{\mathrm{dv}}{\mathrm{d}x}=\mathrm{v}+\sec\mathrm{v} \quad \text{... [From (i)]}$$

$$\Rightarrow \cos\mathrm{v}d\mathrm{v}=\frac{1}{x}\,\mathrm{d}x$$

Integrating both sides, we get

$\sin\mathrm{v}=\log(x)+\mathrm{c}$

$\Rightarrow \sin\left(\frac{y}{x}\right)=\log(x)+\mathrm{c} \quad \text{.... (ii)}$

The curve passes through $\left(1,\frac{\pi}{6}\right)$.

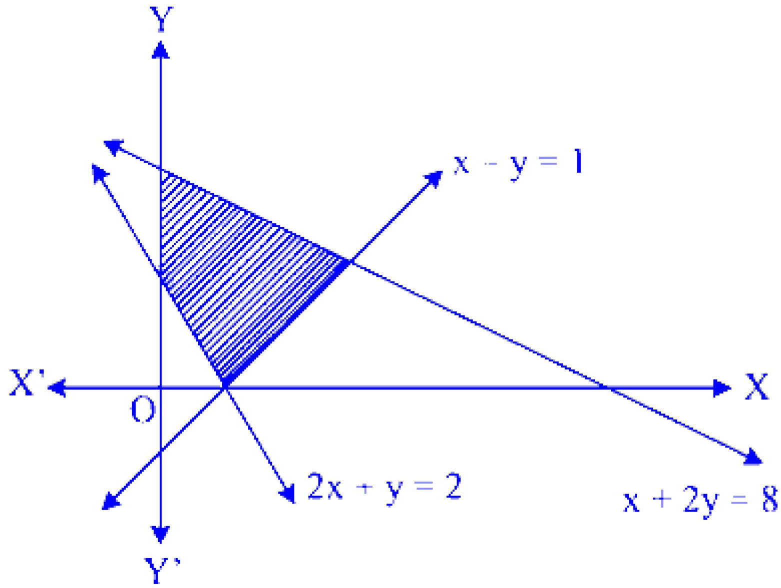
$\sin\left(\frac{\pi}{6}\right)=\log(1)+c\Rightarrow c=\frac{1}{2}$

$\therefore$ Equation (ii) becomes

$\sin\left(\frac{y}{x}\right)=\log(x)+\frac{1}{2}$

Question 35

For the following shaded area, the linear constraints except $x, y \geq 0$ are



Options:

A. $2x + y \leq 2, x - y \leq 1, x + 2y \leq 8$

B. $2x + y \geq 2, x - y \leq 1, x + 2y \leq 8$

C. $2x + y \geq 2, x - y \geq 1, x + 2y \leq 8$

D. $2x + y \geq 2, x - y \geq 1, x + 2y \geq 8$

Answer: B

Solution:

Shaded region lies on non-origin side of $2x + y = 2$ and on origin side of the lines $x - y = 1$ and $x + 2y = 8$

$\therefore \quad 2x + y \geq 2, x - y \leq 1, x + 2y \leq 8$

Question 36

Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of a, b and c is either ω or ω^2, then the number of distinct matrices in the set S is

Options:

A. 2

B. 6

C. 4

D. 8

Answer: A

Solution:

Let $A=\begin{bmatrix}1 & a & b\\ \omega & 1 & c\\ \omega^2 & \omega & 1\end{bmatrix}$

For non-singular matrix $|A|\neq 0$

$$\Rightarrow \begin{vmatrix}1 & a & b\\ \omega & 1 & c\\ \omega^2 & \omega & 1\end{vmatrix}\neq 0$$

$$\begin{aligned}&\Rightarrow 1(1-\omega c)-a\left(\omega-\omega^2 c\right)+b(0)\neq 0\\ &\Rightarrow 1(1-\omega c)-a\omega(1-\omega c)\neq 0\\ &\Rightarrow (1-\omega c)(1-a\omega)\neq 0\\ &\Rightarrow c\neq\frac{1}{\omega}\text{ and }a\neq\frac{1}{\omega}\\ &\Rightarrow c\neq\omega^2\text{ and }a\neq\omega^2\quad\ldots[\because\omega^3=1]\end{aligned}$$

So possible value of a and c is ω only and b can take values ω or ω^2.

$\therefore$ The possible number of distinct matrices = 2.

Question 37

The value of $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

Options:

A. $\tan^{-1}\left(\frac{17}{31}\right)$

B. $\tan^{-1}\left(\frac{19}{31}\right)$

C. $\tan^{-1}\left(\frac{31}{17}\right)$

D. $\tan^{-1}\left(\frac{31}{19}\right)$

Answer: C

Solution:

$$\begin{aligned}
&2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}\\
&=\tan^{-1}\left(\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}\right)+\tan^{-1}\left(\frac{1}{7}\right)\quad\ldots\left[2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]\\
&=\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)\\
&=\tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\left(\frac{4}{3}\right)\left(\frac{1}{7}\right)}\right)\\
&=\tan^{-1}\left(\frac{31}{17}\right)
\end{aligned}$$

Question 38

$\int\frac{\mathrm{e}^x(1+x)}{\cos^2(\mathrm{e}^x\cdot x)}\mathrm{d}x=$

Options:

A. $-\cot\left(\mathrm{e}^x\right)+\mathrm{c}$, where c is a constant of integration.

B. $\tan\left(x\cdot\mathrm{e}^x\right)+\mathrm{c}$, where c is a constant of integration.

C. $\tan\left(\mathrm{e}^x\right)+\mathrm{c}$, where c is a constant of integration.

D. $-\cot\left(x\cdot\mathrm{e}^x\right)+\mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

Let $\mathrm{I}=\int \frac{\mathrm{e}^x(1+x)}{\cos^2(\mathrm{e}^x \cdot x)} \mathrm{d}x$

Put $\mathrm{e}^x \cdot x=\mathrm{t} \Rightarrow \mathrm{e}^x(x+1) \mathrm{d}x=\mathrm{dt}$

$\mathrm{I}=\int \frac{\mathrm{dt}}{\cos^2 \mathrm{t}}$

$\quad=\int \sec^2 \mathrm{t} d\mathrm{t}$

$\quad=\tan \mathrm{t}+\mathrm{c}$

$\therefore \quad \mathrm{I}=\tan(x \cdot \mathrm{e}^x)+\mathrm{c}$

Question 39

In a triangle ABC, with usual notations, if $\mathrm{m}\angle \mathrm{A}=60^{\circ}, \mathrm{b}=8, \mathrm{a}=6$ and $\mathrm{B}=\sin^{-1} x$, then x has the value

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{2}{\sqrt{3}}$

C. $2\sqrt{3}$

D. $\frac{1}{2\sqrt{3}}$

Answer: B

Solution:

By sine rule, we get

$$\frac{\sin A}{a}=\frac{\sin B}{b}$$
$$\Rightarrow \frac{\sin 60^{\circ}}{6}=\frac{x}{8} \quad \ldots[\because B=\sin^{-1}(x)]$$
$$\Rightarrow x=\frac{\sqrt{3}}{2} \times \frac{8}{6}$$
$$\Rightarrow x=\frac{2}{\sqrt{3}}$$

Question 40

If variance of $x_1, x_2 \ldots\ldots, x_n$ is σ_x^2, then the variance of $\lambda x_1, \lambda x_2, \ldots\ldots, \lambda x_{\mathrm{n}}(\lambda \neq 0)$ is

Options:

A. $\lambda \cdot \sigma_x$

B. $\lambda \cdot \sigma_x^2$

C. $\lambda^2 \cdot \sigma_x$

D. $\lambda^2 \cdot \sigma_x^2$

Answer: D

Solution:

When each item of a data is multiplied by λ, variance is multiplied by λ^2.

New variance $= \lambda^2 \cdot \sigma_x^2$

Question 41

If $\overline{\mathrm{a}} = \hat{\mathrm{i}} + \hat{\mathrm{j}}$, $\overline{\mathrm{b}} = 2\hat{\mathrm{j}} - \hat{\mathrm{k}}$ and $\overline{\mathrm{r}} \times \overline{\mathrm{a}} = \overline{\mathrm{b}} \times \overline{\mathrm{a}}, \overline{\mathrm{r}} \times \overline{\mathrm{b}} = \overline{\mathrm{a}} \times \overline{\mathrm{b}}$, then the value $\frac{\overline{\mathrm{r}}}{|\overline{\mathrm{r}}|}$ is

Options:

A. $\frac{\hat{\mathrm{i}}+3\hat{\mathrm{j}}+\hat{\mathrm{k}}}{\sqrt{11}}$

B. $\frac{\hat{\mathrm{i}}-3\hat{\mathrm{j}}+\hat{\mathrm{k}}}{\sqrt{11}}$

C. $\frac{\hat{\mathrm{i}}-3\hat{\mathrm{j}}-\hat{\mathrm{k}}}{\sqrt{11}}$

D. $\frac{\hat{\mathrm{i}}+3\hat{\mathrm{j}}-\hat{\mathrm{k}}}{\sqrt{11}}$

Answer: D

Solution:

Let $\overline{\mathrm{r}}=x\hat{\mathrm{i}}+y\hat{\mathrm{j}}+\mathrm{z}\hat{\mathrm{k}}$ then

$$\overline{\mathrm{r}}\times\overline{\mathrm{a}}=\overline{\mathrm{b}}\times\overline{\mathrm{a}} \quad \Rightarrow(\overline{\mathrm{r}}-\overline{\mathrm{b}})\times\overline{\mathrm{a}}=\overline{0}$$
$$\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ x & y-2 & \mathrm{z}+1\\ 1 & 1 & 0\end{vmatrix}=\overline{0}$$
$$\Rightarrow(-\mathrm{z}-1)\hat{\mathrm{i}}-(-\mathrm{z}-1)\hat{\mathrm{j}}+(x-y+2)\hat{\mathrm{k}}=\overline{0}$$
$$\Rightarrow \mathrm{z}=-1, x-y=-2 \quad \ldots.\text{(i)}$$

Now, $\overline{\mathrm{r}}\times\overline{\mathrm{b}}=\overline{\mathrm{a}}\times\overline{\mathrm{b}}=(\overline{\mathrm{r}}-\overline{\mathrm{a}})\times\overline{\mathrm{b}}=\overline{0}$

$$\therefore \quad \begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ x-1 & y-1 & z\\ 0 & 2 & -1\end{vmatrix}=\overline{0}$$
$$\Rightarrow(1-y-2\mathrm{z})\hat{\mathrm{i}}-(1-x)\hat{\mathrm{j}}+(2x-2)\hat{\mathrm{k}}=\overline{0}$$
$$\Rightarrow 1-y-2\mathrm{z}=0, x=1 \quad \ldots.\text{(ii)}$$

Solving (i) and (ii), we get

$$x=1, y=3, z=-1$$
$$\therefore \quad \overline{\mathrm{r}}=\hat{\mathrm{i}}+3\hat{\mathrm{j}}-\hat{\mathrm{k}}$$
$$|\overline{\mathrm{r}}|=\sqrt{1+9+1}=\sqrt{11}$$
$$\therefore \quad \frac{\overline{\mathrm{r}}}{|\overline{\mathrm{r}}|}=\frac{\hat{\mathrm{i}}+3\hat{\mathrm{j}}-\hat{\mathrm{k}}}{\sqrt{11}}$$

Question 42

If $\mathrm{z}=x+\mathrm{i}y$ and $\mathrm{z}^{1/3}=\mathrm{p}+\mathrm{iq}$, where $x, y, \mathrm{p}, \mathrm{q}\in\mathrm{R}$ and $\mathrm{i}=\sqrt{-1}$, then value of $\left(\frac{x}{\mathrm{p}}+\frac{y}{\mathrm{q}}\right)$ is

Options:

A. $\mathrm{p}^2-\mathrm{q}^2$

B. $4\left(\mathrm{p}^2-\mathrm{q}^2\right)$

C. $\mathrm{p}^2+\mathrm{q}^2$

D. $4\left(\mathrm{p}^2+\mathrm{q}^2\right)$

Answer: B

Solution:

$$\begin{aligned}
& \mathrm{z}^{\frac{1}{3}}=\mathrm{p}+\mathrm{iq} \\
\Rightarrow & \mathrm{z}=(\mathrm{p}+\mathrm{iq})^3 \\
\Rightarrow & x+\mathrm{i} y=\mathrm{p}^3+3 \mathrm{p}^2 \mathrm{qi}+3 \mathrm{p}(\mathrm{iq})^2+(\mathrm{iq})^3 \\
\Rightarrow & x+\mathrm{i} y=\left(\mathrm{p}^3-3 \mathrm{pq}^2\right)+\left(3 \mathrm{p}^2 \mathrm{q}-\mathrm{q}^3\right) \mathrm{i} \\
\Rightarrow & x=\mathrm{p}^3-3 \mathrm{pq}^2 \text { and } y=3 \mathrm{p}^2 \mathrm{q}-\mathrm{q}^3 \\
\Rightarrow & \frac{x}{\mathrm{p}}=\mathrm{p}^2-3 \mathrm{q}^2 \text { and } \frac{y}{\mathrm{q}}=3 \mathrm{p}^2-\mathrm{q}^2 \\
\therefore & \left(\frac{x}{\mathrm{p}}+\frac{y}{\mathrm{q}}\right)=4 \mathrm{p}^2-4 \mathrm{q}^2=4\left(\mathrm{p}^2-\mathrm{q}^2\right)
\end{aligned}$$

Question 43

If $\int \frac{\mathrm{d} x}{x \sqrt{1-x^3}}=\mathrm{k} \log \left(\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right)+\mathrm{c}$, (where c is a constant of integration), then value of k is

Options:

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. $\frac{1}{3}$

D. $-\frac{1}{3}$

Answer: C

Solution:

$$\text{Let I} = \int \frac{dx}{x\sqrt{1-x^3}}$$

$$= \int \frac{x^2\, dx}{x^3\sqrt{1-x^3}}\, dx$$

$$\text{Put } 1 - x^3 = t^2$$

$$\Rightarrow -3x^2\, dx = 2tdt$$

$$\Rightarrow x^2\, dx = \frac{-2tdt}{3}$$

$$\therefore \quad I = \int \frac{-\left(\frac{2tdt}{3}\right)}{(1-t^2)t}$$

$$= -\frac{2}{3}\int \frac{1}{1-t^2}dt$$

$$= \frac{2}{3}\int \frac{1}{t^2-1}dt$$

$$= \frac{2}{3} \times \frac{1}{2} \cdot \log\left|\frac{t-1}{t+1}\right| + c$$

$$= \frac{1}{3}\log\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + c$$

$$\therefore \quad k = \frac{1}{3}$$

Question 44

The statement pattern $p \rightarrow \sim (p \wedge \sim q)$ is equivalent to

Options:

A. q

B. $(\sim p) \vee q$

C. $(\sim p) \wedge q$

D. $(\sim p) \vee (\sim q)$

Answer: B

Solution:

$p \rightarrow \sim (p \wedge \sim q)$
$\equiv \sim p \vee \sim (p \wedge \sim q) \quad \ldots [\because p \rightarrow q \equiv \sim p \vee q]$
$\equiv \sim p \vee (\sim p \vee q)$ ...[De Morgan's law]
$\equiv (\sim p \vee \sim p) \vee q$ ...[Associative law]
$\equiv \sim p \vee q$ ...[Idempotent law]

Question 45

a and b are the intercepts made by a line on the co-ordinate axes. If $3\mathrm{a} = \mathrm{b}$ and the line passes through $(1, 3)$, then the equation of the line is

Options:

A. $x + 3y = 10$

B. $3x + y = 6$

C. $x - 3y + 8 = 0$

D. $3x - 2y + 3 = 0$

Answer: B

Solution:

Let the equation of the line be

$\frac{x}{\mathrm{a}} + \frac{y}{\mathrm{b}} = 1$ .... (i)

This line passes through the point $(1, 3)$

$$\begin{aligned} \therefore \quad & \frac{1}{a} + \frac{3}{b} = 1 \\ & \Rightarrow b + 3a = ab \\ & \Rightarrow 2b = ab \quad \ldots [\because 3a = b] \\ & \Rightarrow a = 2 \\ \therefore \quad & b = 3a = 6 \end{aligned}$$

Equation (i) becomes,

$$\frac{x}{2} + \frac{y}{6} = 1$$
$$3x + y = 6$$

Question 46

Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three unit vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\sqrt{3}}{2}(\bar{b} + \bar{c})$. If $\bar{b}$ is not parallel to $\bar{c}$, then the angle between $\bar{a}$ and $\bar{b}$ is

Options:

A. $\frac{5\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer: A

Solution:

$$\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\sqrt{3}}{2}(\bar{b} + \bar{c})$$

$$\Rightarrow (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c} = \frac{\sqrt{3}}{2}\bar{b} + \frac{\sqrt{3}}{2}\bar{c}$$

On comparing, we get $\bar{a} \cdot \bar{c} = \frac{\sqrt{3}}{2}$ and $\bar{a} \cdot \bar{b} = -\frac{\sqrt{3}}{2}$

$$\Rightarrow |\bar{a}||\bar{b}| \cos\theta = -\frac{\sqrt{3}}{2} \quad \ldots [\because |\bar{a}| = |\bar{b}| = 1]$$

$$\Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

Question 47

The vector equation of the line $2x + 4 = 3y + 1 = 6z - 3$ is

Options:

A. $\overline{\mathrm{r}} = \left(2\hat{\mathrm{i}} + \frac{1}{3}\hat{\mathrm{j}} + \frac{1}{2}\hat{\mathrm{k}}\right) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \overline{\mathrm{k}})$

B. $\overline{\mathrm{r}} = \left(-2\hat{\mathrm{i}} - \frac{1}{3}\hat{\mathrm{j}} + \frac{1}{2}\hat{\mathrm{k}}\right) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}})$

C. $\overline{\mathrm{r}} = (2\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \overline{\mathrm{k}})$

D. $\overline{\mathrm{r}} = (-2\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}})$

Answer: B

Solution:

The equation of line is

$$2x + 4 = 3y + 1 = 6z - 3$$
$$\Rightarrow 2(x+2) = 3\left(y + \frac{1}{3}\right) = 6\left(z - \frac{1}{2}\right)$$
$$\Rightarrow \frac{x+2}{\frac{1}{2}} = \frac{y+\frac{1}{3}}{\frac{1}{3}} = \frac{z-\frac{1}{2}}{\frac{1}{6}}$$
$$\Rightarrow \frac{x+2}{3} = \frac{y+\frac{1}{3}}{2} = \frac{z-\frac{1}{2}}{1}$$

$\therefore$ The given line passes through $\left(-2, -\frac{1}{3}, \frac{1}{2}\right)$ and has direction ratios proportional to $3, 2, 1$.

$\therefore$ Vector equation of the line is

$$\bar{r} = \left(-2\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{2}\hat{k}\right) + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$$

Question 48

If $\overline{\mathrm{a}}$ and $\overline{\mathrm{b}}$ are two unit vectors such that $\overline{\mathrm{a}} + 2\overline{\mathrm{b}}$ and $5\bar{a} - 4\bar{b}$ are perpendicular to each other, then the angle between $\bar{a}$ and $\bar{b}$ is

Options:

A. $\left(\frac{\pi}{4}\right)$

B. $\left(\frac{\pi}{3}\right)$

C. $\cos^{-1}\left(\frac{1}{3}\right)$

D. $\cos^{-1}\left(\frac{2}{7}\right)$

Answer: B

Solution:

Since $\bar{a}+2\bar{b}$ and $5\bar{a}-4\bar{b}$ are perpendicular to each other

$$\therefore\quad (\overline{\mathrm{a}}+2\overline{\mathrm{b}})\cdot(5\overline{\mathrm{a}}-4\overline{\mathrm{b}})=0$$
$$\Rightarrow 5|\overline{\mathrm{a}}|^2-8|\overline{\mathrm{b}}|^2+6\overline{\mathrm{a}}\cdot\overline{\mathrm{b}}=0$$
$$\Rightarrow -3+6|\overline{\mathrm{a}}||\overline{\mathrm{b}}|\cos\theta=0\quad \ldots[\because |\overline{\mathrm{a}}|=|\overline{\mathrm{b}}|=1]$$
$$\Rightarrow \cos\theta=\frac{1}{2}$$
$$\Rightarrow \theta=\frac{\pi}{3}$$

Question 49

$$\int\frac{\log(\cot x)}{\sin 2x}dx=$$

Options:

A. $-\log(\cot x)^2+c$, where c is constant of integration.

B. $2(\log(\cot x))^2+c$, where c is constant of integration.

C. $\frac{-1}{4}(\log(\sin x))^2+c$, where c is constant of integration.

D. $\frac{-1}{4}(\log(\cot x))^2+\mathrm{c}$, where c is constant of integration.

Answer: D

Solution:

Let $\mathrm{I}=\int\frac{\log(\cot x)}{\sin 2x}\,\mathrm{d}x$

Put $\log(\cot x)=\mathrm{t}$

$$\Rightarrow \left[\frac{1}{\cot x} \cdot \left(-\operatorname{cosec}^2 x\right)\right] dx = dt$$

$$\Rightarrow \left[\frac{-\sin x}{\sin^2 x \cdot \cos x}\right] dx = dt$$

$$\Rightarrow \frac{dx}{\sin 2x} = \frac{-dt}{2}$$

$$\therefore \quad I = \int t \cdot \left(-\frac{dt}{2}\right)$$

$$= \frac{-1}{4} t^2 + c$$

$$= \frac{-1}{4} [\log(\cot x)]^2 + c$$

Question 50

If $y = \sqrt{\frac{1-\sin^{-1}(x)}{1+\sin^{-1}(x)}}$, then $\frac{dy}{dx}$ at $x = 0$ and $y = 1$ is

Options:

A. -2

B. -1

C. 1

D. 2

Answer: B

Solution:

$y = \sqrt{\frac{1-\sin^{-1}(x)}{1+\sin^{-1}(x)}}$

Taking log on both sides, we get

$\log y = \frac{1}{2}\left[\log\left(1 - \sin^{-1} x\right) - \log\left(1 + \sin^{-1} x\right)\right]$

Differentiating w. r. t. x, we get

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left[\frac{1}{1-\sin^{-1}x}\cdot\frac{-1}{\sqrt{1-x^2}}-\frac{1}{1+\sin^{-1}x}\cdot\frac{1}{\sqrt{1-x^2}}\right]$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-y}{2\sqrt{1-x^2}}\left(\frac{1}{1-\sin^{-1}x}+\frac{1}{1+\sin^{-1}x}\right)$$

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(0,1)}=\frac{-1}{2(1)}\left(\frac{1}{1-0}+\frac{1}{1+0}\right)=-1$$

Chemistry

Question 51

Name the accelerator used to introduce network of crosslink in elastomer.

Options:

A. Zinc butyl xanthate

B. Zinc ethyl xanthate

C. Zinc butyl stearate

D. Zinc propyl xanthate

Answer: A

Question 52

Which of the following is a property of alkali metals?

Options:

A. High density

B. Compounds are paramagnetic

C. Form dipositive ions only

D. Most electropositive elements

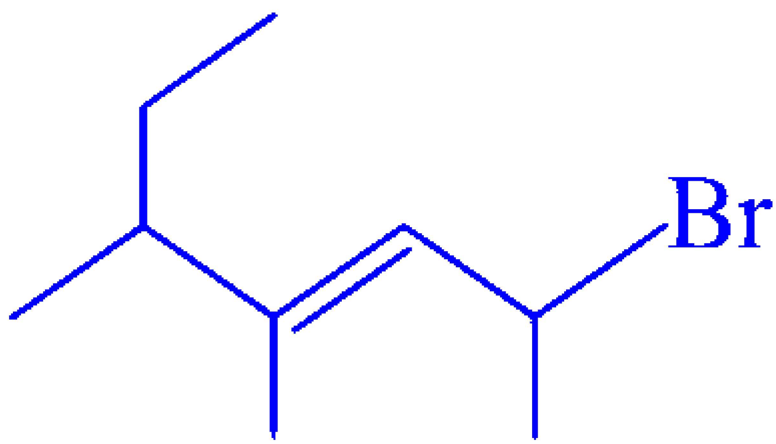
Answer: D

Solution:

Alkali metals are the most electropositive elements and have low density. Alkali metals can lose their one valence shell electron and form unipositive ions with no unpaired electrons. Hence, they form diamagnetic compounds.

Question 53

The IUPAC name of following compound is:



Options:

A. 1-Bromo-1,3,4-trimethylhex-2-ene

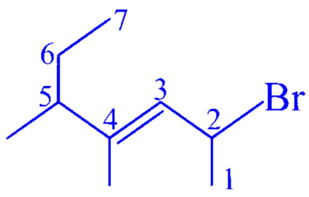
B. 2-Bromo-4,5-dimethylhept-3-ene

C. 2-Bromo-5-ethyl-4-methylhex-3-ene

D. 1-Bromo-4-ethyl-1,3-dimethylpent-2-ene

Answer: B

Solution:

Compound	IUPAC name
	2-Bromo-4,5-dimethylhept-3-ene

----

Question 54

If $\mathrm{K_b}$ denote molal elevation constant of water, then boiling point of an aqueous solution containing 36 g glucose (molar mass $= 180$) per dm^3 is:

Options:

A. $(100+\mathrm{K_b})^{\circ}\mathrm{C}$

B. $(100+2\ \mathrm{K_b})^{\circ}\mathrm{C}$

C. $\left(100+\frac{\mathrm{K_b}}{10}\right)^{\circ}\mathrm{C}$

D. $\left(100+\frac{2\ \mathrm{K_b}}{10}\right)^{\circ}\mathrm{C}$

Answer: D

Solution:

The aqueous solution contains 36 g glucose per dm^3, so mass of solute $\mathrm{W_2}$ is 36 g.

Assuming that the density of solution is $1\ \mathrm{g/dm^3}$, the mass of solvent (water) is 1000 g.

$$\Delta T_b = \frac{1000\ K_b W_2}{M_2\ W_1}$$
$$\Delta T_b = \frac{1000\ g\ kg^{-1} \times K_b \times 36\ g}{180\ g \times 1000\ g}$$
$$\Delta T_b = \frac{2\ K_b}{10}$$
$$\Delta T_b = T_b - T_b^{\circ}$$
$$T_b = T_b^{\circ} + \Delta T_b$$
$$\therefore\ T_b = \left(100 + \frac{2\ K_b}{10}\right)^{\circ} C$$

Question 55

Which metal halide from following has lowest ionic character ($M =$ metal atom)?

Options:

A. MF

B. MCl

C. MBr

D. MI

Answer: D

Solution:

Ionic character of metal halides decreases in the order $MF > MCl > MBr > MI$, where, M is a monovalent metal. Smaller the size of anion, greater is the ionic character (Fajan's rule). Therefore, MI has the lowest ionic character.

Question 56

Which among the following reagents is called as Hinsberg's reagent?

Options:

A. Benzenesulphonyl chloride

B. Sodium nitroprusside

C. Chromyl chloride

D. Hydraine

Answer: A

Question 57

Which among the following amines has highest value of $\mathrm{pK_b}$?

Options:

A. $(\mathrm{CH_3})_2\mathrm{NH}$

B. $(\mathrm{CH_3})_3\mathrm{N}$

C. $\mathrm{CH_3NH_2}$

D. $\mathrm{C_6H_5NH_2}$

Answer: D

Solution:

In general, arylamines are weaker bases than aliphatic amines.

Hence, aniline is the weakest base and its pK_b value is the highest among the given amines.

Question 58

What is vapour pressure of a solution containing $1\ \mathrm{mol}$ of a nonvolatile solute in $36\ \mathrm{g}$ of water $\left(\mathrm{P}_1^0 = 32\ \mathrm{mm\ Hg}\right)$?

Options:

A. 8.14 mm Hg

B. 12.31 mm Hg

C. 16.08 mm Hg

D. 21.44 mm Hg

Answer: D

Solution:

$$\begin{aligned} & \mathrm{n}_2=1 \mathrm{~mol} \\ & \mathrm{n}_1=\frac{36}{18}=2 \mathrm{~mol} \end{aligned}$$

Relative lowering of vapour pressure

$=\frac{\mathrm{P}_1^0-\mathrm{P}_1}{\mathrm{P}_1^0}=x_2=\frac{\mathrm{n}_2}{\mathrm{n}_1+\mathrm{n}_2}$

$$\begin{aligned} \therefore \quad & \frac{32 \mathrm{~mmHg}-\mathrm{P}_1}{32 \mathrm{~mmHg}}=\frac{1}{3} \\ & 96 \mathrm{~mm} \mathrm{~Hg}-3 \mathrm{P}_1=32 \mathrm{~mm} \mathrm{~Hg} \\ & 64 \mathrm{~mm} \mathrm{~Hg}=3 \mathrm{P}_1 \\ \therefore \quad & \mathrm{P}_1=21.33 \mathrm{~mm} \mathrm{~Hg} \approx 21.44 \mathrm{~mm} \mathrm{~Hg} \end{aligned}$$

Question 59

Identify the product obtained when phenol is reacted with dilute nitric acid at low temperature.

Options:

A. ortho-Nitrophenol

B. para-Nitrophenol

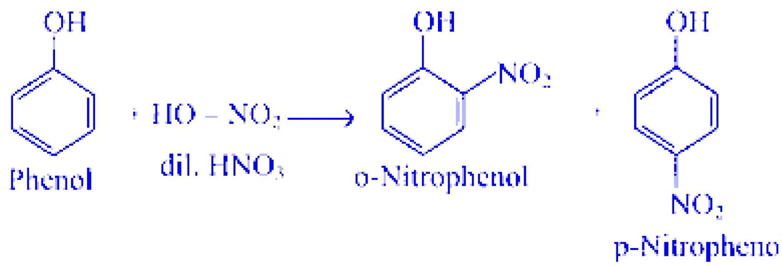
C. Mixture of ortho and para-nitrophenols

D. 2,4,6-Trinitrophenol

Answer: C

Solution:

Phenol reacts with dilute nitric acid at low temperature to give mixture of ortho- and para-nitrophenols.



Question 60

For an elementary reaction

$2\ \mathrm{A} + \mathrm{B} \longrightarrow 3\mathrm{C}$

rate of appearance of C is $1.3 \times 10^{-4}\ \mathrm{mol\ L^{-1}\ s^{-1}}$, the rate of disappearance of A is:

Options:

A. $1.3 \times 10^{-4}\ \mathrm{mol\ L^{-1}\ s^{-1}}$

B. $2.6 \times 10^{-4}\ \mathrm{mol\ L^{-1}\ s^{-1}}$

C. $5.2 \times 10^{-4}\ \mathrm{mol\ L^{-1}\ s^{-1}}$

D. $8.66 \times 10^{-5}\ \mathrm{mol\ L^{-1}\ s^{-1}}$

Answer: D

Solution:

$$\text{Rate of reaction } = -\frac{1}{2}\frac{d[A]}{dt} = -\frac{d[B]}{dt} = \frac{1}{3}\frac{d[C]}{dt}$$

$$\begin{aligned}\text{Rate of reaction } &= \frac{1}{3}\frac{d[C]}{dt} \\ &= \frac{1}{3} \times 1.3 \times 10^{-4} \text{ mol L}^{-1}\text{ s}^{-1} \\ &= 0.433 \times 10^{-4} \text{ mol L}^{-1}\text{ s}^{-1}\end{aligned}$$

$\therefore$ Rate of disappearance of A

$$\begin{aligned}&= 2 \times 0.433 \times 10^{-4} \text{ mol L}^{-1}\text{ s}^{-1} \\ &= 0.866 \times 10^{-4} \text{ mol L}^{-1}\text{ s}^{-1} \\ &= 8.66 \times 10^{-5} \text{ mol L}^{-1}\text{ s}^{-1}\end{aligned}$$

Question 61

Which among the following alkenes is most stable?

Options:

A. $R_2C = CH_2$

B. $H_2C = CH_2$

C. $RCH = CHR$

D. $R_2C = CR_2$

Answer: D

Solution:

Order of stability of alkyl substituted alkenes:

$R_2C = CR_2 > R_2C = CHR > R_2C = CH_2$,
$RCH = CHR > RCH = CH_2 > CH_2 = CH_2$.

Greater is the number of alkyl groups attached to the doubly bonded carbon atoms, greater is the stability of alkene.

Question 62

Identify a CORRECT formula for spin only magnetic moment.

Options:

A. $\mu = \left(n^2 + 2\right) BM$

B. $\mu = \sqrt{n^2 + 2} BM$

C. $\mu = \sqrt{n(n+2)}\ BM$

D. $\mu = (n+2)^2 BM$

Answer: C

Question 63

Which of the following salts turns red litmus blue in its aqueous solution?

Options:

A. NH_4NO_3

B. NH_4Cl

C. NH_4CN

D. NH_4F

Answer: C

Solution:

NH_4CN is the salt of weak acid HCN $\left(K_a = 4.0 \times 10^{-10}\right)$ and weak base NH_4OH $\left(K_b = 1.8 \times 10^{-5}\right)$ showing that $K_a < K_b$. If $K_a < K_b$, the aqueous solution of the salt will be basic. Thus, the solution of NH_4CN is basic and turns red litmus blue.

Question 64

What is the value of increase in internal energy when system does $8 \mathrm{~J}$ of work on surrounding by supplying $40 \mathrm{~J}$ of heat to it?

Options:

A. 23 J

B. 32 J

C. 40 J

D. 48 J

Answer: B

Solution:

Heat $(Q)=+40 \mathrm{~J}$

Work $(W)=-8 \mathrm{~J}$

According to first law of thermodynamics,

$\Delta \mathrm{U}=\mathrm{Q}+\mathrm{W}$
$\Delta \mathrm{U}=(+40-8) \mathrm{J} \quad \therefore \quad \Delta \mathrm{U}=+32 \mathrm{~J}$

Question 65

Benzonitrile on reduction with stannous chloride in presence of hydrochloric acid followed by acid hydrolysis forms:

Options:

A. Benzal chloride

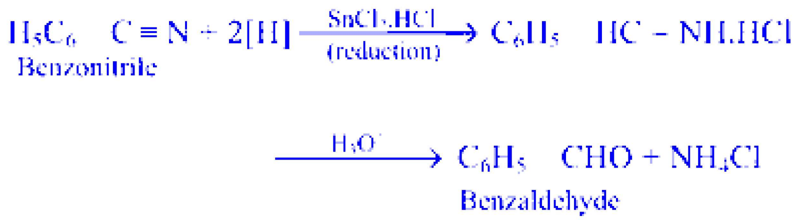
B. Benzoyl chloride

C. Benzophenone

D. Benzaldehyde

Answer: D

Solution:



Question 66

Which among the following is TRUE for isobaric process?

Options:

A. $\Delta U = 0$

B. $-\Delta U = -W$

C. $\Delta U = Q$

D. $Q_P = \Delta U + P_{ext} \Delta V$

Answer: D

Solution:

When chemical reactions are carried out in the open containers under constant atmospheric pressure (isobaric process), $\Delta V \neq 0$.

$W = -P_{ext} \quad \Delta V$

The mathematical expression of first law of thermodynamics is $\Delta U = Q + W$.

$$\therefore \quad \Delta U = Q_P - P_{ext} \quad \Delta V$$
$$\therefore \quad Q_P = \Delta U + P_{ext} \quad \Delta V$$

Question 67

Which among the following statements is TRUE about gammexane?

Options:

A. It is an isomer of BHC.

B. It is one of the herbicides.

C. It is obtained from benzene by bromination.

D. It is monochloro derivative of benzene.

Answer: A

Solution:

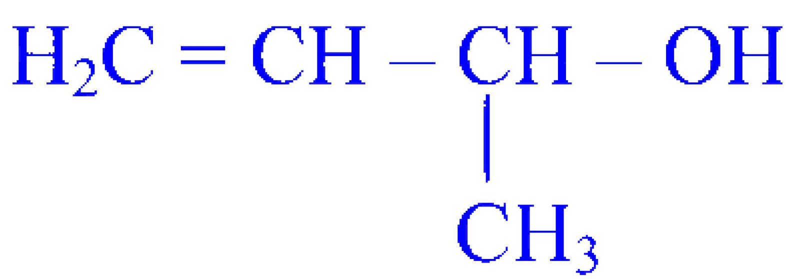
The γ-isomer of benzene hexachloride (BHC), which is used as insecticide, is called as gammexane or lindane.

Question 68

Which of the following is primary allylic alcohol?

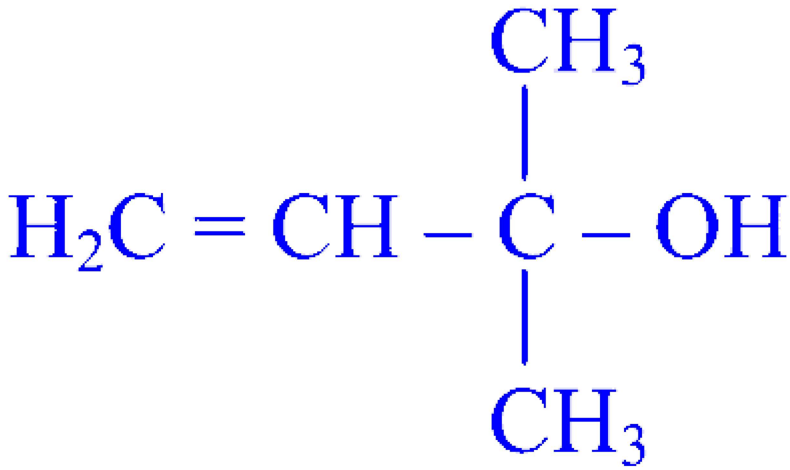
Options:

A.

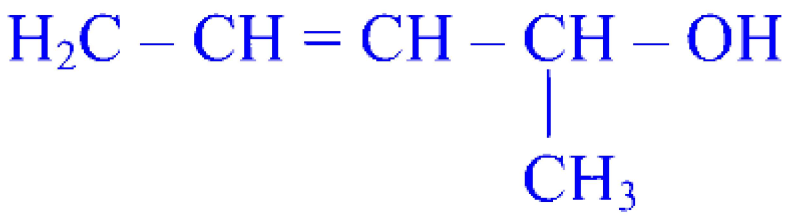


B. $H_2C = CH - CH_2OH$

C.



D.



Answer: B

Solution:

In primary allylic alcohols, hydroxyl group is bonded to a sp^3 hybridized primary carbon atom next to a carbon-carbon double bond.

Hence, $H_2C = CH - CH_2OH$ is a primary allylic alcohol.

Question 69

Which of following is NOT correct about fructose?

Options:

A. It is ketohexose.

B. It is reducing sugar.

C. It is laevorotatory.

D. Its ring structure is hemiacetal.

Answer: D

Solution:

Ring structure of fructose is a hemiketal not hemiacetal.

Fructose $(C_6H_{12}O_6)$ is a laevorotatory ketohexose. It is a reducing sugar.

Question 70

Which among following salts shows decrease in solubility with increase in temperature?

Options:

A. Na_2SO_4

B. KNO_3

C. $NaNO_3$

D. KBr

Answer: A

Solution:

Dissolution of Na_2SO_4 in water is an exothermic process. When a substance dissolves in water by an exothermic process, its solubility decreases with an increase in temperature. Hence, solubility of Na_2SO_4 in water decreases with increase in temperature.

Question 71

Find the number of faradays of electricity required to produce 45 g of Al from molten Al_2O_3.

(At. mass of $Al = 27$)

Options:

A. 1 F

B. 3 F

C. 5 F

D. 7 F

Answer: C

Solution:

$Al^{3+} + 3e^- \longrightarrow Al$

The equation shows that 3 moles of electrons are required to produce 1 mole of Al (i.e., 27 g of Al).

$\therefore$ 3 F of electricity is required to produce 27 g of Al from molten Al_2O_3.

$\therefore$ Faradays of electricity required to produce 45 g of Al $= \frac{3}{27} \times 45 = 5$ F

Question 72

Identify the reagent that confirms the presence of five $-OH$ groups in glucose.

Options:

A. NH_2OH

B. HCN

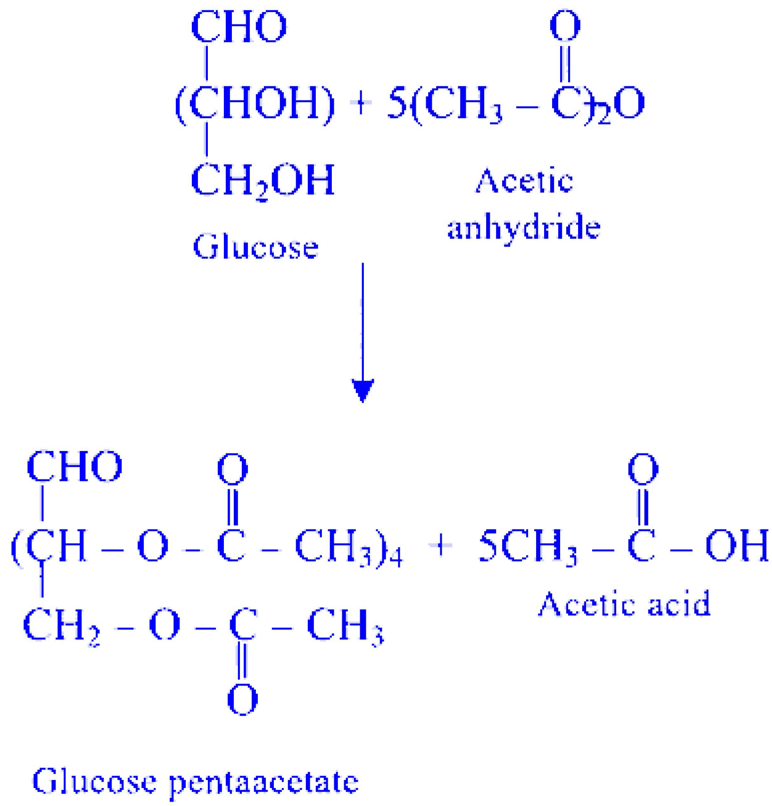
C. dil. HNO_3

D. Acetic anhydride

Answer: D

Solution:

Glucose reacts with acetic anhydride to form glucose pentaacetate. This confirms the presence of five hydroxyl groups in glucose.



Question 73

What is the concentration of OH^{-} ion in a solution containing $0.05\ \mathrm{M}\ \mathrm{H}^{+}$ ions?

Options:

A. $2.5 \times 10^{-13}\mathrm{M}$

B. $5.0 \times 10^{-2}\mathrm{M}$

C. $2.0 \times 10^{-13}\mathrm{M}$

D. $4.2 \times 10^{-12}\mathrm{M}$

Answer: C

Solution:

$$K_w = [H_3O^+][OH^-]$$

$$\therefore \quad 10^{-14} = 0.05 \times [OH^-]$$

$$[OH] = \frac{10^{-14}}{0.05}$$

$$[OH^-] = 2.0 \times 10^{-13} M$$

Question 74

Which formula is used to calculate edge length in bcc structure?

Options:

A. $a = \frac{\sqrt{3}r}{4}$

B. $a = \frac{4}{\sqrt{3}r}$

C. $a = \frac{\sqrt{3}}{4r}$

D. $a = \frac{4r}{\sqrt{3}}$

Answer: D

Question 75

Identify the products of following reaction:

$$\text{Methanal + Benzene carbaldehyde} \xrightarrow[\text{ii. } H_3O^+]{\text{i. conc. NaOH}}$$

Options:

A. Methanoic acid and phenyl methanol

B. Methanol and benzoic acid

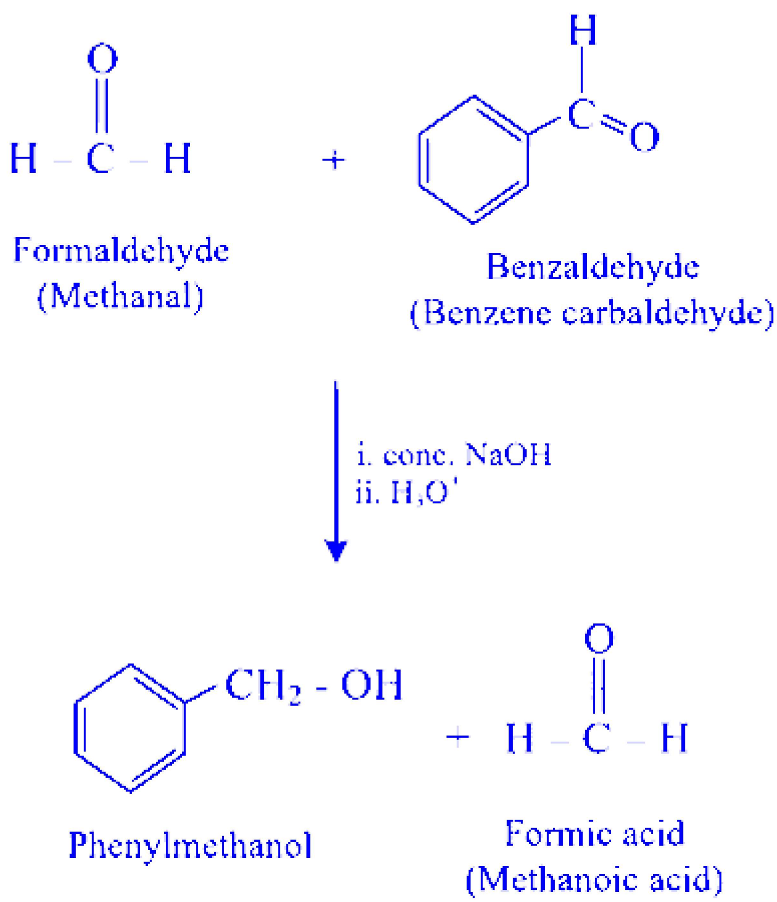
C. Methanol and phenol

D. Methanoic acid and phenol

Answer: A

Solution:

When a mixture of methanal and benzene carbaldehyde (aldehyde with no α-hydrogen) is treated with a strong base, methanal is oxidized to methanoic acid while benzene carbaldehyde is reduced to phenyl methanol.



Question 76

What is the volume in dm^3 occupied by $3\ \mathrm{mol}$ of ammonia gas at STP?

Options:

A. 2.24

B. 22.4

C. 56.0

D. 67.2

Answer: D

Solution:

$$\text{Number of moles of a gas (n)} = \frac{\text{Volume of a gas at STP}}{\text{Molar volume of a gas}}$$

$$\therefore \quad \text{Volume of ammonia gas at STP} = \text{Number of moles of the gas (n)} \times \text{Molar volume of the gas}$$

$$= 3 \text{ mol} \times 22.4 \text{ dm}^3 \text{ mol}^{-1}$$

$$= 67.2 \text{ dm}^3$$

Question 77

Slope of the graph between $\log \frac{[\mathrm{A}]_0}{[\mathrm{A}]_{\mathrm{t}}}$ (y axis) and time (x axis) for first order reaction is equal to:

Options:

A. $+\frac{\mathrm{k}}{2.303}$

B. k

C. $-\mathrm{k}$

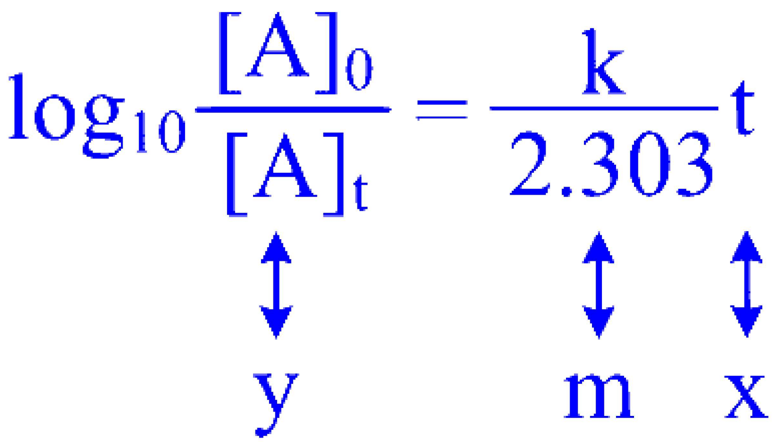
D. $-\frac{2.303}{\mathrm{k}}$

Answer: A

Solution:

The integrated rate law for the first order reaction is

$k = \frac{2.303}{t} \log_{10} \frac{[\mathrm{A}]_0}{[\mathrm{A}]_{\mathrm{t}}}$



The graph of $\log_{10}\frac{[A]_0}{[A]_t}$ versus time (t) is a straight line passing through origin with slope $(m) = +\frac{k}{2.303}$.

Question 78

What is total number of donor atoms in $[Co\,(H_2O)(NH_3)_5]I_3$?

Options:

A. 5

B. 9

C. 6

D. 3

Answer: C

Solution:

In $[Co\,(H_2O)(NH_3)_5]I_3$, five ammonia molecules and one water molecule, that is, total six ligands are attached to cobalt ion. All these ligands are monodentate since each has only one donor atom. Therefore, there are six donor atoms in the complex.

Question 79

Which from following equations is used to express the angular momentum of an electron in a stationary state?

Options:

A. $mvr = \frac{nh}{2\pi}$

B. $\mathrm{mvr} = \frac{2\pi}{\mathrm{nh}}$

C. $r = \frac{mvh}{n2\pi}$

D. $\mathrm{mv} = \frac{2\pi\mathrm{r}}{\mathrm{nh}}$

Answer: A

Question 80

Which emission transition series is obtained when electron jumps from $\mathrm{n}_2 = \infty$ to $\mathrm{n}_1 = 1$?

Options:

A. Balmer

B. Lyman

C. Paschen

D. Bracket

Answer: B

Solution:

When electron jumps from higher energy level to $\mathrm{n} = 1$, the emission line corresponds to Lyman series.

Question 81

What is half life time of a first order reaction if initial conc. of reactant is $0.01 \mathrm{~mol} \mathrm{~L}^{-1}$ and rate of reaction is $0.00352 \mathrm{~mol} \mathrm{~L}^{-1}$ minute $^{-1}$?

Options:

A. 1.969 minute

B. 7.75 minute

C. 16.69 minute

D. 19.69 minute

Answer: A

Solution:

For a first order reaction, rate $= \mathrm{k}[\mathrm{A}]$

$\therefore \quad 0.00352 \mathrm{~mol} \mathrm{~L}^{-1} \text{ minute}^{-1}=\mathrm{k} \times 0.01 \mathrm{~mol} \mathrm{~L}^{-1}$

$\mathrm{k}=0.352$ minute $^{-1}$

Half-life period, $\left(\mathrm{t}_{1/2}\right)=\frac{0.693}{\mathrm{k}}=\frac{0.693}{0.352}=1.969$ minute

----

Question 82

Which among the following pairs of polymers contains both members as copolymers?

Options:

A. Neoprene and Isoprene

B. Orlon and Teflon

C. Bakelite and Orlon

D. SBR and PHBV

Answer: D

Solution:

PHBV - It is a copolymer of two bifunctional β-hydroxy carboxylic acids, namely, β-hydroxybutyric acid (3-hydroxybutanoic acid) and β-hydroxyvaleric acid (3-hydroxypentanoic acid).

SBR - It is a copolymer of styrene with butadiene.

Question 83

What is atomic mass of an element with BCC structure and density $10 \mathrm{~g} \mathrm{~cm}^{-3}$ having edge length $300 \mathrm{~pm}$?

Options:

A. $51.0 \mathrm{~g} \mathrm{~mol}^{-1}$

B. $60.0 \mathrm{~g} \mathrm{~mol}^{-1}$

C. $81.3 \mathrm{~g} \mathrm{~mol}^{-1}$

D. $96.8 \mathrm{~g} \mathrm{~mol}^{-1}$

Answer: C

Solution:

For bcc unit cell, $\mathrm{n}=2$.

$$\text{Density of bcc unit cell } =\rho=\frac{\mathrm{Mn}}{\mathrm{a}^3 \mathrm{~N}_{\mathrm{A}}}$$

$$\mathrm{M}=\frac{\rho \times \mathrm{a}^3 \times \mathrm{N}_{\mathrm{A}}}{\mathrm{n}}$$

$$\mathrm{M}=\frac{10 \mathrm{~g} \mathrm{~cm}^{-3} \times\left(3 \times 10^{-8} \mathrm{~cm}\right)^3 \times 6.022 \times 10^{23} \text{atom mol}^{-1}}{2 \text { atoms }}$$

$$M=81.3 \mathrm{~g} \mathrm{~mol}^{-1}$$

Question 84

Oxidation state of manganese in potassium permanganate is:

Options:

A. +3

B. +7

C. +5

D. +1

Answer: B

Solution:

Oxidation number of $\mathrm{K}=+1$

Oxidation number of $\mathrm{O}=-2$

$\mathrm{KMnO_4}$ is a neutral molecule.

$\therefore$ Sum of the oxidation number of all atoms $=0$

Oxidation number of K + Oxidation number of $\mathrm{Mn}+4 \times$ (Oxidation number of O) $=0$

$(+1)+$ (Oxidation number of Mn) $+4 \times(-2)=0$

$1+$ (Oxidation number of Mn) $=+8$

$\therefore$ Oxidation number of $\mathrm{Mn}=+7$

Question 85

Which among following complexes is a neutral complex?

Options:

A. $[\mathrm{Co}\,(\mathrm{H_2O})(\mathrm{NH_3})_5]\mathrm{I_3}$

B. $[\mathrm{Co}(\mathrm{NO_2})_3(\mathrm{NH_3})_3]$

C. $\mathrm{Na}\,[\mathrm{Co}(\mathrm{NO_2})_6]$

D. $[\mathrm{Fe}(\mathrm{H_2O})_3(\mathrm{NCS})]\mathrm{Cl_2}$

Answer: B

Solution:

The complexes in which the coordination sphere does not carry any charge are called neutral complexes. $[Co(NO_2)_3(NH_3)_3]$ is a neutral complex. $[Co\,(H_2O)(NH_3)_5]I_3$ and $[Fe(H_2O)_3(NCS)]Cl_2$ are cationic complexes whereas $Na\,[Co(NO_2)_6]$ is an anionic complex.

A neutral complex does not have any counter ion (either positive or negative). So, options (A), (C) and (D) can be eliminated.

Question 86

Which among the following halogens combines readily with metals to form metal halides with highest ionic character?

Options:

A. Chlorine

B. Bromine

C. Iodine

D. Fluorine

Answer: D

Solution:

Ionic character of metal halides decreases in the order $MF > MCl > MBr > MI$, where, M is a monovalent metal. Smaller the size of anion, greater is the ionic character (Fajan's rule). Therefore, fluorine forms metal halides with highest ionic character.

Question 87

Which of the following compounds has highest boiling point?

Options:

A. Chloromethane

B. Fluoromethane

C. Iodomethane

D. Bromomethane

Answer: C

Solution:

Molecules with higher molecular mass have higher boiling points.

So, for given alkyl group $(-CH_3)$ boiling point decreases as, $CH_3I > CH_3Br > CH_3Cl > CH_3F$.

Question 88

Identify physisorption from following.

Options:

A. O_2 gas deposited on tungsten

B. H_2 gas deposited on nickel

C. N_2 gas deposited on iron

D. All gases deposited on charcoal

Answer: D

Question 89

Identify the product of following reaction.

Benzoyl chloride $\xrightarrow{H_2O}$ product

Options:

A. Benzyl alcohol

B. Benzaldehyde

C. Benzoic acid

D. Benzophenone

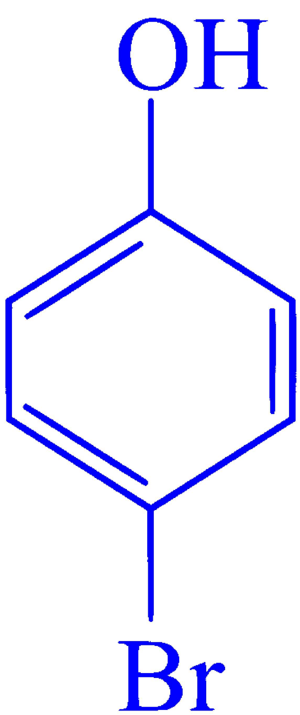
Answer: C

Solution:

$$\underset{\text{Benzoyl chloride}}{C_6H_5-COCl} + H_2O \longrightarrow \underset{\text{Benzoic acid}}{C_6H_5-COOH} + H-Cl$$

Question 90

What is IUPAC name of the compound?



Options:

A. 1-Bromo-4-hydroxybenzene

B. 1-Hydroxy-4-bromobenzene

C. 4-Hydroxybromobenzene

D. 4-Bromophenol

Answer: D

Question 91

Calculate the pH of $1.36 \times 10^{-2}\mathrm{M}$ solution of perchloric acid.

Options:

A. 1.43

B. 1.86

C. 2.43

D. 2.86

Answer: B

Solution:

Perchloric acid is a strong monobasic acid.

Hence, $\left[\mathrm{H_3O^+}\right] = 1.36 \times 10^{-2}\mathrm{M}$

$$\begin{aligned} \therefore \quad \mathrm{pH} &= -\log_{10}\left[\mathrm{H_3O^+}\right] \\ &= -\log_{10}\left[1.36 \times 10^{-2}\right] \\ &= -\log_{10} 1.36 - \log_{10} 10^{-2} \\ &= -\log_{10} 1.36 + 2 \\ &= 2 - 0.1335 \\ \therefore \quad \mathrm{pH} &= 1.86 \end{aligned}$$

Question 92

Which of the following formulae is used to determine compressibility factor for measurement of deviation from ideal behaviour?

Options:

A. $Z = \frac{nRT}{PV}$

B. $Z = \frac{PV}{nRT}$

C. $Z = \frac{nRT}{V}$

D. $Z = \frac{nRT}{P}$

Answer: B

Question 93

The molar conductivity of $0.02\ \mathrm{M\ AgI}$ at $298\ \mathrm{K}$ is $142.3\Omega^{-1}\ \mathrm{cm}^2\ \mathrm{mol}^{-1}$. What is its conductivity?

Options:

A. $1.42 \times 10^{-3}\Omega^{-1}\ \mathrm{cm}^{-1}$

B. $2.41 \times 10^{-3}\Omega^{-1}\ \mathrm{cm}^{-1}$

C. $2.85 \times 10^{-3}\Omega^{-1}\ \mathrm{cm}^{-1}$

D. $7.11 \times 10^{-3}\Omega^{-1}\ \mathrm{cm}^{-1}$

Answer: C

Solution:

$$\wedge = \frac{1000k}{c}$$

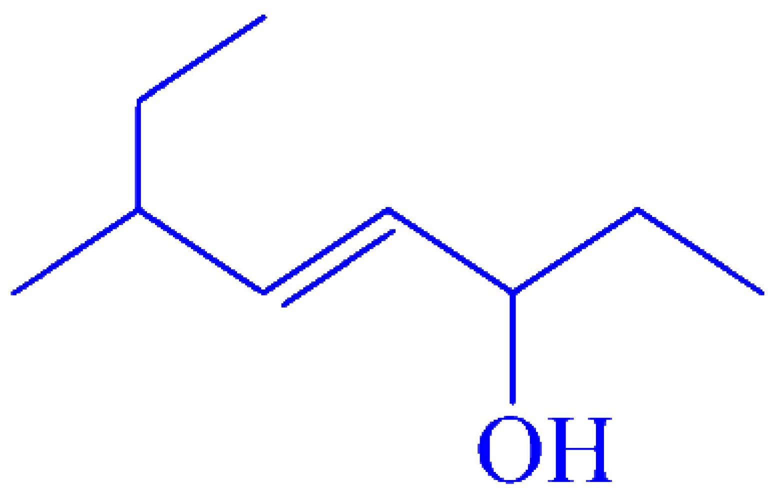
$$k = \frac{\Lambda c}{1000}$$

$$k = \frac{142.3\Omega^{-1}\ cm^2\ mol^{-1} \times 0.02\ mol\ L^{-1}}{1000\ cm^3\ L^{-1}}$$

$$\therefore \quad k = 2.85 \times 10^{-3}\Omega^{-1}\ cm^{-1}$$

Question 94

What is IUPAC name of following compound?



Options:

A. 2-Ethylhept-3-en-5-ol

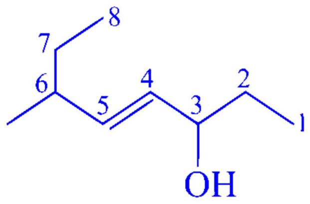
B. 3-Methyloct-4-en-6-ol

C. 6-Methyloct-4-en-3-ol

D. 6-Ethylhept-4-en-3-ol

Answer: C

Solution:

Compound	IUPAC name
	6-Methyloct-4-en-3-ol

---

Question 95

Identify a molecule with incomplete octet from following.

Options:

A. SF_6

B. PCl_5

C. LiCl

D. H_2SO_4

Answer: C

Solution:

Li in LiCl has less than eight electrons in its valence shell. It has only four electrons. Hence, it has an incomplete octet. Molecules like SF_6, PCl_5, H_2SO_4 have an expanded octet. They have more than eight electrons around the central atom.

Question 96

What is the number of unit cells present in a cubic pack crystal lattice having 4 atoms per unit cell and weighing 0.60 g (molar mass 60 g mol^{-1}) ?

Options:

A. 1×10^{21}

B. 1.5×10^{21}

C. 3.0×10^{21}

D. 6.0×10^{21}

Answer: B

Solution:

Number of moles $= \frac{0.6 \text{ g}}{60 \text{ g mol}^{-1}} = 0.01 \text{ mol}$

Total number of atoms $= 0.01 \times 6.022 \times 10^{23}$

Number of atoms per unit cell $= 4$ (Given)

$\therefore$ Number of unit cells present in the given cubic crystal lattice

$$= \frac{0.01 \times 6.022 \times 10^{23}}{4}$$
$$= 1.5 \times 10^{21} \text{ unit cells}$$

Question 97

Identify 'A' in the following reaction.

$$\text{A} + \text{Chloromethane} \xrightarrow[\text{AlCl}_3]{\text{Anhydrous}} \text{2-Chlorotouene} + \text{4-Chlorotoluene}$$

Options:

A. Benzene

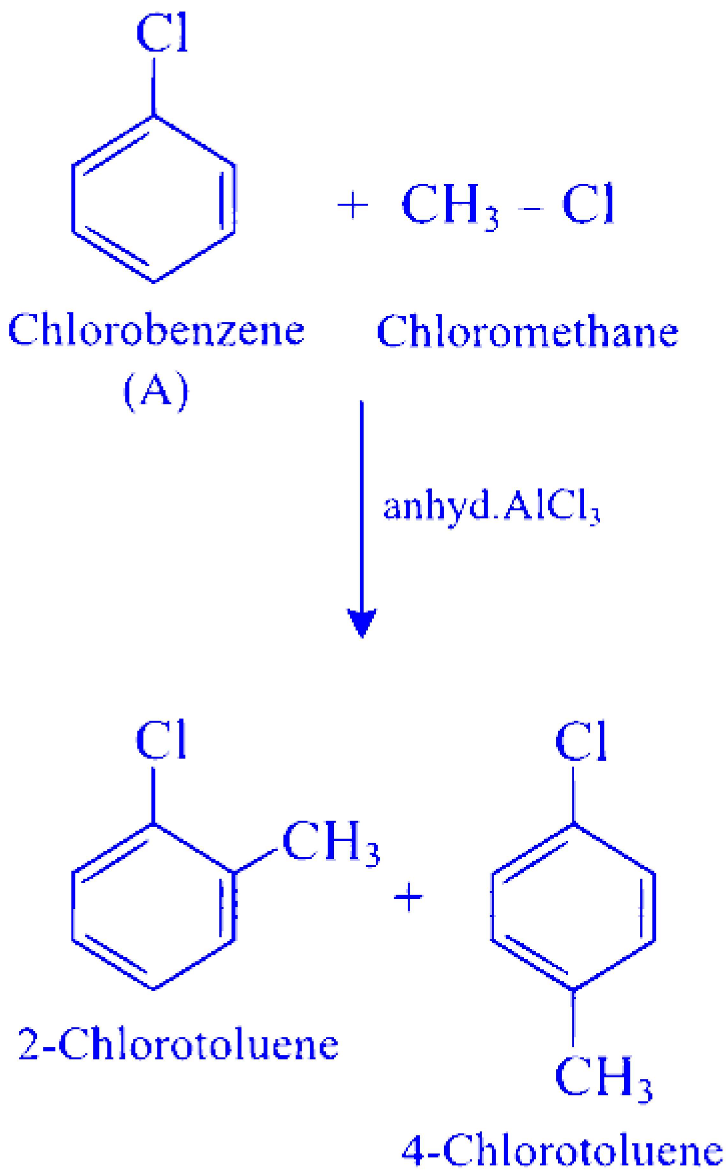
B. Chlorobenzene

C. Toluene

D. Phenol

Answer: B

Solution:



Question 98

Which among the following is CORRECT formula for determination of cell constant?

Options:

A. $\frac{l}{a} = \frac{k}{R}$

B. $\frac{l}{a} = k \cdot R$

C. $\frac{l}{a} = \frac{R}{k}$

D. $\frac{l}{a} = \frac{1}{R}$

Answer: B

Solution:

$$\begin{aligned} & k = G\frac{l}{a} = \frac{1}{R}\frac{l}{a} \\ \therefore \quad & \text{Cell constant } = \frac{l}{a} = k \cdot R \end{aligned}$$

Question 99

Identify the catalyst (A) used in following reaction.

$$CO + H_2O \overset{A}{\rightleftharpoons} CO_2 + H_2$$

Options:

A. Platinised asbestos

B. MnO_2

C. Co-Th alloy

D. $Fe - Cr$

Answer: D

Solution:

Carbon dioxide and hydrogen are formed by reaction of the carbon monoxide and steam at about $500^{\circ}C$ with $Fe - Cr$ catalyst.

Question 100

What is value of PV type of work for following reaction at 1 bar?

$$\underset{(200 \text{ mL})}{C_2H_{4(g)}} + \underset{(150 \text{ mL})}{HCl_{(g)}} \longrightarrow C_2H_5Cl_{(g)}$$

Options:

A. 3.5 J

B. 4.5 J

C. 9.0 J

D. 15 J

Answer: D

Solution:

1 mole of C_2H_4 reacts with 1 mole of HCl to produce 1 mole of C_2H_5Cl.

Hence, 150 mL of HCl would react with only 150 mL of C_2H_4 to produce 150 mL of C_2H_5Cl.

$V_1 = 150 \text{ mL} + 150 \text{ mL} = 300 \text{ mL} = 0.3 \text{ dm}^3$

$V_2 = 150 \text{ mL} = 0.15 \text{ dm}^3$

$W = -P_{ext}\ \Delta V = -P_{ext}\ (V_2 - V_1)$

$\therefore \quad W = -1 \text{ bar} \left(0.15 \text{ dm}^3 - 0.3 \text{ dm}^3\right)$

$\therefore \quad W = 0.15 \text{dm}^3 \text{ bar}$

$\therefore \quad W = 0.15 \text{ dm}^3 \text{ bar} \times 100 \frac{J}{\text{dm}^3 \text{ bar}}$

$\therefore \quad W = 15 \text{ J}$

The PV work in joules is 15 J.

Physics

Question 101

A coil of radius 'r' is placed on another coil (whose radius is R and current flowing through it is changing) so that their centres coincide $(\mathrm{R} \gg \mathrm{r})$. If both the coils are coplanar then the mutual inductance between them is ($\mu_0 =$ permeability of free space)

Options:

A. $\frac{\mu_0 \pi \mathrm{R}^2}{2 \mathrm{r}}$

B. $\frac{\mu_0 \pi r^2}{2 R}$

C. $\frac{\mu_0 \pi r^2}{R}$

D. $\mu_0 \pi R^2$

Answer: B

Solution:

Magnetic field, $B = \frac{\mu_0 I}{2 R}$

Flux passing through the coil,

$$\phi = \mathrm{B} \times \pi \mathrm{r}^2$$
$$\therefore \quad \phi = \frac{\mu_0 \mathrm{I}}{2 \mathrm{R}} \times \pi \mathrm{r}^2$$

Mutual Inductance, $M = \frac{\phi}{I}$

$$\therefore \quad M = \frac{\frac{\mu_0 I}{2 R} \times \pi r^2}{I} = \frac{\mu_0 \pi r^2}{2 R}$$

Question 102

Two capillary tubes of the same diameter are kept vertically in two different liquids whose densities are in the ratio $4 : 3$. The rise of liquid in two capillaries is 'h_1' and 'h_2' respectively. If the surface tensions of liquids are in the ratio $6 : 5$, the ratio of heights $\left(\frac{h_1}{h_2}\right)$ is

(Assume that their angles of contact are same)

Options:

A. 0.4

B. 0.5

C. 0.8

D. 0.9

Answer: D

Solution:

Given: Density ratio: $\frac{\rho_1}{\rho_2} = \frac{4}{3}$

and surface tension ratio: $\frac{T_1}{T_2} = \frac{6}{5}$

$\therefore$ Rise of liquid in a capillary:

$h = \frac{2\ T \cos\theta}{r\rho g}$

For constant θ, r and g,

$$h \propto \frac{T}{\rho}$$

$$\therefore \quad \frac{h_1}{h_2} = \frac{T_1\rho_2}{T_2\rho_1} = \frac{6 \times 3}{5 \times 4} = 0.9$$

Question 103

Two spherical black bodies of radii 'r_1' and 'r_2' at temperature 'T_1' and 'T_2' respectively radiate power in the ratio $1 : 2$ Then $r_1 : r_2$ is

Options:

A. $\frac{1}{2}\left(\frac{T_2}{T_1}\right)^4$

B. $\frac{1}{\sqrt{2}}\left(\frac{T_2}{T_1}\right)^2$

C. $2\left(\frac{\mathrm{T}_1}{\mathrm{T}_2}\right)^4$

D. $2\left(\frac{T_1}{T_2}\right)^2$

Answer: B

Solution:

Power Radiated by black body, $\mathrm{P}=\sigma \mathrm{AT}^4$

$\therefore$ For first black body:

$\mathrm{P}_1=\sigma 4 \pi \mathrm{r}_1^2 \mathrm{~T}_1^4$

$\therefore$ For second black body:

$\mathrm{P}_2=\sigma 4 \pi \mathrm{r}_2{ }^2 \mathrm{~T}_2{ }^4$

$\therefore$ The ratio will be:

$$\frac{\mathrm{P}_1}{\mathrm{P}_2}=\frac{\sigma 4 \pi \mathrm{r}_1^2 \mathrm{~T}_1^4}{\sigma 4 \pi \mathrm{r}_2^2 \mathrm{~T}_2^4}$$
$$\frac{1}{2}=\frac{\mathrm{r}_1^2 \mathrm{~T}_1^4}{\mathrm{r}_2^2 \mathrm{~T}_2^4}$$
$$\frac{\mathrm{r}_1^2}{\mathrm{r}_2^2}=\frac{1}{2} \frac{\mathrm{T}_2^4}{\mathrm{~T}_1^4}$$
$$\frac{\mathrm{r}_1}{\mathrm{r}_2}=\frac{1}{\sqrt{2}}\left(\frac{\mathrm{T}_2}{\mathrm{~T}_1}\right)^2$$

Question 104

For a particle executing S.H.M., its potential energy is 8 times its kinetic energy at certain displacement 'x' from the mean position. If 'A' is the amplitude of S.H.M the value of 'x' is

Options:

A. $\frac{\mathrm{A} \sqrt{2}}{3}$

B. $\mathrm{A} \sqrt{3}$

C. $\frac{2\sqrt{2}\text{ A}}{3}$

D. $\frac{\text{A}}{\sqrt{2}}$

Answer: C

Solution:

Potential energy: $U=\frac{1}{2}m\omega^2x^2$ and

Kinetic energy: $K=\frac{1}{2}m\omega^2\left(A^2-x^2\right)$

Given: Potential energy $=8\times$ Kinetic energy

$$\begin{aligned}
&\therefore \quad \text{U}=8\text{ K}\\
&\frac{1}{2}m\omega^2x^2=8\times\frac{1}{2}m\omega^2\left(A^2-x^2\right)\\
&x^2=8A^2-8x^2\\
&9x^2=8A^2\\
&x^2=\frac{8A^2}{9}\\
&x=\frac{2\sqrt{2}A}{3}
\end{aligned}$$

Question 105

The maximum kinetic energies of photoelectrons emitted are K_1 and K_2 when lights of wavelengths λ_1 and λ_2 are incident on a metallic surface. If $\lambda_1=3\lambda_2$ then

Options:

A. $\text{K}_1=\frac{\text{K}_2}{3}$

B. $\text{K}_1<\frac{\text{K}_2}{3}$

C. $\text{K}_1=3\text{K}_2$

D. $3\text{K}_1=2\text{K}_2$

Answer: B

Solution:

Kinetic energy of the photoelectrons

$K = \frac{hc}{\lambda} - \phi$

$\therefore$ For wavelength λ_1,

$K_1 = \frac{hc}{\lambda_1} - \phi$ ..... (i)

$\therefore$ For wavelength λ_1,

$K_2 = \frac{hc}{\lambda_2} - \phi$ ..... (ii)

Subtracting equation (i) from equation (ii),

$$K_2 - K_1 = \frac{hc}{\lambda_2} - \phi - \frac{hc}{\lambda_1} + \phi$$
$$K_2 - K_1 = \frac{hc}{\lambda_2} - \frac{hc}{3\lambda_2}$$
$$K_2 - K_1 = \frac{2}{3}\frac{hc}{\lambda_2}$$
$$\frac{hc}{\lambda_2} = \frac{3}{2}(K_2 - K_1) \quad ..... \text{(iii)}$$

Substituting equation (iii) in equation (ii),

$K_2 = \frac{3}{2}(K_2 - K_1) - \phi$

$$2K_2 = 3K_2 - 3K_1 - 2\phi$$
$$K_2 - 3K_1 = 2\phi$$
$$\text{As}, \phi > 0$$
$$K_2 - 3K_1 > 0$$
$$K_1 < \frac{K_2}{3}$$

Question 106

A particle moves along a circular path with decreasing speed. Hence

Options:

A. its resultant acceleration is towards the centre.

B. it moves in a spiral path with decreasing radius.

C. the direction of angular momentum remains constant.

D. its angular momentum remains constant

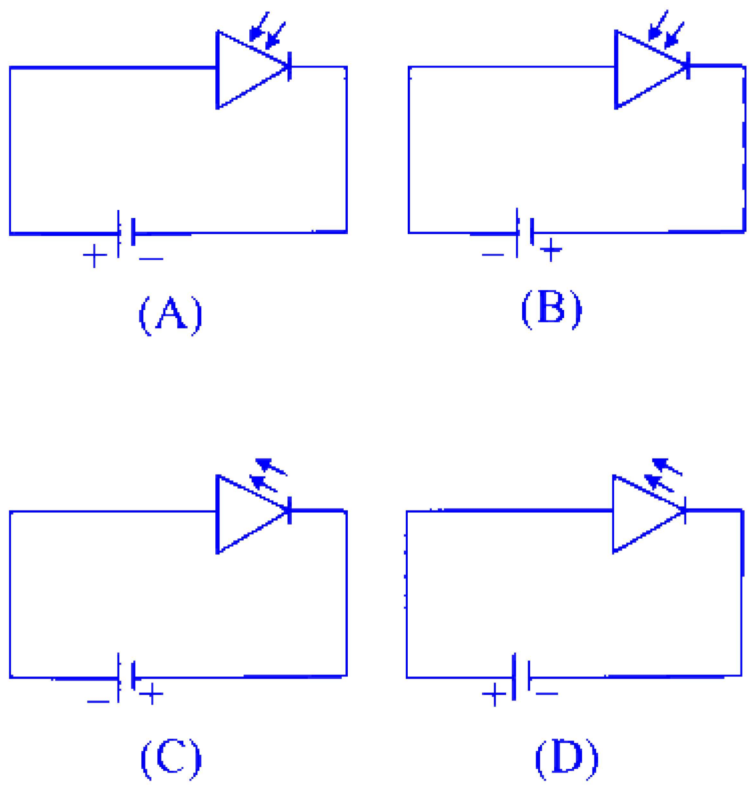
Answer: C

Solution:

The direction of angular moment remains constant as the angular momentum is a vector quantity, and its direction is perpendicular to the plane of motion. As the speed decreases, the linear momentum decreases, but the angular momentum remains constant due to the conservation of angular momentum.

Question 107

Identify the correct circuit diagrams for the normal operation from the following.



Options:

A. (A), (B)

B. (B), (C)

C. (A), (C)

D. (B), (D)

Answer: D

Solution:

Option (B) is photodiode and option (D) is LED with reverse and forward biasing respectively. Therefore both will operate properly.

Question 108

With the gradual increase in frequency of an a. c. source, the impedance of an LCR series circuit

Options:

A. first decreases, becomes minimum and then increases.

B. increases.

C. decreases.

D. remains constant.

Answer: A

Solution:

With increase in frequency current increases initially, becomes maximum and then decreases with further increase in frequency.

Question 109

In energy band diagram of insulators, a band gap and the conduction band is respectively

Options:

A. very low and partially filled.

B. very high and completely filled.

C. very high and empty.

D. very low and empty.

Answer: C

Question 110

Two positively charged identical spheres separated by a distance 'd' exert some force (F) on each other when they are kept in air. If both the spheres are immersed in a liquid of dielectric constant 5 , the force experienced by each is (All other parameters are unchanged.)

Options:

A. 5 F

B. $\frac{F}{3}$

C. $\frac{F}{4}$

D. $\frac{F}{5}$

Answer: D

Solution:

The ratio of force when the medium is changed will be:

$\frac{\mathrm{F}_1}{\mathrm{F}_2}=\frac{1}{\mathrm{k}}$

Here, $\mathrm{k}=5$,

Therefore, the force experience by each will be $\frac{F}{5}$.

Question 111

A string fixed at both the ends forms standing wave with node separation of 5 cm. If the velocity of the wave on the string is 2 m/s, then the frequency of vibration of the string is

Options:

A. 0.2 Hz

B. 10 Hz

C. 20 Hz

D. 40 Hz

Answer: C

Solution:

Separation between consecutive nodes, $\frac{\lambda}{2} = 5 \text{ cm}$

$\therefore \quad \lambda = 10 \text{ cm} = 0.1 \text{ m}$

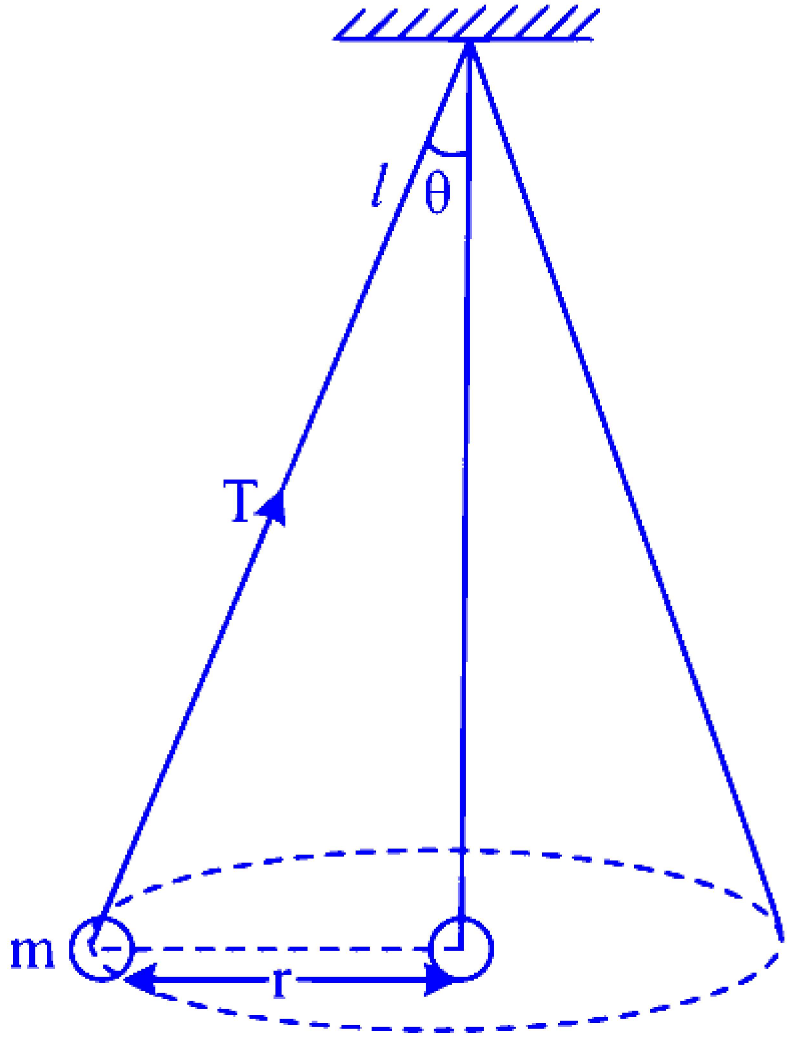
The frequency of vibration is given as:

$\text{n} = \frac{\text{v}}{\lambda} = \frac{2}{0.1} = 20 \text{ Hz}$

Question 112

A ball of mass 'm' is attached to the free end of a string of length 'l'. The ball is moving in horizontal circular path about the vertical axis as shown in the diagram.

The angular velocity 'ω' of the ball will be [$\text{T} =$ Tension in the string.]



Options:

A. $\sqrt{\frac{Tl}{m}}$

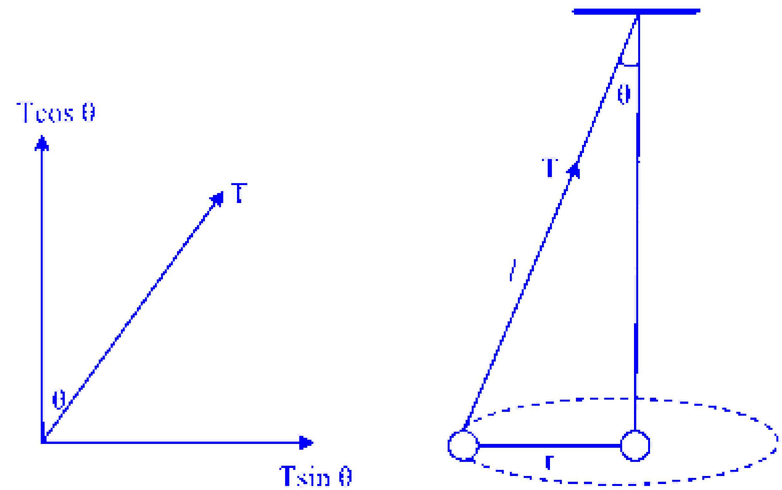
B. $\sqrt{\frac{Tm}{l}}$

C. $\sqrt{\frac{ml}{T}}$

D. $\sqrt{\frac{T}{mll}}$

Answer: D

Solution:



The tension in the string can be resolved in two components along the perpendicular axis. The gravitational force is acting downwards and the centrifugal force is acting in $-x$ direction $T\sin\theta = mr\omega^2$

$\therefore \quad \omega^2 = \frac{T\sin\theta}{\mathrm{mr}}$

$\therefore \quad \omega = \sqrt{\frac{T\sin\theta}{\mathrm{mr}}}$

From figure, $\sin\theta = \frac{r}{l}$

$$\therefore \quad \omega = \sqrt{\frac{\mathrm{Tr}}{\mathrm{mr}l}}$$

$$\therefore \quad \omega = \sqrt{\frac{\mathrm{T}}{\mathrm{m}l}}$$

Using dimensional analysis, dimensions of only option (D) matches with the dimensions of angular velocity.

Question 113

Radius of gyration of a thin uniform circular disc about the axis passing through its centre and perpendicular to its plane is $\mathrm{K_c}$. Radius of gyration of the same disc about a diameter of the disc is K_d. The ratio $K_c : K_d$ is

Options:

A. $\sqrt{2} : 1$

B. $1 : \sqrt{2}$

C. $2 : 1$

D. 1 : 4

Answer: A

Solution:

Let the radius of the disc be R

$$\therefore \quad \mathrm{K_c} = \frac{\mathrm{R}}{\sqrt{2}}$$
$$\therefore \quad \mathrm{K_d} = \frac{\mathrm{R}}{2}$$

Taking the ratio,

$$\therefore \quad \frac{\mathrm{K_c}}{\mathrm{K_d}} = \frac{\frac{\mathrm{R}}{\sqrt{2}}}{\frac{\mathrm{R}}{2}} = \frac{\sqrt{2}}{1}$$

Question 114

When a current of $1\ \mathrm{A}$ is passed through a coil of 100 turns, the flux associated with it is $2.5 \times 10^{-5}\ \mathrm{Wb}/$ turn. The self inductance of the coil in millihenry is

Options:

A. 40

B. 25

C. 4

D. 2.5

Answer: D

Solution:

Given: $N = 100, I = 1\ \mathrm{A}, \phi = 2.5 \times 10^{-5}\ \mathrm{Wb}/$ turn

Self-inductance of coil,

$$L = \frac{N\phi}{I}$$

$$\therefore \quad L = \frac{100 \times 2.5 \times 10^{-5}}{1} = 2.5 \times 10^{-3} H = 2.5 \text{ mH}$$

Question 115

A spherical liquid drop of radius R is divided into 8 equal droplets. If surface tension is S, then the work done in this process will be

Options:

A. $2\pi R^2 S$

B. $3\pi R^2 S$

C. $4\pi R^2 S$

D. $2\pi R S^2$

Answer: C

Solution:

Work done, $W = S \times \Delta A$

Where, ΔA is a change in surface area.

Since, the radius of the big drop be R

$A_{\text{initial}} = 4\pi R^2$

Let the radius of the small drops be r

$A_{\text{final}} = 8 \times 4\pi r^2$

The volume is constant

$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

$\therefore \quad R^3 = 8r^3$

$\therefore \quad R = 2r$

Substituting the values in the formula.

$$W = S \times \Delta A$$
$$W = S\left(A_{\text{final}} - A_{\text{initial}}\right)$$
$$W = S\left(8 \times 4\pi r^2 - 4\pi R^2\right)$$
$$W = S\left(32\pi\left(\frac{R}{2}\right)^2 - 4\pi R^2\right)$$
$$W = S\left(8\pi R^2 - 4\pi R^2\right)$$
$$W = 4\pi R^2\, S$$

Question 116

The rate of flow of heat through a metal rod with temperature difference $40^{\circ}\mathrm{C}$ is $1600\ \mathrm{cal/s}$. The thermal resistance of metal rod in $^{\circ}\mathrm{Cs/cal}$ is

Options:

A. 0.025

B. 0.25

C. 2.5

D. 40

Answer: A

Solution:

Given: rate of flow of heat (conduction rate) $P_{\text{cond}} = 1600\ \mathrm{cal/s}$

Thermal resistance,

$$R_T = \frac{\Delta T}{P_{\text{cond}}}$$
$$R_T = \frac{40}{1600}$$
$$R_T = 0.025^{\circ}\mathrm{Cs/cal}$$

Question 117

Two charges of equal magnitude 'q' are placed in air at a distance '$2r$' apart and third charge '$-2\mathrm{q}$' is placed at mid point. The potential energy of the system is $(\varepsilon_0 =$ permittivity of free space)

Options:

A. $-\frac{q^2}{8\pi\varepsilon_0 r}$

B. $-\frac{3\mathrm{q}^2}{8\pi\varepsilon_0 \mathrm{r}}$

C. $-\frac{5q^2}{8\pi\varepsilon_0 r}$

D. $-\frac{7q^2}{8\pi\varepsilon_0 r}$

Answer: D

Solution:

Potential energy of 'n' point charges,

$$\mathrm{U} = \frac{1}{4\pi\varepsilon_0} \sum_{\text{all pairs}} \frac{\mathrm{q_j q_k}}{\mathrm{r_{jk}}}$$

For 3 point charges,

$$\begin{aligned} \therefore \quad U &= -\frac{q(2q)}{4\pi\varepsilon_0 r} - \frac{q(2q)}{4\pi\varepsilon_0 r} + \frac{q(q)}{4\pi\varepsilon_0(2r)} \\ U &= -\frac{2q^2}{4\pi\varepsilon_0 r} - \frac{2q^2}{4\pi\varepsilon_0 r} + \frac{q^2}{4\pi\varepsilon_0(2r)} \\ U &= -\frac{4q^2}{8\pi\varepsilon_0 r} - \frac{4q^2}{8\pi\varepsilon_0 r} + \frac{q^2}{8\pi\varepsilon_0 r} \\ U &= -\frac{7q^2}{8\pi\varepsilon_0 r} \end{aligned}$$

Question 118

The size of the real image produced by a convex lens of focal length F is 'm' times the size of the object. The image distance from the lens is

Options:

A. $\frac{F}{(m-1)}$

B. $\frac{(\mathrm{m}-1)}{\mathrm{F}}$

C. $F(m+1)$

D. $\mathrm{F}(\mathrm{m}-1)$

Answer: C

Solution:

From lens formula,

$\frac{1}{\mathrm{v}}-\frac{1}{\mathrm{u}}=\frac{1}{\mathrm{F}}$

For a convex lens, u is negative, v and F are positive.

$$\therefore \quad \frac{1}{\mathrm{v}}-\frac{1}{-\mathrm{u}}=\frac{1}{\mathrm{F}}$$
$$\therefore \quad \frac{1}{\mathrm{v}}=\frac{1}{\mathrm{F}}-\frac{1}{\mathrm{u}}$$

Multiplying by v,

$$1=\frac{\mathrm{v}}{\mathrm{F}}-\frac{\mathrm{v}}{\mathrm{u}}$$
$$\therefore \quad 1=\frac{\mathrm{v}}{\mathrm{F}}-\mathrm{m} \quad \ldots\left(\because \frac{v}{u}=m\right)$$
$$\therefore \quad \frac{\mathrm{v}}{\mathrm{F}}=1+\mathrm{m}$$
$$\therefore \quad \mathrm{v}=\mathrm{F}(1+\mathrm{m})$$

Question 119

A double slit experiment is immersed in water of refractive index 1.33. The slit separation is $1 \mathrm{~mm}$, distance between slit and screen is $1.33 \mathrm{~m}$ The slits are illuminated by a light of wavelength $6300 \stackrel{o}{A}$. The fringe width is

Options:

A. $4.9 \times 10^{-4} \mathrm{~m}$

B. 5.8×10^{-4} m

C. 6.3×10^{-4} m

D. 8.6×10^{-4} m

Answer: C

Solution:

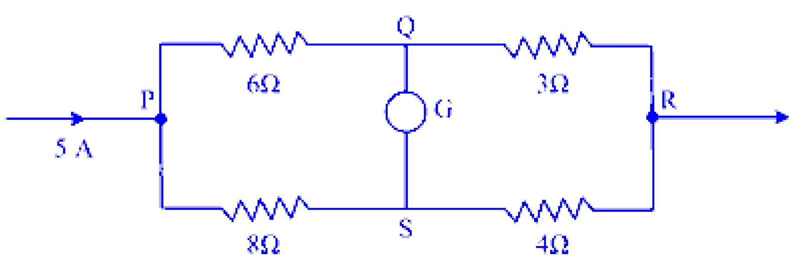
$$\lambda_{\text{liquid}} = \frac{\lambda_{\text{air}}}{\mu}$$
$$\lambda_{\text{liquid}} = \frac{6300 \times 10^{-10}}{1.33}$$

Fringe width,

$$W = \frac{\lambda_{\text{liquid}} \times D}{d} = \frac{6300 \times 10^{-10} \times 1.33}{1.33 \times 0.001} = 6.3 \times 10^{-4} \text{ m}$$

Question 120

Potential difference between the points P and Q is nearly



Options:

A. 6 V

B. 8 V

C. 17 V

D. 21 V

Answer: C

Solution:

As, the resistors across each branch are in series.

$R_1 = 6 + 3 = 9\Omega$
$R_2 = 8 + 4 = 12\Omega$

According to KCL, the current (i) will get divided into two parts I_1 and I_2

$I_1 = \frac{R_2}{(R_1+R_2)i} = \frac{12}{9+12} \times 5 = 2.86\text{ A}$

Potential difference between P and Q is $V = I_1 R = 2.86 \times 6 = 17.14\text{ V}$

Question 121

An electron in the hydrogen atom jumps from the first excited state to the ground state. What will be the percentage change in the speed of electron?

Options:

A. 25%

B. 50%

C. 75%

D. 100%

Answer: B

Solution:

Velocity of electron in the n^{th} orbit is

$$v_n = \frac{e^2}{2\varepsilon_0 nh}$$
$$\Rightarrow v_n \propto \frac{1}{n}$$

Taking the ratio,

$\therefore \quad \frac{v_2}{v_1} = \frac{n_1}{n_2} = \frac{1}{2} \Rightarrow v_2 = \frac{v_1}{2}$

Change in velocity,

$$\begin{aligned}\Delta v &= |v_2 - v_1| \\ &= \left|\frac{v_1}{2} - v_1\right| = 0.5v_1\end{aligned}$$

Therefore, change in percentage is 50%

Question 122

In series LCR circuit, the voltage across the inductance and the capacitance are not

Options:

A. out of phase with the voltage across the resistance by 90°.

B. equal in magnitude at resonance.

C. out of phase with each other by 180°.

D. in phase with the source voltage.

Answer: D

Solution:

In an LCR circuit, the inductor (L) and capacitor (C) introduce phase shifts in the current compared to the source voltage due to their reactive properties. The inductor causes a phase shift of +90 degrees, leading the voltage, while the capacitor causes a phase shift of -90 degrees, lagging the voltage. Thus, voltage across L and C are not in phase with the source voltage.

Question 123

If the temperature of a hot body is increased by 50%, then the increase in the quantity of emitted heat radiation will be approximately

Options:

A. 125%

B. 200%

C. 300%

D. 400%

Answer: D

Solution:

$$\text{Given: } T_2 = T_1 + \frac{50}{100} T_1$$
$$T_2 = 1.5\ T_1$$

According to Stefan's law,

$\frac{Q}{At} = \sigma T^4$

Percentage change in radiation is,

$$\Delta E\% = \frac{T_2^4 - T_1^4}{T_1^4} \times 100$$
$$\Delta E\% = \frac{(1.5)^4 T_1^4 - T_1^4}{T_1^4} \times 100$$
$$\Delta E\% \approx 400\%$$

Question 124

The second overtone of an open pipe has the same frequency as the first overtone of a closed pipe of length 'L'. The length of the open pipe will be

Options:

A. $\frac{L}{2}$

B. L

C. 2 L

D. 4 L

Answer: C

Solution:

The length of closed pipe is denoted using L.

Let l be the length of open pipe and v be the velocity.

Frequency of second overtone of an open organ pipe is $\mathrm{n_o} = \frac{3\mathrm{v}}{2l}$

Frequency of first overtone of a closed pipe is $\mathrm{n_c} = \frac{3\mathrm{v}}{4\,\mathrm{L}}$

Given: $\mathrm{n_0} = \mathrm{n_c}$

$$\frac{3\mathrm{v}}{2l} = \frac{3\mathrm{v}}{4\,\mathrm{L}}$$
$$\mathrm{L} = \frac{l}{2}$$
$$\therefore \quad l = 2\,\mathrm{L}$$

Question 125

The radius of earth is $6400\ \mathrm{km}$ and acceleration due to gravity $\mathrm{g} = 10\ \mathrm{ms}^{-2}$. For the weight of body of mass $5\ \mathrm{kg}$ to be zero on equator, rotational velocity of the earth must be (in $\mathrm{rad/s}$)

Options:

A. $\frac{1}{80}$

B. $\frac{1}{400}$

C. $\frac{1}{800}$

D. $\frac{1}{1600}$

Answer: C

Solution:

At equator, for the weight to be zero, the gravitational force must be equal to centrifugal force.

$\mathrm{mR}\omega^2 = \mathrm{mg}$
$\omega^2 = \frac{\mathrm{g}}{\mathrm{R}}$
$\omega = \sqrt{\frac{\mathrm{g}}{\mathrm{R}}}$
$\omega = \sqrt{\frac{10}{6.4 \times 10^6}}$
$\omega = \frac{1}{800} \frac{\mathrm{rad}}{\mathrm{s}}.$

Question 126

A car sounding a horn of frequency 1000 Hz passes a stationary observer. The ratio of frequencies of the horn noted by the observer before and after passing the car is $11 : 9$. If the speed of sound is 'v', the speed of the car is

Options:

A. V

B. $\frac{\mathrm{v}}{2}$

C. $\frac{\mathrm{v}}{5}$

D. $\frac{\mathrm{v}}{10}$

Answer: D

Solution:

Frequency of a source moving towards a stationary listener is $\mathrm{n_b} = \left(\frac{\mathrm{v}}{\mathrm{v}-\mathrm{v_s}}\right)\mathrm{n}$

Frequency of a source moving towards a stationary listener is $\mathrm{n_a} = \left(\frac{\mathrm{v}}{\mathrm{v}+\mathrm{v_s}}\right)\mathrm{n}$

Taking the ratio

$\frac{n_b}{n_2} = \left(\frac{v+v_s}{v-v_s}\right)$

$$\frac{11}{9}=\left(\frac{v+v_s}{v-v_s}\right)$$
$$11v+11v_s=9v-9v_s$$
$$2v=20v_s$$
$$v_s=\frac{1}{10}v$$

Question 127

The time period of a simple pendulum inside a stationary lift is 'T'. When the lift starts accelerating upwards with an acceleration $\left(\frac{\mathrm{g}}{3}\right)$, the time period of the pendulum will be

Options:

A. $\frac{\sqrt{5}}{2}$ T

B. $\frac{\sqrt{3}}{2}$ T

C. $\frac{2\ \mathrm{T}}{\sqrt{3}}$

D. $\frac{2\ \mathrm{T}}{\sqrt{5}}$

Answer: B

Solution:

Time period of a simple pendulum is $\mathrm{T}=2\pi\sqrt{\left(\frac{l}{\mathrm{a}}\right)}$

In stationary lift, the value of acceleration is $\mathrm{a}=\mathrm{g}$

$$\mathrm{T}=2\pi\sqrt{\left(\frac{l}{\mathrm{g}}\right)}$$

When the lift is accelerating in an upward direction, there is a pseudo force acting in a downward direction.

$$\therefore\quad \mathrm{ma}=\mathrm{mg}+\frac{\mathrm{mg}}{3}$$
$$\therefore\quad \mathrm{a}=\mathrm{g}+\frac{\mathrm{g}}{3}$$
$$\therefore\quad \mathrm{a}=\frac{4\ \mathrm{g}}{3}$$

$\therefore$ Period for a pendulum in accelerating lift is

$$T' = 2\pi\sqrt{\frac{l}{a}} = 2\pi\sqrt{\frac{3l}{4\,g}}$$

$$\therefore \quad T' = \frac{\sqrt{3}}{2}\left(2\pi\sqrt{\frac{l}{g}}\right)$$

$$\therefore \quad T' = \frac{\sqrt{3}}{2}\,T$$

Question 128

The mutual inductance of a pair of coils, each of 'N' turns, is 'M' henry. If a current of 'I' ampere in one of the coils is brought to zero in 't' second, the e. m. f. induced per turn in the other coil in volt is

Options:

A. $\frac{MI}{t}$

B. $\frac{NMI}{t}$

C. $\frac{NM}{It}$

D. $\frac{MI}{Nt}$

Answer: A

Solution:

Emf induced due to mutual inductance per coil,

$e = \frac{M\Delta I}{\Delta t}$

Here, the current changes from I to zero in time t seconds.

$\therefore \quad e = \frac{MI}{t}$

Question 129

To manufacture a solenoid of length $1 \mathrm{~m}$ and inductance $1 \mathrm{mH}$, the length of thin wire required is

(cross - sectional diameter of a solenoid is considerably less than the length)

Options:

A. 0.10 m

B. 0.10 km

C. 1 km

D. 10 km

Answer: B

Solution:

Inductance of solenoid, $\mathrm{L}=\frac{\mu_0 \mathrm{~N}^2 \mathrm{~A}}{l}$ .....(i)

where, $l=$ length of solenoid.

$\mathrm{A}=\pi \mathrm{r}^2=$ area of solenoid

Let 'x' be length of wire required.

$$\begin{aligned} \therefore \quad \mathrm{x} & = \text{circumference of solenoid} \times \text{no. of turns} \\ & = 2 \pi \mathrm{rN}. \end{aligned}$$

$$\therefore \quad \mathrm{N}=\frac{\mathrm{x}}{2 \pi \mathrm{r}}$$

Substituting in equation (i),

$$\mathrm{L}=\frac{\mu_0\left(\frac{\mathrm{x}^2}{4 \pi^2 \mathrm{r}^2}\right) \times \pi \mathrm{r}^2}{l}$$

$$\therefore \quad \mathrm{L}=\frac{\mu_0 \mathrm{x}^2}{4 \pi l}$$

$$\therefore \quad \mathrm{x}^2=\frac{4 \pi \mathrm{L} l}{\mu_0}$$

$$\therefore \quad \mathrm{x}=\sqrt{\frac{4 \pi \mathrm{L} l}{\mu_0}}$$

Substituting the values,

$$x=\sqrt{\frac{4\times\pi\times10^{-3}\times1}{4\pi\times10^{-7}}}$$
$$x=\sqrt{10^4}\ \mathrm{m}$$
$$x=0.10\ \mathrm{km}$$

Question 130

In a radioactive disintegration, the ratio of initial number of atoms to the number of atoms present at time $t=\frac{1}{2\lambda}$ is [$\lambda=$ decay constant]

Options:

A. $\frac{1}{\mathrm{e}}$

B. $\sqrt{\mathrm{e}}$

C. e

D. 2e

Answer: B

Solution:

According to radioactive disintegration law,

$$\mathrm{N}=\mathrm{N}_0\mathrm{e}^{-\lambda\mathrm{t}}$$
$$\frac{\mathrm{N}}{\mathrm{N}_0}=\mathrm{e}^{-\lambda\mathrm{t}}$$
$$\frac{\mathrm{N}}{\mathrm{N}_0}=\mathrm{e}^{-\lambda\times\frac{1}{2\lambda}}\quad\ldots\left(\because\mathrm{t}=\frac{1}{2\lambda}\right)$$
$$\frac{\mathrm{N}}{\mathrm{N}_0}=\mathrm{e}^{-\frac{1}{2}}$$
$$\frac{\mathrm{N}_0}{\mathrm{N}}=\mathrm{e}^{\frac{1}{2}}$$
$$\frac{\mathrm{N}_0}{\mathrm{N}}=\sqrt{\mathrm{e}}$$

Question 131

A bullet is fired on a target with velocity 'V'. Its velocity decreases from 'V' to '$\mathrm{V}/2$' when it penetrates $30\ \mathrm{cm}$ in a target. Through what thickness it will penetrate further in the target before coming to rest?

Options:

A. 5 cm

B. 8 cm

C. 10 cm

D. 20 cm

Answer: C

Solution:

When the velocity of the bullet changes from V to $\frac{\mathrm{V}}{2}$ the distance travelled by the bullet is $30\ \mathrm{cm}$.

Using 3^{rd} equation of motion,

$$\mathrm{v}^2=\mathrm{u}^2+2\mathrm{as}$$
$$\left(\frac{\mathrm{V}}{2}\right)^2=\mathrm{V}^2+2\mathrm{a}(30)$$
$$\frac{\mathrm{V}^2}{4}=\mathrm{V}^2+60\mathrm{a}$$
$$\frac{-3\ \mathrm{V}^2}{4}=60\mathrm{a}$$
$$\mathrm{a}=\frac{-\mathrm{V}^2}{80}$$

Further, when a bullet penetrates it comes to rest. So, the final velocity of the bullet becomes zero.

Using the relation,

$$\mathrm{v}^2=\mathrm{u}^2+2\mathrm{as}$$
$$0=\left(\frac{\mathrm{V}}{2}\right)^2+2\left(-\frac{\mathrm{V}^2}{80}\right)\mathrm{s}$$
$$\frac{\mathrm{V}^2}{4}=\left(\frac{\mathrm{V}^2}{40}\right)\mathrm{s}$$
$$\mathrm{s}=\frac{40}{4}$$
$$\mathrm{s}=10\ \mathrm{cm}$$

Question 132

In the experiment of diffraction due to a single slit, if the slit width is decreased, the width of the central maximum

Options:

A. becomes zero.

B. does not change.

C. increases.

D. decreases.

Answer: C

Solution:

Width of the central maxima $= W = \frac{\lambda\theta}{d}$

Thus, the slit width is inversely proportional to the width of the central maximum.

Hence, when the slit width is decreased, the width of the central maxima increases.

Question 133

A cylindrical magnetic rod has length 5 cm and diameter 1 cm. It has uniform magnetization $5.3 \times 10^3 \text{ A/m}^3$. Its net magnetic dipole moment is nearly

Options:

A. $1 \times 10^{-2} \text{ J/T}$

B. $0.5 \times 10^{-2} \text{ J/T}$

C. $2.5 \times 10^{-2} \text{ J/T}$

D. $2 \times 10^{-2} \text{ J/T}$

Answer: D

Solution:

$$\text{Magnetisation, } \mathrm{M} = \frac{\mathrm{m_{net}}}{\mathrm{V}}$$

$$\begin{aligned} \therefore \quad \mathrm{m_{net}} &= \mathrm{M} \times \mathrm{V} = \mathrm{M} \times \left(\pi \mathrm{r}^2 l\right) = \mathrm{M} \times \pi \times \frac{\mathrm{d}^2}{4} \times l \\ &= 5.3 \times 10^3 \times 3.142 \times \left(\frac{1 \times 10^{-2}}{2}\right)^2 \times 5 \times 10^{-2} \\ &= 2.08 \times 10^{-2}\ \mathrm{J/T} \end{aligned}$$

Question 134

In biprism experiment, if 5^{th} bright band with wavelength λ_1' coincides with 6^{th} dark band with wavelength λ_2' then the ratio $\left(\frac{\lambda_2}{\lambda_1}\right)$ is

Options:

A. $\frac{9}{7}$

B. $\frac{7}{9}$

C. $\frac{10}{11}$

D. $\frac{11}{10}$

Answer: C

Solution:

The fifth bright band will be:

$\mathrm{y_5} = \frac{5\lambda_1 \mathrm{D}}{\mathrm{d}}$

$\therefore$ The sixth dark band will be:

$$y_6{}' = \frac{11\lambda_2 D}{2\,d}$$

Given: $y_5 = y_6'$

$$\therefore \quad \frac{5\lambda_1 D}{d} = \frac{11\lambda_2 D}{2\,d}$$

$$5\lambda_1 = \frac{11\lambda_2}{2}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{10}{11}$$

Question 135

A body of mass 'm' kg starts falling from a distance 3R above earth's surface. When it reaches a distance 'R' above the surface of the earth of radius 'R' and Mass 'M', then its kinetic energy is

Options:

A. $\frac{2}{3}\frac{GMm}{R}$

B. $\frac{1}{3}\frac{GMm}{R}$

C. $\frac{1}{2}\frac{GMm}{R}$

D. $\frac{1}{4}\frac{GMm}{R}$

Answer: D

Solution:

Initial height:

$h = 3R + R = 4R$

The potential energy of the body initially will be:

$U_1 = -\frac{1}{4}\frac{GMm}{R}$

$\therefore$ At the height R,

$h = R + R = 2R$

Potential energy:

$U_2 = -\frac{1}{2}\frac{GMm}{R}$

Gain in kinetic energy is equal to loss in potential energy.

$$\therefore \quad KE = U_1 - U_2$$
$$= -\frac{1}{4}\frac{GMm}{R} - \left(-\frac{1}{2}\frac{GMm}{R}\right) = \frac{1}{4}\frac{GMm}{R}$$

Question 136

In the case of NAND gate, if A and B are the inputs and Y is the output then

Options:

A. $Y = A \cdot B$

B. $Y = \overline{A - B}$

C. $Y = \overline{A + B}$

D. $Y = \overline{A \cdot B}$

Answer: D

Question 137

A monoatomic gas at pressure 'P', having volume 'V' expands isothermally to a volume '$2\ V$' and then adiabatically to a volume '$16\ V$'. The final pressure of the gas is (Take $\gamma = 5/3$)

Options:

A. $P/64$

B. $P/32$

C. $16P$

D. 32P

Answer: A

Solution:

After isothermal expansion:

$$P_1V_1 = P_2V_2$$
$$P_2 = P_1\frac{V_1}{V_2}$$
$$P_2 = P_1\frac{V}{2V}$$
$$P_2 = \frac{P}{2}$$

After adiabatic expansion:

$$\mathrm{P}_2\ \mathrm{V}_2^\gamma = \mathrm{P}_3\ \mathrm{V}_3^\gamma$$
$$\mathrm{P}_3 = \mathrm{P}_2\left(\frac{\mathrm{V}_2}{\mathrm{V}_3}\right)^\gamma$$
$$\mathrm{P}_3 = \frac{\mathrm{P}}{2}\left(\frac{2\ \mathrm{V}}{16\ \mathrm{V}}\right)^{5/3}$$
$$\mathrm{P}_3 = \frac{\mathrm{P}}{2}\left(\frac{1}{8}\right)^{5/3}$$
$$\mathrm{P}_3 = \frac{\mathrm{P}}{64}$$

Question 138

In potentiometer experiments, two cells of e. m. f. 'E_1' and 'E_2' are connected in series $(E_1 > E_2)$, the balancing length is $64\ \mathrm{cm}$ of the wire. If the polarity of E_2 is reversed, the balancing length becomes $32\ \mathrm{cm}$. The ratio $\mathrm{E}_1/\mathrm{E}_2$ is

Options:

A. 3 : 2

B. 2 : 3

C. 1 : 3

D. 3 : 1

Answer: D

Solution:

For potentiometer,

$$\frac{E_1}{E_2}=\frac{l_1+l_2}{l_1-l_2}$$
$$\therefore \quad \frac{E_1}{E_2}=\frac{64+32}{64-32} \quad =\frac{96}{32}=\frac{3}{1}=3:1$$

Question 139

A diatomic gas $\left(\gamma=\frac{7}{5}\right)$ is compressed adiabatically to volume $\frac{V_i}{32}$ where V_i is its initial volume. The initial temperature of the gas is T_i in Kelvin and the final temperature is 'xT_i'. The value of 'x' is

Options:

A. 5

B. 4

C. 3

D. 2

Answer: B

Solution:

For adiabatic process: $TV^{\gamma-1}=$ Constant

$\therefore$ Initially:

$T_iV^{\frac{7}{5}-1}=$ Constant

$\therefore$ Final condition:

$xT_fV^{\frac{7}{5}-1}=$ Constant

So,

$T_i V^{\frac{7}{5}-1} = x T_f V^{\frac{7}{5}-1}$

$T^{\frac{2}{5}} = xT\left(\frac{V}{32}\right)^{\frac{2}{5}}$

$x = 4$

Question 140

A disc has mass M and radius R. How much tangential force should be applied to the rim of the disc, so as to rotate with angular velocity 'ω' in time t ?

Options:

A. $\frac{MR\omega}{4t}$

B. $\frac{\mathrm{MR}\omega}{2\mathrm{t}}$

C. $\frac{\mathrm{MR}\omega}{\mathrm{t}}$

D. $\mathrm{MR}\omega\mathrm{t}$

Answer: B

Solution:

Torque: $\tau = \mathrm{I}\alpha = \frac{\mathrm{MR}^2}{2} \times \frac{\omega}{\mathrm{t}}$

$\therefore \quad \tau = \frac{\mathrm{MR}^2\omega}{2\mathrm{t}}$

But $\tau = \mathrm{R} \times \mathrm{F}$

$\therefore \quad \mathrm{F} = \frac{\tau}{\mathrm{R}} = \frac{\mathrm{MR}\omega}{2\mathrm{t}}$

Question 141

An electron moving with velocity $1.6 \times 10^7\ \mathrm{m/s}$ has wavelength of $0.4\overset{o}{A}$. The required accelerating voltage for the electron motion is

[charge on electron $= 1.6 \times 10^{-19}$C, mass of electron $= 9 \times 10^{-31}$ kg]

Options:

A. 7.2×10^3 V

B. 7.2×10^2 V

C. 7.2 V

D. 7.2×10^{-2} V

Answer: B

Solution:

When an electron is accelerated through a voltage its kinetic energy is converted into electric potential energy:

$$\begin{aligned} & K = U \\ & \frac{1}{2} m v^2 = e V \\ \therefore \quad & V = \frac{m v^2}{2 e} \\ & V = \frac{\left(9 \times 10^{-31}\right)\left(1.6 \times 10^7\right)^2}{2 \times 1.6 \times 10^{-19}} = 7.2 \times 10^2 \text{ V} \end{aligned}$$

Question 142

The prism has refracting angle 'A'. The second refracting surface of the prism is silvered. Light ray falling on first refracting surface with angle of incidence '2 A', reaches the second surface and returns back through the same path due to reflection at the silvered surface. The refractive index of the material of the prism is

Options:

A. $\frac{1}{2} \sin A$

B. $\frac{1}{2} \cos A$

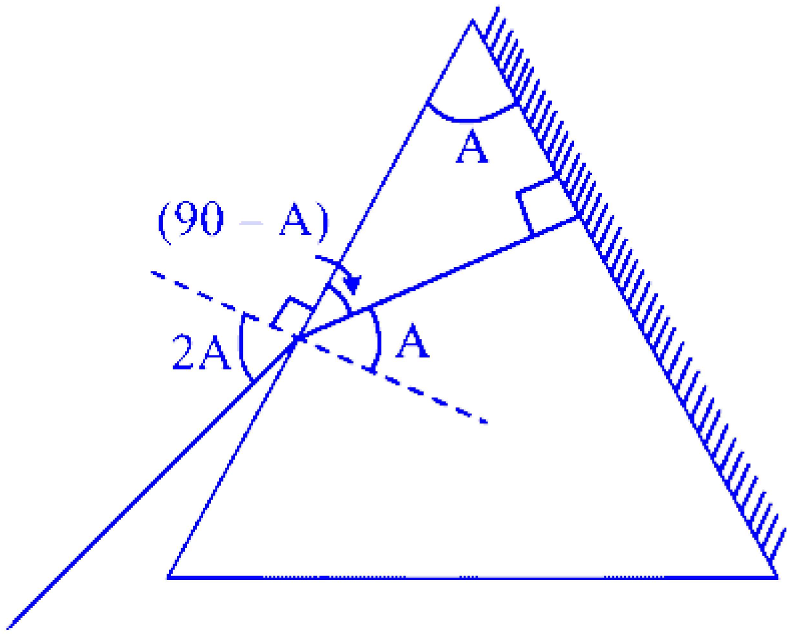
C. $2 \sin A$

D. $2\cos A$

Answer: D

Solution:

Normal incidence at silvered surface



$\therefore \quad \mu=\frac{\sin i}{\sin r}=\frac{\sin 2A}{\sin A}=\frac{2\sin A\cos A}{\sin A}=2\cos A$

Question 143

Two parallel wires of equal lengths are separated by a distance of $3\ \mathrm{m}$ from each other. The currents flowing through 1^{st} and 2^{nd} wire is $3\ \mathrm{A}$ and 4.5 A respectively in opposite directions. The resultant magnetic field at mid point between the wires ($\mu_0=$ permeability of free space)

Options:

A. $\frac{\mu_0}{2\pi}$

B. $\frac{3\mu_0}{2\pi}$

C. $\frac{7\mu_0}{2\pi}$

D. $\frac{5\mu_0}{2\pi}$

Answer: D

Solution:

Using Biot Savart law,

$B = \frac{\mu_0 I}{2\pi r}$

$\therefore$ Magnetic field due to first wire:

$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{\mu_0 \times 3}{2\pi \times 1.5} = \frac{2\mu_0}{2\pi}$

$\therefore$ Magnetic field due to second wire:

$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{\mu_0 \times 4.5}{2\pi \times 1.5} = \frac{3\mu_0}{2\pi}$

$\therefore$ Net field,

$B = B_1 + B_2 = \frac{2\mu_0}{2\pi} + \frac{3\mu_0}{2\pi} = \frac{5\mu_0}{2\pi}$

Question 144

Three point charges $+q$, $+2q$ and $+Q$ are placed at the three vertices of an equilateral triangle. If the potential energy of the system of three charges is zero, the value of Q in terms of q is

Options:

A. $Q = -\frac{2q}{3}$

B. $Q = -\frac{1}{3}q$

C. $Q = \frac{3q}{2}$

D. $Q = \frac{q}{2}$

Answer: A

Solution:

Potential energy due to +q and +2q,

$U_1 = \frac{K_q(2q)}{r}$

$\therefore$ Potential energy due to +q and +Q,

$U_2 = \frac{KqQ}{r}$

$\therefore$ Potential energy due to $+Q$ and $+2q$,

$U_3 = \frac{KQ(2q)}{r}$

Given: Potential energy of system = 0

$$\therefore \quad U_1 + U_2 + U_3 = 0$$
$$\frac{Kq(2q)}{r} + \frac{KqQ}{r} + \frac{KQ(2q)}{r} = 0$$
$$2q + Q + 2Q = 0$$
$$3Q = -2q$$
$$Q = \frac{-2}{3}q$$

Question 145

If a gas is compressed isothermally then the r.m.s. velocity of the molecules

Options:

A. decreases.

B. increases.

C. remains the same.

D. first decreases and then increases.

Answer: C

Solution:

The root-mean-square (rms) velocity of gas molecules is,

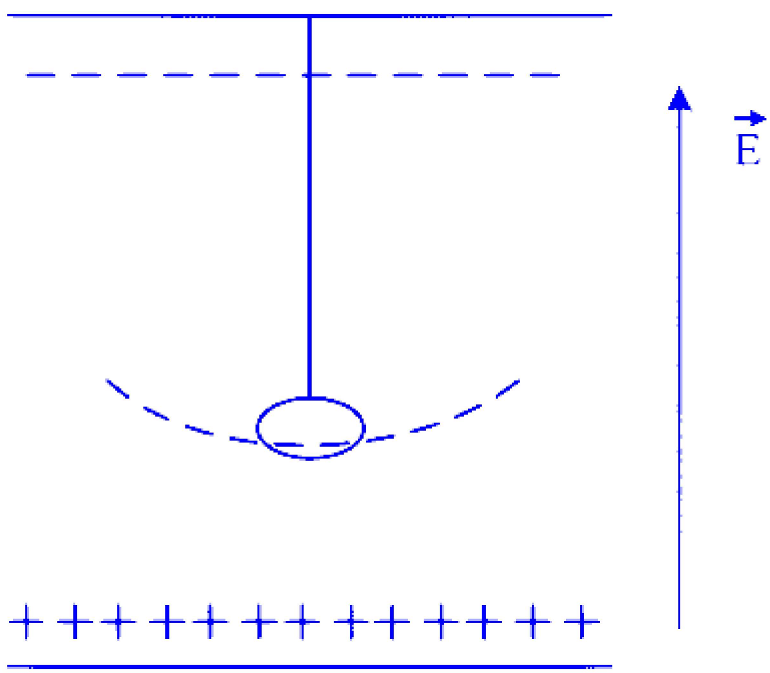
$v_{r,m.s} = \sqrt{\frac{3kT}{m}}$

$\Rightarrow v_{rms} \propto \sqrt{T}$ for a given gas.

As the temperature is constant in isothermal process, the r.m.s velocity remains the same.

Question 146

The bob of a simple pendulum of length 'L' has a mass 'm' and charge 'q'. The pendulum is suspended between the plates of a charged parallel plate capacitor. The direction of electric field is shown in figure. The period of oscillations of the simple pendulum is (acceleration due to gravity $\mathrm{g} > \mathrm{qE}/\mathrm{m}$)



Options:

A. $2\pi\sqrt{\frac{L}{g}}$

B. $2\pi\left[\frac{L}{\frac{qE}{m}-g}\right]^{\frac{1}{2}}$

C. $2\pi\left[\frac{L}{g-\frac{qE}{m}}\right]^{\frac{1}{2}}$

D. $2\pi\left[\frac{L}{g+\frac{qE}{m}}\right]^{\frac{1}{2}}$

Answer: C

Solution:

Electric force, $F_{\text{electric}} = qE$

The effective force,

$$mg_{\text{eff}} = mg - F_{\text{electric}}$$
$$g_{\text{eff}} = g - \frac{qE}{m}$$

The period of oscillation for a simple pendulum,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Time period when pendulum is suspended between the plates,

$$T = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$$
$$T = 2\pi\left[\frac{L}{g - \frac{qE}{m}}\right]^{\frac{1}{2}}$$

Question 147

An electron is projected along the axis of circular conductor carrying current I. Electron will experience

Options:

A. no force.

B. a force along the axis.

C. a force at angle 30° with the axis.

D. a force perpendicular to axis.

Answer: A

Solution:

The magnetic force experienced by the electron, $\mathrm{F} = \mathrm{qvB} \sin \theta$

As electron is projected along the axis, $\theta = 0$

$\therefore \quad \mathrm{F} = 0$

Question 148

Two identical capacitors have the same capacitance 'C'. One of them is charged to a potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

Options:

A. $\frac{1}{4} \mathrm{C}\left(\mathrm{V}_1^2 - \mathrm{V}_2^2\right)$

B. $\frac{1}{4} \mathrm{C}\left(\mathrm{V}_1^2 + \mathrm{V}_2^2\right)$

C. $\frac{1}{4} \mathrm{C}(\mathrm{V}_1 - \mathrm{V}_2)^2$

D. $\frac{1}{4} \mathrm{C}(\mathrm{V}_1 + \mathrm{V}_2)^2$

Answer: C

Solution:

Initial energy of the combined system

$\mathrm{U}_1 = \frac{1}{2} \mathrm{CV}_1^2 + \frac{1}{2} \mathrm{CV}_2^2 = \frac{\mathrm{C}}{2}\left(\mathrm{V}_1^2 + \mathrm{V}_2^2\right)$

On joining the two condensers in parallel, common potential,

$\mathrm{V} = \frac{\mathrm{V}_1 + \mathrm{V}_2}{2}$

$\therefore \quad$ Final energy of the combined system:

$\mathrm{U}_2 = \frac{1}{2}(\mathrm{C} + \mathrm{C})\left(\frac{\mathrm{V}_1 + \mathrm{V}_2}{2}\right)^2$

$\therefore \quad$ Decrease in energy will be:

$$\begin{aligned}\Delta U &= U_1 - U_2 \\ &= \frac{C}{2}\left(V_1^2 + V_2^2\right) - \frac{1}{2}(C + C)\left(\frac{V_1 + V_2}{2}\right)^2 \\ &= \frac{1}{4}C(V_f - V_2)^2\end{aligned}$$

Question 149

A transverse wave $\mathrm{Y} = 2\sin(0.01\mathrm{x} + 30\mathrm{t})$ moves on a stretched string from one end to another end in 0.5 second. If x and y are in cm and t in second, then the length of the string is

Options:

A. 5 m

B. 10 m

C. 15 m

D. 20 m

Answer: C

Solution:

Given $Y = 2\sin(0.01x + 30t)$

Comparing with standard equation,

$$\begin{aligned} & \mathrm{Y} = \mathrm{A}\sin(\mathrm{kx} + \omega\mathrm{t}), \\ \therefore \quad & \mathrm{k} = 0.01/\mathrm{cm} \\ & \omega = 30\mathrm{rad/s}\end{aligned}$$

Velocity $\mathrm{v} = \frac{\omega}{\mathrm{k}} = \frac{30}{0.01} = 3000\ \mathrm{cm/s}$

$$\begin{aligned}\therefore \quad \text{Length } \mathrm{L} &= \mathrm{v} \times \mathrm{t} = 30 \times 0.5 \\ &= 15\ \mathrm{m}\end{aligned}$$

Question 150

A body of density 'ρ' is dropped from rest at a height 'h' into a lake of density '$\sigma'(\sigma > \rho)$. The maximum depth to which the body sinks before returning to float on the surface is (neglect air dissipative forces)

Options:

A. $\frac{\mathrm{h}\rho}{(\sigma-\rho)}$

B. $\frac{h\rho}{(\sigma+\rho)}$

C. $\frac{\mathrm{h}\rho}{(\rho-\sigma)}$

D. $\frac{2\ \mathrm{h}\rho}{(\sigma-\rho)}$

Answer: C

Solution:

Initial velocity of the ball $= \sqrt{2gh}$
Upward force:

$$F = \sigma V g - \rho V g$$

$$(\rho V)a = Vg(\sigma - \rho)$$

$$\therefore \quad a = \frac{g(\sigma - \rho)}{\rho}$$

Final velocity is zero when it sinks.

$$\mathrm{v}^2 - \mathrm{u}^2 = 2\mathrm{as}$$

$$0 - (\sqrt{2gh})^2 = 2\frac{\mathrm{g}(\sigma - \rho)}{\rho}\mathrm{H}$$

$$\mathrm{H} = \frac{\mathrm{h}\rho}{(\rho - \sigma)}$$

Final velocity is zero when it sinks.
