MHT CET 2023 : 13th May Evening Shift

Mathematics

Question 1

If $xdy = y(dx + ydy), y(1) = 1, y(x) > 0$, then $y(-3)$ is

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

$xdy = ydx + y^2dy$

$\Rightarrow \quad -y^2dy = ydx - xdy$

$\Rightarrow \quad dy = \dfrac{ydx - xdy}{-y^2}$

$\Rightarrow \quad dy = -\left\{d\left(\dfrac{x}{y}\right)\right\}$

On integrating both sides w.r.t. x,

$y = -\frac{x}{y} + c_1$

Now, $y(1) = 1$, If $x = 1, y = 1$

$\Rightarrow \quad 1 = \frac{-1}{1} + c_1 \Rightarrow c_1 = 2$

$y = \frac{-x}{y} + 2$

If $x = -3, y = \frac{3}{y} + 2$

$\Rightarrow \quad y^2 - 2y - 3 = 0$

$\Rightarrow \quad (y-3)(y+1) = 0$

$\Rightarrow \quad y = 3, -1$

Question 2

Slope of the tangent to the curve $y = 2e^x \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$, where $0 \le x \le 2\pi$ is minimum at $x =$

Options:

A. 0

B. π

C. 2π

D. 1

Answer: B

Solution:

The slope of the tangent to the curve

$y = 2e^x \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$= e^x \cos x$

$S = \frac{dy}{dx} = e^x(-\sin x + \cos x)$

Now

$$\text{Now, } \frac{dS}{dx} = e^x(-\sin x + \cos x - \cos x - \sin x)$$
$$= -2e^x \sin x$$
$$\frac{dS}{dx} = 0 \Rightarrow -2e^x \sin x = 0$$
$$\Rightarrow \quad x = 0, \pi, 2\pi \quad (\because 0 \leq x \leq 2\pi)$$

Value of S at $x = 0$ is 1

Value of S at $x = \pi$ is $-e^{\pi}$.

Value of S at $x = 2\pi$ is $e^{2\pi}$.

$\therefore$ S is minimum at $x = \pi$

Question 3

If the vectors $p\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + q\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} + r\hat{\mathbf{k}} (p \neq q \neq r \neq 1)$ are coplanar, then the value of $pqr - (p + q + r)$ is

Options:

A. -2

B. 2

C. 0

D. -1

Answer: A

Solution:

If vector $p\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + q\hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{j}} + r\hat{\mathbf{k}}$ are coplanar, then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$
$$pqr - p - 1(r - 1) + 1(1 - q) = 0$$
$$pqr - p - q - r = -2$$
$$pqr - (p + q + r) = -2$$

Question 4

A tetrahedron has vertices at $P(2,1,3), Q(-1,1,2), R(1,2,1)$ and $O(0,0,0)$, then angle between the faces OPQ and PQR is

Options:

A. $\cos^{-1}\left(\frac{5}{7\sqrt{59}}\right)$

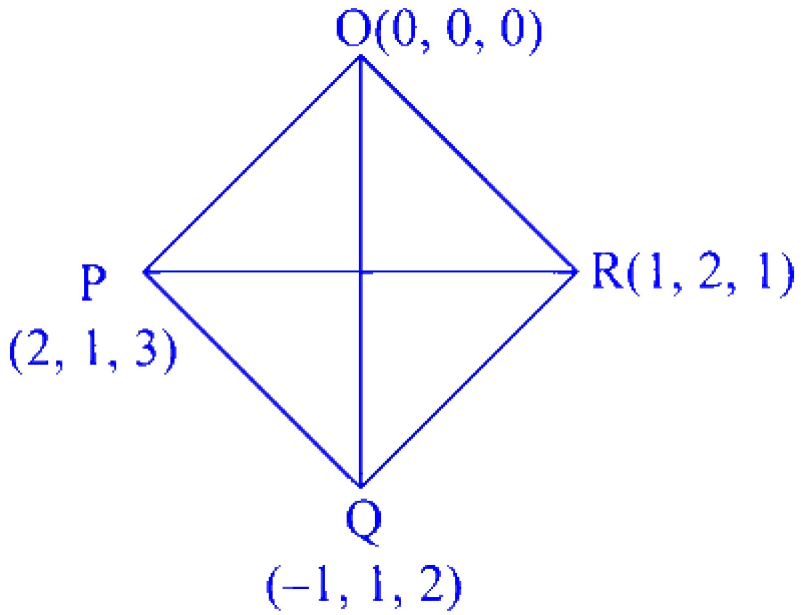
B. $\cos^{-1}\left(\frac{\sqrt{25}}{\sqrt{59}\cdot\sqrt{35}}\right)$

C. $\cos^{-1}\left(\frac{5}{413}\right)$

D. $\cos^{-1}\left(\frac{25}{\sqrt{59}\sqrt{35}}\right)$

Answer: D

Solution:



$$\begin{aligned}
\mathbf{OP} \times \mathbf{OQ} &= (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \times (-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \\
&= -\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}} = a(\text{ let }) \\
\mathbf{PQ} \times \mathbf{PR} &= (-3\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \hat{\mathbf{k}}) \times (-\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \\
&= \hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}} = b \text{ (let)} \\
\cos\theta &= \left[\frac{a \cdot b}{|a||b|}\right] \\
\cos\theta &= \left|\frac{-1 + 35 - 9}{\sqrt{59}\sqrt{35}}\right| \\
\theta &= \cos^{-1}\left(\frac{25}{\sqrt{59}\sqrt{35}}\right)
\end{aligned}$$

Question 5

Two dice are rolled. If both dice have six faces numbered $1, 2, 3, 5, 7, 11$, then the probability that the sum of the numbers on upper most face is prime, is

Options:

A. $1/4$

B. $3/4$

C. $1/9$

D. $2/7$

Answer: A

Solution:

Two dice are rolled.

The total number of outcomes in sample space $n(S)$ are given as

$\Rightarrow \quad n(S) = 6^2 = 36$

The numbers on the faces of the dice are $1, 2, 3, 5, 7, 11$

The possible scenarios where the sum of the numbers on upper most face is prime
$(1, 1), (1, 2), (2, 1), (2, 3), (2, 5), (2, 11), (3, 2), (5, 2), (11, 2)$

Number of ordered pairs $= 9$

Required probability $= \frac{9}{36} = \frac{1}{4}$

Question 6

The domain of the definition of the function $y(x)$ is given by the equation $2^x + 2^y = 2$ is

Options:

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

Solution:

$2^x + 2^y = 2$
$2^y = 2 - 2^x$

$\because \quad 2^y > 0$ [exponential function]
$\therefore \quad 2 - 2^x > 0$
$\quad\quad 2^x < 2$
$\quad\quad x < 1$
$\therefore \quad x \in (-\infty, 1)$

Question 7

If $\mathbf{a} = \frac{1}{\sqrt{10}}(3\hat{\mathbf{i}} + \hat{\mathbf{k}}), \mathbf{b} = \frac{1}{7}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$, then the value of $(2\mathbf{a} - \mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} + 2\mathbf{b})]$ is

Options:

A. -3

B. 5

C. 3

D. -5

Answer: D

Solution:

Here $(2\mathbf{a}-\mathbf{b})\cdot[(\mathbf{a}\times\mathbf{b})\times(\mathbf{a}+2\mathbf{b})]$
$=(2\mathbf{a}-\mathbf{b})\cdot\{(\mathbf{a}\times\mathbf{b})\times\mathbf{a}+2(\mathbf{a}\times\mathbf{b})\times\mathbf{b}\}$
$=(2\mathbf{a}-\mathbf{b})\cdot\{(\mathbf{a}\cdot\mathbf{a})\cdot\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{a}+2(\mathbf{a}\cdot\mathbf{b})\mathbf{b}-2(\mathbf{b}\cdot\mathbf{b})\cdot\mathbf{a}\}$
$=(2\mathbf{a}-\mathbf{b})\cdot(\mathbf{b}-2\mathbf{a})$
$=-4\mathbf{a}\cdot\mathbf{a}-\mathbf{b}\cdot\mathbf{b}=-5$

Question 8

$y=\frac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$. Then, $\frac{dy}{dx}$ at $x=0$ **is**

Options:

A. 3

B. -1

C. 1

D. 2

Answer: C

Solution:

$y=\frac{\sqrt[3]{1+3x}\sqrt[4]{1+4x}\sqrt[5]{1+5x}}{\sqrt[7]{1+7x}\sqrt[8]{1+8x}}$

Taking log on both sides,

$$\log y = \frac{1}{3}\log(1+3x) + \frac{1}{4}\log(1+4x) + \frac{1}{5}\log(1+5x) - \frac{1}{7}\log(1+7x) - \frac{1}{8}\log(1+8x)$$

Differentiating both sides w.r.t. x,

$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{1+3x} + \frac{1}{1+4x} + \frac{1}{1+5x} - \frac{1}{1+7x} - \frac{1}{1+8x}$

At $x = 0, y = 1$

$\therefore \frac{dy}{dx} = 1 + 1 + 1 - 1 - 1 = 1$

Question 9

Five students are selected from n students such that the ratio of number of ways in which 2 particular students are selected to the number of ways 2 particular students not selected is $2:3$. Then, the value of n is

Options:

A. 5

B. 6

C. 11

D. not possible

Answer: C

Solution:

Here, 5 students are to be selected from n students.

When 2 specified students are included, then remaining 3 students can be selected $(n-2)$ students

$\therefore$ Number of ways of selecting 3 student from n students $= {}^{(n-2)}C_3$

Now, when 2 specified students are not included, then 5 students can be selected from the remaining $(n-2)$ students.

$\therefore$ Number of ways of selecting 5 students from $(n-2)$ students $= {}^{n-2}C_5$

According to question,

$$^{n-2}C_3 : {}^{n-2}C_5 = 2:3$$

$$\Rightarrow \quad \frac{(n-2)!}{3!(n-5)!} \times \frac{5!(n-7)!}{(n-2)!} = 2:3$$

$$\Rightarrow \quad \frac{5!}{3!}\frac{(n-7)!}{(n-5)!} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{5\times 4}{(n-5)(n-6)} = \frac{2}{3}$$

$\therefore$ Then, value of $n = 11$

Question 10

If $\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt = \frac{1}{2}[g(t)]^2 + c$, (where c is a constant of integration), then $g(2)$ is

Options:

A. $\frac{1}{\sqrt{5}}\log(2+\sqrt{5})$

B. $\frac{1}{2}\log(2+\sqrt{5})$

C. $2\log(2+\sqrt{5})$

D. $\log(2+\sqrt{5})$

Answer: D

Solution:

Let $I = \int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt$

Now assume $y = \log\left(t+\sqrt{1+t^2}\right)$

$$\Rightarrow \quad dy = \frac{1}{t+\sqrt{1+t^2}}\left(1+\frac{t}{\sqrt{1+t^2}}\right)dt$$

$$= \frac{1}{\sqrt{1+t^2}}dt$$

$$\therefore \quad I = \int y dy = \frac{y^2}{2} + c$$
$$= \frac{\left(\log\left(t + \sqrt{1+t^2}\right)\right)^2}{2} + c$$

Thus, $g(t) = \log\left(t + \sqrt{1+t^2}\right)$

$$\Rightarrow \quad g(2) = \log(2 + \sqrt{5})$$

Question 11

If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^T \cdot A^{-1} =$

Options:

A. $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

B. $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

C. $\begin{bmatrix} \cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

D. $\begin{bmatrix} \cos 2x & -\sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

Answer: B

Solution:

Here, $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

$$\Rightarrow \quad |A| = \sec^2 x$$

$$\text{adj } A = C^T = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}^T$$

$$\Rightarrow \quad \text{adj } A = \begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cdot \cos x & \cos^2 x \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} \cos^2 x & -\sin x \cos x \\ \sin x \cdot \cos x & \cos^2 x \end{bmatrix}$$

Now,

$$A^T A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos^2 x & -\sin x \cdot \cos x \\ \sin x \cdot \cos x & \cos^2 x \end{bmatrix}$$

$$A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

Question 12

$\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}, \mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, \mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ are three vectors. For a vector $\mathbf{r}$ with $\mathbf{r} \times \mathbf{a} = \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{c} = 3$, $|\mathbf{r}|$ is

Options:

A. $\sqrt{55}$

B. $\sqrt{138}$

C. $\sqrt{155}$

D. $\sqrt{170}$

Answer: C

Solution:

We have, $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

$\mathbf{b} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

Let $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Since, $\mathbf{r} \times \mathbf{a} = \mathbf{b}$

$$\Rightarrow \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & z \\ 2 & 3 & 4 \end{vmatrix} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\Rightarrow \hat{\mathbf{i}}(4y - 3z) - \hat{\mathbf{j}}(4x - 2z) + \hat{\mathbf{k}}(3x - 2y) = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\left.\begin{aligned} \Rightarrow \quad & 4y - 3z = 1 \\ & 4x - 2z = 2 \\ \text{and} \quad & 3x - 2y = 1 \end{aligned}\right\} \quad \text{... (i)}$$

$$\begin{aligned} \text{Also,} \quad & \mathbf{r} \cdot \mathbf{c} = 3 \\ \Rightarrow \quad & (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) = 3 \\ \Rightarrow \quad & x + y - z = 3 \quad \text{.... (ii)} \end{aligned}$$

On solving Eqs. (i) and (ii), we get

$$\begin{aligned} & x = 5, y = 7 \text{ and } z = 9 \\ \therefore \quad & |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2} \\ & = \sqrt{25 + 49 + 81} \\ & = \sqrt{155} \end{aligned}$$

Question 13

Let PQR be a right angled isosceles triangle, right angled at $Q(2, 1)$. If the equation of the line PR is $2x + y = 3$, then the combined equation representing the pair of lines PQ and QR is

Options:

A. $3x^2 + 8xy - 3y^2 - 20x - 10y + 25 = 0$

B. $3x^2 - 8xy - 3y^2 - 20x - 10y - 25 = 0$

C. $3x^2 + 8xy - 3y^2 + 20x + 10y + 25 = 0$

D. $3x^2 - 8xy - 3y^2 + 20x + 10y - 25 = 0$

Answer: A

Solution:

The equations of PQ and PR are given by

$y-1=\frac{-2\mp\tan 45^\circ}{1\pm(-2)\tan 45^\circ}(x-2)$

$\Rightarrow \quad y-1=\left(\frac{-2\mp 1}{1\pm 2}\right)(x-2)$

$\Rightarrow \quad y-1=-\frac{1}{3}(x-2)$

and $\quad y-1=3(x-2)$

$\Rightarrow \quad x+3y=5$ and $3x-y=5$

The combined equation of these two lines is

$$(x+3y-5)(3x-y-5)=0$$

$$\Rightarrow 3x^2-3y^2+8xy-20x-10y+25=0$$

Question 14

If a curve $y=a\sqrt{x}+bx$ passes through the point $(1,2)$ and the area bounded by the curve, line $x=4$ and X-axis is 8 sq units, then

Options:

A. $a=3, b=-1$

B. $a=3, b=1$

C. $a=-3, b=1$

D. $a=-3, b=-1$

Answer: A

Solution:

Given curve $y=a\sqrt{x}+bx$ passes through $(1,2)$.

The area formed by curve line $x=4$ and X-axis is 8 sq units.

$\because$ Since, the curves $y=a\sqrt{x}+bx$ passes through $(1,2)$.

$\therefore \quad 2=a\sqrt{1}+b\cdot 1$

$a+b=2 \quad$ .... (i)

Now, the area enclosed by the curve line $x=4$ and X-axis

$$= \int_0^4 y dx = \int_0^4 (a\sqrt{x} + bx) dx$$

$$= \left[a \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + b \cdot \frac{x^2}{2} \right]_0^4$$

$$= \frac{2a}{3} \cdot 4^{\frac{3}{2}} + \frac{b}{2} \cdot 4^2 = \frac{16a}{3} + 8b$$

From the problem

$$\frac{16a}{3} + 8b = 8$$

$\therefore \quad 2a + 3b = 3 \quad$ .... (ii)

From Eqs. (i) and (ii), we get

a = 3, b = -1

Question 15

A plane is parallel to two lines whose direction ratios are $2, 0, -2$ and $-2, 2, 0$ and it contains the point $(2, 2, 2)$. If it cuts coordinate axes at A, B, C, then the volume of the tetrahedron $OABC$ (in cubic units) is

Options:

A. 216

B. 6

C. 36

D. 9

Answer: C

Solution:

Let the equation of the plane through $(2, 2, 2)$ be $a(x-2) + b(y-2) + c(z-2) = 0$

Since, it is parallel to the straight lines having Dr's $(1, 0, -1)$ and $(-1, 1, 0)$, therefore $a - c = 0$ and $-a + b = 0$

$\Rightarrow \quad a = b = c$

Therefore, equation of plane is

$x-2+y-2+z-2=0$

$x+y+z=6 \Rightarrow \frac{x}{6}+\frac{y}{6}+\frac{z}{6}=1$

Its intercepts on coordinate axes are $A(6,0,0), B(0,6,0)$ and $C(0,0,6)$.

Hence, the volume of tetrahedron $OABC$

$= \frac{1}{6}[abc] = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$= \frac{1}{6} \times 6 \times 36 = 36$ cube units

Question 16

If q is false and $p \wedge q \leftrightarrow r$ is true, then ..... is a tautology.

Options:

A. $p \vee r$

B. $(p \wedge r) \rightarrow p \vee r$

C. $(p \vee r) \rightarrow p \wedge r$

D. $p \wedge r$

Answer: B

Solution:

$\because q$ is F and $(p \wedge q) \leftrightarrow r$ is T

$\Rightarrow p \wedge q$ is F which implies that r is F

$\Rightarrow q$ is F and r is F

$\Rightarrow (p \wedge r)$ is always F

$\Rightarrow (p \wedge r) \rightarrow (p \vee r)$ is tautology.

Question 17

Negation of contrapositive of statement pattern $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ is

Options:

A. $(\sim p \wedge q) \vee (p \wedge \sim q)$

B. $(\sim p \vee q) \wedge (p \vee \sim q)$

C. $(p \wedge \sim q) \vee (\sim p \wedge \sim q)$

D. $(\sim p \vee \sim q) \wedge (p \vee q)$

Answer: B

Solution:

Contrapositive of $x \rightarrow y$ is $\sim y \rightarrow \sim x$ negation of $(p \vee \sim q)$ is $(\sim p \wedge q)$ and the negation of $(p \wedge \sim q)$ is $(\sim p \vee q)$

So, we have contrapositive of

$(p \vee \sim q) \rightarrow (p \wedge \sim q) = (\sim p \wedge q) \rightarrow (\sim p \vee q)$

Now, negation of contrapositive statement is

$(\sim p \vee q) \wedge (p \vee \sim q)$

Question 18

The value of $\tan\left(\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right)$ is

Options:

A. $6/17$

B. $17/6$

C. $16/7$

D. $7/16$

Answer: B

Solution:

Here,

$$\begin{aligned}
&\tan(\sin^{-1}\left(\frac{3}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\Big)\\
&\quad=\tan\left[\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]\\
&\quad=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)\\
&\quad=\tan\left\{\tan^{-1}\left(\frac{17}{6}\right)\right\}=\frac{17}{6}
\end{aligned}$$

Question 19

Variance of first $2n$ natural numbers is

Options:

A. $\frac{4n^2+1}{12}$

B. $\frac{(2n-1)^2}{12}$

C. $\frac{n^2}{3}-1$

D. $\frac{4n^2-1}{12}$

Answer: D

Solution:

We know that, mean of the first $2n$ natural numbers $=\frac{1+2+3+\ldots+2n}{2n}=\frac{2n+1}{2}$

The variance is defined as the average of the squared differences from the mean.

Variance $=\left[\frac{(1-mean)^2+(2-mean)^2+\ldots+(2n-mean)^2}{2n}\right]$

$$= \left[\frac{n(2n+1)(4n+1) - 6n(2n+1) + 6n^2}{12n}\right]$$
$$= \frac{4n^2 - 1}{12}$$

Question 20

$\int \frac{x-3}{(x-1)^3} e^x dx =$

Options:

A. $e^x \left(\frac{1}{(x-1)^2}\right) + c$, where c is constant of integration

B. $e^x \left(\frac{1}{x+1}\right) + c$, where c is constant of integration

C. $e^x \left((x-1)^2\right) + c$, where c is constant of integration

D. $e^x \left((x-1)^3\right) + c$, where c is constant of integration

Answer: A

Solution:

$$\text{Let } I = \int e^x \left(\frac{x-3}{(x-1)^3}\right) dx$$
$$= \int e^x \left\{\frac{x-1-2}{(x-1)^3}\right\} dx$$
$$= \int e^x \left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\} dx$$

Let $f(x) = \frac{1}{(x-1)^2}$, then $f'(x) = \frac{-2}{(x-1)^3}$

Now, we know that,

$$\int e^x \{f(x) + f'(x)\} dx$$
$$= e^x f(x) + c$$
$$\therefore \quad I = \frac{e^x}{(x-1)^2} + c$$

Question 21

If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar unit vectors such that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\mathbf{b}+\mathbf{c}}{\sqrt{2}}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

Options:

A. $3\pi/4$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: A

Solution:

Here,

$$\frac{\mathbf{b}+\mathbf{c}}{\sqrt{2}} = \mathbf{a} \times \mathbf{b} \times \mathbf{c}$$
$$\frac{\mathbf{b}+\mathbf{c}}{\sqrt{2}} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$\Rightarrow \quad \left(\mathbf{a} \cdot \mathbf{c} - \tfrac{1}{\sqrt{2}}\right)\mathbf{b} - \left(\mathbf{a} \cdot \mathbf{b} - \tfrac{1}{\sqrt{2}}\right)\mathbf{c} = \mathbf{0}$$

Since, a, b, c are non-coplanar so, a, b, c are linearly independent.

Hence, $a \cdot b = -\frac{1}{\sqrt{2}}$

$\therefore \cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \mathbf{a} \cdot \mathbf{b} = -\frac{1}{\sqrt{2}}$

(θ is the angle between $\mathbf{a}$ and $\mathbf{b}$)

$\Rightarrow \quad \theta = \frac{3\pi}{4}$

Question 22

$\int \frac{2+\cos\frac{x}{2}}{x+\sin\frac{x}{2}}dx =$

Options:

A. $2\log\left(x+\sin\frac{x}{2}\right)+c$, where c is constant of integration

B. $\frac{1}{2}\log\left(x+\sin\frac{x}{2}\right)+c$, where c is constant of integration

C. $4\log\left(x+\sin\frac{x}{2}\right)+c$, where c is constant of integration

D. $\log\left(x+\sin\frac{x}{2}\right)+c$, where c is constant of integration

Answer: A

Solution:

Let $I=\int\frac{2+\cos\frac{x}{2}}{x+\sin\frac{x}{2}}dx$

Put $x+\sin\frac{x}{2}=t$

$$\left(1+\cos\frac{x}{2}\times\frac{1}{2}\right)dx=dt$$
$$\Rightarrow\quad\left(2+\cos\frac{x}{2}\right)dx=2dt$$
$$\therefore\quad I=\int\frac{2dt}{t}=2\log t+c=2\log\left(x+\sin\frac{x}{2}\right)+c$$

Question 23

In $\triangle ABC$, with usual notations, if $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then the value of $\cos A+\cos B+\cos C$ is

Options:

A. $17/35$

B. $51/35$

C. $5/7$

D. $19/35$

Answer: B

Solution:

$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{2(a+b+c)}{36}$

or $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = \frac{a+b+c}{18} = k$

$\therefore \quad b + c = 11k, c + a = 12k, a + b = 13k$

$a + b + c = 18k$

Substituting $b + c = 11k$ in $a + b + c = 18k$, we get $a + 11k = 18k$ or $a = 7k$

Substituting $a = 7k$ in $c + a = 12k$, we get

$c + 7k = 12k$ or $c = 5k$

Substituting $c = 5k$ in $b + c = 11k$, we get

$b + 5k = 11k$ or $b = 6k$

$\therefore \quad a = 7k, b = 6k$ and $c = 5k$

Using cosine rule, we get

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(6k)^2 + (5k)^2 - (7k)^2}{2 \times 6k \times 5k} = \frac{1}{5},$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$= \frac{(5k)^2 + (7k)^2 - (6k)^2}{2 \times 5k \times 7k} = \frac{19}{35}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(7k)^2 + (6k)^2 - (5k)^2}{2 \times 7k \times 6k} = \frac{5}{7}$$

$$\cos A + \cos B + \cos C = \frac{1}{5} + \frac{19}{35} + \frac{5}{7}$$

$$= \frac{7 + 19 + 25}{35} = \frac{51}{35}$$

Question 24

If slope of the tangent to the curve $xy + ax + by = 0$ at the point $(1, 1)$ on it is 2, then the value of $3a + b$ is

Options:

A. 3

B. 1

C. 2

D. -1

Answer: B

Solution:

Given curve is $xy + ax + by = 0$

Slope of tangent at $(1, 1)$ is

$$x \cdot \frac{dy}{dx} + y + a + b \cdot \frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_{1,1} = -\left[\frac{a+y}{x+b}\right]_{(1,1)} = \frac{-(a+1)}{1+b}$$

$\therefore \quad \frac{-(a+1)}{1+b} = 2$

So, $\quad 2 + 2b = -a - 1$

$a + 2b = -3 \quad$ .... (i)

$\because (1, 1)$ lies on curve $xy + ax + by = 0$.

$\therefore \quad a + b = -1$ .... (ii)

From Eqs. (i) and (ii), we get

$b = -2, a = 1$

$\therefore \quad 3a + b = 3 \times 1 + (-2) = 1$

Question 25

If $(3x + 2) - (5y - 3)i$ and $(6x + 3) + (2y - 4)i$ are conjugates of each other, then the value of $\frac{x-y}{x+y}$ is (where $i = \sqrt{-1}, x, y \in R$)

Options:

A. -1

B. 0

C. 1

D. 2

Answer: B

Solution:

$(3x+2)-(5y-3)i$ and $(6x+3)+(2y-4)i$ are conjugates of each other.

$(3x+2)-(5y-3)i=(6x+3)-(2y-4)i$

On comparing real parts and imaginary parts, we get

$$\begin{aligned} 3x+2=6x+3, &\quad 5y-3=2y-4 \\ 3x=-1, &\quad 3y=-1 \\ x=-1/3, &\quad y=-1/3 \\ \therefore \quad \frac{x-y}{x+y} &= \frac{-\frac{1}{3}+\frac{1}{3}}{-\frac{1}{3}-\frac{1}{3}}=0 \end{aligned}$$

Question 26

If $\left(\tan^{-1}x\right)^2+\left(\cot^{-1}x\right)^2=\frac{5\pi^2}{8}$, then the value of x is

Options:

A. -2

B. -1

C. 1

D. 2

Answer: B

Solution:

We have, $\tan^{-1}x+\cot^{-1}x=\pi/2$

The given equation can be written as

$\left(\tan^{-1} x + \cot^{-1} x\right)^2 - 2\tan^{-1} x\left(\frac{\pi}{2} - \tan^{-1} x\right) = \frac{5\pi^2}{8}$

$2\left(\tan^{-1} x\right)^2 - 2\left(\frac{\pi}{2}\right)\tan^{-1}(x) - \frac{3\pi^2}{8} = 0$

$\tan^{-1} x = -\frac{\pi}{4}$ or $\frac{3\pi}{4}$

Hence, $\tan^{-1} x = 1 \Rightarrow x = -1$

Question 27

The solution of $(1 + xy)ydx + (1 - xy)xdy = 0$ is

Options:

A. $\log\left(\frac{x}{y}\right) + \frac{1}{xy} = k$, where k is constant of integration

B. $\log\left(\frac{x}{y}\right) = \frac{1}{xy} + k$, where k is constant of integration

C. $\log\left(\frac{x}{y}\right) + xy = k$, where k is constant of integration

D. $\log\left(\frac{x}{y}\right) = xy + k$, where k is constant of integration

Answer: B

Solution:

$(1 + xy)ydx + (1 - xy)xdy = 0$
$ydx + xdy + xy^2dx - x^2ydy = 0$

On dividing $\frac{1}{x^2y^2}$ both sides, we get

$\int \frac{ydx+xdy}{x^2y^2} + \int \frac{dx}{x} - \int \frac{dy}{y} = 0$

$\Rightarrow \quad -\frac{1}{xy} + \log x - \log y = k$

$\Rightarrow \quad -\frac{1}{xy} + \log\left(\frac{x}{y}\right) = k$

$\Rightarrow \quad \log\left(\frac{x}{y}\right) = \frac{1}{xy} + k$

Question 28

The integral $\int_{\pi/6}^{\pi/3} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x dx$ is equal to

Options:

A. $3^{\frac{5}{6}} - 3^{\frac{2}{3}}$

B. $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$

C. $3^{\frac{5}{3}} - 3^{\frac{1}{3}}$

D. $3^{\frac{4}{3}} - 3^{\frac{1}{3}}$

Answer: B

Solution:

We have,

$$\begin{aligned}\int_{\pi/6}^{\pi/3} \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x dx &= \int_{\pi/6}^{\pi/3} \left(\frac{1}{\cos x}\right)^{\frac{2}{3}} \left(\frac{1}{\sin x}\right)^{\frac{4}{3}} dx \\ &= \int_{\pi/6}^{\pi/3} \frac{1}{\cos^{\frac{2}{3}} x \sin^{\frac{4}{3}} x} dx \\ &= \int_{\pi/6}^{\pi/3} \frac{\sin^{\frac{2}{3}} x}{\cos^{\frac{2}{3}} x \sin^2 x} dx \\ &= \int_{\pi/6}^{\pi/3} \tan^{\frac{2}{3}} x \operatorname{cosec}^2 x dx\end{aligned}$$

Put $\tan x = t$, then $\operatorname{cosec}^2 x dx = dt$. When $x = \frac{\pi}{6}, t = \frac{1}{\sqrt{3}}$ and when $x = \frac{\pi}{3}, t = \sqrt{3}$. Therefore,

$$\int_{\pi/6}^{\pi/3} \tan^{\frac{2}{3}} x \operatorname{cosec}^2 x dx = \int_{1/\sqrt{3}}^{\sqrt{3}} t^{\frac{2}{3}} dt$$

$$= \left[\frac{3}{5} t^{\frac{5}{3}}\right]_{1/\sqrt{3}}^{\sqrt{3}}$$

$$= \frac{3}{5}\left[(\sqrt{3})^{\frac{5}{3}} - \left(\frac{1}{\sqrt{3}}\right)^{\frac{5}{3}}\right]$$

$$= \frac{3}{5}\left[3^{\frac{5}{6}} - 3^{-\frac{5}{6}}\right]$$

$$= \frac{3}{5}\left[3^{\frac{5}{6}} - \frac{1}{3^{\frac{5}{6}}}\right]$$

$$= \frac{3}{5}\left[\frac{3^{\frac{5}{6}} \cdot 3^{\frac{5}{6}} - 1}{3^{\frac{5}{6}}}\right]$$

$$= \frac{3}{5}\left[\frac{3^{\frac{5}{3}} - 1}{3^{\frac{5}{6}}}\right]$$

$$= \frac{3^{\frac{5}{3}} - 1}{3^{\frac{1}{6}}}$$

$$= \frac{3^{\frac{5}{3}} - 1}{3^{\frac{1}{6}}} \cdot \frac{3^{\frac{1}{6}}}{3^{\frac{1}{6}}}$$

$$= 3^{\frac{7}{6}} - 3^{\frac{1}{6}}$$

Hence, the correct answer is **Option B**.

Question 29

$$\lim_{x \to 0} \frac{\sqrt{1+x\sin x} - \sqrt{\cos x}}{\tan^2 \frac{x}{2}} =$$

Options:

A. 1

B. 2

C. 3

D. -1

Answer: C

Solution:

$$\lim_{x\to 0} = \frac{\sqrt{1+x\sin x}-\sqrt{\cos x}}{\tan^2\frac{x}{2}}$$
$$= \lim_{x\to 0} \frac{1+x\sin x-\cos x}{\tan^2\frac{x}{2}[\sqrt{1+x\sin x}+\sqrt{\cos x}]}$$
$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}+x\sin x}{\tan^2\frac{x}{2}[\sqrt{1+x\sin x}+\sqrt{\cos x}]}$$

$$= \lim_{x\to 0} \frac{\frac{2\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}+\frac{x\sin x}{\frac{x^2}{4}}}{\frac{\tan^2\frac{x}{2}[\sqrt{1+x\sin x}+\sqrt{\cos x}]}{\frac{x^2}{4}}}$$

$$= \frac{2+4}{1[1+1]} = \frac{6}{2} = 3$$

Question 30

$A(1,-3), B(4,3)$ are two points on the curve $y = x - \frac{4}{x}$. The points on the curve, the tangents at which are parallel to the chord AB, are

Options:

A. $(1,2),(-1,-2)$

B. $(2,0),(-2,0)$

C. $(0,2),(1,-2)$

D. $(3,2),(-3,1)$

Answer: B

Solution:

Given, $A = (1,-3)$ and $B(4,3)$ and $y = x - \frac{4}{x}$

We know that,

Slope of tangent = Slope of AB

$m_{AB} = \frac{3-(-3)}{4-1} = \frac{6}{3} = 2$

$\Rightarrow \quad m_T = 2$

$y = x - \frac{4}{x}$

$\frac{dy}{dx} = 1 + \frac{4}{x^2}$ [slope of tangent]

$\Rightarrow \quad \frac{dy}{dx} = 1 - 4\left(-\frac{1}{x^2}\right) = 2$
$\Rightarrow \quad 1 + \frac{4}{x^2} = 2$
$\Rightarrow \quad \frac{4}{x^2} = 1$
$\Rightarrow \quad x^2 = 4 \Rightarrow x = \pm 2$

So, $y = 0$, at $x = 2$ and $y = 0$, at $x = -2$

$P(2, 0), Q(-2, 0)$

Question 31

Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in R,$ then $f(2)$ equals

Options:

A. 30

B. -2

C. -4

D. 8

Answer: B

Solution:

$\quad f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$
$\Rightarrow \quad f'(x) = 3x^2 + 2x f'(1) + f''(2) \quad$ .... (i)
$\Rightarrow \quad f''(x) = 6x + 2f'(1) \quad$ .... (ii)
$\Rightarrow \quad f'''(x) = 6 \quad$ .... (iii)

Put $x = 1$, in Eq. (i)

$f'(1) = 3 + 2f'(1) + f''(2)$ .... (iv)

Put $x = 2$ in Eq. (ii)

$f''(2) = 12 + 2f'(1)$ ..... (v)

From Eqs. (iv) and (v),

$$\begin{aligned} & -3 - f'(1) = 12 + 2f'(1) \\ \Rightarrow \quad & 3f'(1) = -15 \\ \Rightarrow \quad & f'(1) = -5 \\ \Rightarrow \quad & f''(2) = 2 \quad \text{[from Eq. (v)]} \end{aligned}$$

Put $x = 3$ in Eq. (iii), we get

$$\begin{aligned} & f'''(3) = 6 \\ & f(x) = x^3 - 5x^2 + 2x + 6 \\ & f(2) = 8 - 20 + 4 + 6 = -2 \end{aligned}$$

Question 32

A radioactive substance, with initial mass m_0, has a half-life of h days. Then, its initial decay rate is given by

Options:

A. $\frac{m_0}{h} \log 2$

B. $m_0 h \log 2$

C. $-\frac{m_0}{h} \log 2$

D. $-m_0 h \log \cdot 2$

Answer: C

Solution:

Mass of substance at time t is m

Rate of change of mass $= \frac{dm}{dt} \propto m$

$$\begin{aligned} \frac{dm}{dt} &= -km; k \rightarrow \text{ constant} \\ \int \frac{dm}{m} &= -\int k dt \\ \log m &= -kt + c \quad \text{.... (i)} \end{aligned}$$

$t = 0, m = m_0$

At $\log m_0 = c$

$\log m = -kt + \log(m_0)$ .... (ii)

At time $t = h, m = \frac{1}{2}m_0$

From Eq. (ii), $\log\left(\frac{m_0}{2}\right) = -kh + \log(m_0)$

$\Rightarrow \quad \log(m_0) - \log\left(\frac{m_0}{2}\right) = kh$

$\Rightarrow \quad \log 2 = kh \Rightarrow k = \frac{1}{h}\log 2$

Put value of k in Eq. (ii), we get

$\log m = \left(-\frac{1}{h}\log 2\right)t + \log m_0$ ..... (iii)

$$\Rightarrow \quad \log(m) - \log m_0 = -\frac{t}{h}\log 2$$

$$\Rightarrow \quad \log\frac{m}{m_0} = \log\left(2^{-\frac{t}{h}}\right)$$

$$\frac{m}{m_0} = 2^{-\frac{t}{h}} \Rightarrow m = 2^{-t/h} \cdot m_0$$

Initial decay rate

$$\frac{dm}{dt} = -km = -\frac{1}{h}\log 2 \times m$$

$$\frac{dm}{dt} = -\frac{m}{h}\log 2$$

So, initial mass $= m_0$ then

$\frac{dm}{dt} = -\frac{m_0}{h}\log 2$

Question 33

The abscissae of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are roots of the equation $y^2 + 2py - q^2 = 0$. Then, the equation of the circle with AB as diameter is given by

Options:

A. $x^2 + y^2 - 2ax - 2py + \left(b^2 + q^2\right) = 0$

B. $x^2 + y^2 - 2ax - 2py - \left(b^2 + q^2\right) = 0$

C. $x^2 + y^2 + 2ax + 2py + \left(b^2 + q^2\right) = 0$

D. $x^2 + y^2 + 2ax + 2py - \left(b^2 + q^2\right) = 0$

Answer: D

Solution:

Here, $x^2 + 2ax - b^2 = 0$

Let roots be x_1 and x_2

$x_1 + x_2 = \dfrac{-b}{a} = -2a$
$x_1 \cdot x_2 = \dfrac{c}{a} = -b^2$

and $y^2 + 2py - q^2 = 0$

Let roots be y_1 and y_2

$y_1 + y_2 = -2p$
$y_1 \cdot y_2 = -q^2$

Equation in diametric form is

$x^2 + y^2 - (x_1 + x_2)x - (y_1 + y_2)y + x_1 \cdot x_2 + y_1 \cdot y_2 = 0$
$x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

Question 34

The incentre of the $\triangle ABC$, whose vertices are $A(0, 2, 1)$, $B(-2, 0, 0)$ and $C(-2, 0, 2)$, is

Options:

A. $\left(-\frac{3}{2}, \frac{1}{2}, 1\right)$

B. $\left(\frac{3}{2}, \frac{1}{2}, 1\right)$

C. $\left(-\frac{3}{2}, -\frac{1}{2}, -1\right)$

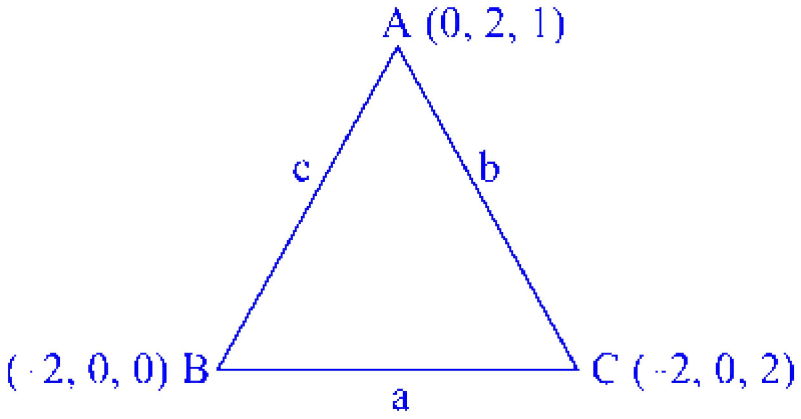
D. $\left(\frac{3}{2}, -\frac{1}{2}, -1\right)$

Answer: A

Solution:

We have, $\triangle ABC$ with $A(0,2,1), B(-2,0,0)$ and $C(-2,0,2)$.

So, $AB = \sqrt{9} = 3 = c$,
$BC = \sqrt{4} = 2 = a$
and $AC = \sqrt{9} = 3 = b$



We know that,

$$\text{incentre } = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$$
$$= \left(\frac{0-6-6}{8}, \frac{4+0+0}{8}, \frac{2+0+6}{8}\right)$$
$$= \left(\frac{-3}{2}, \frac{1}{2}, 1\right)$$

Question 35

The acute angle between the line joining the points $(2,1,-3), (-3,1,7)$ and a line parallel to $\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$ through the point $(-1,0,4)$ is

Options:

A. $\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$

B. $\cos^{-1}\left(\frac{5}{7\sqrt{10}}\right)$

C. $\cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$

D. $\cos^{-1}\left(\frac{3}{5\sqrt{10}}\right)$

Answer: C

Solution:

Direction ratio of the line joining the points $(2, 1, -3)$ and $(-3, 1, 7)$ are (a_1, b_1, c_1) i.e. $(-5, 0, 10)$.

Direction ratio of the line parallel to line $\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$ are (a_2, b_2, c_2) i.e., $(3, 4, 5)$

Angle between two lines is given by

$$\cos\theta = \frac{[(-5 \times 3) + (0 \times 4) + (10 \times 5)]}{\sqrt{25 + 0 + 100}\sqrt{9 + 16 + 25}} = \frac{35}{25\sqrt{10}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$$

Question 36

A random variable X has the probability distribution

$X = x$	1	2	3	4	5	6	7	8
$P(X = x)$	0.15	0.23	0.12	0.20	0.08	0.10	0.05	0.07

For the events $E = \{X$ is a prime number $\}$ and $F = \{x < 5\}, P(EUF)$ is

Options:

A. 0.63

B. 0.75

C. 0.83

D. 0.90

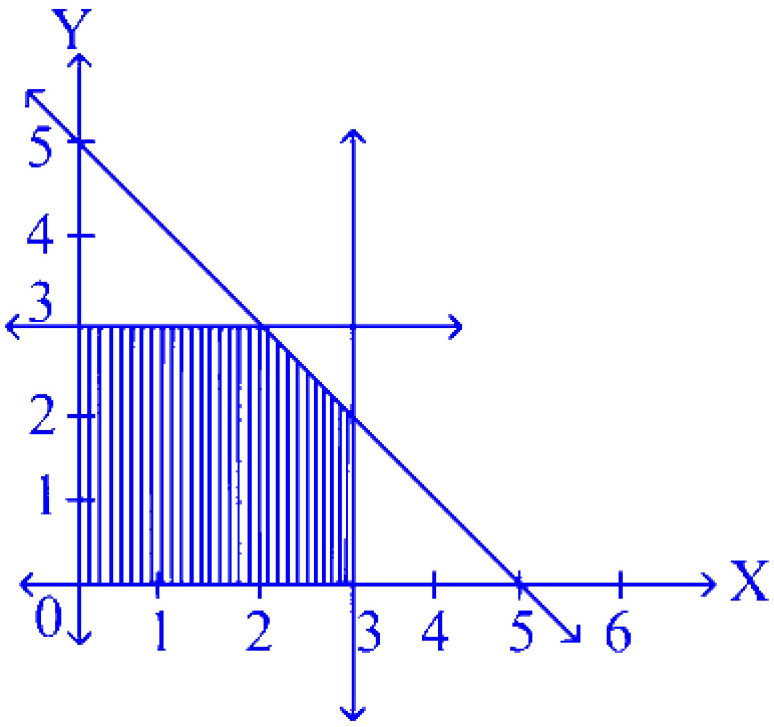
Answer: C

Solution:

$E = \{x \text{ is a prime number }\}$
$P(E) = P(2) + P(3) + P(5) + P(7)$
$= 0.23 + 0.12 + 0.08 + 0.05 = 0.48$
$F = \{x < 5\}$
$P(F) = P(1) + P(2) + P(3) + P(4)$
$= 0.15 + 0.23 + 0.12 + 0.20 = 0.70$
$P(E \cap F) = P(2) + P(3) = 0.35$
$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= 0.48 + 0.70 - 0.35 = 0.48 + 0.35 = 0.83$

Question 37

The shaded area in the given figure is a solution set for some system of inequations. The maximum value of the function $z = 10x + 25y$ subject to the linear constraints given by the system is



Options:

A. 80

B. 100

C. 95

D. 105

Answer: C

Solution:

We have, $z = 10x + 25y$

The shaded area in the given figure is a solution system set for some system of inequations.

The constraints are $x \leq 3, y \leq 3, x + y \leq 5, x \geq 0$ and $y \geq 0$.

The vertices of the feasible region are $(0, 0), (0, 3), (2, 3), (3, 2)$ and $(3, 0)$

At $(0, 0)$, $\quad z = 10 \times 0 + 25 \times 0 = 0$

At $A(0, 3)$, $\quad z = 10(0) + 25(3) = 75$

At $B(2, 3)$, $\quad z = 10(2) + 25(3) = 95$

At $C(3, 2)$, $\quad z = 10(3) + 25(2) = 80$

At $D(3, 0)$, $\quad z = 10(3) + 25(0) = 30$

$\therefore$ maximum value of $z = 95$, where $x = 2$ and $y = 3$.

Question 38

The foot of the perpendicular from the point $(1, 2, 3)$ on the line $\mathbf{r} = (6\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) + \lambda(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ has the coordinates

Options:

A. $(3, 5, 9)$

B. $(5, -3, 9)$

C. $(3, -5, -9)$

D. $(5, -9, 3)$

Answer: A

Solution:

The given point is $P(1, 2, 3)$.

and given line is $\mathbf{r} = (6\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) + \lambda(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$\Rightarrow \quad \frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = k$ (say) ..... (i)

So, any point on this line is given as $Q(3k + 6, 2k + 7, -2k + 7)$

Now, direction ratios of PQ are $3k + 5, 2k + 5, -2k + 4$

Also, $PQ \perp \mathbf{r}$

$\Rightarrow \quad 3(3k + 5) + 2(2k + 5) - 2(-2k + 4) = 0$
$\Rightarrow \quad 17k + 17 = 0 \Rightarrow k = -1$

Hence, foot of the perpendicular drawn on the given line is $(3, 5, 9)$.

Question 39

Water is running in a hemispherical bowl of radius $180\ \text{cm}$ at the rate of 108 cubic decimeters per minute. How fast the water level is rising when depth of the water level in the bowl is $120\ \text{cm}$? (1 decimeter $= 10\ \text{m}$)

Options:

A. $16\pi \text{cm/sec}$

B. $\frac{16}{\pi} \text{cm/sec}$

C. $\frac{1}{16\pi} \text{cm/sec}$

D. $\frac{\pi}{16}\ \text{cm/sec}$

Answer: C

Solution:

We know that

Volume of hemispherical bowl $= \frac{\pi}{3}\left(3rh^2 - h^3\right)$

where r is the radius of the bowl, h is the height of the bowl.

$$\mathbf{V} = \frac{\pi}{3}\left(3rh^2 - h^3\right)$$
$$\frac{dV}{dt} = \frac{d}{dt}\left\{\frac{\pi}{3}\left(3rh^2 - h^3\right)\right\}$$
$$\frac{dV}{dt} = \frac{\pi}{3}\left[\frac{d}{dt}\left(3rh^2\right) - \frac{d}{dt}\left(h^3\right)\right]$$
$$\frac{dV}{dt} = \frac{\pi}{3}\left(6rh\frac{dh}{dt} - 3h^2\frac{dh}{dt}\right)$$
$$\frac{dV}{dt} = \frac{\pi}{3}\left(6rh - 3h^2\right)\frac{dh}{dt}$$

$V = 108$ cubic decimeter per minute

$V = 108000$ cubic centimeter per minute

$\therefore \quad \frac{dh}{dt} = \frac{108000}{\frac{\pi}{3}(6rh - 3h^2)}$

$$\frac{dh}{dt} = \frac{108000}{28800\pi}\left[\because \frac{\pi}{3}\left(6rh - 3h^2\right) = 28800\pi\right]$$
$$= \frac{180}{48\pi}\text{cm/min}$$
$$= \frac{180}{48\pi} \times \frac{1}{60}\text{ cm/sec}$$
$$= \frac{1}{16\pi}\text{cm per sec}$$

Question 40

The solution of $\sin x + \sin 5x = \sin 3x$ in $(0, \pi/2)$ are

Options:

A. $\frac{\pi}{4}, \frac{\pi}{10}$

B. $\frac{\pi}{6}, \frac{\pi}{3}$

C. $\frac{\pi}{4}, \frac{\pi}{12}$

D. $\frac{\pi}{8}, \frac{\pi}{16}$

Answer: B

Solution:

Here, $\sin x + \sin 5x = \sin 3x$

We know that,

$$\sin A + \sin B = 2 \sin \frac{(A+B)}{2} \cos \frac{(A-B)}{2}$$

$$\therefore \quad \sin x + \sin 5x = \sin 3x$$

$$\Rightarrow \quad 2 \sin \left(\frac{5x + x}{2} \right) \cos \left(\frac{5x - x}{2} \right) = \sin 3x$$

$$\Rightarrow \quad 2 \sin 3x \cos 2x = \sin 3x$$

$\therefore$ Either, $\sin 3x = 0$ or $2 \cos 2x - 1 = 0$

$\sin 3x = \sin 0$ or $\cos 2x = \frac{1}{2} = \cos \frac{\pi}{3}$

Comparing obtained equation with standard equation, we have

$$3x = n\pi \text{ or } 2x = 2m\pi \pm \frac{\pi}{3}$$

$$\therefore \quad x = \frac{n\pi}{3} \text{ or } x = m\pi + \frac{\pi}{6}, \text{ where } m, n \in Z$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}$$

Question 41

If $(1 + \sqrt{1+x}) \tan x = 1 + \sqrt{1-x}$, then $\sin 4x$ is

Options:

A. x

B. $-x$

C. $4x$

D. $-4x$

Answer: A

Solution:

$\because \tan x = \frac{1+\sqrt{1-x}}{1+\sqrt{1+x}}$

Let $x = \cos\theta$

Then, $\sqrt{1-\cos\theta} = \sqrt{2}\sin\frac{\theta}{2}$

$\sqrt{1+x} = \sqrt{2}\cos\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad \tan x = \frac{\sqrt{2}\left|\frac{1}{\sqrt{2}}+\sin\frac{\theta}{2}\right|}{\sqrt{2}\left|\frac{1}{\sqrt{2}}+\cos\frac{\theta}{2}\right|} = \frac{\sin\frac{\pi}{4}+\sin\frac{\theta}{2}}{\cos\frac{\pi}{4}+\cos\frac{\theta}{2}}$

$\Rightarrow \quad \tan x = \tan\left(\frac{\pi}{8}+\frac{\theta}{4}\right)$
$\Rightarrow \quad 4x = \frac{\pi}{2}+\theta$
$\Rightarrow \quad \sin 4x = \cos\theta = x$

Question 42

If the sum of the mean and the variance of a binomial distribution for 5 trials is 1.8 , then the value of p is

Options:

A. 0.4

B. 0.8

C. 0.18

D. 0.2

Answer: D

Solution:

Mean $= np$
Variance $= np(1-p)$
So, $np + np(1-p) = 1.8$
$5p + 5p(1-p) = 1.8$
$\Rightarrow \quad 10p - 5p^2 - 1.8 = 0$
$\Rightarrow \quad 5p^2 - 10p + 1.8 = 0$
$\Rightarrow \quad 5p^2 - 9p - p + 1.8 = 0$
$\Rightarrow \quad 5p(p-1.8) - 1(p-1.8) = 0$
$\Rightarrow \quad (p-1.8)(5p-1) = 0$
$\therefore \quad p = 1.8, 0.2$
$\because \quad p = 1.8$

(It is not possible because it is not greater than 1)

Hence, p = 0.2

Question 43

If $I = \int \frac{e^x}{e^{4x}+e^{2x}+1}dx$ and $J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$, then for any arbitrary constant C, than the value of $J - I$ equals

Options:

A. $\frac{1}{2}\log\left|\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)\right| + C$

B. $\frac{1}{2}\log\left|\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)\right| + C$

C. $\frac{1}{2}\log\left|\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)\right| + C$

D. $\frac{1}{2}\log\left|\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)\right| + C$

Answer: C

Solution:

$$\because I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx \text{ and}$$

$$J = \int \frac{e^{3x}}{1 + e^{2x} + e^{4x}} dx$$

$$\therefore \quad J - I = \int \frac{e^{3x} - e^x}{1 + e^{2x} + e^{4x}} dx$$

By substituting $e^x = u \Rightarrow e^x dx = du$

$$\therefore \quad J - I = \int \frac{u^2 - 1}{1 + u^2 + u^4} du = \int \frac{1 - \frac{1}{u^2}}{1 + \frac{1}{u^2} + u^2} du$$

By substituting $u + \frac{1}{u} = t$, we get

$$\left(1 - \frac{1}{u^2}\right) du = dt$$

$$J - I = \int \frac{dt}{t^2 - 1}$$

$$= \frac{1}{2} \log \left| \frac{t - 1}{t + 1} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{u^2 - u + 1}{u^2 + u + 1} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + C$$

Question 44

The principal value of $\sin^{-1}(\sin(3\pi/4))$ is

Options:

A. $\pi/4$

B. $-\pi/4$

C. $3\pi/4$

D. $5\pi/4$

Answer: A

Solution:

$$\sin^{-1}\left(\sin\frac{3\pi}{4}\right)=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$=\sin^{-1}\left(\sin\frac{\pi}{4}\right)=\frac{\pi}{4}\quad\left[\because\frac{-\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}\right]$$

$\therefore$ Principle value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$ is $\frac{\pi}{4}$.

Question 45

If $x=\log_e\left(\frac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}\right),\tan\frac{y}{2}=\sqrt{\frac{1-t}{1+t}}$ Then, $(y_1)_{t=1/2}$ has the value

Options:

A. $1/2$

B. $-1/2$

C. $1/4$

D. $-1/4$

Answer: B

Solution:

$$x=\log_e\left(\frac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}\right)$$
$$=\log_e\left(\frac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}\right)$$

[On dividing $\cos y/2$ in both numerator and denominator]

$$=\log_e\left(\frac{1-\sqrt{\frac{1-t}{1+t}}}{1+\sqrt{\frac{1-t}{1+t}}}\right)=\log_e\left(\frac{\sqrt{1+t}-\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}\right)$$
$$=\log_e\left(\frac{1+t+1-t-2\sqrt{1-t^2}}{1+t-1+t}\right)$$

$x=\log_e\left(\frac{1-\sqrt{1-t^2}}{t}\right)$

$$\frac{dx}{dt} = \frac{t}{1-\sqrt{1-t^2}}\left[\frac{\frac{t^2}{\sqrt{1-t^2}} - \left(1-\sqrt{1-t^2}\right)}{t^2}\right]$$

$$\frac{dx}{dt} = \frac{t}{1-\sqrt{1-t^2}}\left[\frac{t^2-\sqrt{1-t^2}+1-t^2}{\sqrt{1-t^2}t^2}\right]$$

$$\frac{dx}{dt} = \frac{1}{t\sqrt{1-t^2}} \quad \text{..... (i)}$$

$$\because \quad \tan\frac{y}{2} = \sqrt{\frac{1-t}{1+t}} \Rightarrow \frac{y}{2} = \tan^{-1}\left(\sqrt{\frac{1-t}{1+t}}\right)$$

$$\Rightarrow \quad \frac{1}{2}\frac{dy}{dt} = \left(\frac{t+1}{2}\right)\left(\frac{\sqrt{1+t}}{2\sqrt{1-t}}\right)\left(\frac{-1-t-1+t}{(1+t)^2}\right)$$

$$\Rightarrow \quad \frac{1}{2}\frac{dy}{dt} = \frac{-1}{2\sqrt{1-t^2}}$$

$$\Rightarrow \quad \frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}} \quad \text{..... (ii)}$$

$\frac{dy}{dx}\Big|_{t=\frac{1}{2}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-1}{\sqrt{1-t^2}} \times \frac{t\sqrt{1-t^2}}{1} = -\frac{1}{2}$

Question 46

The c.d.f. $F(x)$ associated with p.d.f. $f(x)$

$$f(x) = \begin{cases} 12x^2(1-x), & \text{if } 0 < x < 1 \\ 0; & \text{otherwise} \end{cases} \textbf{ is}$$

Options:

A. $F(x) = 4x^3 + 3x^4$

B. $F(x) = 4x^3 - 3x^4$

C. $F(x) = -4x^3 - 3x^4$

D. $F(x) = -4x^3 + 3x^4$

Answer: B

Solution:

We have, $f(x) = \begin{cases} 12x^2(1-x), & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$F(x) = \int\limits_{-\infty}^{x} f(x)dx = \int\limits_{-\infty}^{0} 0 \cdot dx + \int\limits_{0}^{x} f(x)dx = \int\limits_{0}^{x} \left(12x^2(1-x)\right)dx$

$F(x) = 4x^3 - 3x^4$

Question 47

If $f(x)$ is continuous on its domain $[-2, 2]$, where

$$f(x) = \begin{cases} \frac{\sin ax}{x} + 3 & , \text{ for } -2 \leq x < 0 \\ 2x + 7 & , \text{ for } 0 \leq x \leq 1 \\ \sqrt{x^2 + 8} - b & , \text{ for } 1 < x \leq 2 \end{cases}$$

then the value of $2a + 3b$ is

Options:

A. -12

B. -10

C. 10

D. 12

Answer: B

Solution:

We have, $f(x) = \begin{cases} \frac{\sin ax}{x} + 3, & \text{for } -2 \leq x < 0 \\ 2x + 7, & \text{for } 0 \leq x \leq 1 \\ \sqrt{x^2 + 8} - b, & \text{for } 1 < x \leq 2 \end{cases}$

$\because f(x)$ is continuous on its domain $[-2, 2]$.

So it must be continuous at $x = 0$ and $x = 1$

$\therefore \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$ and $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

$\lim_{x\to 0^-} \frac{\sin ax}{x} + 3 = \lim_{x\to 0^+} 2x + 7$
$a + 3 = 7 \Rightarrow a = 4$

Now, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\lim_{x\to 1^-} 2x + 7 = \lim_{x\to 1^+} \sqrt{x^2+8} - b$

$$2 + 7 = \sqrt{1+8} - b$$
$$\sqrt{9} - b = 9 \Rightarrow b = -6$$

Hence, $2a + 3b = 2 \times 4 + 3(-6) = -10$

Question 48

PS is the median of the triangle with vertices at $P(2, 2), Q(6, -1)$ and $R(7, 3)$, then the intercepts on the coordinate axes of the line passing through point $(1, -1)$ and parallel to PS are respectively

Options:

A. $\frac{7}{2}, \frac{-7}{9}$

B. $\frac{2}{7}, \frac{9}{7}$

C. $\frac{-7}{2}, \frac{-7}{9}$

D. $-2, -9$

Answer: C

Solution:

Since, S is mid-point of Q and R.

$\therefore \quad S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Slope of $PS = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$

$\therefore$ Equation of line passing through point $(1, -1)$ and parallel to PS is

$y+1=-\frac{2}{9}(x-1)$

$9y+9=-2x+2 \Rightarrow 2x+9y=-7$

$\Rightarrow \quad \frac{x}{(\frac{-7}{2})}+\frac{y}{(\frac{-7}{9})}=1$

Hence, x-intercept $=-\frac{7}{2}$ and y-intercept $=-\frac{7}{9}$

Question 49

If Rolle's theorem holds for the function $f(x)=x^3+bx^2+ax+5$ on $[1,3]$ with $c=2+\frac{1}{\sqrt{3}}$, then the values of a and b respectively are

Options:

A. $-11,-6$

B. $11,6$

C. $11,-6$

D. $6,11$

Answer: C

Solution:

We have, $f(x)=x^3+bx^2+ax+5$

If Rolle's theorem holds for $f(x)$ on $[1,3]$ with $c=2+\frac{1}{\sqrt{3}}$, then

$f'(c)=0$ and $f(1)=f(3)$
$f'(x)=3x^2+2bx+a \quad$ ..... (i)

$\because f'(x)$ is quadratic and one root of $f'(x)$ is $\left(2+\frac{1}{\sqrt{3}}\right)$, then other root must be equal to $\left(2-\frac{1}{\sqrt{3}}\right)$.

So, from Eq. (i),

Sum of roots $= \frac{-2b}{3} = 2 + \frac{1}{\sqrt{3}} + 2 - \frac{1}{\sqrt{3}}$

$\Rightarrow \quad -2b = 12 \Rightarrow b = -6$

and product of roots $= \frac{a}{3} = \left(2 + \frac{1}{\sqrt{3}}\right)\left(2 - \frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad \frac{a}{3} = 4 - \frac{1}{3} = \frac{11}{3} \Rightarrow a = 11$

Hence, $a = 11$ and $b = -6$

Question 50

The distance of the point $(1, 6, 2)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$ is

Options:

A. 11 units

B. 12 units

C. 13 units

D. 14 units

Answer: C

Solution:

We have, equation of line.

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and plane : $x - y + z = 16$

any point on line is $(3t + 2, 4t - 1, 12t + 2)$ and this point will satisfy the plane.

$\therefore 3t + 2 - 4t + 1 + 12t + 2 = 16 \Rightarrow 11t = 11 \Rightarrow t = 1$

So, point will be $(5, 3, 14)$

Hence, distance between $(5, 3, 14)$ and $(1, 6, 2)$ is

$= \sqrt{(1-5)^2 + (6-3)^2 + (2-14)^2}$

$= \sqrt{16 + 9 + 144} = \sqrt{169} = 13$ units

Chemistry

Question 51

What is the pH of solution containing $4.62 \times 10^{-4} \mathrm{MH}^{+}$ ions?

Options:

A. 8.62

B. 4.64

C. 5.66

D. 3.34

Answer: D

Solution:

$\left[\mathrm{H}^{+}\right]=4.62 \times 10^{-4} \mathrm{M}$

$\mathrm{pH}=\ ?$

Using, $\mathrm{pH}=-\log \left[\mathrm{H}^{+}\right]=-\log \left[4.62 \times 10^{-4}\right]$

$=-\log [4.62]-\log \left[10^{-4}\right]=-0.66+4$

$=3.34$

Question 52

Which of the following is vinylic alcohol?

Options:

A. Prop-2-en-1-ol

B. But-2-en-2-ol

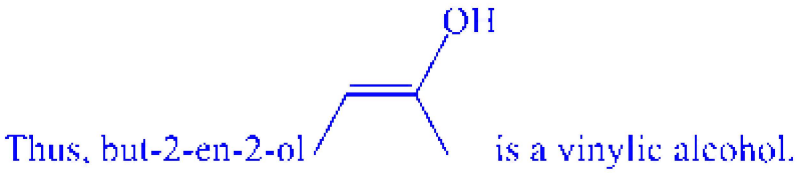
C. But-3-en-2-ol

D. 2-methylbut-3-en-2-ol

Answer: B

Solution:

A compound where the hydroxy group is bonded to a sp^2-hybridised carbon-carbon double bond ($\mathrm{C}=\mathrm{C}$) is called vinylic alcohol. They contains vinyl group i.e. $\mathrm{H}-\mathrm{C}=\mathrm{CH}_2$.



Question 53

Which among the following is benzylic halide?

Options:

A. $C_6H_5 - CH_2 - X$

B. $C_6H_5 - X$

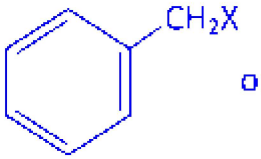
C. $C_6H_5 - CH = CH - X$

D. $C_6H_5 - CH_2 - CH_2 - X$

Answer: A

Solution:

Benzylic halides are the compounds in which the halogen atom is bonded to an sp^3-hybridised carbon atom next to an aromatic ring.



or $C_6H_5 - CH_2 - X$ is a benzylic halide.

Question 54

Which among the following is a simple ketone?

Options:

A. Acetophenone

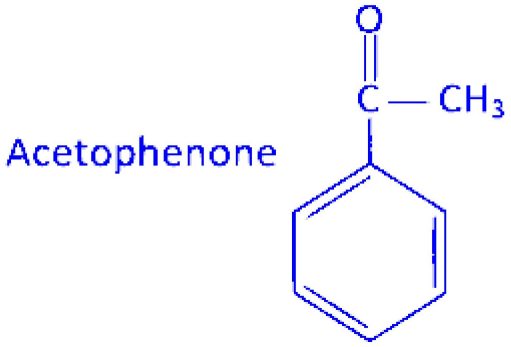
B. Butanone

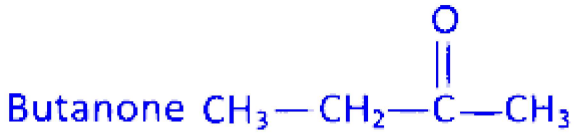
C. Benzophenone

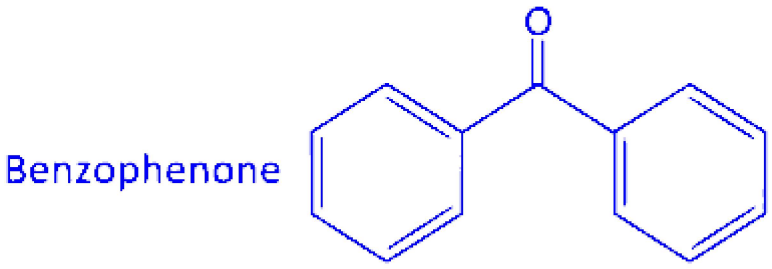
D. Pentan-2-one

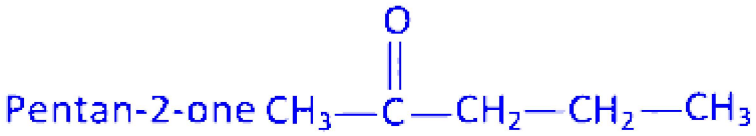
Answer: C

Solution:









Simple ketones are the ones in which both alkyl and aryl groups linked to carbonyl carbon are identical. Thus, among the given options benzophenone is a simple ketone.

Question 55

Identify glycosidic linkage present in maltose.

Options:

A. $\beta - 1,4$

B. $\alpha - 1,6$

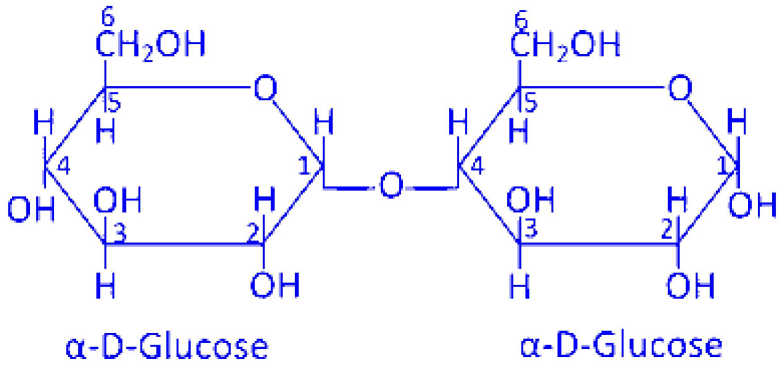
C. $\beta - 1,6$

D. $\alpha - 1,4$

Answer: D

Solution:

Maltose is composed of two $\alpha - D$-glucose units in which C_1 of one glucose is linked to C_4 of another glucose unit.



Question 56

Which statement from following about nano-material is NOT correct?

Options:

A. As the particle size decreases the total surface area of particles increases.

B. Nano-material based catalysts exhibit increased catalytic activities.

C. Nano-sized Cu and Pd clusters have very less hardness than bulk metal.

D. Carbon nano-tube can act as electrical conductor.

Answer: C

Solution:

The nano-sized Cu and Pd clusters have more hardness than bulk metal. Thus, statement given in option (c) is not true about nano-material.

Question 57

Which from following species is NOT a monodentate ligand?

Options:

A. CN^-

B. $H_2N(CH_2)_2NH_2$

C. OH^-

D. Cl^-

Answer: B

Solution:

Among the given options $H_2N(CH_2)_2NH_2$ is not a monodentate ligand. Instead it is a bidentate ligand.

Question 58

Two moles of an ideal gas expand freely and isothermally from $5\ dm^3$ to $50\ dm^3$. What is the value of ΔH ?

Options:

A. Zero kJ

B. 20 kJ

C. 100 kJ

D. 150 kJ

Answer: A

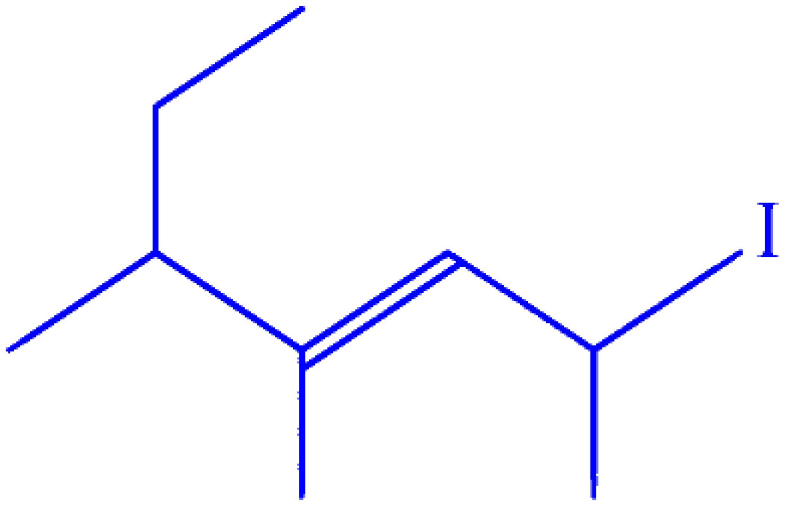
Solution:

For isothermal expansion of an ideal gas, $p_{ext} = 0$. Therefore, $w = 0$. More over, enthalpy is a function of only temperature.

Thus, in an isothermal process involving only ideal gases, the value of $\Delta H = 0$.

Question 59

What is IUPAC name of following compound?



Options:

A. 1-iodo-1,3,4-trimethylhex-2-ene

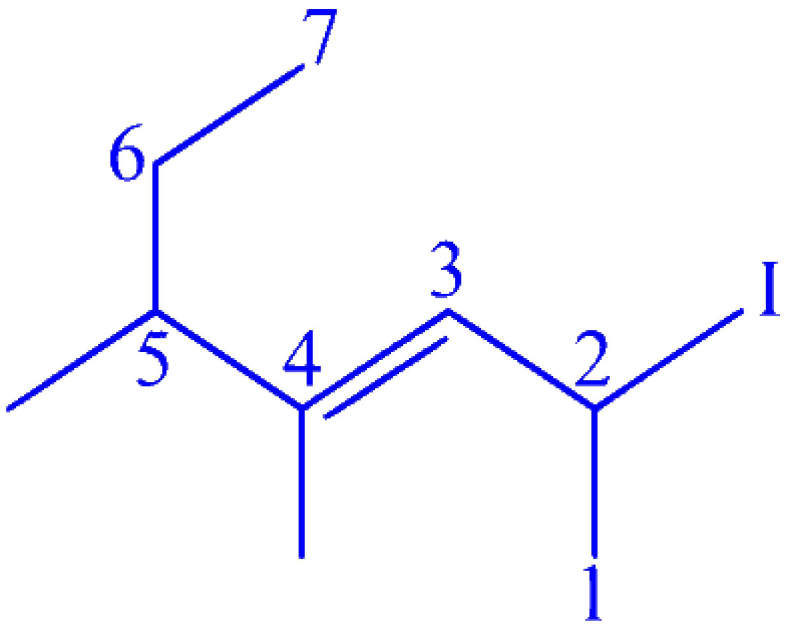
B. 2-iodo-4,5-dimethylhept-3-ene

C. 2-iodo-5-ethyl-4-methylhex-3-ene

D. 1-iodo-4-ethyl-1,3-dimethylpent-2-ene

Answer: B

Solution:



The IUPAC name of the given compound is 2-iodo-4,5-dimethyl hept-3-ene.

Question 60

What is the molecular formula of an alkane if it exhibits three structural isomers?

Options:

A. C_3H_8

B. C_4H_{10}

C. C_5H_{12}

D. C_6H_{14}

Answer: C

Solution:

Among the given options, pentane with molecular formula C_5H_{12} has three structural isomers. These are *n*-pentane, iso-pentane and neo-pentane.

Question 61

What is the bond order in N_2^+?

Options:

A. 0

B. 1

C. 2

D. 2.5

Answer: D

Solution:

Total number of electrons in N_2^+ is 13.

So, its electronic configuration is

$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \pi 2p_x^2 = \pi 2p_y^2 < \sigma 2p_z^1$

So, BO $= \frac{N_b - N_a}{2} = \frac{9-4}{2} = 2.5$

Question 62

Identify alkaline earth metal from following.

Options:

A. Rb

B. Sr

C. Fr

D. Cs

Answer: B

Solution:

Group 2 elements are commonly called as alkaline earth metals. Among the given options only strontium (Sr) belongs to group 2. So, Sr is an alkaline earth metal.

Question 63

What is the molality of solution of a non-volatile solute having boiling point elevation $7.15\ \text{K}$ and molal elevation constant $2.75\ \text{K kg mol}^{-1}$?

Options:

A. 3.2 m

B. 2.0 m

C. 2.6 m

D. 3.8 m

Answer: C

Solution:

ΔT_b =7.15 K

$K_b = 2.75\ \text{K kg mol}^{-1}$

m =?

Using, $\Delta T_b = K_b m$

$\Rightarrow m = \frac{\Delta T_b}{K_b} = \frac{7.15}{2.75} = 2.6\ \text{m} \Rightarrow m = 2.6$ molal

---

Question 64

Which from following statements about rate constant is NOT true?

Options:

A. It is independent of concentration.

B. It varies with temperature.

C. It is equal to rate of reaction at unit concentration of reactants.

D. It's unit is independent of order of reaction.

Answer: D

Solution:

Incorrect statement about rate constant is that it's unit is independent of order of reaction. It is because the unit of rate constant depends on the order of reaction. For nth order reaction, unit of rate constant is given by $\frac{\left[\text{mol L}^{-1}\ \text{s}^{-1}\right]}{\left[\text{mol L}^{-1}\right]^{n}}$.

Question 65

Identify 'B' in the following conversions

$$\text{CH}_3\text{Br} \xrightarrow{\text{KCN}} A \xrightarrow{\text{Na/C}_2\text{H}_5\text{OH}} B$$

Options:

A. CH_3CN

B. $\text{CH}_3\text{OC}_2\text{H}_5$

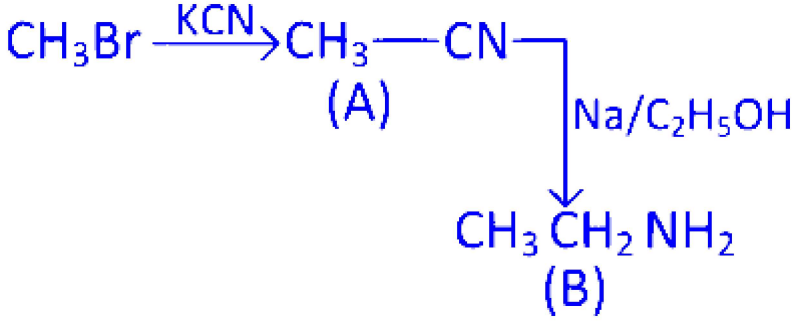
C. CH_3ONa

D. $\text{CH}_3\text{CH}_2\text{NH}_2$

Answer: D

Solution:

The complete reaction sequence is as follows :



Thus, the compound B is $CH_3CH_2NH_2$.

Question 66

Find the radius of metal atom in bcc unit cell having edge length 450 pm.

Options:

A. 225.04 pm

B. 194.85 pm

C. 159.08 pm

D. 90.05 pm

Answer: B

Solution:

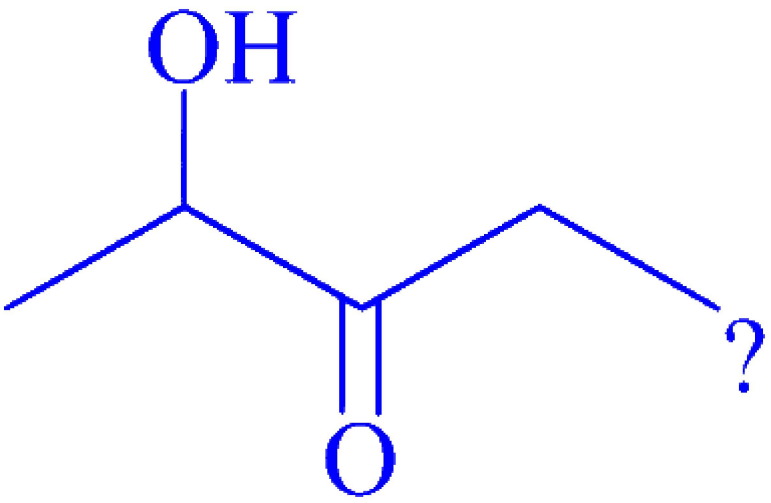
Given, $a = 450$ pm

For bcc unit cell, radius is given by

$r = \frac{\sqrt{3}a}{4} = \frac{\sqrt{3}}{4} \times 450 \text{ pm} \Rightarrow r = 194.85 \text{ pm}$

Question 67

Which from following statements is true for the molecule



Options:

A. $-OH$ is considered as principal functional group.

B. Molecular formula of this compound is $C_4H_9O_2$.

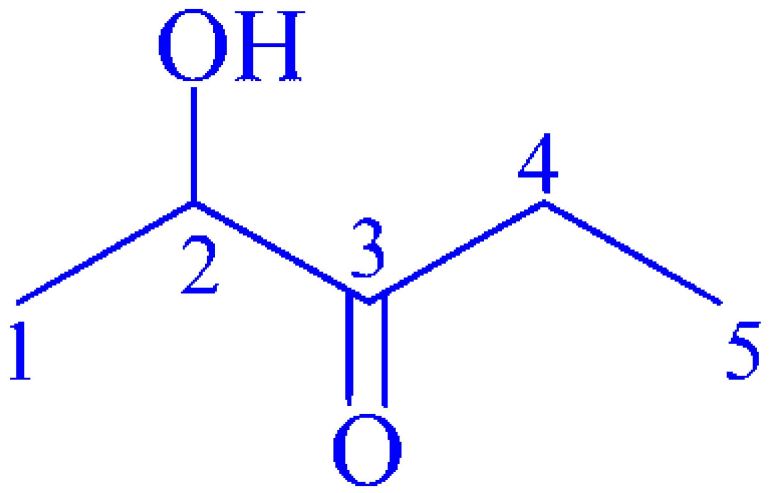
C. $-OH$ is attached with sp^2 hybrid carbon.

D. IUPAC name of this compound is 2-hydroxypentane-3-one.

Answer: D

Solution:

The correct statement for given compound



is that it's IUPAC name, is 2-hydroxy pentan-3-one.

The correct form of remaining incorrect statements are as follows :

Carbonyl group is consider as principal functional group.

Molecular formula of this compound is $C_5H_9O_2$. $-OH$ is attached with sp^3 hybrid carbon.

Question 68

Which among the following salt solution in water is acidic in nature?

Options:

A. $CuCl_2$

B. NH_4CN

C. KCN

D. CH_3COONa

Answer: A

Solution:

$CuCl_2$ is an acidic in water as Cl^- do not hydrolyse. Since, it is an anion of strong acid, HCl. Cu^{2+} will hydrolyse to form $Cu(OH)^-$.

$Cu^{2+} + H_2O(aq) \rightleftharpoons Cu(OH)^-(aq) + H^+(aq)$ which is an acidic solution. So, the solution of $CuCl_2$ in water is acidic.

Question 69

Which among the following is intensive and extensive properties respectively?

Options:

A. Boiling point and heat capacity

B. Heat capacity and boiling point

C. Internal energy and pressure

D. Heat capacity and surface tension

Answer: A

Solution:

An intensive property is a property of a system that does not depend on the size and amount of matter, for example boiling point. While extensive property depends on the amount of matter in a system, for example heat capacity.

Thus option (a) is the correct answer.

Question 70

Which of the following compounds has lowest boiling point?

Options:

A. Chloromethane

B. Fluoromethane

C. Iodomethane

D. Bromomethane

Answer: B

Solution:

Generally, boiling points increase with increasing molecular mass due to greater magnitude of van der Waals' force of attraction. Thus, fluoromethane with least molecular mass will have lowest boiling point.

Question 71

Which of the following statements is NOT true about polymorphism?

Options:

A. The existence of substance in more than one crystalline form.

B. Polymorphism occuring in element is called allotropy.

C. Polymorphic forms of a substance are formed under different conditions.

D. The crystal shape of polymorphic substances is identical to each other.

Answer: D

Solution:

The concept of polymorphism in chemistry refers to the ability of a substance to exist in more than one form or crystal structure. Allotropy is a specific type of polymorphism that occurs in elements rather than compounds. Polymorphic forms emerge under varying conditions such as temperature and pressure, and each form is distinct in its intermolecular arrangement. However, polymorphic substances do not have an identical crystal shape, which is what distinguishes the different forms. Therefore, the statement that 'The crystal shape of polymorphic substances is identical to each other' is not true about polymorphism.

Question 72

Which from the following expression represents molar conductivity of an electrolyte A_2B_3 type?

Options:

A. $2\lambda^0_{A^{+++}} + 3\lambda^0_{B^{--}}$

B. $3\lambda^0_{A^{+++}} + 2\lambda^0_{B^{--}}$

C. $2\lambda^\circ_{A^{+++}} + \lambda^\circ_{B^{--}}$

D. $\lambda^\circ_{A^{+++}} + 3\lambda^\circ_{B^{--}}$

Answer: A

Solution:

For A_2B_3 type electrolyte molar conductivity is given by Kohlrausch's law as.

$\Lambda^\circ_{\mathrm{m}}\left(A_2B_3\right) = 2\lambda^\circ_{A^{+++}} + 3\lambda^\circ_{B^{--}}$

Question 73

Which of the following electromagnetic radiations possesses lowest energy?

Options:

A. Radio waves

B. Microwaves

C. Infrared

D. Ultraviolet

Answer: A

Solution:

In electromagnetic radiations, radio waves have longest wavelength and least frequency. So by $E = hv$ equation, the radiation with least frequency i.e. radio waves will have lowest energy.

Question 74

Which among the following is an example of branched chain polymer?

Options:

A. High density polythene

B. Low density polythene

C. Bakelite

D. Melamine

Answer: B

Solution:

Among the given options, low density polythene (LDP) is an example of branched chain polymer. It is obtained by the polymerisation of ethene under high pressure of 1000 to 2000 atm at a temperature of 350 to 570 K in presence of traces dioxygen or peroxide catalyst.

Question 75

For the reaction,

$$\mathrm{CH_3Br}(aq) + \mathrm{OH^-}(aq) \longrightarrow \mathrm{CH_3OH}(aq) + \mathrm{Br^-}(aq),$$

The rate law is rate $= k\left[\mathrm{CH_3Br}\right]\left[\mathrm{OH^-}\right]$. What is change in rate of reaction if concentration of both reactants is doubled?

Options:

A. Rate increases by factor 2

B. Rate increases by factor 4

C. Rate remains same

D. Rate decreases by factor 2

Answer: B

Solution:

Given, $\mathrm{CH_3Br}(aq) + \mathrm{OH^-}(aq) \longrightarrow \mathrm{CH_3OH}(aq) + \mathrm{Br^-}(aq)$

Rate $= k\left[2\mathrm{CH_3Br}\right]\left[2\mathrm{OH^-}\right]$

When the concentration of both reactants are doubled, then new rate will be $= k'\left[2\mathrm{CH_3Br}\right]\left[2\mathrm{OH^-}\right]$

Thus, the rate increases by factor 4.

Question 76

For the reaction, $2A + 2B \longrightarrow 2C + D$, the rate law is expressed as rate $= k[A]^2[B]$. Calculate the rate constant if rate of reaction is $0.24\ \mathrm{mol\ dm^{-3}\ s^{-1}}$.

[[A]$= 0.5\mathrm{M}$ and $[B] = 0.2\mathrm{M}$]

Options:

A. $4.8\ \text{mol}^{-2}\ \text{dm}^{6}\ \text{s}^{-1}$

B. $9.6\ \text{mol}^{-2}\ \text{dm}^{6}\ \text{s}^{-1}$

C. $12.1\ \text{mol}^{-2}\ \text{dm}^{6}\ \text{s}^{-1}$

D. $14.4\ \text{mol}^{-2}\ \text{dm}^{6}\ \text{s}^{-1}$

Answer: A

Solution:

Given,

$2A + 2B \longrightarrow 2C + D$

$\text{Rate } = k[A]^2[B]$ ..... (i)

$[A] = 0.5\text{M}, [B] = 0.2\text{M}$

Rate of reaction $= 0.24\ \text{mol dm}^{-3}\ \text{s}^{-1}$

On substituting the given values in Eq. (i)

$0.24 = k[0.5]^2[0.2] \Rightarrow k = 4.8\ \text{mol}^{-2}\ \text{dm}^{6}\ \text{s}^{-1}$

Question 77

What is the molar conductivity of $0.005\ \text{M NaI}$ solution if it's conductivity is $6.065 \times 10^{-4} \Omega^{-1}\ \text{cm}^{-1}$?

Options:

A. $121.3 \Omega^{-1}\ \text{cm}^{2}\ \text{mol}^{-1}$

B. $115.1 \Omega^{-1}\ \text{cm}^{2}\ \text{mol}^{-1}$

C. $126.5 \Omega^{-1}\ \text{cm}^{2}\ \text{mol}^{-1}$

D. $131.2 \Omega^{-1}\ \text{cm}^{2}\ \text{mol}^{-1}$

Answer: A

Solution:

The molar conductivity, often denoted as Λ_m, can be calculated by the formula:

$\Lambda_m = \frac{\kappa}{C}$

where:

- κ represents the conductivity of the solution.
- C is the concentration of the solution in moles per liter.

In the given problem, the conductivity κ is provided as $6.065 \times 10^{-4} \Omega^{-1} cm^{-1}$, and the concentration C of the NaI solution is given as $0.005 M$.

For calculating molar conductivity, we must also ensure that the units are consistent, converting concentration to moles per cubic centimeter (since the conductivity unit involves cm) if necessary.

Here is how the calculation is performed:

$\Lambda_m = \frac{6.065 \times 10^{-4} \ \Omega^{-1} \text{cm}^{-1}}{0.005 \ mol \ L^{-1}} \times \frac{1000 \ \text{cm}^3}{1 \ \text{L}}$

After multiplying and dividing as required, the result is:

$\Lambda_m = 121.3 \Omega^{-1} \text{cm}^2 \text{mol}^{-1}$

The answer reflects the molar conductivity of the NaI solution.

Question 78

Which of the following is not a disaccharide?

Options:

A. Sucrose

B. Maltose

C. Lactose

D. Raffinose

Answer: D

Solution:

Among the given options, raffinose is not a disaccharide. It is because it is a trisaccharide, made up of three monosaccharides namely galactose, glucose and fructose.

Question 79

Which from following is a correct decreasing order of ionisation enthalpy for different elements?

Options:

A. $\mathrm{Ar} > \mathrm{Ne} > \mathrm{S} > \mathrm{Cl}$

B. $\mathrm{Ne} > \mathrm{Ar} > \mathrm{Cl} > \mathrm{S}$

C. $\mathrm{Ne} > \mathrm{S} > \mathrm{Cl} > \mathrm{Ar}$

D. $\mathrm{Cl} > \mathrm{S} > \mathrm{Ne} > \mathrm{Ar}$

Answer: B

Solution:

Ionisation enthalpy decreases on moving down the group while it increases along the period. So, the correct order of ionization enthalpy for the given elements is $\mathrm{Ne} > \mathrm{Ar} > \mathrm{Cl} > \mathrm{S}$.

Question 80

Which among the following salts exhibits inverse relation between it's solubility and temperature?

Options:

A. NaBr

B. $\mathrm{NaNO_3}$

C. $\mathrm{KNO_3}$

D. $\mathrm{Na_2SO_4}$

Answer: D

Solution:

The dissolution process for Na_2SO_4 is exothemic while for $NaBr, NaNO_3$ and KNO_3 is endothermic in nature. Therefore, solubility decrease with increase in temperature for Na_2SO_4 salt. Thus, it exhibits inverse relation between its solubility and temperature.

Question 81

Which from following series of elements is correctly arranged according to their decreasing order of ionisation enthalpy (IE_1) ?

Options:

A. $Zn > Fe > Cr > Sc$

B. $Cr > Fe > Zn > Sc$

C. $Sc > Fe > Cr > Zn$

D. $Cr > Zn > Sc > Fe$

Answer: A

Solution:

In general ionisation enthalpy increases along the period and decreases on moving down the group. So, the correct order for ionisation enthalpy is as follows :

$Zn > Fe > Cr > Sc$

Question 82

Identify the product formed by the action of $H_2(g)$ with $CO(g)$ in presence of Ni ?

Options:

A. Methane and water

B. Methyl alcohol

C. Acetaldehyde

D. Ethylene oxide

Answer: A

Solution:

The reaction of $H_2(g)$ with $CO(g)$ in presence of Ni is as follows :

$$\underset{\text{Hydrogen}}{3H_2(g)} + \underset{\substack{\text{Carbon}\\\text{monoxide}}}{CO(g)} \xrightarrow[\text{Methane}]{\text{Ni}} CH_4(g) + \underset{\text{Water}}{H_2O} \quad (g)$$

Thus, products formed are methane and water.

Question 83

Calculate E° cell for following.

$\mathrm{Zn}(s)\left|\mathrm{Zn}^{++}(1\mathrm{M})\right|\left|\mathrm{Pb}^{++}(1\mathrm{M})\right|\mathrm{Pb}(s)$ if $E^\circ_{\mathrm{Zn}} = -0.763\ \mathrm{V}$ and $E^\circ_{\mathrm{Pb}} = -0.126\ \mathrm{V}$

Options:

A. $0.637\ \mathrm{V}$

B. $-0.530\ \mathrm{V}$

C. $-0.889\ \mathrm{V}$

D. $0.789\ \mathrm{V}$

Answer: A

Solution:

For the given reaction,

$\mathrm{Zn}(s) \mid \mathrm{Zn}^{2+}(1\mathrm{M}) \| \mathrm{Pb}^{2+}(1\mathrm{M}) \mid \mathrm{Pb}(s)$
$E^{\circ}_{\mathrm{Zn}} = -0.763$ V and $E^{\circ}_{\mathrm{Pb}} = -0.126$ V
$E^{\circ}_{\text{cell}} = ?$

Using formula,

$$\begin{aligned} E^{\circ}_{\text{cell}} &= E_{\text{cathode}} - E_{\text{anode}} \\ &= -0.126 - (-0.763) \\ E^{\circ}_{\text{cell}} &= 0.637 \text{ V} \end{aligned}$$

Question 84

Which from following elements exhibits usual tendency to undergo reduction?

Options:

A. Mg

B. Ni

C. O

D. Cu

Answer: C

Solution:

Among the given species oxygen has six electrons in its outermost shell. So, to attain stable electronic configuration it easily gains electrons due to which its oxidation number decreases and it undergoes reduction.

Question 85

According to reaction,

$$\mathrm{Mg}(s) + 2\mathrm{HCl}(aq) \longrightarrow \mathrm{MgCl_2}(aq) + \mathrm{H_2}(g) \uparrow.$$

Calculate the mass of Mg required to liberate $4.48\ \mathrm{dm}^3$ $\mathrm{H_2}$ at STP?

(Molar mass of $\mathrm{Mg} = 24\ \mathrm{g\ mol^{-1}}$)

Options:

A. 12 g

B. 4.8 g

C. 6 g

D. 2.4 g

Answer: B

Solution:

For the given reaction

$\mathrm{Mg}(s) + 2\mathrm{HCl(aq)} \longrightarrow \mathrm{MgCl_2(aq)} + \mathrm{H_2}(aq) \uparrow$

Mass of $\mathrm{Mg} = ?$

Molar mass of $\mathrm{Mg} = 24\ \mathrm{g/mol}$ at STP

Using the formula of number of moles,

$$n = \frac{V}{22.4}$$

$$\frac{\text{Mass of Mg}}{\text{Molar mass of Mg}} = \frac{4.48}{22.4}$$

$$\frac{\text{Mass of Mg}}{24} = \frac{4.48}{22.4}$$

$$\Rightarrow \text{ Mass of Mg} = \frac{4.48 \times 24}{22.4} = 4.8\ \mathrm{g}$$

Question 86

A closed container contains mixture of non-reacting gases A and B. Partial pressure of A and B are 4.5 bar and 5.5 bar respectively. Find mole fractions of A and B respectively?

Options:

A. 0.035 and 0.065

B. 0.055 and 0.045

C. 0.45 and 0.55

D. 0.55 and 0.45

Answer: C

Solution:

From Dalton's law of particle pressure

$$\begin{aligned} p_A = \chi_A p_{\text{total}} \quad &\text{and } p_B = \chi_B p_{\text{total}} \\ p_{\text{total}} \quad &= p_A + p_B \\ &= 4.5 + 5.5 \text{ bar} = 10 \text{ bar} \end{aligned}$$

Substituting the values in above formula,

$$\begin{aligned} 4.5 &= \chi_A \times 10 \\ \chi_A &= 0.45 \\ 5.5 &= \chi_B \times 10 \\ \chi_B &= 0.55 \end{aligned}$$

So, the mole fraction of A and B are 0.45 and 0.55 respectively.

Question 87

Which from following compounds is obtained when toluene is treated with $\mathrm{CrO_2Cl_2}$ in presence of $\mathrm{CS_2}$ followed by acid hydrolysis?

Options:

A. Chlorobenzene

B. Benzal chloride

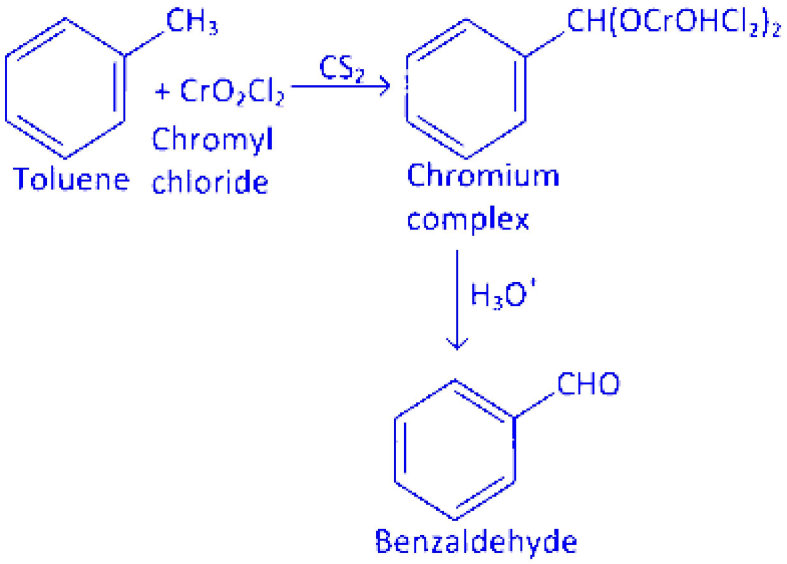
C. Benzaldehyde

D. Benzoic acid

Answer: C

Solution:

When chromyl chloride reacts with toluene, it oxidises methyl group to a chromium complex, which on hydrolysis gives corresponding benzaldehyde. This reaction is called Etard reaction.



Question 88

Which of the following is NOT prepared by the action of Grignard's reagent on methanal?

Options:

A. Ethanol

B. Propan-1-ol

C. Propan-2-ol

D. Butan-1-ol

Answer: C

Solution:

Reaction of methanal with Grignard reagent is as follows :

$$\mathrm{HCHO} + R\mathrm{Mg}X \longrightarrow R\mathrm{CH_2OMg}X \xrightarrow{\mathrm{H_2O}} R\mathrm{CH_2OH} + \mathrm{Mg(OH)}X$$

From the above reaction, we can see that primary alcohol is produced as a product. Thus, among the given options propan-2-ol is not prepared by the action of Grignard’s reagent on methanal as it is a secondary alcohol.

Question 89

What is the number of elements present in each series of transition element?

Options:

A. 8

B. 10

C. 14

D. 24

Answer: B

Solution:

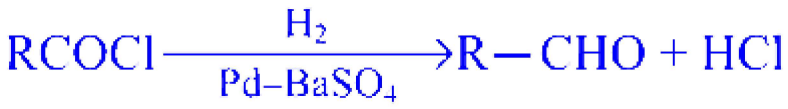
Each series of transition elements contains 10 elements.

Question 90

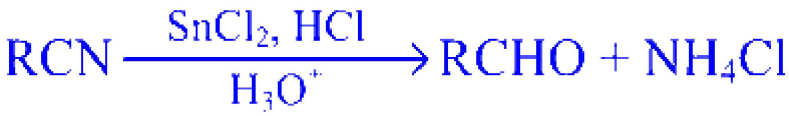
Which of the following is Wolf-Kishner reduction?

Options:

A.



B.



C.

$$RCHO \xrightarrow[\Delta]{Zn - Hg,\ conc.\ HCl} R-CH_3 + H_2O$$

D.

$$RCOR \xrightarrow[(ii)\ KOH,\ HO(CH_2)_2OH]{(i)\ H_2N\ NH_2,\ \Delta} R-CH_2-R + N_2$$

Answer: D

Solution:

The reduction of carbonyl group of aldehydes and ketones to CH_2 group when treated with hydrazine followed by heating with sodium or potassium hydroxide in high boiling solvent such as ethylene glycol is called Wolf-Kishner reduction. It's reaction is as follows:

$$RCOR \xrightarrow[(ii)\ KOH,\ HO(CH_2)_2OH]{(i)\ H_2N\ NH_2,\ \Delta} R-CH_2-R + N_2$$

Question 91

Calculate the wave number of photon emitted during transition from the orbit of $n = 3$ to $n = 2$ in hydrogen atom $\left(R_H = 109677\ \text{cm}^{-1}\right)$.

Options:

A. $15354.8\ \text{cm}^{-1}$

B. $82257.8\ \text{cm}^{-1}$

C. $30515.4\ \text{cm}^{-1}$

D. $41128.5\ \text{cm}^{-1}$

Answer: A

Solution:

Given,

$n_2 = 3$
$n_1 = 2$
$R_{\mathrm{H}} = 109677\ \mathrm{cm}^{-1}$
$\bar{v}$ (wave number) $=?$

Using formula,

$$\bar{v} = 109677 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \mathrm{cm}^{-1}$$
$$\bar{v} = 109677 \left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] \mathrm{cm}^{-1}$$
$$\bar{v} = 109677 \left[\frac{9-4}{36}\right] \mathrm{cm}^{-1}$$
$$\bar{v} = 109677 \times \frac{5}{36}\ \mathrm{cm}^{-1}$$
$$\bar{v} = 15232.9\ \mathrm{cm}^{-1}$$

Question 92

What is the molar concentration of acetic acid if value of it's, dissociation constant is 1.8×10^{-5} and degree of dissociation is 0.02 ?

Options:

A. 4.6×10^{-3} M

B. 4.5×10^{-2} M

C. 4.0×10^{-4} M

D. 3.6×10^{-2} M

Answer: B

Solution:

$k = 1.8 \times 10^{-5}$
$\alpha = 0.02$
$C =?$

For acetic acid, dissociation is given as

$$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$$

Initial conc.	C	0	0
Conc.at equilibrium	C – Cα	Cα	Cα

$$\therefore k = \frac{\alpha^2 C^2}{(1-\alpha)C} = \frac{\alpha^2 C}{(1-\alpha)} \quad [\alpha <<< 1 \Rightarrow (1-\alpha) \approx 1]$$

$$1.8 \times 10^{-5} = \frac{(0.02)^2 C}{1}$$

$$\text{or} \quad C = 4.5 \times 10^{-2} \text{M}$$

Question 93

An ideal gas absorbs 210 J of heat and undergoes expansion from 3 L to 6 L against a constant external pressure of 10^5 Pa. What is the value of ΔU ?

Options:

A. 310 J

B. −90 J

C. −210 J

D. 190 J

Answer: B

Solution:

Given,

$q = 210$ J
$V_2 = 6$ L
$V_1 = 3$ L

$p = 10^5$ Pa $= 10^5$ J/cm^3
$\Delta U = ?$

Using, $\Delta U = q + w \quad [\because w = p\Delta V]$

$$= 210 + \left(-10^5(6-3) \times 10^{-3}\right)$$
$$= 210 - (300)\text{J}$$
$$\Delta U = -90 \text{ J}$$

Question 94

What is the molecular formula of p-toluidine?

Options:

A. $C_8H_{10}N_2$

B. C_7H_9N

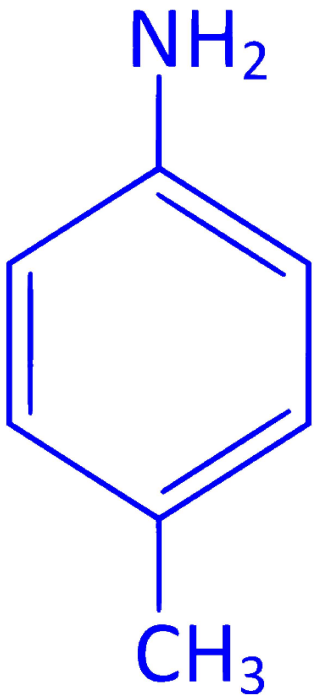
C. $C_{10}H_{14}N_2$

D. $C_{10}H_{14}N$

Answer: B

Solution:

The structure of p-toluidine is



Thus, its molecular formula is C_7H_9N.

Question 95

If $0.01\ \text{m}$ aqueous solution of an electrolyte freezes at $-0.056\ \text{K}$. Calculate van't Hoff factor for an electrolyte (cryoscopic constant of water $= 1.86\ \text{K kg mol}^{-1}$)

Options:

A. 1.30

B. 2.33

C. 3.00

D. 4.11

Answer: C

Solution:

$$\begin{aligned}\text{Given, molality } &= 0.01\ \text{m}\\ \Delta T_f &= -0.056\ \text{K}\\ K_f &= 1.86\ \text{K kg mol}^{-1}\\ i &= ?\end{aligned}$$

Using formula,

$$\begin{aligned}\Delta T_f &= iK_f m\\ \Rightarrow \quad i &= \frac{0.056}{1.86 \times 0.01} = 3\\ \therefore \quad i &= 3\end{aligned}$$

Question 96

Which element from following does NOT belong to chalcogen family?

Options:

A. At

B. Po

C. Se

D. Te

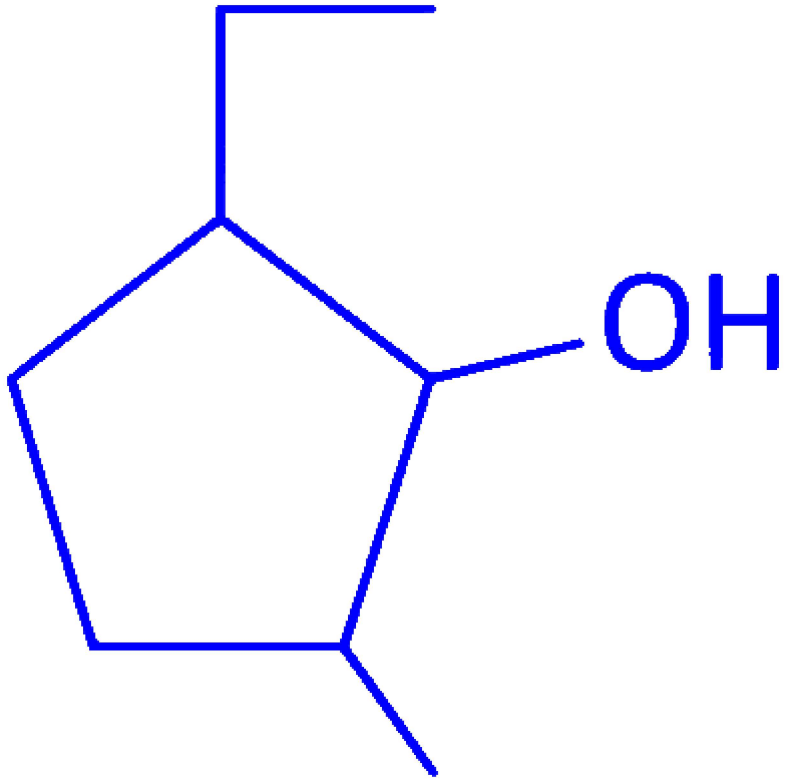
Answer: A

Solution:

Members of oxygen family are commonly called as chalcogen family. Their members are oxygen (O), sulphur (S), selenium (Se), tellurium (Te), polonium(Po). Thus, among the given options (At) astatine does not belongs to group 16 or chalcogen family.

Question 97

What is IUPAC name of following compound?



Options:

A. 2-ethyl-3-methylcyclopentanol

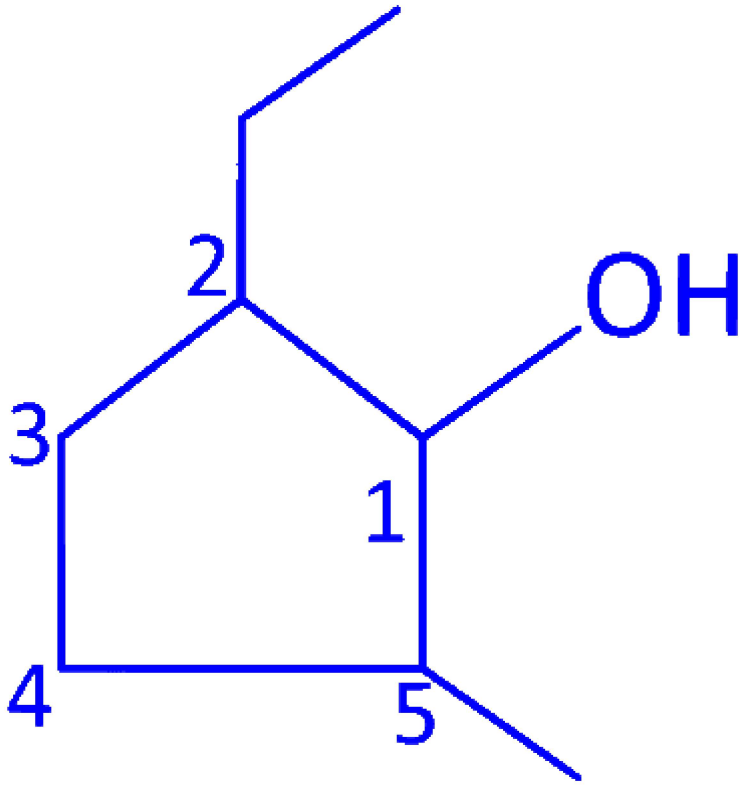
B. 2-methyl-3-ethylcyclopentanol

C. 1-methyl-3-ethylcyclopentan-2-ol

D. 2-ethyl-5-methylcyclopentanol

Answer: D

Solution:



The IUPAC name of the given compound is 2-ethyl-5-methylcyclopentanol.

Question 98

Identify the monomers used to prepare novolac.

Options:

A. Isoprene

B. 1,3-butadiene

C. Phenol and formaldehyde

D. Melamine

Answer: C

Solution:

Monomers of novolac are phenol and formaldehyde. Novolac is obtained by condensation reaction of monomers in the presence of an acid or base as a catalyst.

Question 99

Calculate the volume of unit cell if an element having molar mass $180\ \text{g mol}^{-1}$ forms fcc unit cell. $\left[\rho \cdot N_A = 120 \times 10^{21}\ \text{g cm}^{-3}\ \text{mol}^{-1}\right]$

Options:

A. $6.00 \times 10^{-21}\ \text{cm}^3$

B. $5.00 \times 10^{-21}\ \text{cm}^3$

C. $4.00 \times 10^{-21}\ \text{cm}^3$

D. $7.00 \times 10^{-21}\ \text{cm}^3$

Answer: A

Solution:

To find the volume of the unit cell, we can use the given formula which relates density ρ, molar mass M, Avogadro's number N_A, and the volume of the unit cell V as follows:

$\rho = \frac{M}{N_A \cdot V}$

Given,

$\rho \cdot N_A = 120 \times 10^{21}\ \text{g/cm}^3\ \text{mol}^{-1}$ and $M = 180\ \text{g/mol}$.

We are supposed to find V, the volume of the unit cell. We can start by rearranging the formula to solve for V:

$V = \frac{M}{\rho \cdot N_A}$

Substitute the given values into the equation:

$V = \frac{180\ \text{g/mol}}{120 \times 10^{21}\ \text{g/cm}^3\ \text{mol}^{-1}}$

After placing the values in, you proceed with the calculation:

$V = \frac{180}{120 \times 10^{21}}\ \text{cm}^3$

$V = \frac{3}{2} \times 10^{-21}\ \text{cm}^3$

$V = 1.5 \times 10^{-21}\ \text{cm}^3$

This calculated value does not match any of the options provided, suggesting there may have been a mistake in my calculation or in interpreting the given question and options. Let's correct the oversight with the proper calculation, considering that for an FCC unit cell, there are four atoms per unit cell. The correct approach would address this relationship more precisely.

The corrected formula, taking into account the number of atoms in an FCC unit cell ($Z = 4$ for FCC), should be:

$\rho = \frac{Z \cdot M}{N_A \cdot V}$

So, correcting the calculation for V:

$V = \frac{Z \cdot M}{\rho \cdot N_A}$

We know $Z = 4$ for FCC, so we substitute the given values into this corrected formula:

$V = \frac{4 \cdot 180}{120 \times 10^{21}}\ \text{cm}^3$

$V = \frac{720}{120 \times 10^{21}}\ \text{cm}^3$

$V = \frac{6}{10^{21}}\ \text{cm}^3$

$V = 6.00 \times 10^{-21}\ \text{cm}^3$

Therefore, the volume of the unit cell is $6.00 \times 10^{-21}\ \text{cm}^3$, which corresponds to Option A.

Question 100

Identify homoleptic complex from following.

Options:

A. $[Co(NH_3)_6]^{3+}$

B. $[Co(NH_3)_4Cl_2]^{+}$

C. $[Pt(NH_3)_2Cl_2]$

D. $[Co(NH_3)_5Cl]SO_4$

Answer: A

Solution:

Complexes in which a metal is bound to only one kind of donor groups are called as homoleptic complex. Thus, $[Co(NH_3)_6]^{3+}$ is a homoleptic complex.

Physics

Question 101

A sample of oxygen gas and a sample of hydrogen gas both have the same mass, same volume and the same pressure. The ratio of their absolute temperature is

Options:

A. 1 : 4

B. 4 : 1

C. 1 : 16

D. 16 : 1

Answer: D

Solution:

Given,

$p =$ Pressure
$V =$ Volume
$m =$ mass

Now, using Ideal gas law,

$pV = nRT$ .... (i)

Here $n =$ mass of gas $= m/M$

$R =$ Gas constant

$\because$ We know that p, V and m are same for hydrogen and oxygen and from Eq. (i), we have

$T = \frac{pV}{nR} = \frac{pVM}{mR}$ ..... (ii)

For oxygen, $T_{O_2} \propto M_{O_2}$

For hydrogen, $T_{\mathrm{H}} \propto M_{\mathrm{H}}$

Now, the ratio of $T_{\mathrm{O_2}}/T_{\mathrm{H}}$ is

$\frac{T_{\mathrm{O_2}}}{T_{\mathrm{H}}} = \frac{M_{\mathrm{O_2}}}{M_{\mathrm{H}}} = \frac{32}{2} = \frac{16}{1}$

$\therefore$ Ratio of $T_{\mathrm{O_2}} : T_{\mathrm{H}} = 16 : 1$

Question 102

If two planets have their radii in the ratio $x : y$ and densities in the ratio $m : n$, then the acceleration due to gravity on them are in the ratio

Options:

A. $\frac{ny}{mx}$

B. $\frac{my}{nx}$

C. $\frac{nx}{my}$

D. $\frac{mx}{ny}$

Answer: D

Solution:

Acceleration due to gravity of planet A is given below.

$g_A = \frac{GM_A}{R_A^2}$ ..... (i)

$\because$ We know that, (Mass = Density $\times$ Volume) $(M = D \times V)$

So, from Eq. (i)

$$g_A = \frac{GD_A \times \frac{4}{3}\pi R_A^3}{R_A^2} = GD_A \times \frac{4}{3}\pi R_A$$

$$g_A = \frac{4}{3}\pi G R_A D_A \quad \text{.... (ii)}$$

Similarly, Acceleration due to gravity of planet B is

$g_B = \frac{GM_B}{R_B^2}$ .... (iii)

and $g_B = \frac{4}{3}\pi G D_B R_B$ .... (iv)

We have to find the ratio of $g_A : g_B$

So, according to question, we have

$$\frac{R_A}{R_B} = \frac{x}{y} \quad \text{.... (v)}$$

and $$\frac{D_A}{D_B} = \frac{m}{n} \quad \text{.... (vi)}$$

Now, taking the ratio of Eqs. (ii) and (iv), we get

$\frac{g_A}{g_B} = \frac{R_A D_A}{R_B D_B}$

Put the value from Eqs. (v) and (vi) in this equation, we get

$\frac{g_A}{g_B} = \frac{x \cdot m}{y \cdot n}$

or $g_A/g_B = mx/ny$

Question 103

An excited hydrogen atom emits a photon of wavelength λ in returning to ground state. The quantum number n of the excited state is ($R =$ Rydberg's constant)

Options:

A. $\sqrt{\lambda R(\lambda R - 1)}$

B. $\sqrt{\frac{\lambda R}{(\lambda R - 1)}}$

C. $\sqrt{\frac{(\lambda R - 1)}{\lambda R}}$

D. $\sqrt{\frac{1}{\lambda R(\lambda R - 1)}}$

Answer: B

Solution:

The wave number of emitted photon when electron jumps from n_1 to n_2 is given by

$v = \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ .... (i)

Here, $n_1 = 1, n_2 = n$

Put the value of n_1 and n_2 in Eq. (i), we get

$\Rightarrow \quad \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$

$\Rightarrow \quad \frac{1}{\lambda R} = \left[1 - \frac{1}{n^2}\right] \Rightarrow \frac{1}{n^2} = 1 - \frac{1}{\lambda R}$

$\Rightarrow \quad \frac{1}{n^2} = \frac{(\lambda R - 1)}{\lambda R} \Rightarrow n^2 = \frac{\lambda R}{(\lambda R - 1)}$

$\Rightarrow \quad n = \sqrt{\frac{\lambda R}{(\lambda R - 1)}}$

Question 104

There is hole of area A at the bottom of a cylindrical vessel. Water is filled to a height h and water flows out in t second. If water is filled to a height $4h$, it will flow out in time (in second)

Options:

A. t

B. $4t$

C. $2t$

D. $t/4$

Answer: C

Solution:

$\because$ We know that, time required to empy the tank is

$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$

and $t \propto \sqrt{H}$

$\therefore \quad \frac{t_2}{t_1} = \sqrt{\frac{H_2}{H_1}}$ .... (i)

and we know that (given data)

$H_1 = h$ and $t_1 = t$
$H_2 = 4h$ and $t_2 =?$

Put H_1 and H_2 value in Eq. (i), we get

$$\frac{t_2}{t_1} = \sqrt{\frac{4h}{h}} = \sqrt{4} = 2$$
$$\Rightarrow \quad \frac{t_2}{t} = 2 \Rightarrow t_1 = 2t$$

Question 105

The number of turns in the primary and the secondary of a transformer are 1000 and 3000 , respectively. If $80\ \mathrm{V\ AC}$ is applied to the primary coil of the transformer, then the potential difference per turn of the secondary coil would be

Options:

A. 240 V

B. 2400 V

C. 0.24 V

D. 0.08 V

Answer: D

Solution:

∵ We know that, Transformation ratio is

$k = \frac{V_2}{V_1}$ .... (i)

and Turn ratio is

$\alpha = \frac{1}{k} = \frac{V_1}{V_2} = \frac{N_1}{N_2}$ ..... (ii)

Here, $N_1 =$ Primary winding turns

$N_2 =$ Secondary winding turns

V_1 = Primary winding voltage

V_2 = Secondary winding voltage

$$\begin{aligned} \text{Given : } N_1 &= 1000 \\ N_2 &= 3000 \text{ and } V_1 = 80 \text{ V} \\ V_2 &= ? \\ \therefore \quad \frac{V_2}{V_1} &= \frac{N_2}{N_1} \\ \Rightarrow \quad V_2 &= \frac{N_2}{N_1} \cdot V_1 \quad \text{..... (iii)} \end{aligned}$$

put the values of N_1, N_2 and V_1 in above equation, we get

$V_2 = \frac{3000}{1000} \times 80 = 240$ V

Now, potential difference across each turn

$= \frac{240}{3000} = 0.08$ V

Question 106

The ratio of magnetic field at the centre of the current carrying circular loop and magnetic moment is X. When both the current and radius are doubled, then the ratio will be

Options:

A. $2X$

B. $\frac{X}{2}$

C. $\frac{X}{4}$

D. $\frac{X}{8}$

Answer: D

Solution:

The magnetic field at the centre of a current carrying loop is

$B = \frac{\mu_0}{4\pi}\left(\frac{2\pi I}{a}\right) = \frac{\mu_0 l}{2a}$ ..... (i)

and magnetic moment M is

$M = I\left(\pi a^2\right)$ .... (ii)

Thus, $X = \frac{B}{M} = \frac{\mu_0 I}{2a} \times \frac{I}{I\pi a^2} = \frac{\mu_0}{2\pi a^3}$ .... (iii)

Now, according to question, when both the current and radius are doubled, then the ratio will be

$\Rightarrow \frac{\mu_0}{2\pi(2a)^3} = \frac{\mu_0}{8(2\pi a^3)}$

From Eq. (iii), $\frac{\mu_0}{2\pi a^3} = X$

So, $\frac{\mu_0}{8(2\pi a^3)} = \frac{X}{8}$

Question 107

Light of wavelength $5000\overset{o}{A}$ is incident normally on a slit. The first minimum of the diffraction pattern is observed to lie at a distance of $5\ \mathrm{mm}$ from the central maximum on a screen placed at a distance of $2\ \mathrm{m}$ from the slit. The width of the slit is

Options:

A. 2 cm

B. 0.2 cm

C. 0.02 cm

D. 0.01 cm

Answer: C

Solution:

Position of the nth minima is

$x_n = \frac{n\lambda D}{a}$ .... (i)

where, $a =$ slit width

$D =$ Distance between screen

and slit $\lambda =$ wavelength

Given, $\lambda = 5000\overset{o}{A} = 5000 \times 10^{-10}$ m

$$x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$
$$D = 2 \text{ m}$$
$$\Rightarrow \quad n = 1$$

Put all these values in Eq. (i), we get

$$5 \times 10^{-3} = \frac{1 \times 5000 \times 10^{-10} \times 2}{a}$$
$$\Rightarrow \quad a = \frac{5 \times 10^{-7} \times 2}{5 \times 10^{-3}}$$
$$\Rightarrow \quad a = 2 \times 10^{-4} \text{ m}$$
$$= 0.02 \text{ cm}$$

Therefore, the slit width is 0.02 cm.

Question 108

When two tuning forks are sounded together, 5 beats per second are heard. One of the forks is in unison with 0.97 m length of sonometer wire and the other is in unison with 0.96 m length of the same wire. The frequencies of the two tuning forks are

Options:

A. 383 Hz, 388 Hz

B. 475 Hz, 480 Hz

C. 388 Hz, 39z Hz

D. 480 Hz, 485 Hz

Answer: D

Solution:

Let, f_1 and f_2 be the frequency of the two turning forks, given

$f_1 - f_2 = 5$ ..... (i)

Length of wire in sonometer is

$L_1 = 0.96$ m and $L_2 = 0.97$ m

Frequency of vibration of the wire is given by

$f = \frac{1}{2l}\sqrt{T/m}$

According to the question,

$$\frac{f_1}{f_2} = \frac{l_2}{l_1} = \frac{0.97}{0.96} > 1$$

$\therefore \quad f_1 > f_2$

Now, from Eq. (i)

$\Rightarrow \quad f_1 - f_2 = 5$
$\Rightarrow \quad \frac{0.97}{0.96} f_2 - f_2 = 5$
$\Rightarrow \quad \left[\frac{0.97}{0.96} - 1\right] f_2 = 5$
$\Rightarrow \quad \left[\frac{0.97-0.96}{0.96}\right] f_2 = 5$
$\Rightarrow \quad 0.01 f_2 = 5 \times 0.96$
$\Rightarrow \quad 0.01 f_2 = 4.8$
$\Rightarrow \quad f_2 = 480 \text{ Hz}$
$\Rightarrow \quad f_1 = 5 + f_2 = 485 \text{ Hz}$

Question 109

The work done in rotating a dipole placed parallel to the electric field through 180° is W. So, the work done in rotating it through 60° is $\left(\cos 0^\circ = 1, \cos 60^\circ = \frac{1}{2}, \cos 180^\circ = -1\right)$

Options:

A. $4W$

B. 3 W

C. $W/2$

D. $W/4$

Answer: D

Solution:

$\because$ Work done in rotating dipole by θ angle from equilibrium,

$W = pE[1 - \cos\theta]$ ..... (i)

$\Rightarrow \quad W = pE\,[1 - \cos 180^\circ]$

$\Rightarrow \quad W = pE[1 + 1] = 2pE$

or $\quad pE = \dfrac{W}{2}$ ..... (ii)

and $\quad W' = pE\,(1 - \cos 60^\circ]$

$\Rightarrow \quad W' = pE\left[1 - \dfrac{1}{2}\right] = \dfrac{pE}{2}$

$\Rightarrow \quad W' = \dfrac{pE}{2}$ ..... (iii)

From Eq. (ii) put the value of pE in Eq. (iii), we get

$\Rightarrow \quad W' = \frac{W}{2\times 2} = \frac{W}{4} \Rightarrow W' = \frac{W}{4}$

Question 110

When an electron is excited from its 4 th orbit to 5 th stationary orbit, the change in the angular momentum of electron is approximately.

(Planck's constant $= h = 6.63 \times 10^{-34}$ J $-$ s)

Options:

A. 2×10^{-34} J $-$ s

B. 6.63×10^{-34} J $-$ s

C. 1×10^{-34} J $-$ s

D. 3.14×10^{-34} J $-$ s

Answer: C

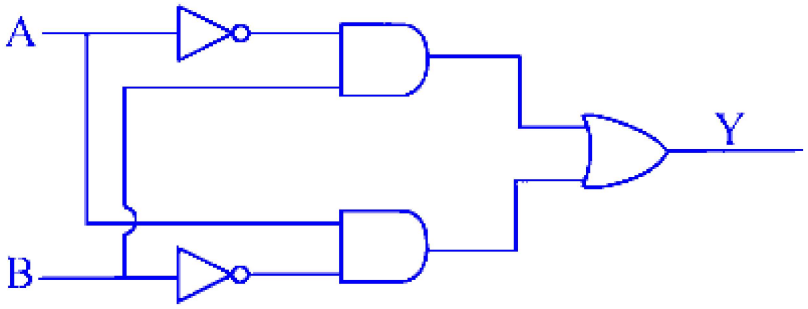
Solution:

Charge in angular momentum is

$$\Delta L = L_2 - L_1 = n_2 h - n_1 h$$
$$\Rightarrow \quad \Delta L = h\left(n_2 - n_1\right)$$
$$\Rightarrow \quad \Delta L = \frac{h}{2\pi}\left(n_2 - n_1\right)$$
$$\Rightarrow \quad \Delta L = \frac{6.6\times10^{-34}}{2\times3.14}(5-4)$$
$$\Rightarrow \quad \Delta L = 1\times10^{-34}\ \mathrm{J-s}$$

Question 111

The Boolean expression for the following combination is



Options:

A. $(\overline{A+B})\cdot(A+B)$

B. $(\overline{A\cdot B})+(\bar{A}\cdot B)$

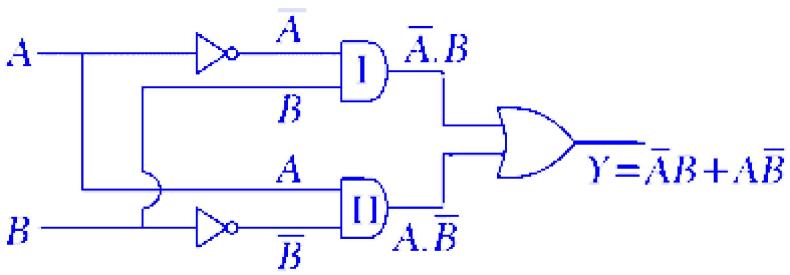
C. $(\bar{A}\cdot B)+(A\cdot\bar{B})$

D. $(\bar{A}+B)\cdot(A+\bar{B})$

Answer: C

Solution:

Let, obtain the output at each step first



Output at I AND gate $= \bar{A} \cdot B$

Output at II AND gate $= A \cdot \bar{B}$

$\therefore$ Output at OR gate $= Y = \bar{A} \cdot B + A \cdot \bar{B}$

Question 112

Under the influence of force F_1 the body oscillates with a period T_1 and due to another force F_2 body oscillates with period T_2. If both forces acts simultaneously, then the resultant period is (consider displacement is same in all three cases)

Options:

A. $T = \sqrt{\frac{T_1^2+T_2^2}{T_1^2 T_2^2}}$

B. $T = \sqrt{\frac{T_1^2 T_2^2}{T_1^2+T_2^2}}$

C. $T = \sqrt{\frac{T_1^2}{T_2^2}}$

D. $T = \sqrt{T_1^2 + T_2^2}$

Answer: B

Solution:

Time period for the force $F_1 = T_1$

Time period for the force $F_2 = T_2$

If both forces acts simultaneously then the resultant force is F,

$F = F_1 + F_2$

Let, time period for the force F be T

Now, $F = F_1 + F_2$

$m\omega^2 y = m\omega_1^2 y + m\omega_2^2 y \quad (\because F = m\omega^2 y)$
$\omega^2 = \omega_1^2 + \omega_2^2$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{T_1}\right)^2 + \left(\frac{2\pi}{T_2}\right)^2$$
$$\Rightarrow \quad \frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$$
$$\Rightarrow \quad \frac{1}{T^2} = \frac{T_2^2 + T_1^2}{T_1^2 T_2^2}$$
$$\Rightarrow \quad T^2 = \frac{T_1^2 T_2^2}{T_1^2 + T_2^2} \Rightarrow T = \sqrt{\frac{T_1^2 T_2^2}{T_1^2 + T_2^2}}$$

Question 113

The internal energy of a monoatomic ideal gas molecule is

Options:

A. partly kinetic and partly potential

B. totally kinetic

C. totally potential

D. Neither kinetic nor potential

Answer: B

Solution:

The internal energy of the system is sum of kinetic and potential energy of the particles.

For an ideal gas, we assume that the particles do not intract with each other. Hence, there is no potential energy associated with the particles and the internal energy of the system will only be due to the kinetic energy of the system.

Note For a real gas, the internal energy of the system is the sum of kinetic and potential energy as the gas molecules are interacting with each other.

Question 114

A gas at pressure p_0 is contained in a vessel. If the masses of all the molecules are halved and their velocities are doubled, then the resulting pressure would be equal to

Options:

A. $4p_0$

B. $2p_0$

C. p_0

D. $p_0/2$

Answer: B

Solution:

Pressure p of the gas is given by,

$p = \frac{1}{3}\frac{mN}{V}v_{\text{rms}}^2$

Where, m is mass, N is the number of moles, V is the volume and v_{rms} is the root mean square velocity of the particles of the gas.

$\therefore \quad p_1 = p_0 = \frac{1}{3}\frac{m_1 N}{V_1}\left(v_{\text{rms}}^2\right)_1$ ..... (i)

If $m_2 = \frac{m_1}{2}$ and $(v_{\text{rms}})_2 = 2(v_{\text{rms}})_1$

For simplicity write $v_2 = 2v_1$

Hence, $p_2 = \frac{1}{3}\frac{m_2 N}{V}v_2^2$ ..... (ii)

Divide Eq. (i) by Eq. (ii)

$$\frac{p_0}{p_2} = \frac{\frac{1}{3}\frac{m_1 N}{V}v_1^2}{\frac{1}{3}\frac{m_2 N}{V}v_2^2}$$

$$\Rightarrow \quad \frac{p_0}{p_2} = \frac{m_1 v_1^2}{\frac{m_1}{2}(2v_1)^2} \Rightarrow \frac{p_0}{p_2} = \frac{1}{2}$$

$$\Rightarrow \quad p_2 = 2p_0$$

$\therefore$ Resulting pressure is $2p_0$

Question 115

The equation of a progressive wave is $Y = a\sin 2\pi\left(nt - \frac{x}{5}\right)$. The ratio of maximum particle velocity to wave velocity is

Options:

A. $\frac{\pi a}{5}$

B. $\frac{2\pi a}{5}$

C. $\frac{3\pi a}{5}$

D. $\frac{4\pi a}{5}$

Answer: B

Solution:

General equation of a wave is given by $y = A\sin(\omega t - kx)$

Equation for progressive wave is

$y = a\sin 2\pi\left(nt - \frac{x}{5}\right)$

On comparing both equations,

$A = a, \omega = 2\pi n, k = \frac{2\pi}{5}$

Maximum particle velocity is given by

$v_m = A\omega$
$\Rightarrow \quad v_m = a(2\pi n)$

Wave velocity is given by

$$v_\omega = \frac{\omega}{k} \Rightarrow v_\omega = \frac{2\pi n}{\frac{2\pi}{5}} = 5n$$

$$\therefore \quad \frac{v_m}{v_\omega} = \frac{a(2\pi n)}{2n} \Rightarrow \frac{v_m}{v_\omega} = \frac{2\pi a}{5}$$

Question 116

For an adiabatic process, which one of the following is wrong statement?

Options:

A. Equation of state is $pV = \text{constant}$

B. There is exchange of heat with surrounding

C. All the work is utilised to change the internal energy of the system

D. Temperature of the system changes i.e. $\Delta T \neq 0$

Answer: B

Solution:

For an adiabatic process, there is no transfer of heat.

Since, $\Delta Q = 0$

From first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$
or $$\Delta W = -\Delta U$$

Hence, all work done is utilised to change in internal energy.

Question 117

A mine is located at depth $R/3$ below earth's surface. The acceleration due to gravity at that depth in mine is ($R =$ radius of earth, $g =$ acceleration due to gravity)

Options:

A. g

B. $3g$

C. $\frac{2g}{3}$

D. $\frac{g}{3}$

Answer: C

Solution:

Here, g' be the acceleration due to gravity in mine.

Then, $g' = g\left(1 - \frac{d}{R}\right)$ ..... (i)

Here,

$d(\text{ depth }) = \frac{R}{3}$

$R =$ Radius of earth

Now, from Eq. (i), we get

$$\Rightarrow \quad g' = g\left(1 - \frac{R}{3 \times R}\right)$$

$$g' = \frac{2g}{3}$$

Question 118

A $10\ \mathrm{m}$ long wire of resistance 20Ω is connected in series with a battery of emf $3\ \mathrm{V}$ (negligible internal resistance) and a resistance of 10Ω. The potential gradient along the wire is

Options:

A. $3\ \mathrm{V/m}$

B. $0.1\ \mathrm{V/m}$

C. $0.2\ \mathrm{V/m}$

D. $0.3\ \mathrm{V/m}$

Answer: C

Solution:

Total length of wire $= 10$ m

The equivalent resistance of the circuit, $R_{\text{eq}} = 10 + 20 = 30\Omega$

The current flowing through the wire, $i = V/R$

$= \frac{3V}{30\Omega} = 0.1$ A

$\therefore$ Potential gradient along the wire,

$$= \frac{\text{potential difference across } 20\Omega}{\text{length of } 20\Omega \text{ wire}}$$
$$= \frac{0.1 \times 20\Omega}{10 \text{ m}} = 0.20 \text{ V/m}$$

Question 119

A group of lamps having total power rating of 1000 W is supplied by an AC voltage of $E = 200 \sin(310t + 60^\circ)$, the rms value of current flowing through the circuit is

Options:

A. 10 A

B. $5\sqrt{2}$ A

C. 20 A

D. $10\sqrt{2}$ A

Answer: D

Solution:

Power of lamp $(P) = 1000$ W

AC voltage $(E) = 200 \sin(310t + 60^\circ)$

Comparing with $E = E_0 \sin(\omega t + \phi)$,

we get, $E_0 =$ peak value of voltage

$\phi =$ phase difference

rms value of supply voltage,

$E_{\text{rms}} = \frac{E_0}{\sqrt{2}}$

$E_{\text{rms}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}$

Average power is given by,

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$$1000 = (100\sqrt{2}) I_{\text{rms}} \left(\frac{1}{2}\right) \quad \left(\because \cos\phi = \cos 60^\circ = \frac{1}{2}\right)$$

$$I_{\text{rms}} = 10\sqrt{2}\ \text{A}$$

Question 120

At a particular angular frequency, the reactance of capacitor and that of inductor is same. If the angular frequency is doubled, the ratio of the reactance of the capacitor to that of the inductor will be

Options:

A. 1/4

B. 1/2

C. 2

D. 4

Answer: A

Solution:

At particular angular frequency $\omega_0, X_L = 1/X_C$

$\Rightarrow \quad \omega_0 L = \frac{1}{\omega_c} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

when $\omega_0' = 2\omega_0$

$$\therefore \quad \frac{X_C'}{X_L'} = \frac{1}{\frac{\omega_0' C}{\omega_0' L}} = \frac{1}{\left(\omega_0'\right)^2 LC}$$
$$= \frac{1}{(2\omega_0)^2 LC} = \frac{1}{4\omega_0^2 LC} = \frac{1}{4 \cdot \frac{1}{LC} \times LC} = \frac{1}{4}$$

Question 121

A metal disc of radius R rotates with an angular velocity ω about an axis perpendicular to its plane passing through its centre in a magnetic field of induction B acting perpendicular to the plane of the disc. The magnitude of induced emf between the rim and axis of the disc is

Options:

A. $\pi B R^2$

B. $\frac{2\pi^2 B R^2}{\omega}$

C. $\pi B R^2 \omega$

D. $\frac{B R^2 \omega}{2}$

Answer: D

Solution:

Total magnetic flux linked

$$\phi = BA$$
$$\therefore \text{ induced emf, } |e| = \frac{\Delta\phi}{\Delta t} = \frac{\Delta BA}{\Delta t} = B \cdot \frac{\Delta A}{\Delta T}$$
$$= B \cdot \frac{\pi R^2}{\left(\frac{2\pi}{\omega}\right)} = \frac{B\omega R^2}{2}$$

Question 122

A mass M is suspended from a light spring. An additional mass M_1 added extends the spring further by a distance x. Now, the combined

mass will oscillate on the spring with period $T =$

Options:

A. $2\pi\left[\left(\frac{M_1 g}{x(M+M_1)}\right)\right]^{\frac{1}{2}}$

B. $2\pi\left[\frac{(M+M_1)x}{M_1 g}\right]^{\frac{1}{2}}$

C. $\left(\frac{\pi}{2}\right)\left[\left(\frac{M_1 g}{x(M+M_1)}\right)\right]^{\frac{1}{2}}$

D. $2\pi\left[\left(\frac{M+M_1}{M_1 g x}\right)\right]^{\frac{1}{2}}$

Answer: B

Solution:

Time period of oscillation when mass M hangs by spring of spring constant k is given by,

$T = 2\pi\sqrt{M/k}$

From Hooke's law, $kx = mg$

$\Rightarrow \quad \frac{1}{k} = \frac{x}{mg}$

When mass M_1 is added to the system

$T' = 2\pi\sqrt{\frac{(M_1+M)}{k}} = 2\pi\sqrt{\frac{x(M_1+M)}{mg}}$

Question 123

The path difference between two identical light waves at a point Q on the screen is $3\mu\text{m}$. If wavelength of the waves is $5000\overset{o}{A}$, then at point Q there is

Options:

A. 3rd dark band

B. 4th bright band

C. 5th dark band

D. 6 th bright band

Answer: D

Solution:

Given, path difference $(\Delta x) = 3\mu\text{m} = 3 \times 10^{-6}$ m

Wavelength of length, $\lambda = 5000\overset{o}{A} = 5 \times 10^{-7}$ m

angular position is given by

$\theta = \frac{\Delta x}{\lambda} = \frac{3 \times 10^{-6}}{5 \times 10^{-7}} = 6$

Since, $n =$ integer (6) is even, therefore, we will have 6th order maxima.

Question 124

If the maximum efficiency of a full wave rectifier is $x\%$ and that of half-wave rectifier is $y\%$, then the relation between x and y is

Options:

A. $x = y$

B. $x = 2y$

C. $y = 2x$

D. $y = 4x$

Answer: B

Solution:

We know that, efficiency of full wave rectifier $= 0.8$

efficiency of half wave rectifier $= 0.4$

According to question, $0.8 = x, 0.4 = y$

$\Rightarrow x = 2y$

Alternate solution We know that, efficiency of full wave rectifier is twice the efficiency of half wave rectifier. Therefore, $x = 2y$.

Question 125

Two bodies A and B start from the same point at the same instant and move along a straight line. body A moves with uniform acceleration a and body B moves with uniform velocity v. They meet after time t. The value of t is

Options:

A. $\frac{2v}{a}$

B. $\frac{a}{2v}$

C. $\frac{v}{2a}$

D. $\sqrt{\frac{v}{a}}$

Answer: A

Solution:

Distance travelled by body B, $s_B = vt$

Distance travelled by A, $s_A = ut + \frac{1}{2}at^2 = at^2$

When they meet, $s_A = s_B$

$$\Rightarrow \quad vt = \frac{1}{2}at^2$$
$$\Rightarrow \quad t = \frac{2v}{a}$$

Question 126

A small steel ball is dropped from a height of $1.5\ \mathrm{m}$ into a glycerine jar. The ball reaches the bottom of the jar $1.5\ \mathrm{s}$ after it was dropped. If the retardation is $2.66\ \mathrm{m/s^2}$, the height of the glycerine in the jar is about (acceleration due to gravity $g = 9.8\ \mathrm{m/s^2}$)

Options:

A. 7.0 m

B. 7.5 m

C. 5.5 m

D. 3.2 m

Answer: C

Solution:

The ball executes free fall until it hits the upper surface of glycerine.

$\therefore$ Velocity of ball when it reaches the upper surface of glycerine is given as,

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2 \times 9.8 \times 1.5} \\ \Rightarrow \quad v^2 &= 29.4\ \mathrm{m/s^2} \end{aligned}$$

Using the formula, $v^2 - u^2 = 2as$ inside the glycerine.

We have,

$0 - 29.4 = -2 \times (2.66) \times h$

($\because h =$ distance travelled inside glycerine)

$$\Rightarrow \quad h = 5.5\ \mathrm{m}$$

$\therefore$ The height of glycerine in jar will be $5.5\ \mathrm{m}$.

Question 127

In a parallel plate capacitor with air between the plates, the distance d between the plates is changed and the space is filled with dielectric

constant 8. The capacity of the capacitor is increased 16 times, the distance between the plates is

Options:

A. 2d

B. 4d

C. d/2

D. d/4

Answer: C

Solution:

Capacitance of a parallel plate capacitor is given by, $C = \frac{\varepsilon_0 A}{d}$

When capacitor is completely filled with dielectric K then, $C' = \frac{\varepsilon_0 K A}{d'}$

According to question, $\frac{C'}{C} = \frac{16}{1} = \frac{\frac{\varepsilon_0 \times 8 \times A}{d'}}{\frac{\varepsilon_0 \times A}{d}}$

$$\Rightarrow \quad \frac{16}{1} = \frac{8d}{d'}$$
$$\Rightarrow \quad d' = \frac{d}{2}$$

Question 128

A ray of light passes through an equilateral prism such that the angle of incidence (i) is equal to angle of emergence (e). The angle of emergence is equal to $\left(\frac{3}{4}\right)$th the angle of prism. The angle of deviation is

Options:

A. 20°

B. 30°

C. 39°

D. 45°

Answer: B

Solution:

Given, angle of incidence = angle of emergence $= \frac{3}{4}$ A

Here, $A =$ angle of prism

$\because$ prism is equilateral, $A = 60^\circ$

$\therefore \quad i = e = 60^\circ \times \frac{3}{4} = 45^\circ$

From prism formula,

Angle of deviation, $\delta = i + e - A$

$= 45 + 45 - 60 = 30^\circ$

Question 129

The radii of curvature of both the surfaces of a convex lens of focal length f and power P are equal. One of the surfaces is made by plane grinding. The new focal length and focal power of the lens is

Options:

A. $\frac{2}{3}f, \frac{2}{3}P$

B. $\sqrt{\frac{2}{f}}, \sqrt{\frac{P}{2}}$

C. $\frac{f}{2}, 2P$

D. $2f, \frac{P}{2}$

Answer: D

Solution:

Since, the radius of curvature of both the surface is same,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (\mu - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\therefore \quad \frac{1}{f} = \frac{2(\mu - 1)}{R} \quad \text{.... (i)}$$

So, when one surface is made plane by grinding, we have

$R_1 = R$ and $R_2 = \infty$

Then, $\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R} - 0\right)$

$$\Rightarrow \quad \frac{1}{f'} = \frac{(\mu - 1)}{R} \quad \text{..... (ii)}$$

From Eqs. (i) and (ii), we get

So,

$\frac{f'}{f} = 2$

$\therefore \quad f' = 2f$

and power will be $\frac{P}{2}$. $\left(\because P = \frac{1}{f}\right)$

Question 130

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring balance reads $49\ \text{N}$, when the lift is stationary. If the lift moves downward with an acceleration of $5\ \text{m/s}^2$, the reading of the spring balance will be $\left(g = 9.8\ \text{m/s}^2\right)$

Options:

A. 15 N

B. 24 N

C. 49 N

D. 74 N

Answer: B

Solution:

When lift is stationary, reading of spring balance

$= 49$ N

i.e., weight of bag, $= 49$ N

$\Rightarrow \quad m = \frac{49}{g}$

When lift moves downward, $a = 5\ \mathrm{m/s^2}$

Weight (reading on spring balance) $= (g - a)\mathrm{m}$

$= (9.8 - 5) \times \frac{49}{9.8} = 24$ N

Question 131

In a L-R circuit the inductive reactance is equal to the resistance R in the circuit. An emf $E = E_0 \cos \omega t$ is applied to the circuit. The power consumed in the circuit is

Options:

A. $\frac{E_0^2}{\sqrt{2}R}$

B. $\frac{E_0^2}{4R}$

C. $\frac{E_0^2}{2R}$

D. $\frac{E_0^2}{8R}$

Answer: B

Solution:

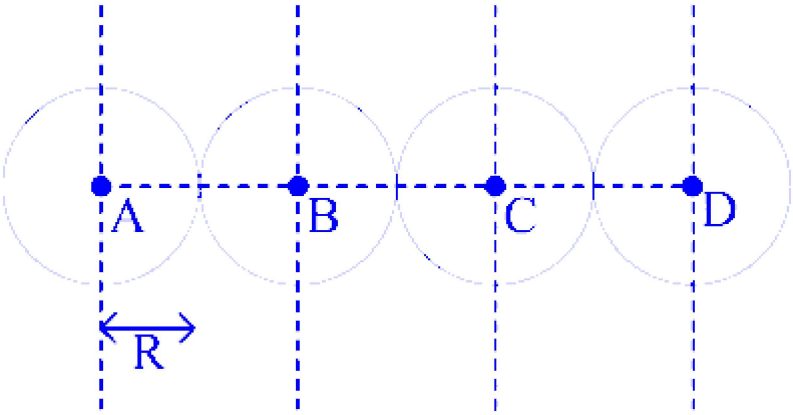
Given, inductive reactance $= R$

Power consumed $= E_{\mathrm{rms}} I_{\mathrm{rms}} \cos \phi$

$$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \frac{R}{Z} \quad \left(\because \cos\phi = \frac{R}{Z}\right)$$
$$= \frac{E_0}{\sqrt{2}} \times \frac{E_0}{Z\sqrt{2}} \times \frac{R}{Z} \quad \left(\because I_0 = \frac{E_0}{Z}\right)$$
$$= \frac{E_0^2 R}{2Z^2} \quad \left(\because X_L = R \Rightarrow Z = \sqrt{2}R\right)$$
$$= \frac{E_0^2}{4R}$$

Question 132

Four identical uniform solid spheres each of same mass M and radius R are placed touching each other as shown in figure with centres A, B, C, D. I_A, I_B, I_C, I_D are the moment of inertia of these spheres respectively about an axis passing through centre and perpendicular to the plane, then



Options:

A. $I_A > I_B > I_C > I_D$

B. $I_D > I_C > I_B > I_A$

C. $I_A = I_D > I_B = I_C$

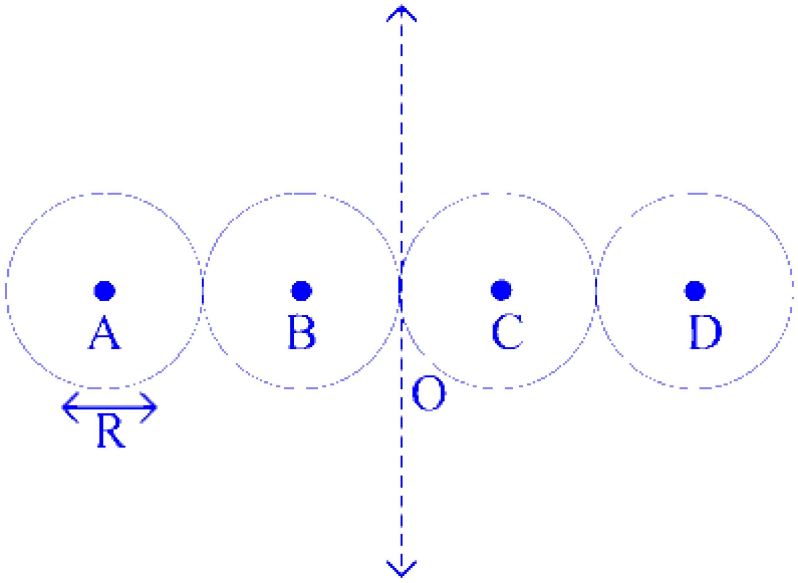
D. $I_A = I_D < I_B = I_C$

Answer: C

Solution:

The four identical uniform solid spheres are shown below. The radius of each sphere is R.

Lets take centre of the system at O.



Then, distance from the centre for $r_B = r_C$ and $r_A = r_D$

$\therefore$ Moment of inertial, $M_0 I = Mr^2$, for each spheres will be I_A, I_B, I_C and I_D.

$\therefore$ $I_A = I_D$ and $I_B = I_C$

Since, $r_B = r_C < r_A = r_D$

Therefore, $I_A = I_D > I_B = I_C$

Question 133

A transverse wave strike against a wall,

Options:

A. its phase changes by 180° but velocity does not change

B. its phase does not change but velocity changes

C. its velocity changes and phase changes by 180°

D. nothing can be predicted about changes in its velocity and phase

Answer: A

Solution:

When a traverse wave strikes a wall, it gets reflected back, the reflection causes of change in phase $= \pi$ or 180°.

The speed of a transverse wave remain constant in a medium.

Question 134

A circular current carrying coil has radius R. The magnetic induction at the centre of the coil is B_C. The magnetic induction of the coil at a distance $\sqrt{3}R$ from the centre along the axis is B_A. The ratio $B_A : B_C$ is

Options:

A. $1 : 3$

B. $1 : 8$

C. $8 : 1$

D. $27 : 1$

Answer: B

Solution:

Magnetic field on the axis of a circular coil,

$B_A = \frac{\mu_0 i R^2}{2(R^2+Z^2)^{3/2}}$

Magnetic field at the centre of the circular coil,

$B_C = \frac{\mu_0 i}{2R}$

According to question,

$$B_A = \frac{\mu_0 i R^2}{2(R^2+3R^2)^{3/2}} \quad (\because Z = \sqrt{3}R)$$

$$\Rightarrow \quad B_A = \frac{\mu_0 i R^2}{2 \times (4R^2)^{3/2}}$$

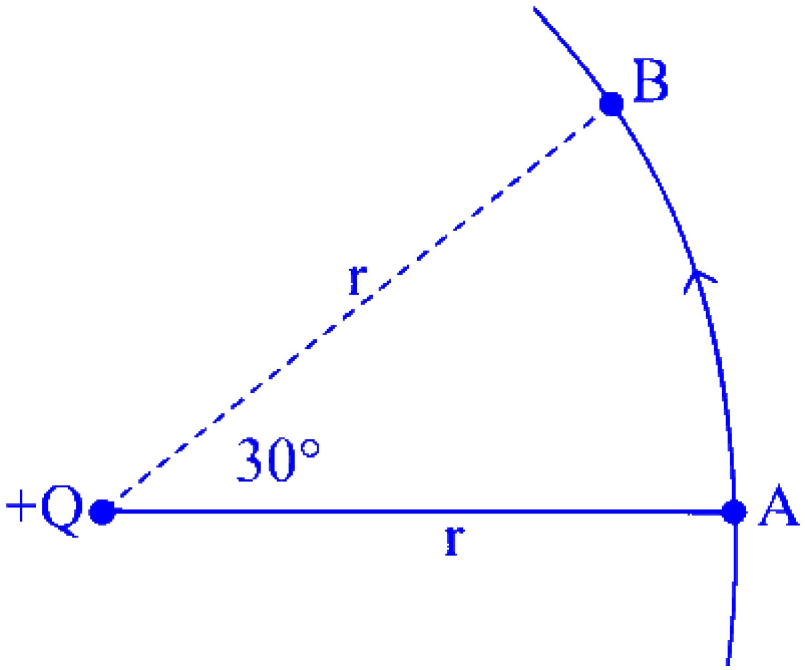
$$\Rightarrow \quad \frac{\mu_0 i R^2}{2 \times 8 \times R^3} = \frac{\mu_0 i R^2}{8 \times 2R^3}$$

$$= \frac{\mu_0 i}{8 \times 2R}$$

Thus, the ratio, $B_A : B_C = 1 : 8$

Question 135

In the electric field due to a charge Q, a charge q moves from point A to B. The work done is (ε_0 = permittivity of vacuum)



Options:

A. $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$

B. $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} \times \frac{\pi}{6}$

C. $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$

D. Zero

Answer: D

Solution:

The electric field due to a single charge has point symmetry, or single charged forms spherical equipotential surface.

The work done in moving a charge on this spherical surface is always zero.

Question 136

Of the two slits producing interference in Young's experiment, one is covered with glass so that light intensity passing is reduced to 50%. Which of the following is correct?

Options:

A. Intensity of fringes remains unaltered.

B. Intensity of bright fringe decreases and that of dark fringe increases.

C. Intensity of bright fringe increases and that of dark fringe decreases.

D. Intensity of both bright and dark fringes decreases.

Answer: B

Solution:

Initial intensity of bright fringes, $(\sqrt{I}+\sqrt{I})^2$

Final intensity of bright fringes after covering one slit with glass sheet $=\left(\sqrt{I}+\sqrt{\frac{I}{2}}\right)^2$

Clearly, the intensity of bright fringes has decreased. For dark fringes initial intensity $=(\sqrt{I}-\sqrt{I})^2$

But, new intensity $=\left(\sqrt{I}-\frac{\sqrt{I}}{2}\right)^2 \neq 0$

Hence, intensity of dark fringes increases.

Question 137

Magnetic shielding is done by surrounding the instrument to be protected from magnetic field by

Options:

A. soft ferromagnetic substance soft iron.

B. diamagnetic substance fine copper gauge.

C. paramagnetic substance aluminium.

D. paramagnetic material tantalum.

Answer: A

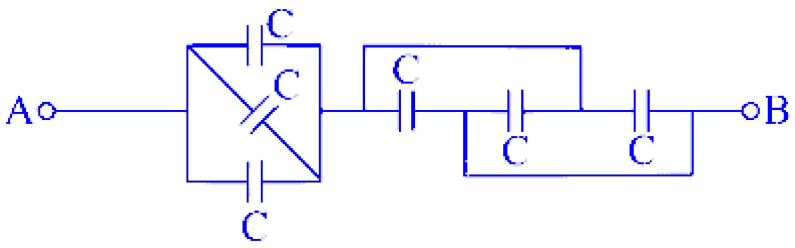
Solution:

For shielding, the magnetic field lines / flux should transfer via the shielding material not the object inside it.

Therefore, ferromagnetic, soft iron which attracts all field line (or the magnetic field lines prefer to more towards) towards itself, it used for magnetic shielding.

Question 138

In the given figure, the equivalent capacitance between points A and B is



Options:

A. 1.5 C

B. 2 C

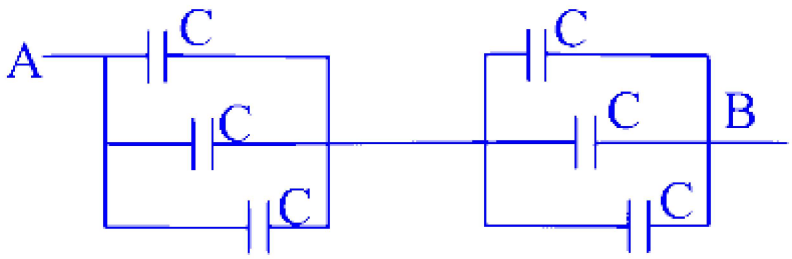
C. 3 C

D. 6 C

Answer: A

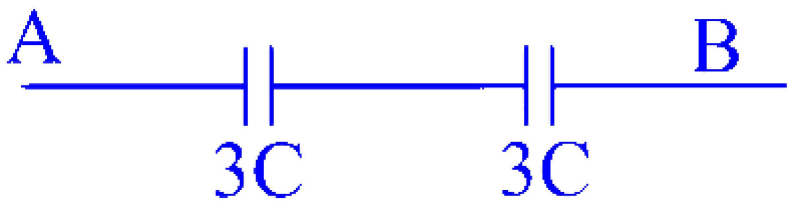
Solution:

The given circuit can also be redrawn as,



Thus, the parallel capacitors are calculated as $C_p = C + C + C = 3C$

Hence,



The equivalent capacitance between A and B is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{3C} + \frac{1}{3C}$$

$$\Rightarrow \quad \frac{1}{C_{\text{eq}}} = \frac{2}{3C}$$

$$\Rightarrow \quad C_{\text{eq}} = \frac{3}{2}C$$

$$\Rightarrow \quad C_{\text{eq}} = 1.5C$$

Question 139

If work done in blowing a soap bubble of volume V is W, then the work done in blowing the bubble of volume $2\ \text{V}$ from same soap solution is

Options:

A. $W/2$

B. $\sqrt{2}\ \text{W}$

C. $(2)^{1/3}W$

D. $(4)^{1/3}W$

Answer: D

Solution:

Work done in blowing a soap bubble, W = Surface Tension $\times$ change in area

$\Rightarrow \quad T \times \Delta A$

As, we know that,

$V = \frac{4}{3}\pi r^3$

and $A = 4\pi r^2$

$\Rightarrow \quad A \propto (V)^{\frac{2}{3}}$

or $\quad \Delta A \propto (\Delta V)^{\frac{2}{3}}$

According to the question,

$$\frac{W_1}{W_2} = \frac{\Delta A_1}{\Delta A_2} = \left(\frac{\Delta V_1}{\Delta V_2}\right)^{2/3}$$

$$\Rightarrow \quad W_2 = W_1\left(\frac{2V}{V}\right)^{2/3} = W \cdot (2)^{\frac{2}{3}} = (4)^{\frac{1}{3}} \cdot W$$

Question 140

Which one of the following is based on convection?

Options:

A. Heating of a copper utensil

B. Heating a room by heater

C. Heating of iron rod

D. Heat transferred from sun to earth

Answer: B

Solution:

Convection is a method of heat transfer, in which the molecules/particle travel from one place to another to transmit heat energy. Heating of copper and iron rod is an example of conduction and heat transfer from sun is due to radiation.

Question 141

A simple spring has length l and force constant K. It is cut in to two springs of length l_1 and l_2 such that $l_1 = nl_2$(n is an integer). The force constant of spring of length l_1 is

Options:

A. $K(1+n)$

B. $\frac{K(n+1)}{n}$

C. K

D. $\frac{K}{(n+1)}$

Answer: B

Solution:

Let, two parts of the springs have spring constants K_1 and K_2, respectively.

$$\because \quad \begin{matrix} l_1 = nI_2 \\ k_2 = nK_1 \end{matrix}$$

Now, before cutting into two parts l_1 and I_2, we have springs in series.

$$\begin{aligned} \text{i.e.,} \quad & \frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} \\ \Rightarrow \quad & \frac{1}{K} = \frac{1}{K_1} + \frac{1}{nK_1} \\ \Rightarrow \quad & \frac{1}{K} = \frac{n+1}{nK_1} \\ \Rightarrow \quad & K = \frac{nK_1}{(n+1)} \\ \Rightarrow \quad & K_1 = \frac{K(n+1)}{n} \end{aligned}$$

Question 142

A closed pipe and an open pipe have their first overtone equal in frequency. Then, the lengths of these pipe are in the ratio

Options:

A. $1:2$

B. $2:3$

C. $3:4$

D. $4:5$

Answer: C

Solution:

First overtone for open pipe, $V_{01}=\frac{2\mathrm{v}}{2I_0}$

First overtone for close pipe, $V_{C1}=\frac{3V}{4I_C}$

According to question, $V_{01}=V_{C1}$

$\Rightarrow \quad \frac{2\ \mathrm{V}}{2I_0}=\frac{3}{4}\frac{\mathrm{V}}{I_C}$

$\Rightarrow \quad \frac{I_C}{I_0}=\frac{3}{4}$

Question 143

A conductor $10\ \mathrm{cm}$ long is moves with a speed $1\ \mathrm{m/s}$ perpendicular to a field of strength $1000\ \mathrm{A/m}$. The emf induced in the conductor is (Given : $\mu_0=4\pi\times10^{-7}\ \mathrm{Wb/Am}$)

Options:

A. $\pi\ \mathrm{mV}$

B. $2\pi\ \mathrm{mV}$

C. $40\pi\mu$ V

D. $4\pi\mu$ V

Answer: C

Solution:

Given, length of conductor $(I) = 10$ cm $= 0.1$ m

Speed of conductor perpendicular to field, $v = 1$ m/s

Field strength, $H = 1000$ A/m

$\therefore$ Magnetic field, $B = \mu_0 H$

$= 4\pi \times 10^{-7} \times 1000 = 4\pi \times 10^{-4}$ T

Induced emf $(\varepsilon) = vBI$

$= 1 \times 4\pi \times 10^{-4} \times 0.1 = 4\pi \times 10^{-5}$ V
$= 40\pi\mu$V

Question 144

Dual nature of light is exhibited by

Options:

A. diffraction as well as photoelectric effect

B. diffraction as well as reflection

C. refraction as well as interference

D. photoelectric effect

Answer: A

Solution:

Both diffraction and photoelectric effect can be explained by the help of wave nature and particle nature of light.

Refraction and reflection is explained by the help of wave nature only.

Question 145

A carnot engine operates with source at $227^\circ\mathrm{C}$ and sink at $27^\circ\mathrm{C}$. If the source supplies $50\ \mathrm{kJ}$ of heat energy, the work done by the engine is

Options:

A. 2 kJ

B. 5 kJ

C. 10 kJ

D. 20 kJ

Answer: D

Solution:

Given, $T_{\text{source}} = 227^\circ\mathrm{C}$

$= 227 + 273 = 500\ \mathrm{K}$

$T_{\text{sink}} = 27^\circ\mathrm{C} = 27 + 273 = 300\ \mathrm{K}$

Efficiency of carnot engine, $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

$= 1 - \frac{300}{500} = \frac{2}{5}$

Also, efficiency, $\eta = \frac{\text{Work done by engine}}{\text{Heat supplied}}$

$$\frac{2}{5} = \frac{W}{50}$$

$$\Rightarrow \quad W = 20\ \mathrm{kJ}$$

Question 146

Doping of a semiconductor (with small impurity atoms) generally changes the resistivity as follows.

Options:

A. Decreases

B. Does not change

C. May increase or decrease depending on the dopant

D. Increase

Answer: A

Solution:

Doping increases the charge concentration of an intrinsic semiconductor. This increases conductivity, and so resistivity decreases.

Question 147

A stone is projected at angle θ with velocity u. If it executes nearly a circular motion at its maximum point for short time, then the radius of the circular path will be ($g =$ acceleration due to gravity)

Options:

A. $\frac{u^2}{g}$

B. $\frac{u^2 \cos^2 \theta}{g}$

C. $\frac{u^2 \sin^2 \theta}{g}$

D. $\frac{u^2 \cos^2 \theta}{2g}$

Answer: B

Solution:

Since, the particle experiences a downward acceleration due to gravity at peak (highest point), and traces a circular arc, it has no component of acceleration in x-direction.

Let, radius of circle be R.

Horizontal velocity of the particle of height point.

$v_x = u\cos\theta \quad (\because a_x = 0)$

$\therefore a = \frac{v_x^2}{R}$ (for circular motion centripetal acceleration $= \frac{v^2}{R}$)

$\Rightarrow R = \frac{v_x^2}{a} = \frac{u^2\cos^2\theta}{g} \quad (\because a = g)$

Question 148

When radiations of wavelength λ is incident on a metallic surface the stopping potential required is 4.8 V. If same surface is illuminated with radiations of double the wavelength, then required stopping potential becomes 1.6 V, then the value of threshold wavelength for the surface is

Options:

A. 2λ

B. 4λ

C. 6λ

D. 8λ

Answer: B

Solution:

From Einstein's photoelectric equation,

$$\begin{aligned} & \mathrm{KE}_{\max} &&= h\nu - h\nu_0 \\ \text{or} \quad & eV &&= h\nu - h\nu_0 \\ \Rightarrow \quad & eV &&= \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \end{aligned}$$

Here, V is stopping potential and λ is the incident wavelength, whereas λ_0 its threshold wavelength.

For case $1e(4.8) = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ..... (i)

For case $2e(1.6) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$ ..... (ii)

From Eqs. (i) and (ii), we get

$$e \times (3.2) = \frac{hc}{2\lambda}$$

$$\Rightarrow \quad e(1.6) = \frac{hc}{4\lambda} \quad \text{.... (iii)}$$

Putting the Eq. (iii) in Eq. (ii), we get

$$\Rightarrow \quad \frac{hc}{4\lambda} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$$

$$\Rightarrow \quad \frac{hc}{\lambda_0} = \frac{hc}{2\lambda} - \frac{hc}{4\lambda}$$

$$\Rightarrow \quad \frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$$

$$\Rightarrow \quad \lambda_0 = 4\lambda$$

Question 149

A thin uniform rod AB of mass m and length l is hinged at one end A to the ground level. Initially the rod stands vertically and is allowed to fall freely to the ground in the vertical plane. The angular velocity of the rod when its end B strikes the ground is (g = acceleration due to gravity)

Options:

A. $\sqrt{\frac{g}{l}}$

B. $\sqrt{\frac{mg}{l}}$

C. $\sqrt{\frac{3g}{l}}$

D. $\sqrt{\frac{mg}{3l}}$

Answer: C

Solution:

Let the length of the rod AB be l and mass of the rod be m.

Moment of inertia of the uniform rod about an axis passing through one of its end V given as,

$\text{MI} = \frac{ml^2}{3}$

Rotational kinetic energy, $(\text{KE})_{\text{rod}} = \frac{1}{2}l\omega^2$

Initially the rod is held, vertical, so potential energy of rod is given by $(\text{PE})_i = mg\frac{I}{2} \quad \left(\because \frac{l}{2} = \text{COM}\right)$

Initial kinetic energy, $(\text{KE})_i = 0$

The rod is allowed to fall, when it just touches the ground its potential energy becomes zero, i.e., $(\text{PE})_f = 0$

Let, angular velocity of rod at that instant be ω, then kinetic energy at that instant is

$(\text{KE})_f = \frac{1}{2}/\omega^2$

From conservation of energy,

$(\text{KE})_i + (\text{PE})_i = (\text{KE})_f + (\text{PE})_f$

$\Rightarrow \quad 0 + mg\frac{l}{2} = \frac{1}{2}l\omega^2 + 0$
$\Rightarrow \quad mgl = l\omega^2$
$\Rightarrow \quad mgl = \frac{ml^2}{3}\omega^2$
$\Rightarrow \quad \omega = \sqrt{\frac{3g}{l}}$

Question 150

Only 4% of the total current in the circuit passes through a galvanometer. If the resistance of the galvanometer is G, then the shunt resistance connected to the galvanometer is

Options:

A. $\frac{G}{25}$

B. $\frac{G}{24}$

C. 24G

D. 25G

Answer: B

Solution:

From the given figure, we have $(I - I_G)S = I_G G$

Image

$\Rightarrow \quad S = \frac{I_G G}{(I - I_G)}$

Here, $I_G = \frac{4I}{100} = \frac{I}{25}$

$$\therefore \quad S = \frac{\frac{1}{25} \times G}{I - \frac{1}{25}}$$
$$= \frac{\frac{G}{25}}{\frac{24}{25}} = \frac{1}{24} \times G$$
$$= \frac{G}{24}$$