MHT CET 2023 : 14th May Evening Shift

Mathematics

Question 1

Let $z \in C$ with $\operatorname{Im}(z)=10$ and it satisfies $\frac{2z-n}{2z+n}=2i-1, i=\sqrt{-1}$ for some natural number n, then

Options:

A. $\mathrm{n}=20$ and $\operatorname{Re}(\mathrm{z})=-10$

B. $\mathrm{n}=40$ and $\operatorname{Re}(\mathrm{z})=-10$

C. $\mathrm{n}=40$ and $\operatorname{Re}(\mathrm{z})=10$

D. $\mathrm{n}=20$ and $\operatorname{Re}(\mathrm{z})=10$

Answer: B

Solution:

$$
\begin{aligned}
& \operatorname{Im}(\mathrm{z})=10 \\
& \text { Let } \mathrm{z}=x+10 \mathrm{i} \\
& \frac{2 \mathrm{z}-\mathrm{n}}{2 \mathrm{z}+\mathrm{n}}=2 \mathrm{i}-1 \\
& \Rightarrow \frac{2(x+10 \mathrm{i})-\mathrm{n}}{2(x+10 \mathrm{i})+\mathrm{n}}=2 \mathrm{i}-1 \\
& \Rightarrow(2 x-\mathrm{n})+20 \mathrm{i}=(2 \mathrm{i}-1)(2 x+20 \mathrm{i}+\mathrm{n}) \\
& \Rightarrow(2 x-\mathrm{n})+20 \mathrm{i}=(-2 x-\mathrm{n}-40)+(4 x+2 \mathrm{n}-20) \mathrm{i}
\end{aligned}
$$

Equating real and imaginary parts, we get

$$
\begin{aligned}
& 2 x-\mathrm{n}=-2 x-\mathrm{n}-40 \text { and } 20=4 x+2 \mathrm{n}-20 \\
& \Rightarrow x=-10 \text { and } 20=4(-10)+2 \mathrm{n}-20 \\
& \Rightarrow x=-10 \text { and } \mathrm{n}=40
\end{aligned}
$$

Question 2

Let $\bar{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{b} = \hat{i} + \hat{j}$. If $\bar{c}$ is a vector such that $\bar{a} \cdot \bar{c} = |\bar{c}|, |\bar{c} - \bar{a}| = 2\sqrt{2}$ and the angle between $\bar{a} \times \bar{b}$ and $\bar{c}$ is $\frac{2\pi}{3}$, then $|(\bar{a} \times \bar{b}) \times \bar{c}| =$

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{3\sqrt{3}}{2}$

C. $3\sqrt{3}$

D. $4\sqrt{3}$

Answer: B

Solution:

$|\bar{c} - \vec{a}| = 2\sqrt{2}$
$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2(\bar{a} \cdot \bar{c}) = 8$
$\Rightarrow |\vec{c}|^2 + 9 - 2|\vec{c}| = 8 \quad \ldots [\because \bar{a} \cdot \bar{c} = |\vec{c}| \text{ (given) }]$
$\Rightarrow (|\vec{c}| - 1)^2 = 0$
$\Rightarrow |\vec{c}| = 1$

Now, $|(\bar{a} \times \bar{b}) \times \bar{c}| = |\bar{a} \times \bar{b}||\vec{c}| \sin \frac{2\pi}{3}$

$= |\bar{a} \times \bar{b}|(1) \left(\frac{\sqrt{3}}{2}\right)$

$= \frac{3\sqrt{3}}{2}$

$\ldots [\because \bar{a} \times \bar{b} = 2\hat{i} - 2\hat{j} + \hat{k}]$

Question 3

If both mean and variance of 50 observations $x_1, x_2, \ldots, x_{50}$ are equal to 16 and 256 respectively, then mean of

$(x_1-5)^2, (x_2-5)^2, \ldots\ldots, (x_{50}-5)^2$ **is**

Options:

A. 357

B. 367

C. 377

D. 387

Answer: C

Solution:

$$\text{Mean } = \frac{\sum x_\text{i}}{\text{n}}$$
$$\Rightarrow 16 = \frac{\sum x_\text{i}}{50}$$
$$\Rightarrow \sum x_\text{i} = 800$$
$$\text{Variance } = \frac{\sum x_\text{i}^2}{\text{n}} - (\bar{x})^2$$
$$\Rightarrow 256 = \frac{\sum x_\text{i}^2}{50} - (16)^2$$
$$\Rightarrow \frac{\sum x_\text{i}^2}{50} = 512$$
$$\Rightarrow \sum x_\text{i}^2 = 25600$$
$$\begin{aligned} \text{New mean } &= \frac{\sum (x_\text{i}-5)^2}{\text{n}} \\ &= \frac{\sum x_\text{i}^2 - 10\sum x_\text{i} + \sum 25}{50} \\ &= \frac{25600 - 10(800) + 25(50)}{50} \\ &= \frac{18850}{50} \\ &= 377 \end{aligned}$$

Question 4

If the statement $\text{p} \leftrightarrow (\text{q} \rightarrow \text{p})$ is false, then true statement/statement pattern is

Options:

A. p

B. $\mathrm{p} \rightarrow (\mathrm{p} \vee \sim \mathrm{q})$

C. $\mathrm{p} \wedge (\sim \mathrm{p} \wedge \mathrm{q})$

D. $(p \vee \sim q) \rightarrow p$

Answer: B

Solution:

$\mathrm{p} \leftrightarrow (\mathrm{q} \rightarrow \mathrm{p})$ is false

$\Rightarrow \mathrm{p} \equiv \mathrm{F}$ and $\mathrm{q} \equiv \mathrm{F}$

Consider option (B),

$$\begin{aligned} \mathrm{p} \rightarrow (\mathrm{p} \vee \sim \mathrm{q}) &\equiv \mathrm{F} \rightarrow (\mathrm{F} \vee \sim \mathrm{F}) \\ &\equiv \mathrm{F} \rightarrow (\mathrm{F} \vee \mathrm{T}) \\ &\equiv \mathrm{F} \rightarrow \mathrm{T} \\ &\equiv \mathrm{T} \end{aligned}$$

Question 5

If $|\bar{a}| = 2, |\bar{b}| = 3, |\bar{c}| = 5$ and each of the angles between the vectors $\bar{a}$ and $\bar{b}, \bar{b}$ and $\bar{c}$, $\bar{c}$ and $\bar{a}$ is 60°, then the value of $|\bar{a} + \bar{b} + \bar{c}|$ is

Options:

A. $\sqrt{69}$

B. $\sqrt{70}$

C. $\sqrt{80}$

D. $\sqrt{39}$

Answer: A

Solution:

$$\begin{aligned}\bar{a}\cdot\bar{b} &= |\vec{a}||\vec{b}|\cos 60^\circ\\ &= (2)(3)\left(\frac{1}{2}\right)\\ &= 3\end{aligned}$$

$$\begin{aligned}\bar{b}\cdot\bar{c} &= |\bar{b}||\bar{c}|\cos 60^\circ\\ &= (3)(5)\left(\frac{1}{2}\right) = \frac{15}{2}\end{aligned}$$

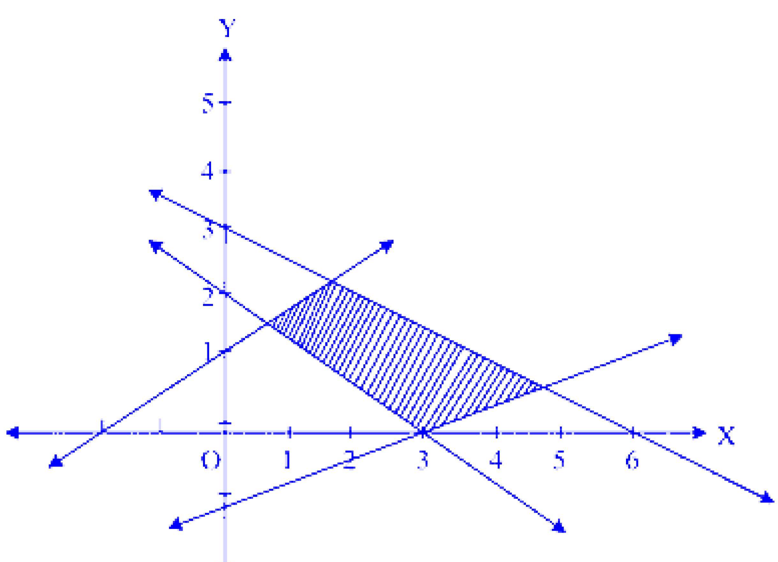
$$\begin{aligned}\bar{a}\cdot\bar{c} &= |\vec{a}|\vec{c}\cos 60^\circ\\ &= (2)(5)\left(\frac{1}{2}\right)\\ &= 5\end{aligned}$$

$$\begin{aligned}\therefore\quad |\bar{a}+\bar{b}+\bar{c}|^2 &= |\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2(\bar{a}\cdot\bar{b}+\bar{b}\cdot\bar{c}+\bar{c}\cdot\bar{a})\\ &= 2^2+3^2+5^2+2\left(3+\frac{15}{2}+5\right)\\ &= 4+9+25+31\\ &= 69\end{aligned}$$

$$\therefore\quad |\bar{a}+\bar{b}+\bar{c}| = \sqrt{69}$$

Question 6

The shaded region in the following figure represents the solution set for a certain linear programming problem. Then linear constraints for this region are given by



Options:

A. $2x+3y\geq 6, -x+2y\geq 2, 3x+6y\leq 18, x-3y\geq 3, x\geq 0, y\geq 0$

B. $2x+3y\geq 6, -x+2y\leq 2, x-3y\leq 3x+2y\geq 18, x\geq 0, y\geq 0$

C. $2x+3y\leq 6, -x+2y\geq 2, 3x+6y\leq 18x-3y\leq 3, x\geq 0, y\geq 0$

D. $2x+3y\geq 6, 3x+6y\leq 18, x-3y\leq 3-x+2y\leq 2, x\geq 0, y\geq 0$

Answer: D

Solution:

Shaded region lies on origin side of $3x+6y=18, x-3y=3, -x+2y=2$ and on non-origin side of $2x+3y=6$.

$\therefore 2x+3y\geq 6, 3x+6y\leq 18, x-3y\leq 3, -x+2y\leq 2, x\geq 0, y\geq 0$

Question 7

The function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $\mathrm{f}(x)=\begin{cases}\frac{1}{x}\log\left(\frac{1+3x}{1-2x}\right) & , \quad x\neq 0\\ \mathrm{k} & , \quad x=0\end{cases}$ is continuous at $x=0$, then k is

Options:

A. 6

B. 1

C. 5

D. -5

Answer: C

Solution:

f is continuous at $x=0$.

$$\therefore \quad \mathrm{f}(0) = \lim_{x \to 0} \mathrm{f}(x)$$

$$\therefore \quad \mathrm{k} = \lim_{x \to 0} \left(\frac{1}{x} \log(1+3x) - \frac{1}{x} \log(1-2x) \right)$$

$$= \lim_{x \to 0} \left(\frac{3 \log(1+3x)}{3x} + \frac{2 \log(1-2x)}{-2x} \right)$$

$$= 3 + 2 = 5$$

Question 8

The mirror image of $\mathrm{P}(2, 4, -1)$ in the plane $x - y + 2z - 2 = 0$ is $(\mathrm{a}, \mathrm{b}, \mathrm{c})$, then the value of $a + b + c$ is

Options:

A. 4

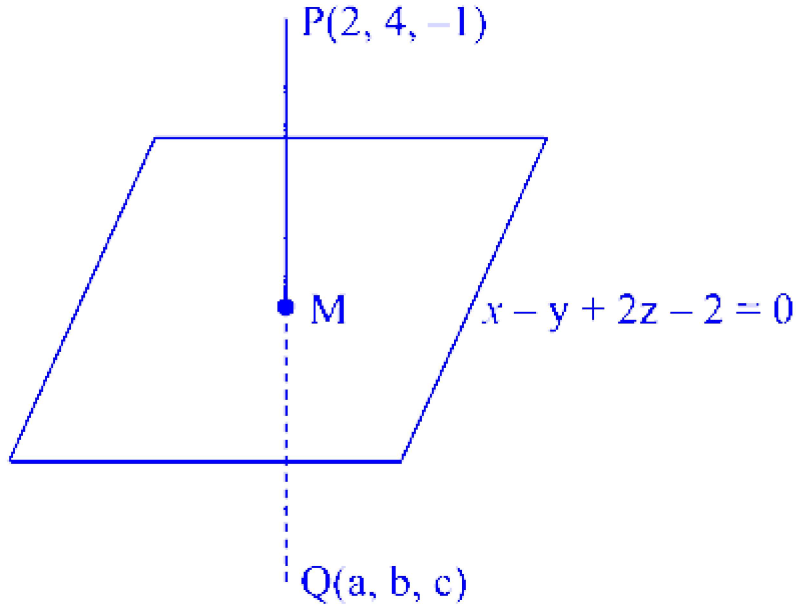
B. 5

C. 7

D. 9

Answer: D

Solution:



The d.r.s. of the normal to the plane are $1, -1, 2$.

$\therefore$ The equation of line PM is

$$\frac{x-2}{1}=\frac{y-4}{-1}=\frac{z+1}{2}=\lambda\text{ (say)}$$
$\Rightarrow x=\lambda+2, y=-\lambda+4, z=2\lambda-1$

Let $M\equiv(\lambda+2,-\lambda+4,2\lambda-1)$

$\therefore$ Equation of plane becomes

$\therefore$ Equation of plane becomes
$1(\lambda+2)-1(-\lambda+4)+2(2\lambda-1)-2=0$
$\Rightarrow \lambda=1$
$\therefore$ $M\equiv(3,3,1)$

Since M is the mid-point of PQ.

$\therefore \frac{2+a}{2}=3, \frac{4+b}{2}=3, \frac{-1+c}{2}=1$
$\Rightarrow a=4, b=2, c=3$
$\Rightarrow a+b+c=9$

Question 9

If the slope of the tangent of the curve at any point is equal to $-y+\mathrm{e}^{-x}$, then the equation of the curve passing through origin is

Options:

A. $y + x\mathrm{e}^{x} = 0$

B. $y\mathrm{e}^{x} + x = 0$

C. $y\mathrm{e}^{x} - x = 0$

D. $y - x\mathrm{e}^{x} = 0$

Answer: C

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -y + \mathrm{e}^{-x}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} + y = \mathrm{e}^{-x}$$
$\therefore$ I.F. $= \mathrm{e}^{\int \mathrm{dx}} = \mathrm{e}^{x}$

$\therefore$ Solution of the given equation is

$$y\mathrm{e}^{x} = \int \mathrm{e}^{x} \cdot \mathrm{e}^{-x}\, \mathrm{d}x + c$$
$$\Rightarrow y\mathrm{e}^{x} = \int \mathrm{d}x + c$$
$$\Rightarrow y\mathrm{e}^{x} = x + \mathrm{c} \quad \text{.... (i)}$$

Since the curve passes through $(0, 0)$.

$\therefore \quad 0 = 0 + \mathrm{c}$

$\Rightarrow \mathrm{c} = 0$

$\therefore \quad y\mathrm{e}^{x} = x$ ... [From (i)]

$\Rightarrow y\mathrm{e}^{x} - x = 0$

Question 10

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$, then $(A+B)^{-1}$ is

Options:

A. $\begin{bmatrix} \frac{-1}{2} & 0 \\ \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$

B. $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$

C. $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$

D. $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

Answer: B

Solution:

$$A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 6 & -2 \end{bmatrix}$$

$$\therefore \quad |A + B| = \begin{vmatrix} 2 & 0 \\ 6 & -2 \end{vmatrix} = -4 \neq 0$$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad - bc \neq 0$,

then $A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$\therefore \quad (A + B)^{-1} = \frac{1}{-4} \begin{bmatrix} -2 & 0 \\ -6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

Question 11

The function $\mathrm{f}(x) = x^3 - 6x^2 + 9x + 2$ has maximum value when x is

Options:

A. 1

B. 2

C. 3

D. 6

Answer: A

Solution:

$$\begin{aligned} & f(x)=x^3-6x^2+9x+2 \\ \therefore \quad & f'(x)=3x^2-12x+9 \end{aligned}$$

For maximum or minimum,

$\mathrm{f}'(x)=0$
$\Rightarrow 3x^2-12x+9=0$
$\Rightarrow 3(x-1)(x-3)=0$
$\Rightarrow x=1,3$

Now, $\mathrm{f}''(x)=6x-12$

$\therefore \quad \mathrm{f}''(1)=-6<0$

$\therefore \quad \mathrm{f}(x)$ is maximum at $x=1$.

Question 12

If $I_n=\int\limits_0^{\frac{\pi}{4}} \tan^n \theta d\theta$, then $I_{12}+I_{10}=$

Options:

A. $\frac{1}{8}$

B. $\frac{1}{12}$

C. $\frac{1}{11}$

D. $\frac{1}{10}$

Answer: C

Solution:

$$\int_0^{\frac{\pi}{4}} (\tan^n x + \tan^{n-2} x)\,dx = \frac{1}{n-1}$$

$$\therefore \quad I_{12} + I_{10} = \int_0^{\frac{\pi}{4}} (\tan^{12}\theta + \tan^{10}\theta)\,d\theta$$

$$= \frac{1}{12-1}$$

$$= \frac{1}{11}$$

Question 13

The centre of the circle whose radius is 3 units and touching internally the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point $(-1, -1)$ is

Options:

A. $\left(\frac{4}{5}, \frac{7}{5}\right)$

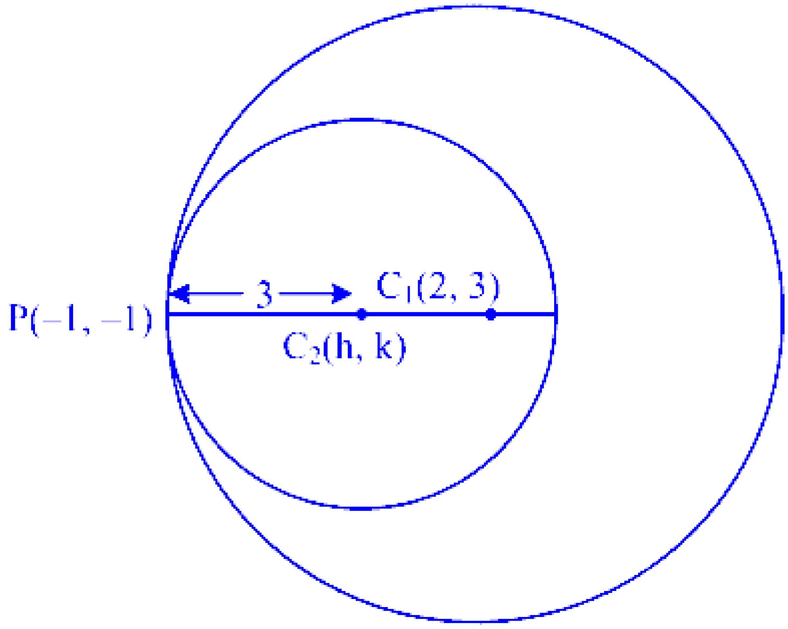
B. $\left(\frac{4}{5}, \frac{-7}{5}\right)$

C. $\left(\frac{-4}{5}, \frac{-7}{5}\right)$

D. $\left(\frac{-4}{5}, \frac{7}{5}\right)$

Answer: A

Solution:



$PC_1 = \sqrt{(2+1)^2 + (3+1)^2}$

$= \sqrt{25}$

$= 5$

P divides C_1C_2 externally in the ratio $r_1 : r_2$ i.e. $5 : 3$

$\therefore \quad -1 = \frac{5(h) - 3(2)}{5 - 3}$ and $-1 = \frac{5(k) - 3(3)}{5 - 3}$

$\Rightarrow -2 = 5h - 6$ and $-2 = 5k - 9$

$\Rightarrow h = \frac{4}{5}$ and $k = \frac{7}{5}$

Question 14

A fair die with numbers 1 to 6 on their faces is thrown. Let X denote the number of factors of the number, on the uppermost face, then the probability distribution of X is

Options:

A.

$X = x$	1	2	3	4
$P(x = x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$

B.

$X = x$	1	2	3	4
$P(x = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

C.

$X = x$	1	2	3	4
$P(x = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

D.

$X = x$	1	2	3	4
$P(x = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$

Answer: A

Solution:

$S = \{1, 2, 3, 4, 5, 6\}$

The values of X for the possible outcomes of the experiment are as follows:

experiment are as follows:
$X(1) = 1, X(2) = 2, X(3) = 2, X(4) = 3$
$X(5) = 2, X(6) = 4$

$P(X = 1) = P[\{1\}] = \frac{1}{6}$

$P(X = 2) = P[\{2, 3, 5\}] = \frac{3}{6} = \frac{1}{2}$

$P(X = 3) = P[\{4\}] = \frac{1}{6}$

$P(X = 4) = P[\{6\}] = \frac{1}{6}$

The probability distribution of X is

X $=x$	1	2	3	4
$\mathrm{P}(\mathrm{x}=x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$

Question 15

Let $\overline{\mathrm{u}}, \overline{\mathrm{v}}$ and $\overline{\mathrm{w}}$ be the vectors such that $|\overline{\mathrm{u}}|=1; |\bar{v}|=2; |\bar{w}|=3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to that of $\overline{\mathrm{w}}$ along $\overline{\mathrm{u}}$ and $\overline{\mathrm{v}}, \overline{\mathrm{w}}$ are perpendicular to each other, then $|\bar{u}-\bar{v}+\bar{w}|$ is equal to

Options:

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: C

Solution:

Projection of $\overline{\mathrm{v}}$ along $\overline{\mathrm{u}}=$ Projection of $\overline{\mathrm{w}}$ along $\bar{u}$

$$\Rightarrow \frac{\overline{\mathrm{v}} \cdot \overline{\mathrm{u}}}{|\overrightarrow{\mathrm{u}}|}=\frac{\overline{\mathrm{w}} \cdot \overline{\mathrm{u}}}{|\overrightarrow{\mathrm{u}}|}$$

$\Rightarrow \overline{\mathrm{v}} \cdot \overline{\mathrm{u}}=\overline{\mathrm{w}} \cdot \overline{\mathrm{u}}$ .... (i)

Also, $\overline{\mathrm{v}}$ and $\overline{\mathrm{w}}$ are perpendicular to each other.

$\therefore \quad \overline{\mathrm{v}} \cdot \overline{\mathrm{w}}=0$ ..... (ii)

Now, $|\overline{\mathrm{u}}-\overline{\mathrm{v}}+\overline{\mathrm{w}}|^2=|\overline{\mathrm{u}}|^2+|\overline{\mathrm{v}}|^2+|\overline{\mathrm{w}}|^2-2(\overline{\mathrm{u}} \cdot \overline{\mathrm{v}})-2(\bar{v} \cdot \bar{w})+2(\bar{u} \cdot \bar{w})$

$\Rightarrow |\overline{\mathrm{u}}-\overline{\mathrm{v}}+\overline{\mathrm{w}}|^2=1+4+9$ ....[From (i) and (ii)]

$\Rightarrow |\overline{\mathrm{u}}-\overline{\mathrm{v}}+\overline{\mathrm{w}}|=\sqrt{14}$

Question 16

If $y = 4x - 5$ is a tangent to the curve $y^2 = \mathrm{p}x^3 + \mathrm{q}$ at $(2, 3)$, then $\mathrm{p} - \mathrm{q}$ is

Options:

A. -5

B. 5

C. 9

D. -9

Answer: C

Solution:

$y^2 = px^3 + q$ .... (i)

Differentiating both sides w.r.t. x, we get

$$2y \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = 3\mathrm{p}x^2$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3\mathrm{p}}{2}\left(\frac{x^2}{y}\right)$$
$$\therefore \quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(2,3)} = \frac{3\mathrm{p}}{2} \times \frac{4}{3} = 2\mathrm{p}$$

Slope of the line $y = 4x - 5$ is 4.

Since the line touches the curve, their slopes are equal.

$\therefore \quad 2\mathrm{p} = 4 \Rightarrow \mathrm{p} = 2$

Since $(2, 3)$ lies on $y^2 = \mathrm{p}x^3 + \mathrm{q}$.

$$\therefore \quad 9 = 2 \times 8 + q \Rightarrow q = -7$$
$$\therefore \quad p - q = 2 + 7 = 9$$

Question 17

If $x=\sqrt{\mathrm{e}^{\sin^{-1}t}}$ and $y=\sqrt{\mathrm{e}^{\cos^{-1}t}}$, then $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is

Options:

A. $\frac{-y}{x^2}$

B. $\frac{y^2}{2x^2}$

C. $\frac{2y}{x^2}$

D. $\frac{-2y}{x^2}$

Answer: C

Solution:

$$\begin{aligned} xy &= \sqrt{\mathrm{e}^{\sin^{-1}t}}\cdot\sqrt{\mathrm{e}^{\cos^{-1}t}} \\ &= \sqrt{\mathrm{e}^{\sin^{-1}t+\cos^{-1}t}} \\ \therefore \quad xy &= \sqrt{\mathrm{e}^{\frac{\pi}{2}}} \end{aligned}$$

Differentiating both sides w.r.t. x, we get

$$x\frac{\mathrm{d}y}{\mathrm{d}x}+y\cdot 1=0$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y}{x} \quad \text{.... (i)}$$

$$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\left(\frac{x\frac{\mathrm{d}y}{\mathrm{d}x}-y\cdot 1}{x^2}\right)$$

$$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\left(\frac{x\left(-\frac{y}{x}\right)-y}{x^2}\right) \quad \text{... [From (i)]}$$

$$=\frac{2y}{x^2}$$

Question 18

If $\sum_{r=1}^{50}\tan^{-1}\frac{1}{2r^2}=p$ then $\tan p$ is

Options:

A. $\frac{100}{101}$

B. $\frac{51}{50}$

C. $\frac{50}{51}$

D. $\frac{101}{102}$

Answer: C

Solution:

$$\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$$
$$\Rightarrow \sum_{r=1}^{50} \tan^{-1} \left(\frac{2}{4r^2}\right) = p$$
$$\Rightarrow \sum_{r=1}^{50} \tan^{-1} \left[\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right] = p$$
$$\Rightarrow \sum_{r=1}^{50} \left[\tan^{-1}(2r+1) - \tan^{-1}(2r-1)\right] = p$$
$$\Rightarrow \tan^{-1}(101) - \tan^{-1}(1) = p$$
$$\Rightarrow \tan^{-1}\left(\frac{101-1}{1+101}\right) = p$$
$$\Rightarrow \frac{100}{102} = \tan p$$
$$\Rightarrow \tan p = \frac{50}{51}$$

Question 19

The value of $\int e^x \left(\frac{x^2+4x+4}{(x+4)^2}\right) dx$ is :

Options:

A. $e^x \left(\frac{x}{x+4}\right) + c$, where c is a constant of integration.

B. $e^x \left(\frac{4}{x+4}\right) + c$, where c is a constant of integration.

C. $\mathrm{e}^{x}\left(\frac{x}{(x+4)^2}\right)+\mathrm{c}$, where c is a constant of integration.

D. $\mathrm{e}^{x}\left(\frac{4}{(x+4)^2}\right)+\mathrm{c}$, where c is a constant of integration.

Answer: A

Solution:

$$\int \mathrm{e}^{x}\left[\frac{x^2+4x+4}{(x+4)^2}\right]\mathrm{d}x$$
$$=\int \mathrm{e}^{x}\left[\frac{x(x+4)+4}{(x+4)^2}\right]\mathrm{d}x$$
$$=\int \mathrm{e}^{x}\left[\frac{x}{x+4}+\frac{4}{(x+4)^2}\right]\mathrm{d}x$$
$$=\mathrm{e}^{x}\left(\frac{x}{x+4}\right)+\mathrm{c}$$

$\ldots\left[\because \int \mathrm{e}^{x}\left[\mathrm{f}(x)+\mathrm{f}'(x)\right]\mathrm{d}x=\mathrm{e}^{x}\mathrm{f}(x)+\mathrm{c}\right]$

Question 20

The diagonal of a square is changing at the rate of $0.5\ \mathrm{cm/sec}$. Then the rate of change of area when the area is $400\ \mathrm{cm}^2$ is equal to

Options:

A. $20\sqrt{2}\ \mathrm{cm}^2/\mathrm{sec}$

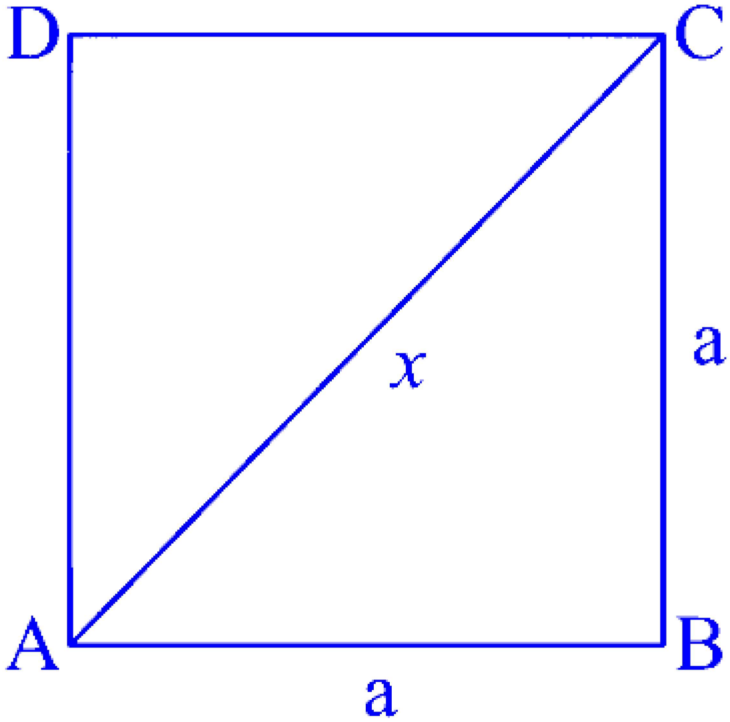
B. $10\sqrt{2}\ \mathrm{cm}^2/\mathrm{sec}$

C. $\frac{1}{10\sqrt{2}}\ \mathrm{cm}^2/\mathrm{sec}$

D. $\frac{10}{\sqrt{2}}\ \mathrm{cm}^2/\mathrm{sec}$

Answer: B

Solution:



$$\frac{\mathrm{d}x}{\mathrm{dt}} = 0.5 \mathrm{~cm/sec}$$

$$\text{Area } = \frac{x^2}{2}$$

$$\frac{\mathrm{dA}}{\mathrm{dt}} = \frac{2x}{2} \cdot \frac{\mathrm{d}x}{\mathrm{dt}} = x\frac{\mathrm{d}x}{\mathrm{dt}} = \frac{1}{2}x$$

$$\left[\frac{\mathrm{dA}}{\mathrm{dt}}\right]_{\mathrm{A}=400} = \frac{1}{2}\sqrt{800} \quad \ldots \left[\begin{array}{l} \because \mathrm{A} = 400 \mathrm{~cm}^2 \\ x = \sqrt{800} \mathrm{~cm} \end{array}\right]$$

$$= 10\sqrt{2} \mathrm{~cm}^2/\mathrm{sec}$$

Question 21

Let $\bar{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\bar{b} = \hat{i} + \hat{j} - \hat{k}$ be two vectors. If $\bar{c}$ is a vector such that $\bar{b} \times \bar{c} = \bar{b} \times \bar{a}$ and $\overline{\mathrm{c}} \cdot \overline{\mathrm{a}} = 0$, then $\overline{\mathrm{c}} \cdot \overline{\mathrm{b}}$ is

Options:

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{-3}{2}$

D. $\frac{-1}{2}$

Answer: D

Solution:

Given, $\overline{\mathrm{b}} \times \overline{\mathrm{c}} = \overline{\mathrm{b}} \times \overline{\mathrm{a}}$
$\Rightarrow \overline{\mathrm{b}} \times (\overline{\mathrm{c}} - \overline{\mathrm{a}}) = \overline{0}$
$\Rightarrow \overline{\mathrm{b}}$ is parallel to $(\overline{\mathrm{c}} - \overline{\mathrm{a}})$.
$\Rightarrow \overline{\mathrm{c}} - \overline{\mathrm{a}} = \lambda \overline{\mathrm{b}}$ for some scalar λ
$\Rightarrow \overline{\mathrm{c}} = \overline{\mathrm{a}} + \lambda \overline{\mathrm{b}}$ .... (i)
$\Rightarrow \overline{\mathrm{c}} \cdot \overline{\mathrm{a}} = \overline{\mathrm{a}} \cdot \overline{\mathrm{a}} + \lambda(\overline{\mathrm{b}} \cdot \overline{\mathrm{a}})$
$\Rightarrow 0 = |\ldots|^2 + \lambda(\overline{\mathrm{b}} \cdot \overline{\mathrm{a}}) \quad \ldots [\because \overline{\mathrm{c}} \cdot \overline{\mathrm{a}} = 0 \text{ (given)}]$
$\Rightarrow 0 = 6 + 4\lambda$
$\Rightarrow \lambda = -\frac{3}{2}$

Substituting the value of λ in (i), we get

$$\begin{aligned} \bar{c} &= (\hat{i} + 2\hat{j} - \hat{k}) - \frac{3}{2}(\hat{i} + \hat{j} - \hat{k}) \\ &= -\frac{1}{2}(\hat{i} - \hat{j} - \hat{k}) \\ \therefore \quad \bar{c} \cdot \bar{b} &= -\frac{1}{2}(\hat{i} - \hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) \\ &= -\frac{1}{2}(1 - 1 + 1) = -\frac{1}{2} \end{aligned}$$

Question 22

Let $P \equiv (-3, 0), Q \equiv (0, 0)$ and $R \equiv (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is

Options:

A. $\frac{\sqrt{3}}{2}x + y = 0$

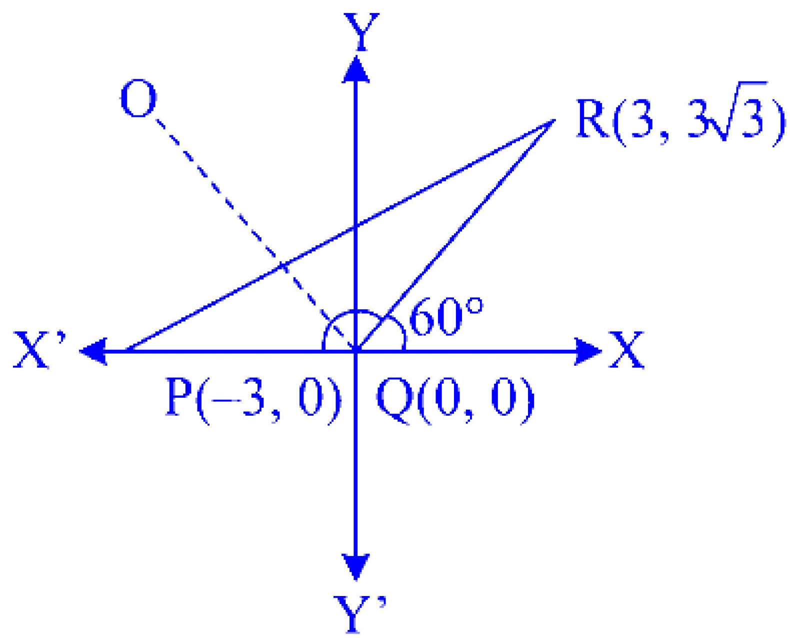
B. $x + \sqrt{3}y = 0$

C. $\sqrt{3}x + y = 0$

D. $x + \frac{\sqrt{3}}{2} y = 0$

Answer: C

Solution:



Slope of $QR = \frac{3\sqrt{3}-0}{3-0} = \sqrt{3}$ i.e., $\theta = 60^{\circ}$

Clearly, $\angle \mathrm{PQR} = 120^{\circ}$

OQ is the angle bisector of the angle PQR, so line OQ makes 120° with the positive direction of X-axis.

Therefore, equation of the bisector of $\angle \mathrm{PQR}$ is $y = \tan 120^{\circ} x \Rightarrow y = -\sqrt{3} x \Rightarrow \sqrt{3} x + y = 0$

Question 23

If in a regular polygon, the number of diagonals are 54, then the number of sides of the polygon are

Options:

A. 10

B. 12

C. 9

D. 6

Answer: B

Solution:

Number of diagonals in a polygon of n sides is ${}^{\mathrm{n}}C_2 - \mathrm{n}$.

$$\begin{aligned} \therefore \quad & {}^{\mathrm{n}}C_2 - \mathrm{n} = 54 \\ & \Rightarrow \frac{\mathrm{n}(\mathrm{n}-1)}{2} - \mathrm{n} = 54 \\ & \Rightarrow \mathrm{n}^2 - 3\mathrm{n} - 108 = 0 \\ & \Rightarrow (n-12)(\mathrm{n}+9) = 0 \end{aligned}$$

But, number of sides cannot be negative,

$\therefore$ n = 12

Question 24

Let x_0 be the point of local minima of $\mathrm{f}(x) = \overline{\mathrm{a}} \cdot (\overline{\mathrm{b}} \times \overline{\mathrm{c}})$ where $\overline{\mathrm{a}} = x\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}, \overline{\mathrm{b}} = -2\hat{\mathrm{i}} + x\hat{\mathrm{j}} - \hat{\mathrm{k}}, \overline{\mathrm{c}} = 7\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + x\hat{\mathrm{k}}$, then value of $\overline{\mathrm{a}} \cdot \overline{\mathrm{b}}$ at $x = x_0$ is

Options:

A. 15

B. -15

C. 12

D. -12

Answer: B

Solution:

$$f(x)=\overline{\mathrm{a}}\cdot(\overline{\mathrm{b}}\times\overline{\mathrm{c}})$$
$$=\begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}$$
$$=x\left(x^2-2\right)+2(-2x+7)+3(4-7x)$$
$$\therefore \quad \mathrm{f}(x)=x^3-27x+26$$

Now, $\mathrm{f}'(x)=0$

$$\Rightarrow 3x^2-27=0$$
$$\Rightarrow x^2=9$$
$$\Rightarrow x=\pm 3$$
$$\mathrm{f}''(x)=6x$$
$$\Rightarrow \mathrm{f}''(x)=18>0$$

$\mathrm{f}(x)$ has local minimum at $x=3$.

$$\therefore \quad \bar{a}=3\hat{i}-2\hat{j}+3\hat{k}$$
$$\bar{b}=-2\hat{i}+3\hat{j}-\hat{k}$$
$$\therefore \quad \bar{a}\cdot\bar{b}=(3\hat{i}-2\hat{j}+3\hat{k})\cdot(-2\hat{i}+3\hat{j}-\hat{k})$$
$$=3(-2)+(-2)(3)+3(-1)$$
$$=-15$$

Question 25

If a body cools from $80^{\circ}\mathrm{C}$ to $50^{\circ}\mathrm{C}$ in the room temperature of $25^{\circ}\mathrm{C}$ in 30 minutes, then the temperature of the body after 1 hour is

Options:

A. $31.36^{\circ}\mathrm{C}$

B. $32.25^{\circ}\mathrm{C}$

C. $36.36^{\circ}\mathrm{C}$

D. $33.25^{\circ}\mathrm{C}$

Answer: C

Solution:

Let θ be the temperature of the body at any time t.

$$\therefore \quad \frac{d\theta}{dt} \propto (\theta - 25)$$
$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - 25), k > 0$$

Integrating on both sides, we get

$$\log|\theta - 25| = -kt + c$$
$$\text{When } t = 0, \theta = 80^\circ$$
$$\therefore \quad \log 55 = 0 + c$$
$$\Rightarrow c = \log 55$$

$$\therefore \quad \log|\theta - 25| = -kt + \log 55 \text{ ..... (i)}$$

$$\therefore \quad \text{When } t = 30, \theta = 50^\circ$$
$$\log 25 = -30k + \log 55$$
$$\Rightarrow \quad k = -\tfrac{1}{30}\log\tfrac{5}{11}$$
$$\therefore \quad \log|\theta - 25| = \tfrac{t}{30}\log\tfrac{5}{11} + \log 55 \quad \text{... [From (i)]}$$

When $t = 1$ hour $= 60$ minutes,

$$\log|\theta - 25| = \frac{60}{30}\log\frac{5}{11} + \log 55$$
$$\Rightarrow \log\left(\frac{\theta - 25}{55}\right) = 2\log\left(\frac{5}{11}\right)$$
$$\Rightarrow \frac{\theta - 25}{55} = \left(\frac{5}{11}\right)^2$$
$$\Rightarrow \frac{\theta - 25}{55} = \frac{25}{121}$$
$$\Rightarrow \theta = 25 + \frac{125}{11} = 25 + 11.36 = 36.36^\circ C$$

Question 26

If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then as x approaches a, $\frac{g(x)f(a) - g(a)f(x)}{(x-a)}$ approaches

Options:

A. 3

B. 5

C. 0

D. 2

Answer: B

Solution:

Applying L-Hospital's rule, we get

$$\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{(x-a)}$$
$$= \lim_{x \to a} \frac{g'(x)f(a) - g(a)f'(x)}{1}$$
$$= g'(a)f(a) - g(a)f'(a)$$
$$= 2(2) - (-1)(1)$$
$$= 4 + 1$$
$$= 5$$

Question 27

The differential equation representing the family of curves $y^2 = 2\mathrm{c}(x + \sqrt{\mathrm{c}})$, where c is a positive parameter, is of

Options:

A. order 1, degree 4

B. order 2, degree 3

C. order 2, degree 4

D. order 1, degree 3

Answer: D

Solution:

$y^2 = 2c(x + \sqrt{c})$ .... (i)

Differentiating w.r.t. x, we get

$2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2c$ ..... (ii)

Substituting (ii) in (i), we get

$$y^2 = 2y\frac{\mathrm{d}y}{\mathrm{d}x}\left(x + \sqrt{y\frac{\mathrm{d}y}{\mathrm{d}x}}\right)$$

$$\Rightarrow y = 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2\frac{\mathrm{d}y}{\mathrm{d}x}\sqrt{y\frac{\mathrm{d}y}{\mathrm{d}x}}$$

$$\Rightarrow \left(y - 2x\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 4y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

This is a differential equation of order 1 and degree 3.

Question 28

If $\cos^{-1} x - \cos^{-1} \frac{y}{3} = \alpha$, where $-1 \leq x \leq 1$$, $$ - 3 \leq y \leq 3, x \leq \frac{y}{3}$, then for all $x, y, 9x^2 - 6xy\cos\alpha + y^2$ is equal to

Options:

A. $\sin^2 \alpha$

B. $3\sin^2 \alpha$

C. $9\sin^2 \alpha$

D. $\frac{4}{9}\sin^2 \alpha$

Answer: C

Solution:

If $\cos^{-1} \frac{x}{a} - \cos^{-1} \frac{y}{b} = \theta$, then

$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$

Given, $\cos^{-1} x - \cos^{-1} \frac{y}{3} = \alpha$

Here, $a = 1, b = 3$

$$\therefore \quad \frac{x^2}{1^2} - \frac{2xy}{(1)(3)}\cos\alpha + \frac{y^2}{3^2} = \sin^2\alpha$$
$$\Rightarrow x^2 - \frac{2xy}{3}\cos\alpha + \frac{y^2}{9} = \sin^2\alpha$$
$$\Rightarrow 9x^2 - 6xy\cos\alpha + y^2 = 9\sin^2\alpha$$

Question 29

In a triangle $\mathrm{ABC}, \mathrm{m}\angle\mathrm{A}, \mathrm{m}\angle\mathrm{B}, \mathrm{m}\angle\mathrm{C}$ are in A.P. and lengths of two larger sides are 10 units, 9 units respectively, then the length (in units) of the third side is

Options:

A. $5+\sqrt{6}$

B. $\sqrt{5}-1$

C. $\sqrt{6}+1$

D. $\sqrt{5}+1$

Answer: A

Solution:

$\angle\mathrm{A}, \angle\mathrm{B}, \angle\mathrm{C}$ are in A.P.

$$\Rightarrow 2B = A + C$$
$$\Rightarrow 3B = A + B + C$$
$$\Rightarrow 3B = 180^\circ$$
$$\Rightarrow B = 60^\circ$$
$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$
$$\Rightarrow \cos 60^\circ = \frac{c^2 + 10^2 - 9^2}{2c(10)} \quad \text{...[Let a = 10, b = 9]}$$

$\Rightarrow \frac{1}{2} = \frac{c^2 + 100 - 81}{20c}$
$\Rightarrow 10c = c^2 + 19$
$\Rightarrow c^2 - 10c + 19 = 0$
$\Rightarrow c = \frac{10 \pm \sqrt{100 - 76}}{2}$
$= \frac{10 \pm 2\sqrt{6}}{2}$
$= 5 \pm \sqrt{6}$

Question 30

The value of $\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right)$ is

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

$$\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)$$
$$=\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\cdot\frac{1}{5}}\right)$$
$$=\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{7}{9}\right)$$
$$=\tan^{-1}\left(\frac{\frac{1}{8}+\frac{7}{9}}{1-\frac{1}{8}\cdot\frac{7}{9}}\right)$$
$$=\tan^{-1}\left(\frac{\frac{65}{72}}{\frac{65}{72}}\right)$$
$$=\tan^{-1}(1)$$
$$=\frac{\pi}{4}$$

Question 31

The p.m.f. of a random variable X is $\mathrm{P}(x)=\begin{cases}\frac{2x}{\mathrm{n}(\mathrm{n}+1)}, & x=1,2,3,\ldots\mathrm{n}\\ 0, & \text{otherwise}\end{cases}$**, then** $\mathrm{E}(\mathrm{X})$ **is**

Options:

A. $\frac{\mathrm{n}+1}{6}$

B. $\frac{2\mathrm{n}+1}{6}$

C. $\frac{\mathrm{n}+1}{3}$

D. $\frac{2n+1}{3}$

Answer: D

Solution:

X	1	2	3	......	n
P(X)	$\frac{2}{\mathrm{n}(\mathrm{n}+1)}$	$\frac{4}{\mathrm{n}(\mathrm{n}+1)}$	$\frac{6}{\mathrm{n}(\mathrm{n}+1)}$	......	$\frac{2\mathrm{n}}{\mathrm{n}(\mathrm{n}+1)}$

$E(X) = \sum x_i \cdot P(x_i)$

$= 1 \cdot \frac{2}{n(n+1)} + 2 \cdot \frac{4}{n(n+1)} + 3 \cdot \frac{6}{n(n+1)}$
$+ \ldots . + n \cdot \frac{2n}{n(n+1)}$

$$= \frac{2}{n(n+1)}\left(1 + 4 + 9 + \ldots + n^2\right)$$
$$= \frac{2}{n(n+1)}\left(1^2 + 2^2 + 3^2 + \ldots + n^2\right)$$
$$= \frac{2}{n(n+1)} \cdot \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{2n+1}{3}$$

Question 32

If the lines $\frac{x-k}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-\frac{9}{2}}{2} = \frac{z}{1}$ intersect, then the value of k is

Options:

A. $\frac{1}{2}$

B. -1

C. 1

D. $\frac{3}{2}$

Answer: C

Solution:

Since the given lines intersect,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \quad \begin{vmatrix} 3 - k & \frac{9}{2} + 1 & 0 - 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 3 - k & \frac{11}{2} & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -5 + 5k = 0$$

$$\Rightarrow k = 1$$

Question 33

If $|\vec{a}| = \sqrt{3}; |\vec{b}| = 5; \bar{b} \cdot \bar{c} = 10$, angle between $\overline{\mathrm{b}}$ and $\overline{\mathrm{c}}$ is $\frac{\pi}{3}$, $\overline{\mathrm{a}}$ is perpendicular to $\overline{\mathrm{b}} \times \overline{\mathrm{c}}$. Then the value of $|\overline{\mathrm{a}} \times (\overline{\mathrm{b}} \times \overline{\mathrm{c}})|$ is

Options:

A. 20

B. 30

C. 60

D. 40

Answer: B

Solution:

To find the value of $|\bar{a} \times (\bar{b} \times \bar{c})|$, we need to utilize given information step by step. First, let's understand the vectors and their relationships stated in the problem.

Given:

- $|\vec{a}| = \sqrt{3}$
- $|\vec{b}| = 5$
- $\vec{b} \cdot \vec{c} = 10$
- The angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$
- $\vec{a}$ is perpendicular to $\vec{b} \times \vec{c}$

To find $|\vec{b} \cdot \vec{c}|$ given the angle between them and the magnitude of $\vec{b}$, we apply the formula for dot product in terms of the angle between the vectors:

$\vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}| \cos(\theta)$

Substituting the known values, we get:

$10 = 5|\vec{c}| \cos\left(\frac{\pi}{3}\right)$

Since $\cos(\frac{\pi}{3}) = \frac{1}{2}$, we have:

$10 = 5|\vec{c}|\frac{1}{2}$

$|\vec{c}| = 4$

So the magnitude of $\vec{c}$ is 4.

Given that $\vec{a}$ is perpendicular to $\vec{b} \times \vec{c}$, we can find the magnitude of the cross product $|\vec{b} \times \vec{c}|$ using the formula for the cross product in terms of magnitudes and angles:

$|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}| \sin(\theta)$

Substituting the known values, we get:

$|\vec{b} \times \vec{c}| = 5 \cdot 4 \cdot \sin\left(\frac{\pi}{3}\right)$

Since $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, we have:

$|\vec{b} \times \vec{c}| = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}$

Now, to find $|\vec{a} \times (\vec{b} \times \vec{c})|$, we use the fact that the magnitude of the cross product of two vectors is given by the product of their magnitudes and the sine of the angle between them. Since $\vec{a}$ is perpendicular to $\vec{b} \times \vec{c}$, the angle between them is $\frac{\pi}{2}$, and $\sin\left(\frac{\pi}{2}\right) = 1$. Therefore, the magnitude of their cross product is:

$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}| \sin\left(\frac{\pi}{2}\right)$

$= \sqrt{3} \cdot 10\sqrt{3} \cdot 1 = 3 \cdot 10 = 30$

Thus, the correct answer is:

Option B: 30

Question 34

If $\int \frac{x^2}{\sqrt{1-x}}\, \mathrm{d}x = \mathrm{p}\sqrt{1-x}\left(3x^2+4x+8\right) + \mathrm{c}$ where c is a constant of integration, then the value of p is

Options:

A. $\frac{-2}{15}$

B. $\frac{2}{15}$

C. $\frac{4}{15}$

D. $\frac{-4}{15}$

Answer: A

Solution:

Let $I = \int \frac{x^2}{\sqrt{1-x}} dx$

Put $1 - x =$ t

$\Rightarrow$ d$x = -$dt

$$\begin{aligned}
\therefore \quad I &= \int \frac{(1-t)^2}{\sqrt{t}} \cdot (-dt) \\
&= -\int \frac{1-2t+t^2}{t^{\frac{1}{2}}} dt \\
&= -\int \left(t^{-\frac{1}{2}} - 2t^{\frac{1}{2}} + t^{\frac{3}{2}}\right) dt \\
&= -\left(2t^{\frac{1}{2}} - \frac{4}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}}\right) + c \\
&= t^{\frac{1}{2}} \left(\frac{-30+20t-6t^2}{15}\right) + c \\
&= \sqrt{1-x} \left(\frac{-30+20(1-x)-6(1-x)^2}{15}\right) + c \\
&= \sqrt{1-x} \left(\frac{-6x^2-8x-16}{15}\right) + c \\
&= -\frac{2}{15}\sqrt{1-x}\left(3x^2+4x+8\right) + c
\end{aligned}$$

$$\therefore \quad p = \frac{-2}{15}$$

Question 35

The centroid of the triangle formed by the lines $x + 3y = 10$ and $6x^2 + xy - y^2 = 0$ is

Options:

A. $\left(\frac{1}{3}, \frac{-7}{3}\right)$

B. $\left(\frac{-1}{3}, \frac{-7}{3}\right)$

C. $\left(\frac{-1}{3}, \frac{7}{3}\right)$

D. $\left(\frac{1}{3}, \frac{7}{3}\right)$

Answer: C

Solution:

Lines represented by the equation $6x^2 + xy - y^2 = 0$ are $y = 3x$ and $y = -2x$ The co-ordinates of the vertices of the triangle formed by above lines with $x + 3y = 10$ are $(0, 0), (1, 3)$ and $(-2, 4)$.

$\therefore \quad \text{Centroid } = \left(\frac{0+1-2}{3}, \frac{0+3+4}{3}\right) = \left(\frac{-1}{3}, \frac{7}{3}\right)$

Question 36

The statement $[\mathrm{p} \wedge (\mathrm{q} \vee \mathrm{r})] \vee [\sim \mathrm{r} \wedge \sim \mathrm{q} \wedge \mathrm{p}]$ is equivalent to

Options:

A. $\sim \mathrm{r}$

B. p

C. $\sim \mathrm{q}$

D. q

Answer: B

Solution:

$[p \wedge (q \vee r)] \vee [\sim r \wedge \sim q \wedge p]$
$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim q \wedge \sim r]$ ...[Commutative law]

$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim (q \vee r)]$ ....[De Morgan's law]

$\equiv p \wedge [(q \vee r) \vee \sim (q \vee r)] \ldots$ [Distributive law]

$\equiv$ p $\wedge$ T ..... [Complement law]

$\equiv p$ .... [Identity law]

Question 37

If $\mathrm{f}(x)=\frac{2x-3}{3x-4}, x \neq \frac{4}{3}$, then the value of $\mathrm{f}^{-1}(x)$ is

Options:

A. $\frac{4x-3}{3x-2}$

B. $\frac{3x-2}{4x+3}$

C. $\frac{3x-4}{4x-2}$

D. $\frac{2x+3}{4x-3}$

Answer: A

Solution:

Let $\mathrm{f}(x)=y \Rightarrow x=\mathrm{f}^{-1}(y)$

$y = \dfrac{2x-3}{3x-4}$

$\Rightarrow 3xy - 4y = 2x - 3$

$\Rightarrow x(3y-2) = 4y - 3$

$\Rightarrow x = \dfrac{4y-3}{3y-2}$

$\Rightarrow \mathrm{f}^{-1}(y) = \dfrac{4y-3}{3y-2}$

$\Rightarrow \mathrm{f}^{-1}(x) = \dfrac{4x-3}{3x-2}$

Question 38

If $\mathrm{f}'(x) = \sin(\log x)$ and $y = \mathrm{f}\left(\frac{2x+3}{3-2x}\right)$, then $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 1$ is

Options:

A. $6\sin(\log 5)$

B. $5\sin(\log 6)$

C. $12\sin(\log 5)$

D. $5\sin(\log 12)$

Answer: C

Solution:

$$y=\mathrm{f}\left(\frac{2x+3}{3-2x}\right)$$

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}'\left(\frac{2x+3}{3-2x}\right)\cdot\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2x+3}{3-2x}\right)$$

$$=\mathrm{f}'\left(\frac{2x+3}{3-2x}\right)\cdot\frac{(3-2x)\cdot 2-(2x+3)(-2)}{(3-2x)^2}$$

$$=\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]\cdot\frac{12}{(3-2x)^2}$$

$$\therefore \quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=1}=\sin(\log 5)\cdot 12=12\sin(\log 5)$$

Question 39

The area bounded by the curve $y=|x-2|, x=1, x=3$ and X-axis is

Options:

A. 3 sq. units

B. 2 sq. units

C. 1 sq. units

D. 4 sq. units

Answer: C

Solution:

$$\text{Required area } =\int_1^3|x-2|\mathrm{d}x$$

$$=\int_1^2(2-x)\mathrm{d}x+\int_2^3(x-2)\mathrm{d}x$$

$$=\left[2x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}-2x\right]_2^3$$

$$=\frac{1}{2}+\frac{1}{2}=1 \text{ sq. unit}$$

Question 40

$\int \frac{\mathrm{d}x}{\cot^2 x-1} = \frac{1}{\mathrm{A}}\log|\sec 2x+\tan 2x| - \frac{x}{\mathrm{B}} + \mathrm{c}$, (where c is constant of integration), then $\mathrm{A}+\mathrm{B}=$

Options:

A. -6

B. 6

C. -5

D. 5

Answer: B

Solution:

$$\begin{aligned}
\int \frac{\mathrm{d}x}{\cot^2 x-1} &= \int \frac{\mathrm{d}x}{\frac{\cos^2 x-\sin^2 x}{\sin^2 x}} \\
&= \int \frac{\sin^2 x}{\cos 2x}\,\mathrm{d}x \\
&= \int \frac{\frac{1-\cos 2x}{2}}{\cos 2x}\,\mathrm{d}x \\
&= \frac{1}{2}\int(\sec 2x-1)\mathrm{d}x \\
&= \frac{1}{2}\left(\frac{\log|\sec 2x+\tan 2x|}{2}-x\right)+\mathrm{c} \\
&= \frac{1}{4}\log|\sec 2x+\tan 2x| - \frac{x}{2}+\mathrm{c}
\end{aligned}$$

$$\begin{aligned}
\therefore\quad & A=4, B=2 \\
& \Rightarrow A+B=6
\end{aligned}$$

Question 41

There are 6 positive and 8 negative numbers. From these four numbers are chosen at random and multiplied. Then the probability, that the product is a negative number, is

Options:

A. $\frac{496}{1001}$

B. $\frac{505}{1001}$

C. $\frac{490}{1001}$

D. $\frac{504}{1001}$

Answer: A

Solution:

Total number of numbers $= 8 + 6 = 14$

4 numbers can be chosen out of 14 numbers in ${}^{14}\mathrm{C}_4$ ways

The product of 4 numbers will be negative, if

i. one is negative and three are positive

OR

ii. three are negative and one is positive.

$$\begin{aligned} \therefore \quad \text{Required probability } &= \frac{\left({}^{8}\mathrm{C}_1 \times {}^{6}\mathrm{C}_3\right) + \left({}^{8}\mathrm{C}_3 \times {}^{6}\mathrm{C}_1\right)}{{}^{14}\mathrm{C}_4} \\ &= \frac{(8 \times 20) + (56 \times 6)}{1001} \\ &= \frac{160 + 336}{1001} = \frac{496}{1001} \end{aligned}$$

Question 42

If $\mathrm{a}\cos 2\theta + \mathrm{b}\sin 2\theta = \mathrm{c}$ has α and β as its roots, then the value of $\tan\alpha + \tan\beta$ is

Options:

A. $\frac{2b}{c+a}$

B. $\frac{2a}{b+c}$

C. $\frac{b}{c+a}$

D. $\frac{a}{b+c}$

Answer: A

Solution:

We have, $a \cos 2\theta + b \sin 2\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$$
$$\Rightarrow a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$$
$$\Rightarrow -(a+c)\tan^2\theta + 2b\tan\theta + (a-c) = 0$$

$\therefore \quad \tan\alpha + \tan\beta = -\frac{2b}{-(c+a)} = \frac{2b}{c+a}$

Question 43

A lot of 100 bulbs contains 10 defective bulbs. Five bulbs are selected at random from the lot and are sent to retail store. Then the probability that the store will receive at most one defective bulb is

Options:

A. $\frac{7}{5}\left(\frac{9}{10}\right)^4$

B. $\frac{7}{5}\left(\frac{9}{10}\right)^5$

C. $\frac{6}{5}\left(\frac{9}{10}\right)^4$

D. $\frac{6}{5}\left(\frac{9}{10}\right)^5$

Answer: A

Solution:

Let X denote the number of defective bulbs.

p = Probability that a bulb is defective

$$= \frac{10}{100} = \frac{1}{10}$$

$$q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$P(X = r) = {}^5C_r \left(\frac{1}{10}\right)^r \left(\frac{9}{10}\right)^{5-r}, r = 0, 1, \ldots, 5$$

$$\therefore \quad P(X \leq 1)$$

$$= P(X = 0) + P(X = 1)$$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + 5 \times \frac{1}{10} \times \left(\frac{9}{10}\right)^4$$

$$= \frac{7}{5}\left(\frac{9}{10}\right)^4$$

Question 44

Given $0 \leq x \leq \frac{1}{2}$, then the value of $\tan\left(\sin^{-1}\left(\frac{x}{\sqrt{2}} + \frac{\sqrt{1-x^2}}{\sqrt{2}}\right) - \sin^{-1} x\right)$ is

Options:

A. 1

B. $\sqrt{3}$

C. -1

D. $\frac{1}{\sqrt{3}}$

Answer: A

Solution:

$$\tan\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right)-\sin^{-1}x\right]$$
$$=\tan\left[\sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)-\sin^{-1}x\right]$$
$$=\tan\left[\sin^{-1}\left(\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right)-\theta\right] \quad \ldots\begin{bmatrix}\text{Put } \sin^{-1}x=\theta \\ \Rightarrow x=\sin\theta\end{bmatrix}$$
$$=\tan\left[\sin^{-1}\left[\sin\left(\theta+\frac{\pi}{4}\right)\right]-\theta\right]$$
$$=\tan\left(\theta+\frac{\pi}{4}-\theta\right)$$
$$=\tan\frac{\pi}{4}=1$$

Question 45

A vector parallel to the line of intersection of the planes $\bar{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\bar{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$ is

Options:

A. $-2\hat{i}+7\hat{j}+13\hat{k}$

B. $2\hat{i}-7\hat{j}+13\hat{k}$

C. $-\hat{i}+4\hat{j}+7\hat{k}$

D. $\hat{\mathrm{i}}-4\hat{\mathrm{j}}+7\hat{\mathrm{k}}$

Answer: A

Solution:

The line of intersection of the planes $\bar{r}\cdot(3\hat{i}-\hat{j}+\hat{k})=1$ and $\bar{r}\cdot(\hat{i}+4\hat{j}-2\hat{k})=2$ is perpendicular to each of the normal vectors $\overline{n_1}=3\hat{i}-\hat{j}+\hat{k}$ and $\overline{n_2}=\hat{i}+4\hat{j}-2\hat{k}$.

$\therefore$ The line is parallel to the vector $\bar{n}_1\times\bar{n}_2$

$$\therefore \quad \bar{\mathrm{n}}_1\times\bar{\mathrm{n}}_2=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ 3 & -1 & 1 \\ 1 & 4 & -2\end{vmatrix}$$
$$=-2\hat{\mathrm{i}}+7\hat{\mathrm{j}}+13\hat{\mathrm{k}}$$

Question 46

The length of the perpendicular drawn from the point $(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

Options:

A. 4 units

B. 5 units

C. 6 units

D. 7 units

Answer: D

Solution:

Let $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$ (say)

Any point on the line is

$\mathrm{P}(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$

Let $\mathrm{A} \equiv (1, 2, 3)$

The d.r.s. of line AP are

$3\lambda + 6 - 1, 2\lambda + 7 - 2, -2\lambda + 7 - 3$
i.e. $3\lambda + 5, 2\lambda + 5, -2\lambda + 4$

Since AP is perpendicular to the given line,

$3(3\lambda + 5) + 2(2\lambda + 5) - 2(-2\lambda + 4) = 0$
$\Rightarrow 17\lambda + 17 = 0$
$\Rightarrow \lambda = -1$
$\therefore \quad \mathrm{P} \equiv (3, 5, 9)$
$\therefore \quad \mathrm{AP} = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2}$
$= \sqrt{49}$
$= 7$ units

Question 47

If $I=\int \frac{dx}{\sin(x-a)\sin(x-b)}$, then I is given by

Options:

A. $\frac{1}{\sin(b-a)}\log|\sin(x-a)\sin(x-b)|+c$, where c is a constant of integration.

B. $\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$, where c is a constant of integration.

C. $\frac{1}{\sin(b-a)}\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|+c$, where c is a constant of integration.

D. $\frac{1}{\sin(b-a)}\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+c$, where c is a constant of integration.

Answer: D

Solution:

$$
\begin{aligned}
I &= \int \frac{dx}{\sin(x-a)\sin(x-b)} \\
&= \frac{1}{\sin(b-a)}\int \frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)}dx \\
&= \frac{1}{\sin(b-a)}\int \frac{1}{\sin(x-a)\sin(x-b)}[\sin(x-a)\cos(x-b)-\cos(x-a)\sin(x-b)]dx \\
&= \frac{1}{\sin(b-a)}\int[\cot(x-b)-\cot(x-a)]dx \\
&= \frac{1}{\sin(b-a)}[\log|\sin(x-b)|-\log|\sin(x-a)|]+c \\
&= \frac{1}{\sin(b-a)}\log\left|\frac{\sin(x-b)}{\sin(x-a)}\right|+c
\end{aligned}
$$

Question 48

Let $\mathrm{P}(x)$ be a polynomial of degree 2, with $\mathrm{P}(2)=-1, \mathrm{P}'(2)=0, \mathrm{P}''(2)=2$, then $\mathrm{P}(1.001)$ is

Options:

A. 0.002

B. -0.002

C. 0.004

D. -0.004

Answer: B

Solution:

$$
\begin{aligned}
&\text{Let } \mathrm{P}(x) = ax^2 + bx + c \\
&\Rightarrow P'(x) = 2ax + b \\
&\Rightarrow P''(x) = 2a \\
&P''(2) = 2a \\
&\Rightarrow 2 = 2a \\
&\Rightarrow a = 1 \\
&P'(2) = 2a(2) + b \\
&\Rightarrow 0 = 4a + b \\
&\Rightarrow 0 = 4(1) + b \\
&\Rightarrow b = -4 \\
&P(2) = a(2)^2 + b(2) + c \\
&\Rightarrow -1 = 4a + 2b + c \\
&\Rightarrow -1 = 4(1) + 2(-4) + c \\
&\Rightarrow c = 3
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad & \mathrm{P}(x) = x^2 - 4x + 3 \\
& \Rightarrow \mathrm{P}'(x) = 2x - 4 \\
& x = 1.001 = 1 + 0.001 = \mathrm{a} + \mathrm{h}
\end{aligned}
$$

Here, $\mathrm{a} = 1,\ \mathrm{h} = 0.001$

$$
\begin{aligned}
&\mathrm{P}(\mathrm{a}) = \mathrm{P}(1) = 1 - 4 + 3 = 0 \\
&\mathrm{P}'(\mathrm{a}) = \mathrm{P}'(1) = 2 - 4 = -2
\end{aligned}
$$

$$\therefore \quad \mathrm{P}(1.001) = 0 + (0.001)(-2) = -0.002$$

Question 49

If $y=\sqrt{(x-\sin x)+\sqrt{(x-\sin x)+\sqrt{(x-\sin x)\ldots}}}$, then $\frac{\mathrm{d}y}{\mathrm{d}x}=$

Options:

A. $\frac{1-\cos x}{2y-1}$

B. $\frac{1+\cos x}{2y-1}$

C. $\frac{1-\cos x}{2y+1}$

D. $\frac{1-\sin x}{2y-1}$

Answer: A

Solution:

To solve this problem, let's first observe the nested square root structure within the equation $y=\sqrt{(x-\sin x)+\sqrt{(x-\sin x)+\sqrt{(x-\sin x)\ldots}}}$. Since the structure of nested square roots repeats indefinitely, we can rewrite this as:

$y=\sqrt{(x-\sin x)+y}$

Now let's square both sides:

$y^2=(x-\sin x)+y$

Isolate y on one side:

$y^2-y=x-\sin x$

$y^2-y-(x-\sin x)=0$

This equation gives us a relationship between x and y that we can differentiate with respect to x. We will use implicit differentiation, differentiating both sides of the equation with respect to x:

$\frac{\mathrm{d}}{\mathrm{d}x}(y^2-y-(x-\sin x))=\frac{\mathrm{d}}{\mathrm{d}x}(0)$

When differentiating the left side, keep in mind that y is a function of x ($y=f(x)$). Applying the chain rule and using the fact that $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x)=\cos x$, we get:

$2y\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{\mathrm{d}y}{\mathrm{d}x}-(1-\cos x)=0$

Reorganize the terms:

$2y\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{\mathrm{d}y}{\mathrm{d}x}=1-\cos x$

$\frac{\mathrm{d}y}{\mathrm{d}x}(2y-1)=1-\cos x$

Now, solving for $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-\cos x}{2y-1}$

This matches Option A:

$$\boxed{\frac{1-\cos x}{2y-1}}$$

Question 50

Let $\mathrm{f}(x)=5-|x-2|$ and $\mathrm{g}(x)=|x+1|, x\in \mathrm{R}$ If $\mathrm{f}(x)$ attains maximum value at α and $\mathrm{g}(x)$ attains minimum value at β, then $\lim\limits_{x\to -\alpha\beta}\frac{(x-1)\left(x^2-5x+6\right)}{x^2-6x+8}$ is equal to

Options:

A. $\frac{1}{2}$

B. $\frac{-3}{2}$

C. $\frac{-1}{2}$

D. $\frac{3}{2}$

Answer: A

Solution:

$|x-2|\geq 0$
$\Rightarrow -|x-2|\leq 0$
$\Rightarrow 5-|x-2|\leq 5$

Maximum value of $\mathrm{f}(x)$ is 5.

$$
\begin{aligned}
\therefore \quad & 5-|x-2|=5 \\
& \Rightarrow |x-2|=0 \\
& \Rightarrow x=2 \\
& \Rightarrow \alpha=2 \\
& |x+1| \geq 0
\end{aligned}
$$

Minimum value of $g(x)$ is 0.

$$
\begin{aligned}
\therefore \quad & |x+1|=0 \\
& \Rightarrow x=-1 \\
& \Rightarrow \beta=-1
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad & \lim_{x \rightarrow-\alpha \beta} \frac{(x-1)\left(x^2-5 x+6\right)}{x^2-6 x+8} \\
& =\lim_{x \rightarrow 2} \frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)} \\
& =\lim_{x \rightarrow 2} \frac{(x-1)(x-3)}{x-4} \\
& =\frac{(1)(-1)}{-2} \\
& =\frac{1}{2}
\end{aligned}
$$

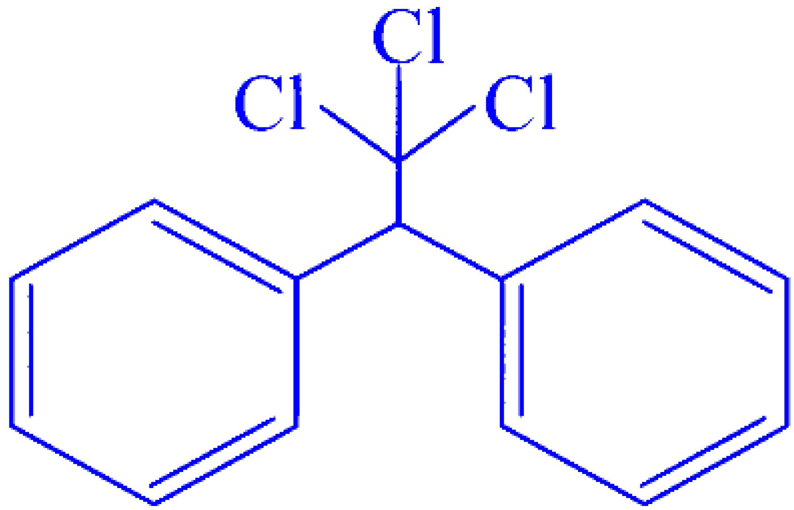
Chemistry

Question 51

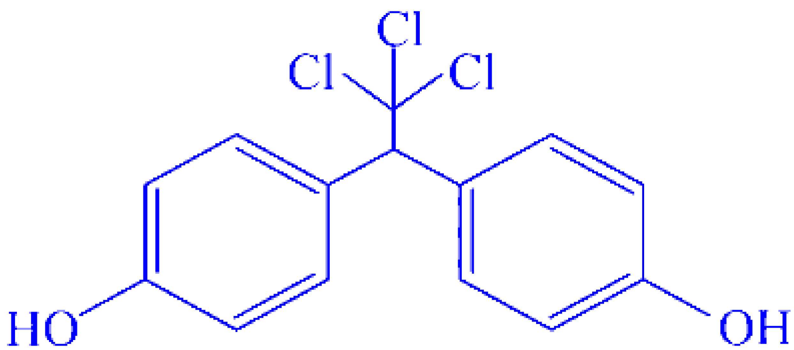
Which of the following is a structural formula of DDT?

Options:

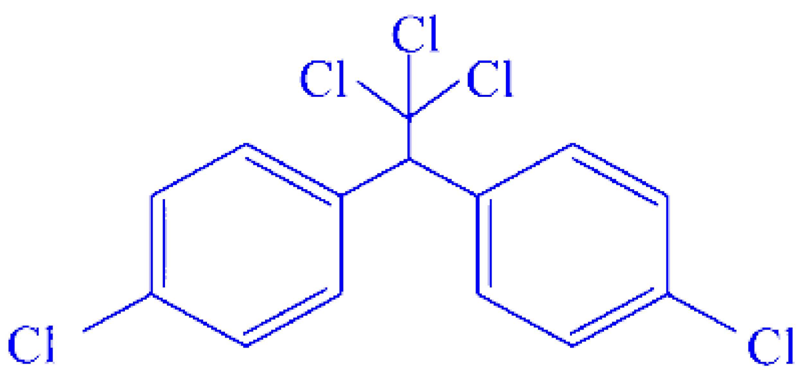
A.



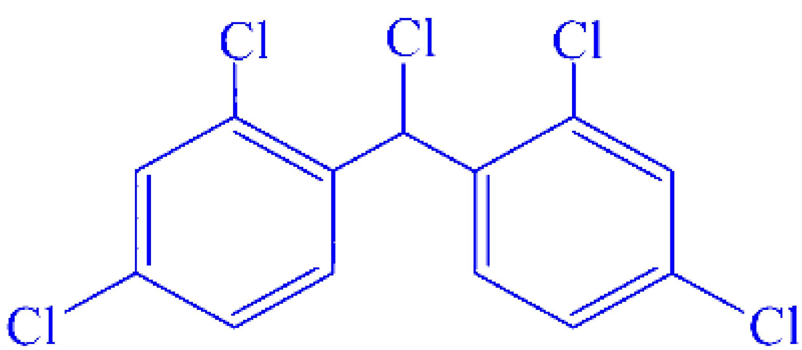
B.



C.



D.



Answer: C

Question 52

Which among the following is haloalkyne?

Options:

A. Chloroethyne

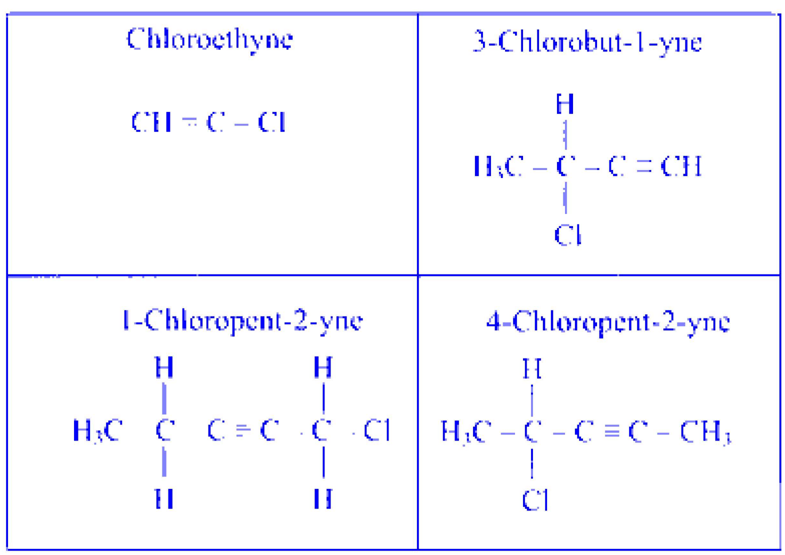
B. 3-Chlorobut-1-yne

C. 1-Chloropent-2-yne

D. 4-Chloropent-2-yne

Answer: A

Solution:



In a haloalkyne, halogen atom is bonded to sp hybridized carbon atom. Hence, among the given options, chloroethyne is a haloalkyne.

Question 53

What type of peptide is the glycylalanine?

Options:

A. Dipeptide

B. Tripeptide

C. Tetrapeptide

D. Pentapeptide

Answer: A

Solution:

Glycylalanine is a peptide formed by the linkage of two amino acids: glycine and alanine. This linkage occurs through a peptide bond, which is a covalent bond that joins the carboxyl group of one amino acid to the amine group of another with the release of a water molecule (a condensation reaction). Since glycylalanine consists of only two amino acids connected by a single peptide bond, it is classified as a dipeptide. Therefore, the correct answer is:

Option A: Dipeptide

Question 54

What is Henry's law constant of a gas if solubility of gas in water at $25^{\circ} \mathrm{C}$ is $0.028 \mathrm{~mol} \mathrm{dm}^{-3}$?

[Partial pressure of gas $=0.346$ bar]

Options:

A. $0.081 \mathrm{~mol} \mathrm{dm}^{-3} \mathrm{bar}^{-1}$

B. $0.075 \mathrm{~mol} \mathrm{dm}^{-3} \mathrm{bar}^{-1}$

C. $0.093 \mathrm{~mol} \mathrm{dm}^{-3} \mathrm{bar}^{-1}$

D. $0.049 \mathrm{~mol} \mathrm{dm}^{-3} \mathrm{bar}^{-1}$

Answer: A

Solution:

According to Henry's Law, the solubility of a gas in a liquid at a particular temperature is directly proportional to the pressure of the gas above the liquid. The law can be mathematically expressed as:

$S = k_H \cdot P$

Where:

- S is the solubility of the gas (in mol dm^{-3}).
- k_H is Henry's law constant (in mol $\mathrm{dm}^{-3}\mathrm{bar}^{-1}$).
- P is the partial pressure of the gas (in bar).

To find Henry's law constant for the given conditions, we need to rearrange the equation to solve for k_H:

$k_H = \frac{S}{P}$

Given that the solubility $S = 0.028$ mol dm^{-3} and the partial pressure $P = 0.346$ bar, we can plug these values into the equation:

$k_H = \frac{0.028 \text{ mol dm}^{-3}}{0.346 \text{ bar}}$

To calculate k_H:

$k_H = \frac{0.028}{0.346}$ mol $\mathrm{dm}^{-3}\mathrm{bar}^{-1}$

$k_H \approx 0.08092$ mol $\mathrm{dm}^{-3}\mathrm{bar}^{-1}$

If you review the options provided, the closest value to 0.08092 is option A, 0.081 mol $\mathrm{dm}^{-3}\mathrm{bar}^{-1}$. Therefore, the correct option is:

Option A 0.081 mol $\mathrm{dm}^{-3}\mathrm{bar}^{-1}$

Question 55

Calculate the rate constant of the first order reaction if 80% of the reactant decomposes in 60 minutes.

Options:

A. 2.68×10^{-2} minute $^{-1}$

B. 5.36×10^{-2} minute $^{-1}$

C. 1.34×10^{-2} minute $^{-1}$

D. 8.1×10^{-2} minute $^{-1}$

Answer: A

Solution:

$$
\begin{aligned}
k &= \frac{2.303}{t}\log_{10}\frac{[A]_0}{[\,A]_t} \\
&= \frac{2.303}{60}\log_{10}\frac{(100)}{(20)} = \frac{2.303}{60}\log_{10}\frac{10}{2} \\
&= \frac{2.303}{60}[\log_{10}(10) - \log_{10}(2)] \\
&= \frac{2.303}{60} \times 0.699 = 2.68 \times 10^{-2}\text{ minute }^{-1}
\end{aligned}
$$

Question 56

Which from following polymers is classified fibres depending on inter molecular forces?

Options:

A. Vulcanized rubber

B. Buna-S

C. Terylene

D. Polystyrene

Answer: C

Solution:

The classification of polymers into fibres is typically based on their ability to form long chains that exhibit strong intermolecular forces like hydrogen bonding, van der Waals forces, and dipole-dipole interactions. These forces must be strong enough to hold the chains together tightly, allowing the material to be drawn into fibers that are both strong and flexible.

- **Vulcanized rubber (Option A)**: Vulcanized rubber is not classified as a fibre. It is a thermoset elastomer, made by adding sulfur to raw rubber which creates cross-linked polymer chains. The resulting material is elastic and resilient, ideal for products like tires and other elastic materials, but not for fibers.

- **Buna-S (Option B)**: Also known as styrene-butadiene rubber, is a copolymer made up of styrene and butadiene. It has good abrasion resistance and is used for items like shoe soles and car tires. However, it does not form fibers and is not classified as a fibrous polymer.

- **Terylene (Option C)**: Terylene, also known as polyester or polyethylene terephthalate (PET), is a polymer that has strong intermolecular forces due to its aromatic rings and ester linkages. These forces allow the molecules to align closely with each other, thereby forming fibers. Hence, Terylene is classified as a fibre and is used in textiles, recording tapes, and other products requiring material in fiber form.

- **Polystyrene (Option D)**: Polystyrene is a thermoplastic polymer with a structure unsuitable for fiber formation. It is commonly used in products like foam packaging, disposable cups, and insulating material. It is not characterized by the same degree of intermolecular forces seen in fibrous materials.

Therefore, the correct answer to the question is **Option C, Terylene**, which is classified as a fiber due to its strong intermolecular forces allowing it to be drawn into fibers.

Question 57

Calculate the frequency if wavelength is 750 nm.

Options:

A. 2×10^{14} Hz

B. 4×10^{14} Hz

C. 6×10^{15} Hz

D. 8×10^{15} Hz

Answer: B

Solution:

To calculate the frequency of light when the wavelength is given, we can use the relationship between speed, frequency, and wavelength. This relationship is given by the equation:

$c = \lambda \cdot \nu$

where:

- c is the speed of light in a vacuum ($c \approx 3 \times 10^8 \text{ m/s}$),
- λ is the wavelength in meters,
- ν (nu) is the frequency in hertz (Hz).

To find the frequency, we can rearrange the equation to solve for ν:

$\nu = \frac{c}{\lambda}$

Given that the wavelength is 750 nm (nanometers), we have to convert the wavelength into meters for our calculations because the speed of light is in meters per second. There are 1×10^9 nanometers in one meter, so:

$750 \text{ nm} = 750 \times 10^{-9} \text{ m}$

Now, let's insert the values for c and λ into our equation:

$\nu = \frac{3 \times 10^8 \text{ m/s}}{750 \times 10^{-9} \text{ m}}$

Calculating the frequency ν:

$\nu = \frac{3 \times 10^8}{750 \times 10^{-9}} = \frac{3 \times 10^8}{0.75 \times 10^{-6}} = \frac{3}{0.75} \times 10^{8+6}$

$\nu = 4 \times 10^{14} \text{ Hz}$

Therefore, the frequency of light with a wavelength of 750 nm is 4×10^{14} Hz, which corresponds to Option B.

Question 58

Calculate the edge length of bcc unit cell if radius of metal atom is 227 pm.

Options:

A. 4.54×10^{-8} cm

B. 5.24×10^{-8} cm

C. 6.42×10^{-8} cm

D. 1.135×10^{-8} cm

Answer: B

Solution:

For bcc unit cell, $r = \frac{\sqrt{3}}{4} a$

$$\therefore \quad a = \frac{4r}{\sqrt{3}} = \frac{4 \times 227}{1.73} = 524.85 \text{ pm}$$

$$= 524.85 \times 10^{-10} \text{ cm}$$

$$= 5.24 \times 10^{-8} \text{ cm}$$

Question 59

Identify the compound 'A' in the following sequence of reactions.

$$A \xrightarrow[\text{Dryether}]{C_2H_3MgBr} B \xrightarrow{H_3O^+} \text{3-Methylpentan-3-0l}$$

Options:

A. Propanal

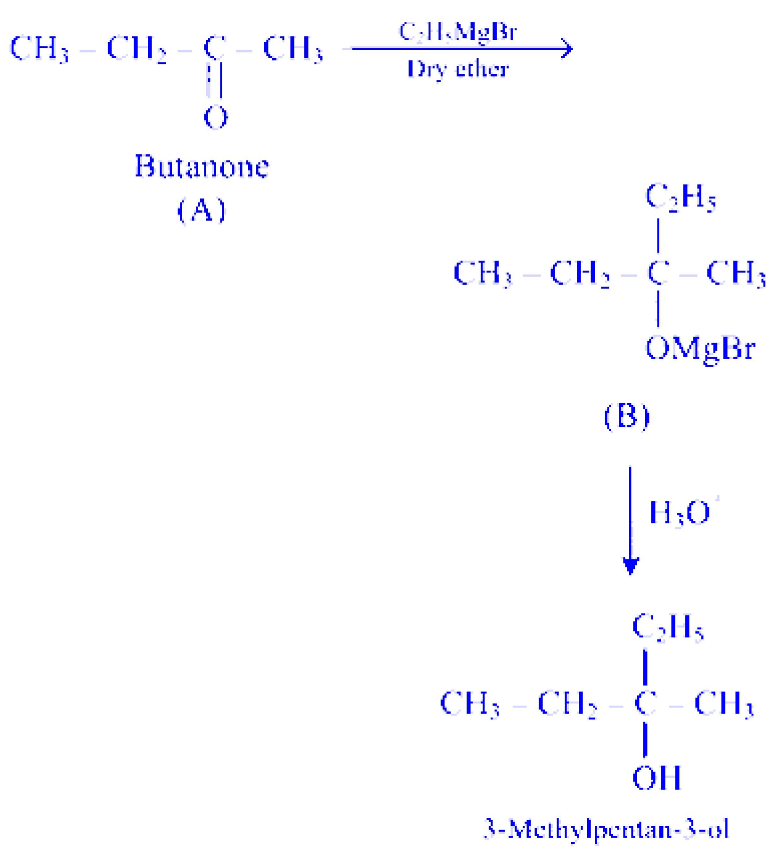
B. Propanone

C. Butanal

D. Butanone

Answer: D

Solution:



Question 60

A solution of nonvolatile solute is obtained by dissolving $15 \mathrm{~g}$ in $200 \mathrm{~mL}$ water has depression in freezing point $0.75 \mathrm{~K}$. Calculate the molar mass of solute if cryoscopic constant of water is $1.86 \mathrm{~K} \mathrm{~kg} \mathrm{~mol}^{-1}$.

Options:

A. $160 \mathrm{~g} \mathrm{~mol}^{-1}$

B. $172 \mathrm{~g} \mathrm{~mol}^{-1}$

C. $186 \mathrm{~g} \mathrm{~mol}^{-1}$

D. $198 \mathrm{~g} \mathrm{~mol}^{-1}$

Answer: C

Solution:

$$
\begin{aligned}
\mathrm{M}_2 & =\frac{1000 \mathrm{~K}_{\mathrm{f}} \mathrm{W}_2}{\Delta \mathrm{T}_{\mathrm{f}} \mathrm{W}_1} \\
& =\frac{1000 \times 1.86 \times 15}{0.75 \times 200} \\
& =186 \mathrm{~g} \mathrm{~mol}^{-1}
\end{aligned}
$$

Question 61

For a reaction $\mathrm{A}+\mathrm{B} \rightarrow$ products $\Delta \mathrm{H}$ is $-84.2 \mathrm{~kJ}$ and $\Delta \mathrm{S}$ is $-200 \mathrm{~J} \mathrm{~K}^{-1}$. Calculate the highest value of temperature so that the reaction will proceed in forward direction.

Options:

A. 421 K

B. 237 K

C. 168 K

D. 273 K

Answer: A

Solution:

$$\begin{aligned} & \mathrm{T}=\frac{\Delta \mathrm{H}}{\Delta \mathrm{S}} \\ \therefore \quad & \mathrm{T}=\frac{-84200 \mathrm{~J}}{-200 \mathrm{~J} \mathrm{~K}^{-1}}=421 \mathrm{~K} \end{aligned}$$

Since ΔH and ΔS are negative, the reaction is spontaneous at low temperatures. Therefore, the highest temperature is 421 K and the reaction will proceed in forward direction spontaneously below 421 K.

Question 62

The solubility product of $\mathrm{PbCl_2}$ at $298 \mathrm{~K}$ is 3.2×10^{-5}. What is its solubility in $\mathrm{mol dm}^{-3}$?

Options:

A. 8×10^{-6}

B. 2×10^{-2}

C. 5.6×10^{-3}

D. 5.0×10^{-2}

Answer: B

Solution:

$K_{sp} = 3.2 \times 10^{-5}$

$PbCl_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2Cl^{-}_{(aq)}$

$\therefore \quad K_{sp} = 4\,S^3$

$\therefore \quad 4\,S^3 = 3.2 \times 10^{-5}$

$$S = \sqrt[3]{\frac{3.2 \times 10^{-5}}{4}}$$

$= \sqrt[3]{8 \times 10^{-6}}$

$= 2 \times 10^{-2}$ mol dm^{-3}

Question 63

Which among the following elements does NOT exhibit ferromagnetic properties?

Options:

A. Cr

B. Fe

C. Co

D. Ni

Answer: A

Solution:

Ferromagnetic materials are those that exhibit strong magnetic properties, where the magnetic moments of the atoms tend to align in the same direction upon an external magnetic field, even remaining magnetized after the external field is removed. Among the elements listed, chromium (Cr) does not exhibit ferromagnetic properties.

Let's consider each element:

- **Option A: Chromium (Cr) -** Chromium is actually antiferromagnetic. In antiferromagnetic materials, adjacent electrons pair up with opposite spins, which cancels out their magnetism at a macroscopic scale. This is why Cr does not exhibit ferromagnetic properties.

- **Option B: Iron (Fe) -** Iron is well-known as a ferromagnetic material, widely used in various applications for its magnetic properties. It is one of the most common ferromagnetic materials.

- **Option C: Cobalt (Co) -** Cobalt is ferromagnetic. It is part of the rare-earth metals and, like iron, is used for making magnets.

- **Option D: Nickel (Ni) -** Nickel is another well-known ferromagnet, although it has weaker magnetic properties than iron.

Therefore, the correct answer is **Option A: Chromium (Cr)**, as it does not exhibit ferromagnetic properties like the other elements listed.

Question 64

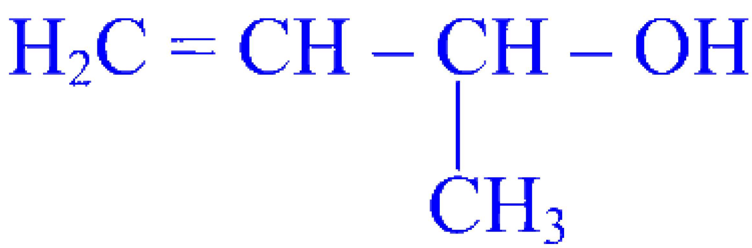
Which of the following is a secondary allylic alcohol?

Options:

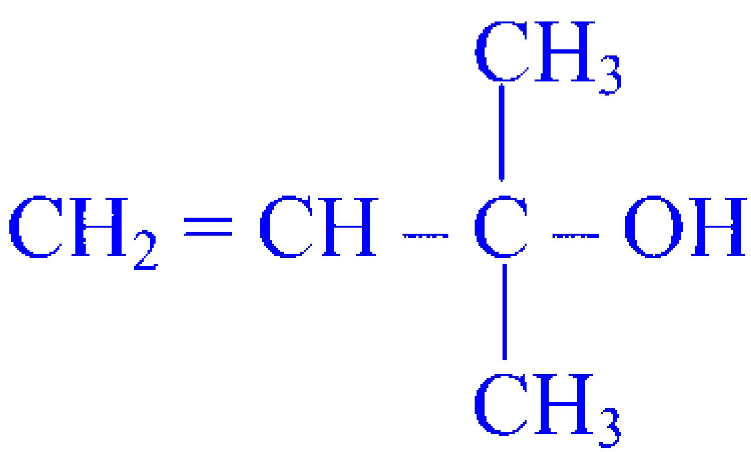
A. $H_2C = CH - CH_2 - OH$

B. $CH_3 - CH = CH - CH_2 - OH$

C.



D.



Answer: C

Question 65

Which from following elements is in liquid state at room temperature?

Options:

A. Se

B. Br

C. I

D. S

Answer: B

Solution:

Among the options given, the element that is in a liquid state at room temperature is:

Option B: Br (Bromine)

Bromine is a chemical element with the symbol Br and atomic number 35. At room temperature (approximately 25 degrees Celsius or 77 degrees Fahrenheit), bromine is one of the only two elements on the periodic table that are in a liquid state, the other being mercury (Hg). Bromine is a halogen, and it appears as a reddish-brown liquid. It has a distinctive, strong smell and is known for its reactive properties.

The other elements listed are:

- **Option A: Se (Selenium)** which is a nonmetal that exists in several allotropes, but the most stable and dense form is a gray metallic-looking solid.
- **Option C: I (Iodine)** which is also a halogen like bromine, but iodine is a solid at room temperature. It sublimates into a purple vapor when heated.
- **Option D: S (Sulfur)** which is a yellow solid under normal conditions.

Question 66

Identify major product formed in the following reaction.

$$\textbf{3-Bromo-2-methylpentane} \xrightarrow[\Delta]{\text{Alc.KOH}} \textbf{Major product}$$

Options:

A. 2-Methylpentan-3-ol

B. 2-Methylpent-2-ene

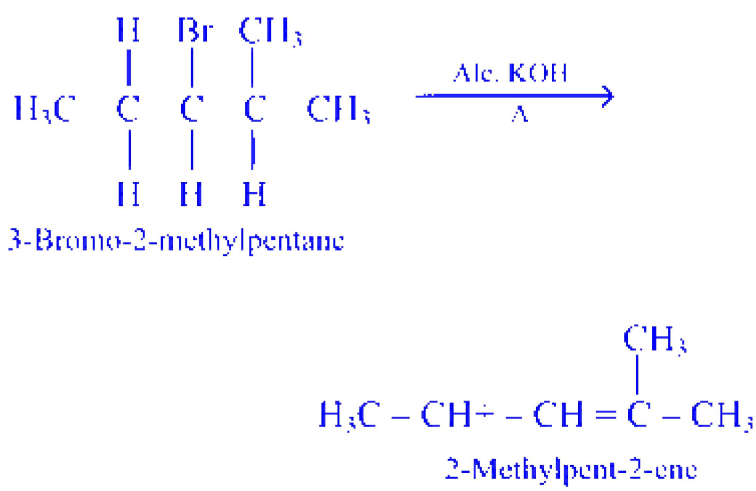
C. 4-Methylpent-3-ene

D. 4-Methylpentan-3-ol

Answer: B

Solution:

This is dehydrohalogenation reaction.



Question 67

What is molecular formula of cyclohexylamine?

Options:

A. C_6H_8N

B. $C_6H_{10}N$

C. $C_6H_{12}N$

D. $C_6H_{13}N$

Answer: D

Solution:

The molecular formula for cyclohexylamine would reference the number of carbon (C), hydrogen (H), and nitrogen (N) atoms in the compound. Cyclohexylamine consists of a cyclohexane ring with an amine (-NH_2) group attached to one of the carbons. Therefore, to determine the correct molecular formula, we can simply add the typical number of hydrogens found in a cyclohexane ring to the number from the amine group.

Cyclohexane is a ring of six carbon atoms, and each carbon atom, being in a non-aromatic ring with single bonds, is bonded to two other carbons and two hydrogens, resulting in the molecular formula $\mathrm{C_6H_{12}}$ for cyclohexane.

However, when one hydrogen on the cyclohexane ring is substituted by an amine group, the nitrogen atom generally has two hydrogens. Thus, removing one hydrogen from cyclohexane and adding an amine group (-NH_2) changes the formula to:

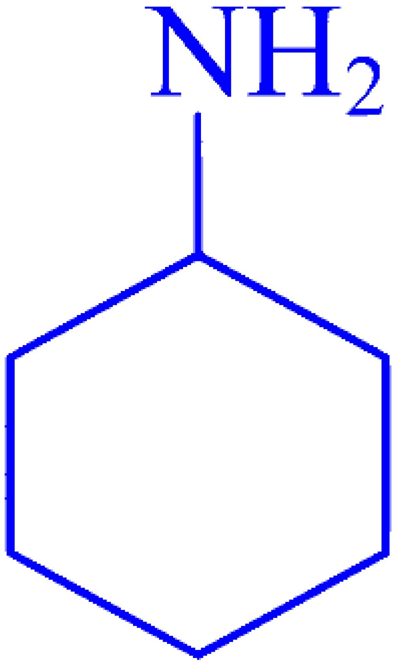
$$\mathrm{C_6H_{12} \rightarrow C_6H_{11}NH_2}$$

You can simplify the expression by combining the number of hydrogen atoms:

$$\mathrm{C_6H_{11}NH_2 = C_6H_{11} + NH_2 = C_6H_{13}N}$$

Therefore, the correct molecular formula for cyclohexylamine is Option D:

$$\mathrm{C_6H_{13}N}$$



Cyclohexylamine
($C_6H_{13}N$)

Question 68

Identify base$_2$ for following equation according to Bronsted-Lowry theory.

$$HCl_{(aq)} + H_2O_{(l)} \rightleftharpoons H_3O^+_{(aq)} + Cl^-_{(aq)}$$

Options:

A. $H_3O^+_{(aq)}$

B. $H_2O_{(l)}$

C. $Cl^-_{(aq)}$

D. $HCl_{(aq)}$

Answer: B

Solution:

The Bronsted-Lowry theory defines an acid as a substance that can donate a proton (H^+ ion), and a base as a substance that can accept a proton. For the given chemical reaction:

$HCl_{(aq)} + H_2O_{(l)} \rightleftharpoons H_3O^+_{(aq)} + Cl^-_{(aq)}$

In this reaction, hydrochloric acid (HCl) donates a proton to water (H_2O), which acts as a base and accepts the proton, forming hydronium ion (H_3O^+) and chloride ion (Cl^-). Here is the role of each species according to the Bronsted-Lowry theory:

- HCl is a Bronsted-Lowry acid because it donates a proton to water.
- H_2O is a Bronsted-Lowry base because it accepts a proton from HCl.
- H_3O^+ is the conjugate acid formed after H_2O accepts the proton.
- Cl^- is the conjugate base formed after HCl donates the proton.

According to these roles, the correct option identifying the base that accepts a proton in this reaction is:

Option B: $H_2O_{(l)}$

Question 69

Which of the following is Clemmensen reduction?

Options:

A.

$$RCOCl \xrightarrow[Pd-BaSO_4]{H_2} RCHO + HCl$$

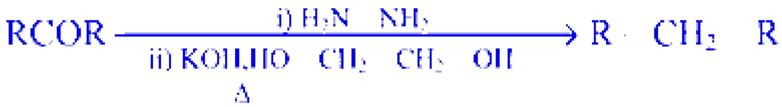
B.

$$RCN \xrightarrow[H_3O^+]{SnCl_2,\ HCl} RCHO + NH_4Cl$$

C.

$$RCHO \xrightarrow[\Delta]{Zn-Hg,\ Conc.\ HCl} RCH_3 + H_2O$$

D.



Answer: C

Question 70

Which of the following gases is readily adsorbed by solid adsorbent?

Options:

A. Cl_2

B. N_2

C. O_2

D. H_2

Answer: A

Solution:

Gases having high critical temperature liquefy easily and can be readily adsorbed. Examples: SO_2, Cl_2, NH_3

Question 71

Which from following is an example of two dimensional nanostructures?

Options:

A. Nanoparticles

B. Thin films

C. Quantum dots

D. Nanowires

Answer: B

Solution:

Among the options provided for examples of two-dimensional nanostructures, the correct answer is:

Option B: Thin films

Explanation of each option:

Option A: Nanoparticles

Nanoparticles are considered zero-dimensional nanostructures because they are tiny objects that behave as a whole unit with respect to their transport and properties. They have all their dimensions measured within the nanoscale (typically less than 100 nanometers) and, therefore, do not fall under the category of two-dimensional nanostructures.

Option B: Thin films

Thin films are indeed an example of two-dimensional nanostructures. They are layers of material ranging from fractions of a nanometer to several micrometers in thickness. Unlike one-dimensional structures that are extended in only one dimension (such as nanowires), or zero-dimensional structures like nanoparticles, thin films are extended in two dimensions: length and width, but their thickness is confined to the nanoscale, which gives them unique properties.

Option C: Quantum dots

Quantum dots are zero-dimensional nanostructures. These are tiny semiconductor particles a few nanometers in size, having electronic properties in between those of bulk semiconductors and discrete molecules. Though they often exhibit quantum behavior due to their size (where their electron energy levels are discrete), they don't qualify as two-dimensional.

Option D: Nanowires

Nanowires are one-dimensional nanostructures. They have a length that is much greater than their diameter, which is typically in the nanometer range. Their properties are unique because electrons in nanowires are confined in two dimensions and can only move freely along one dimension (along the length of the wire).

Question 72

A conductivity cell containing $0.001 \mathrm{~M} \mathrm{AgNO}_3$ solution develops resistance $6530 \mathrm{ohm}$ at $25^{\circ} \mathrm{C}$. Calculate the electrical conductivity of solution at same temperature if the cell constant is $0.653 \mathrm{~cm}^{-1}$.

Options:

A. $1.3 \times 10^{-4} \Omega^{-1} \mathrm{~cm}^{-1}$

B. $1.5 \times 10^{-4} \Omega^{-1} \mathrm{~cm}^{-1}$

C. $1.7 \times 10^{-4} \Omega^{-1} \mathrm{~cm}^{-1}$

D. $1.0 \times 10^{-4} \Omega^{-1} \mathrm{~cm}^{-1}$

Answer: D

Solution:

$$\begin{aligned} & \text{Cell constant } = \mathrm{k} \times \mathrm{R} \\ \therefore \quad & 0.653 = \mathrm{k} \times 6530 \\ \therefore \quad & \mathrm{k} = \frac{0.653}{6530} = 1 \times 10^{-4} \Omega^{-1} \mathrm{~cm}^{-1} \end{aligned}$$

Question 73

What is the number of sp^3 hybrid carbon atoms in $\mathrm{HO}(\mathrm{CH}_2)_3\mathrm{CH}(\mathrm{CH}_3)\mathrm{CH}(\mathrm{CH}_3)_2$?

Options:

A. Five

B. Two

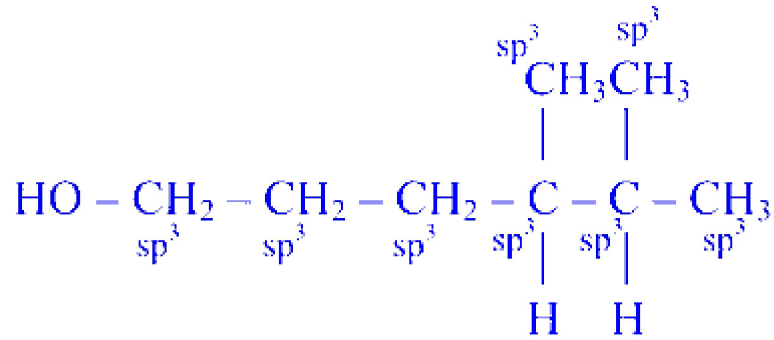
C. Eight

D. Four

Answer: C

Solution:

$HO(CH_2)_3CH(CH_3)CH(CH_3)_2$:



Number of sp^3 hybrid carbon atoms = 8

Question 74

Which from following thermodynamic properties is a path function?

Options:

A. Internal energy

B. Work

C. Entropy

D. Enthalpy

Answer: B

Solution:

Among the options given, the thermodynamic property that is a path function is :

Option B : Work

Path functions depend on the specific way that a system changes from one state to another. They are properties or quantities whose values depend on the transition path of the process rather than just the initial and final states of the system. Work (and also heat) fits this description because the amount of work done by or on a system can vary depending on the process path taken between two states.

In contrast to path functions, state functions (or point functions) are properties that depend only on the state of the system and not on the path taken to reach that state. Internal energy (Option A), entropy (Option C), and enthalpy (Option D) are examples of state functions. Their changes are determined entirely by the initial and final states of a process and do not depend on the path taken to get from one state to another. Therefore, for any thermodynamic system that transitions from state 1 to state 2, the change in a state function (like internal energy ΔU, entropy ΔS, or enthalpy ΔH) is the same no matter what path is taken.

Question 75

Which among the following species is reduced by tin easily?

Options:

A. Iodine

B. Iron

C. Zinc

D. Sodium

Answer: A

Solution:

Among the given species, only iodine has positive E° value. Higher (more positive) E° value for a half reaction indicates its greater tendency to get reduced.

Therefore, iodine is reduced by tin easily.

Question 76

Which from following combinations is an example for construction of n-type semiconductor?

Options:

A. Si doped with B

B. Si doped with P

C. Si doped with Ga

D. Si doped with In

Answer: B

Solution:

An n-type semiconductor is created by doping an intrinsic (pure) semiconductor with a dopant that has more valence electrons than the semiconductor. Silicon (Si) has four valence electrons. For an n-type semiconductor, one would typically dope it with an element from group V of the periodic table (pentavalent), which has five valence electrons.

Here are the options analyzed:

Option A: Si doped with B (Boron)

Boron is a group III element (trivalent), which has three valence electrons. If silicon is doped with boron, it will create a p-type semiconductor because boron will introduce holes (positive charge carriers) into the silicon crystal.

Option B: Si doped with P (Phosphorus)

Phosphorus is a group V element (pentavalent), which has five valence electrons. When silicon is doped with phosphorus, it will create an n-type semiconductor because the extra electron from each phosphorus atom will become a free electron (negative charge carrier) in the silicon crystal.

Option C: Si doped with Ga (Gallium)

Gallium is a group III element (trivalent). Similar to boron, gallium would create a p-type semiconductor when used to dope silicon for the same reasons explained under option A.

Option D: Si doped with In (Indium)

Indium is also a group III element (trivalent). Like gallium and boron, indium would result in the formation of a p-type semiconductor when doping silicon.

Based on the information above, the correct answer is:

Option B: Si doped with P

Because doping silicon with phosphorus introduces extra electrons that increase the concentration of free electrons, making it an n-type semiconductor.

Question 77

Calculate the density of metal having molar mass $210 \mathrm{~g} \mathrm{~mol}^{-1}$ that forms simple cubic unit cell. $\left(\mathrm{a}^3 \cdot \mathrm{N}_{\mathrm{A}}=21.5 \mathrm{~cm}^3 \mathrm{~mol}^{-1}\right)$

Options:

A. 9.77 g cm^{-3}

B. 7.15 g cm^{-3}

C. 8.12 g cm^{-3}

D. 6.94 g cm^{-3}

Answer: A

Solution:

The density (ρ) of a simple cubic unit cell is calculated using:

$\rho = \frac{Z \cdot M}{a^3 \cdot N_A}$

Where:

- Z = Number of atoms per unit cell (1 for simple cubic)
- M = Molar mass of the metal (g/mol)
- a = Edge length of the unit cell (cm)
- N_A = Avogadro's Number (6.022 x 10^{23} mol^{-1})

1. **Finding the Edge Length (a):**

$a^3 = \frac{21.5 \text{ cm}^3 \text{ mol}^{-1}}{N_A}$

$a^3 = \frac{21.5 \text{ cm}^3 \text{ mol}^{-1}}{6.022 \times 10^{23} \text{ mol}^{-1}}$

$a^3 = 3.57 \times 10^{-23} \text{ cm}^3$

$a = 3.29 \times 10^{-8} \text{ cm}$

1. **Calculating Density (ρ):**

$\rho = \frac{1 \cdot 210 \text{ g mol}^{-1}}{(3.29 \times 10^{-8} \text{ cm})^3 \cdot (6.022 \times 10^{23} \text{ mol}^{-1})}$

$\rho = 9.77 \text{ g cm}^{-3}$

Answer:

The density of the metal is 9.77 g/cm^3.

Question 78

Which element from following rapidly loses its luster in air and tarnishes?

Options:

A. Ba

B. Be

C. K

D. Mg

Answer: C

Solution:

The correct answer is Option C, K, which stands for potassium.

Potassium is an alkali metal and is highly reactive, especially with oxygen in the air. It reacts rapidly with atmospheric oxygen to form potassium oxide and other compounds, which causes the metal to tarnish or lose its luster quickly. This reaction can be so rapid that potassium must be stored under oil or in an inert atmosphere to prevent it from tarnishing or becoming a fire hazard.

In contrast, some of the other elements mentioned have a slower reaction with air:

Option A, Ba (Barium), is an alkaline earth metal and does oxidize in air, but not as rapidly as potassium.

Option B, Be (Beryllium), is a relatively inert metal and does not tarnish quickly. It forms an oxide layer that protects it from further corrosion.

Option D, Mg (Magnesium), is another alkaline earth metal that is more reactive than beryllium but forms a protective oxide layer which somewhat slows down further oxidation.

The reactivity of these metals with air increases from beryllium (least reactive), to magnesium, to barium, and then to potassium (most reactive).

Question 79

A neon-dioxygen mixture contains $64 \mathrm{~g} \mathrm{O}_2$ and $160 \mathrm{~g} \mathrm{Ne}$. If the total pressure is $25 \mathrm{bar}$, calculate the partial pressure of dioxygen.

Options:

A. 5 bar

B. 7.5 bar

C. 10 bar

D. 20 bar

Answer: A

Solution:

$$n_{O_2} = \frac{64}{32} = 2 \text{ mol}; \quad n_{N_c} = \frac{160}{20} = 8 \text{ mol}$$
$$x_{O_2} = \frac{n_{O_2}}{n_{Total}} = \frac{2}{2+8} = \frac{2}{10} = 0.2$$
$$P_{O_2} = x_{O_2} \times P_{Total} \quad = 0.2 \times 25 = 5 \text{ bar}$$

Question 80

Identify anionic sphere complex from following.

Options:

A. Hexaamminecobalt(III) chloride

B. Potassium hexacyanoferrate(II)

C. Tetraamminedichlorocobalt(III) ion

D. Pentaamminechlorocobalt(III) sulphate

Answer: B

Solution:

The term "anionic sphere complex" refers to a complex ion that carries a net negative charge. To identify an anionic sphere complex among the options provided, we need to look at the complex ions and see which ones are negatively charged.

Option A: Hexaamminecobalt(III) chloride. This complex is $[Co(NH_3)_6]^{3+}$ and carries a net positive charge. Therefore, it is not an anionic sphere complex.

Option B: Potassium hexacyanoferrate(II). This complex is $[Fe(CN)_6]^{4-}$, which carries a net negative charge. Therefore, it is an anionic sphere complex.

Option C: Tetraamminedichlorocobalt(III) ion. This complex ion is $[Co(NH_3)_4Cl_2]^{+}$, which carries a net positive charge. Thus, it is not an anionic sphere complex.

Option D: Pentaamminechlorocobalt(III) sulphate. This complex itself is $[Co(NH_3)_5Cl]^{2+}$ and carries a net positive charge. So, it is not an anionic sphere complex.

The correct answer is **Option B**: Potassium hexacyanoferrate(II), which features the anionic complex $[Fe(CN)_6]^{4-}$.

Question 81

What is the number of moles of ethane obtained from $2n$ moles of bromomethane using $2n$ moles of sodium atoms in dry ether medium?

Options:

A. 4n

B. 3n

C. 2n

D. n

Answer: D

Solution:

$$2CH_3Br + 2Na \xrightarrow{\text{Dry ether}} \underset{\text{Ethane}}{CH_3 - CH_3} + 2NaBr$$

$$\therefore \quad 2n \text{ moles of } CH_3Br \equiv n \text{ moles of ethane}$$

Question 82

Calculate E^0_{cell} for $Cd_{(s)}\left|Cd^{++}_{(1M)}\right|\left|Ag^{+}_{(1M)}\right|Ag_{(s)}$.

$\left[E^0_{Cd} = -0.403\ V; E^0_{Ag} = 0.799\ V\right]$

Options:

A. $1.202\ V$

B. $-1.202\ V$

C. $0.396\ V$

D. $-0.396\ V$

Answer: A

Solution:

For the given cell, anode is Cd and cathode is Ag.

$$\begin{aligned} E^0_{cell} &= E^0_{cathode} - E^0_{anode} \\ &= 0.799 - (-0.403) \\ &= 1.202\ V \end{aligned}$$

Question 83

Find the number of unpaired electrons for copper in ground state configuration.

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

To find the number of unpaired electrons in copper (Cu), we need to write down its electron configuration. The atomic number of copper is 29, indicating it has 29 electrons.

Copper, a transition metal, can sometimes have an electron configuration that is an exception to the typical filling order due to the relative energies of the atomic orbitals. The generally expected electron configuration would follow Hund's rule and the Aufbau principle, filling orbitals from lowest to highest energy level.

However, copper exhibits a unique electron arrangement where one electron in the 4s orbital is used to complete the 3d subshell, resulting in a more stable configuration. Its electron configuration in the ground state is:

Cu: $[Ar]3d^{10}4s^1$

When we look at this configuration:

- The 3d subshell is completely filled with 10 electrons, which means all five 3d orbitals have paired electrons. There are no unpaired electrons in the 3d subshell.
- The 4s subshell has 1 electron. This means there is just one unpaired electron in the 4s orbital.

Therefore, in its ground state, copper has a total of 1 unpaired electron.

The answer to the question is:

Option A : 1

Question 84

Identify the element having highest ionization enthalpy.

Options:

A. Ti

B. Sc

C. Zn

D. Ni

Answer: C

Solution:

The ionization enthalpy, also known as ionization energy, is the energy required to remove the most loosely bound electron from an isolated gaseous atom to form a cation. Generally speaking, ionization enthalpy increases across a period from left to right in the periodic table and decreases down a group. This is because the effective nuclear charge (which refers to the net positive charge experienced by an electron in a multi-electron atom) increases across a period, pulling the electrons in closer and thus requiring more energy to remove an electron. Conversely, ionization enthalpy decreases down a group as the outermost electrons are farther from the nucleus and more shielded by inner electrons, which makes them easier to remove.

Let's examine the given options in the context of their positions in the periodic table:

- **Titanium (Ti) -** Atomic number 22, located in period 4 and Group 4 of the periodic table (the first of the transition metals in this period).

- **Scandium (Sc) -** Atomic number 21, located in period 4 and is the first element in the transition series, Group 3 of the periodic table.

- **Zinc (Zn) -** Atomic number 30, located at the end of period 4 in the transition metals, specifically Group 12 of the periodic table. It has a completely filled d-subshell in its ground state.

- **Nickel (Ni) -** Atomic number 28, located in period 4, and is in Group 10 of the periodic table.

Now, considering the trends in ionization energy across the periodic table:

- Going from Sc to Zn (left to right) in the same period, generally, the ionization energy should increase because of the increased nuclear charge.

- Scandium has a $3d^1 4s^2$ configuration, so it would have lower ionization energy compared to Zinc, which has a $3d^{10} 4s^2$ configuration, since Zinc's d-subshell is completely filled, adding to the stability and requiring more energy to remove an electron.

- Nickel has the electron configuration $3d^8 4s^2$, so it has higher ionization energy compared to Scandium as well, but less than Zinc since Zinc has a completely filled d-subshell configuration that contributes to stability.

- Titanium with the electron configuration $3d^2 4s^2$ would have higher ionization energy than Scandium as it is to the right of Scandium but lower than that of Zinc for the same reasons discussed above.

Hence, Zinc (Zn) has the highest ionization enthalpy among the given elements due to its filled d-subshell which makes it highly stable relative to the other elements listed.

The correct answer is **Option C: Zn**.

Question 85

What is the oxidation number of Pt in PtCl_6^{2-} ?

Options:

A. +6

B. +4

C. -6

D. -4

Answer: B

Solution:

To determine the oxidation number of platinum (Pt) in the complex ion PtCl_6^{2-}, we need to consider the oxidation numbers of the other atoms in the ion and the overall charge.

Chlorine (Cl) is a halogen and typically has an oxidation number of -1 when it's a part of a compound but not in a neutral elemental state.

The complex ion PtCl_6^{2-} has a total charge of -2. There are six chlorine atoms, each contributing an oxidation number of -1. This means the total contribution of the chlorines to the charge is $6 \times (-1) = -6$.

Now, let's set up an equation to solve for the oxidation number of platinum (Pt), which we'll call x:

$x + (6 \times -1) = -2$

Solve for x:

$x - 6 = -2$

$x = -2 + 6$

$x = 4$

Thus, the oxidation number of platinum in the complex ion PtCl_6^{2-} is +4.

The correct answer is:

Option B: +4

Question 86

What is the value of rate constant for first order reaction if slope for the graph of rate versus concentration is 2.5×10^{-3} ?

Options:

A. 2.5×10^{-3} time^{-1}

B. 5.0×10^{-3} time^{-1}

C. 7.5×10^{-3} time^{-1}

D. 1.25×10^{-3} time^{-1}

Answer: A

Solution:

The rate of a first-order reaction is directly proportional to the concentration of one reactant. The mathematical expression for the rate of a first-order reaction is:

$k[A]$

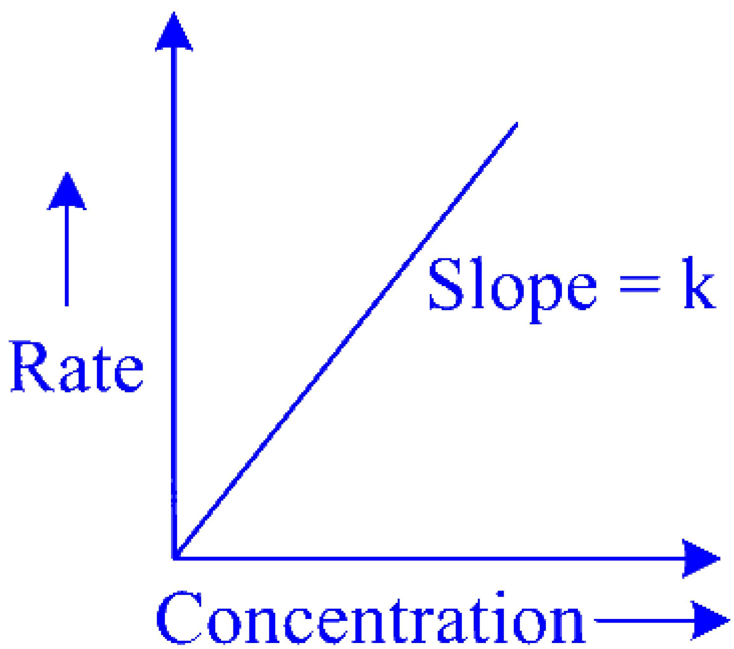
where:

- k is the rate constant
- $[A]$ is the concentration of the reactant.

The rate equation can be expressed as:

Rate $= k[A]$

Given that a graph of rate versus concentration, the slope of this graph for a first-order reaction would be equal to the rate constant, k, because the rate is directly proportional to the concentration $[A]$:



Slope $= k$

Therefore, if the slope for the graph of rate versus concentration is 2.5×10^{-3}, then the value of the rate constant for the first-order reaction is:

$k = 2.5 \times 10^{-3}$ time^{-1}

So, the correct answer is:

Option A 2.5×10^{-3} time^{-1}

Question 87

An organic monobasic acid has dissociation constant 2.25×10^{-6}. What is percent dissociation in its 0.01 M solution?

Options:

A. 1.5%

B. 15%

C. 5%

D. 0.5%

Answer: A

Solution:

The percent dissociation of an acid in solution can be determined by first calculating the concentration of the hydrogen ions ($[H^+]$) that are produced when the acid dissociates. The dissociation constant (K_a) gives us a measure of the extent to which the acid dissociates in solution. For a weak acid, which only partially dissociates, the equilibrium expression can be written as:

$HA \rightleftharpoons H^+ + A^-$

where HA is the weak acid, H^+ is the hydrogen ion, and A^- is the conjugate base. The dissociation constant, K_a, is then given by:

$K_a = \frac{[H^+][A^-]}{[HA]}$

The degree of dissociation (α) in weak acids is quite small. We can therefore assume that the change in concentration of the acid (the amount that dissociates) is relatively small and the equilibrium concentration of the acid $[HA]$ can be approximated as the original concentration (C_0) of the acid. Hence, taking α to represent the degree of dissociation, we get:

$[H^+] = [A^-] = \alpha C_0$

$[HA] = C_0 - \alpha C_0 \approx C_0$

Now we plug in these into the expression for K_a:

$K_a = \frac{\alpha C_0 \cdot \alpha C_0}{C_0}$

$K_a = \alpha^2 C_0$

Solving for α, we get:

$\alpha = \sqrt{\frac{K_a}{C_0}}$

Now let's plug in the given values:

$K_a = 2.25 \times 10^{-6}$

$C_0 = 0.01\text{ M}$

$\alpha = \sqrt{\frac{2.25 \times 10^{-6}}{0.01}}$

$\alpha = \sqrt{2225 \times 10^{-4}}$

$\alpha = 1.5 \times 10^{-2}$

Now to find the percent dissociation, we convert the degree of dissociation into a percentage:

$\text{Percent dissociation} = \alpha \times 100\%$

$\text{Percent dissociation} = 1.5 \times 10^{-2} \times 100\%$

$\text{Percent dissociation} = 1.5\%$

Therefore, the percent dissociation of the acid in its 0.01 M solution is 1.5%, which corresponds to option A.

Question 88

Which of the following pair of compounds demonstrates the law of multiple proportions?

Options:

A. CH_4, CCl_4

B. BF_3, NH_3

C. CO, CO_2

D. NO_2, CO_2

Answer: C

Solution:

The Law of Multiple Proportions states that when two elements form more than one compound, the masses of one element that combine with a fixed mass of the other element are in the ratio of small whole numbers. To identify which pair of compounds demonstrates the law of multiple proportions, let's analyze each option:

Option A: CH_4, CCl_4

This pair does not demonstrate the law of multiple proportions, as the compounds have different elements (hydrogen in CH_4 and chlorine in CCl_4) combining with carbon, without a fixed mass of one element combining with varying masses of the other.

Option B: BF_3, NH_3

This option is also not an example of the law of multiple proportions, as these compounds do not contain the same two elements combining in different ratios; BF_3 contains boron and fluorine whereas NH_3 contains nitrogen and hydrogen.

Option C: CO, CO_2

This option correctly represents the law of multiple proportions. Both CO and CO_2 consist of carbon and oxygen. In carbon monoxide (CO), one atom of carbon combines with one atom of oxygen, whereas in carbon dioxide (CO_2), one atom of carbon combines with two atoms of oxygen. Here, the fixed mass of carbon combines with varying masses of oxygen in a ratio of 1:2 (for the oxygen atoms).

Option D: NO_2, CO_2

Like Option A and B, this choice does not follow the law of multiple proportions because each compound has different elements combining with each other; NO_2 has nitrogen and oxygen, while CO_2 has carbon and oxygen.

The correct answer is therefore Option C: CO, CO_2, as these two compounds consist of the same two elements, carbon and oxygen, which combine in different simple numerical ratios, fulfilling the criteria of the law of multiple proportions.

Question 89

Which of the following amines on heating with chloroform generate foul smelling product?

Options:

A. Ethanamine

B. Ethylmethylamine

C. Ethyldimethylamine

D. Diethylmethylamine

Answer: A

Solution:

Aliphatic or aromatic primary amines on heating with chloroform form foul smelling products (carbylamines).

Question 90

The rate law for the reaction $\mathrm{A}+\mathrm{B} \rightarrow$ product is rate $=\mathrm{k}[\mathrm{A}][\mathrm{B}]$. When will the rate of reaction increase by factor two?

Options:

A. $[\mathrm{A}]$ and $[\mathrm{B}]$ both are doubled

B. $[\mathrm{A}]$ is doubled and $[\mathrm{B}]$ is kept constant

C. $[\mathrm{B}]$ is doubled and $[\mathrm{A}]$ is halved

D. [A] is kept constant [B] is halved

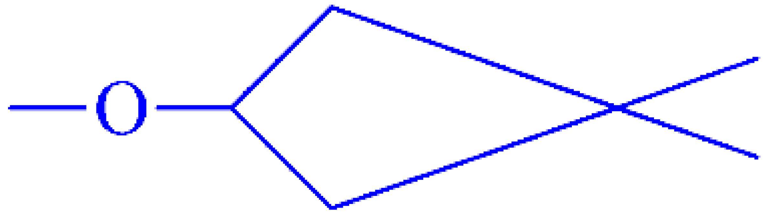
Answer: B

Solution:

$$\begin{aligned}
& \text{Rate } =\mathrm{k}[\mathrm{A}][\mathrm{B}] \\
& (\text{ Rate })_1=\mathrm{k} \times 2[\mathrm{~A}][\mathrm{B}] \\
& \frac{(\text{ Rate })_1}{\text{Rate}}=\frac{\mathrm{k} 2[\mathrm{~A}][\mathrm{B}]}{\mathrm{k}[\mathrm{A}][\mathrm{B}]}=2 \\
& (\text{ Rate })_1=2 \times \text{ Rate }
\end{aligned}$$

Question 91

What is IUPAC name of following compound?



Options:

A. 1-Methoxy-3,3-dimethylcyclobutane

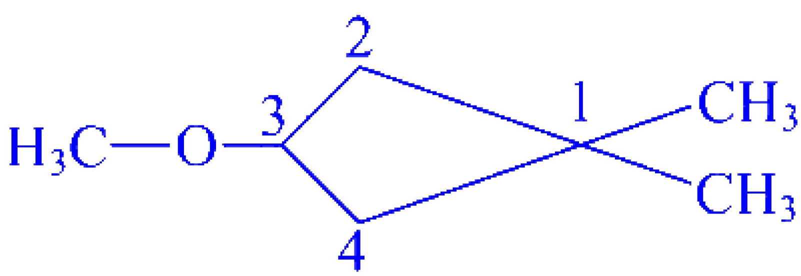
B. 3-Methoxy-1,1-dimethylcyclobutane

C. 3,3-Dimethylcyclobutoxymethane

D. 1-Methoxy-3-isoproylbutane

Answer: B

Solution:



3-Methoxy-1,1-dimethylcyclobutane

Question 92

Identify substrate 'A' in the following reaction.

$$2nA \xrightarrow{\text{Dimethyl cadmium}} \text{2n Propanone + n Cadmium chloride}$$

Options:

A. Ethyl chloride

B. Ethylene dichloride

C. Ethanoyl chloride

D. Ethylidene dichloride

Answer: C

Solution:

$$\underset{\text{Ethanoyl chloride}}{2CH_3COCl} + (CH_3)_2Cd \rightarrow \underset{\text{Propanone}}{2CH_3COCH_3} + CdCl_2$$

Ketones are prepared from acyl chloride by reaction with dialkyl cadmium.

Question 93

Which of the following is NOT a basic amino acid?

Options:

A. Proline

B. Lysine

C. Arginine

D. Histidine

Answer: A

Solution:

The term "basic amino acid" refers to amino acids that have side chains with a net positive charge at physiological pH (approximately pH 7.4). Basic amino acids are typically those that contain nitrogen and can accept a hydrogen ion, making them positively charged. Among the options provided, let's consider each amino acid:

- **Proline (Option A)**: Proline is unique among the twenty standard amino acids because it has a secondary amine group, where the amino nitrogen is bound to two alkyl groups. Its side chain is a cyclic structure. Proline is classified as a nonpolar, aliphatic amino acid and is not basic; it does not carry a positive charge at physiological pH.

- **Lysine (Option B)**: Lysine has a side chain with an aliphatic terminal (straight-chain) amine group, which is positively charged at physiological pH. This makes lysine a basic amino acid.

- **Arginine (Option C)**: Arginine contains a side chain with a complex guanidinium group, which has a high pKa and thus remains positively charged at physiological pH. Therefore, arginine is considered a basic amino acid.

- **Histidine (Option D)**: Histidine has an imidazole side chain, which can be positively charged at slightly acidic to neutral pH. Histidine's side chain has a pKa around 6.0, making it occasionally positively charged at physiological pH, and thus, histidine is sometimes considered a basic amino acid, especially since it can form hydrogen bonds and salt bridges.

From the options provided, **Option A, Proline,** is the correct answer. Proline is not a basic amino acid; it does not have a positive charge on its side chain at physiological pH.

Question 94

Calculate the PV type of work for the following reaction at 1 bar pressure.

$$\underset{(150\ \mathrm{mL})}{\mathrm{C_3H_{6(g)}}} + \underset{(150\ \mathrm{mL})}{\mathrm{HCl_{(g)}}} \longrightarrow \underset{(150\ \mathrm{mL})}{\mathrm{C_3H_7Cl_{(g)}}}$$

Options:

A. 5.2 J

B. 10.21 J

C. 15.00 J

D. 18.2 J

Answer: C

Solution:

$$\begin{aligned}
&W = -P_{\text{ext}}\ \ \Delta V = -P_{\text{ext}}\ \ (V_2 - V_1) \\
&V_1 = 150 + 150 = 300\ \mathrm{mL}^3 = 0.3\ \mathrm{dm}^3 \\
&V_2 = 150\ \mathrm{mL} = 0.15\ \mathrm{dm}^3 \\
&\quad P_{\text{ext}} \quad = 1\ \text{bar} \\
&W = -1(0.15 - 0.3) \\
&\quad = 0.15\ \mathrm{dm}^3\ \text{bar} \\
&\quad = 15.0\ \mathrm{J} \quad \left(\because 100\ \mathrm{J} = 1\ \mathrm{dm}^3\ \text{bar}\right)
\end{aligned}$$

Question 95

Which among the following is NOT dicarboxylic acid?

Options:

A. Malonic acid

B. Caproic acid

C. Glutaric acid

D. Succinic acid

Answer: B

Solution:

Caproic acid: $CH_3(CH_2)_4COOH$

Question 96

Identify the element having positive electron gain enthalpy.

Options:

A. Ne

B. I

C. S

D. O

Answer: A

Solution:

Electron gain enthalpy, sometimes called electron affinity, refers to the amount of energy released or spent when an electron is added to a neutral atom in the gas phase to form a negative ion. If the energy is released during the process, the electron gain enthalpy is negative, indicating that the process is exothermic and the element tends to gain an electron more readily. Conversely, when the energy is required to add an electron, the electron gain enthalpy is positive, and the process is endothermic, meaning that the element does not favor gaining an electron.

Generally, nonmetals have more negative electron gain enthalpies because they are more likely to accept electrons to complete their valence electron shells. However, there are exceptions, particularly among the noble gases.

In the context of noble gases like neon (Ne), they already have a complete valence shell and are quite stable. Therefore, adding an electron would require forcing it into an already complete electron shell, which is an unfavorable process requiring energy. Hence, noble gases generally have positive electron gain enthalpies despite being nonmetals.

Let's analyze the options given:

Option A: Ne (Neon) - Neon is a noble gas with a stable electronic configuration of $2s^2 2p^6$. Therefore, it does not tend to gain electrons, and the process of adding an electron to a neon atom is endothermic, requiring energy. Therefore, neon has a positive electron gain enthalpy.

Option B: I (Iodine) - Iodine is a nonmetal and has a high tendency to gain an electron to complete its valence shell, hence it has large negative electron gain enthalpy.

Option C: S (Sulfur) - Sulfur is another nonmetal which prefers to gain electrons to achieve a stable electron configuration, hence it also has negative electron gain enthalpy.

Option D: O (Oxygen) - Oxygen is highly electronegative and readily gains electrons, resulting in negative electron gain enthalpy.

The element among the options provided that has positive electron gain enthalpy is Ne (Neon), so the correct answer is:

Option A: Ne (Neon)

Question 97

Identify the use of HDP from following.

Options:

A. Insulation of electric cables

B. Manufacture of toys

C. Submarine cable insulation

D. Producing extruded films

Answer: B

Solution:

The correct answer is **B) Manufacture of toys**. Here's why:

- **HDP (High-Density Polyethylene):** HDP is a rigid, strong, and versatile plastic with a wide range of applications.

Let's analyze each option:

- **A) Insulation of electric cables:** LDPE (Low-Density Polyethylene) is more commonly used for electrical insulation due to its flexibility and electrical properties.
- **B) Manufacture of toys:** HDP is perfect for toys because of its durability, chemical resistance, and ability to be molded into various shapes.
- **C) Submarine cable insulation:** Submarine cables require specialized materials with extreme durability and water resistance. HDP is not typically used for this purpose.
- **D) Producing extruded films:** While HDP can be used for film extrusion, it is not the most common choice compared to other types of polyethylene.

Key Point: HDP's properties make it ideal for manufacturing toys that need to withstand rough handling and maintain their shape.

Question 98

Which coordination complex from following contains neutral ligand?

Options:

A. Pentacarbonyl iron(0)

B. Trioxalatocobaltate(III) ion

C. Sodium hexanitrocobaltate(III)

D. Tetracyanonickelate(II) ion

Answer: A

Solution:

The coordination complex that contains a neutral ligand among the given options is:

Option A: Pentacarbonyl iron(0), which has the formula $Fe(CO)_5$

Ligands can be neutral molecules or anions that donate a pair of electrons to the metal center to form coordinate bonds. Carbonyl (CO) is a common example of a neutral ligand, as it contains a carbon atom triple-bonded to an oxygen atom and has a lone pair of electrons that can be donated to a metal atom. In the complex $Fe(CO)_5$, the iron atom is surrounded by five carbonyl ligands, all of which are neutral.

Option B: Trioxalatocobaltate(III) ion contains oxalate ligands ($C_2O_4^{2-}$), which are anionic.

Option C: Sodium hexanitrocobaltate(III), $Na_3[Co(NO_2)_6]$, contains nitro (NO_2^-) ligands, which are also anionic.

Option D: Tetracyanonickelate(II) ion, $[Ni(CN)_4]^{2-}$, contains cyanide ligands (CN^-), which are anionic as well.

Hence, the complex with a neutral ligand is Option A: Pentacarbonyl iron(0).

Question 99

What type of following solutions is the gasoline?

Options:

A. Liquid as solute and liquid as solvent

B. Liquid as solute and solid as solvent

C. Solid as solute and liquid as solvent

D. Gas as solute and liquid as solvent

Answer: A

Solution:

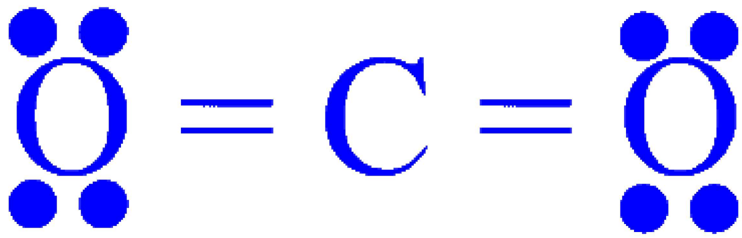
Gasoline is a homogeneous mixture of liquid hydrocarbons. It is primarily used as a fuel in internal combustion engines. Given the nature of its composition, the solution itself is liquid, consisting of many different liquid organic compounds (hydrocarbons and additives) mixed together. Thus, each component can be considered a liquid solute that dissolves in a liquid solvent—other hydrocarbons that make up the bulk of the gasoline. Therefore, the correct option for the type of solution that gasoline represents is:

Option A: Liquid as solute and liquid as solvent

Gasoline does not fit the descriptions for options B, C, or D since it does not contain solid solutes or solvents, nor does it have gases dissolved in it under normal conditions to any significant extent that would classify it as a gas in liquid solution.

Question 100

What is formal charge on carbon in the following Lewis structure?



Options:

A. 0

B. 1

C. -1

D. 2

Answer: A

Solution:

$\text{Formal charge} = \text{VE} - \text{NE} - (\text{BE}/2)$

$\text{Formal charge on C} = 4 - 0 - (8/2) = 0$

Physics

Question 101

The power factor of an R-L circuit is $\frac{1}{\sqrt{2}}$. If the frequency of AC is doubled the power factor will now be

Options:

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{5}}$

C. $\frac{1}{\sqrt{7}}$

D. $\frac{1}{\sqrt{11}}$

Answer: B

Solution:

The power factor of an R-L circuit is given as,

$$\cos\phi = \frac{\mathrm{R}}{\sqrt{\mathrm{R}^2 + \mathrm{X}_\mathrm{L}^2}}$$

$$\therefore \quad \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \frac{1}{\sqrt{1 + \left(\frac{X_L}{R}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \left(\frac{\omega L}{R}\right)^2 + 1 = 2$$

$$\therefore \quad \frac{\omega L}{R} = 1$$

So, when the AC frequency is doubled,

$$\frac{\omega L}{R} = 2$$

$$\therefore \quad \cos\phi = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}} = \frac{1}{\sqrt{1 + (2)^2}}$$

$$\therefore \quad \cos\phi = \frac{1}{\sqrt{5}}$$

Question 102

Ratio of longest wavelength corresponding to Lyman and Balmer series in hydrogen spectrum is

Options:

A. $\frac{7}{29}$

B. $\frac{9}{31}$

C. $\frac{5}{27}$

D. $\frac{3}{23}$

Answer: C

Solution:

Wavelength for Lyman series is,

$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{\mathrm{n}^2}\right]$

For the longest wavelength, $\lambda=\lambda_{\max}$ and $\mathrm{n}=2$

$\frac{1}{\lambda_{\max(\mathrm{L})}}=\mathrm{R}\left[\frac{1}{1^2}-\frac{1}{2^2}\right]=\frac{3}{4}$

Wavelength for Balmer series is,

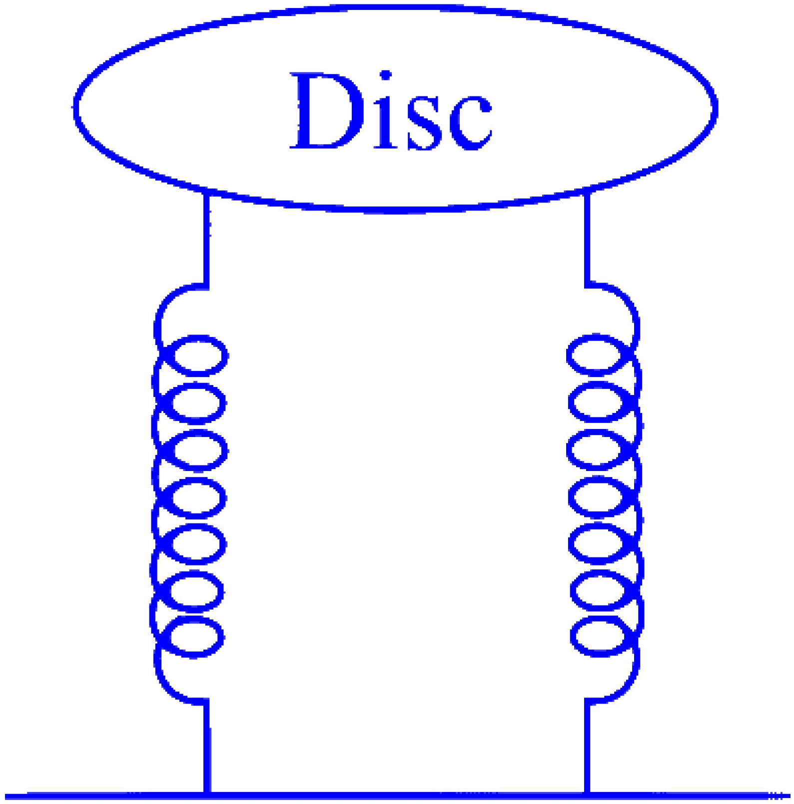
$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$

For the longest wavelength, $\mathrm{n}=3$

$$\frac{1}{\lambda_{\max(\mathrm{B})}}=\mathrm{R}\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=\frac{5}{36}$$
$$\frac{\lambda_{\max(\mathrm{L})}}{\lambda_{\max(\mathrm{B})}}=\frac{4}{3}\times\frac{5}{36}=\frac{5}{27}$$

Question 103

A uniform circular disc of mass $12\ \mathrm{kg}$ is held by two identical springs. When the disc is slightly pressed down and released, it executes S.H.M. of period 2 second. The force constant of each spring is (nearly) (Take $\pi^2=10$)



Options:

A. $230 \ \mathrm{Nm}^{-1}$

B. $120 \ \mathrm{Nm}^{-1}$

C. $60 \ \mathrm{Nm}^{-1}$

D. $30 \ \mathrm{Nm}^{-1}$

Answer: C

Solution:

The two springs are connected in parallel. So, the effective spring constant is, $\mathrm{k}_{\mathrm{eff}} = 2\mathrm{k}$

Time period of the spring system is,

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{2\mathrm{k}}}$$

$$\therefore \quad \mathrm{T}^2 = 4\pi^2 \times \frac{\mathrm{m}}{2\mathrm{k}}$$

$$\therefore \quad \mathrm{k} = 4\pi^2 \times \frac{\mathrm{m}}{2\,\mathrm{T}^2} = 4 \times 10 \times \frac{12}{2 \times 4} = 60 \ \mathrm{N}/\mathrm{m}$$

Question 104

A charge $17.7 \times 10^{-4} \mathrm{C}$ is distributed uniformly over a large sheet of area $200 \mathrm{~m}^2$. The electric field intensity at a distance $20 \mathrm{~cm}$ from it in air will be $\left[\varepsilon_0=8.85 \times 10^{-12} \mathrm{C}^2 / \mathrm{Nm}^2\right]$

Options:

A. $5 \times 10^5 \mathrm{~N} / \mathrm{C}$

B. $6 \times 10^5 \mathrm{~N} / \mathrm{C}$

C. $7 \times 10^5 \mathrm{~N} / \mathrm{C}$

D. $8 \times 10^5 \mathrm{~N} / \mathrm{C}$

Answer: A

Solution:

The surface charge density is given by,

$\sigma=\frac{\mathrm{q}}{\mathrm{A}}=\frac{17.7 \times 10^{-4}}{200}=8.85 \times 10^{-6} \mathrm{C} / \mathrm{m}^2$

The electric field intensity at a distance of 20 cm in air is,

$\mathrm{E}=\frac{\sigma}{2 \varepsilon_0}=\frac{8.85 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}}=5 \times 10^5 \mathrm{~N} / \mathrm{C}$

Hence, option (A).

Question 105

Sound waves of frequency $600 \mathrm{~Hz}$ fall normally on a perfectly reflecting wall. The shortest distance from the wall at which all particles will have maximum amplitude of vibration is (speed of sound $=300 \mathrm{~ms}^{-1}$)

Options:

A. $\frac{1}{4} \mathrm{~m}$

B. $\frac{1}{8}$ m

C. $\frac{3}{8}$ m

D. $\frac{7}{8}$ m

Answer: B

Solution:

The maximum displacement of the wave will occur at antinode of the wave. The first antinode will be a point where, $d = \frac{\lambda}{4}$

The wavelength is given by,

$\lambda = \frac{\mathrm{v}}{\mathrm{f}} = \frac{300}{600}$

So, $d = \frac{0.5}{4} = \frac{1}{8}$ m

Question 106

A long solenoid has 1500 turns. When a current of 3.5 A flows through it, the magnetic flux linked with each turn of solenoid is 2.8×10^{-3} weber. The self-inductance of solenoid is

Options:

A. 1.2 H

B. 2.4 H

C. 3.6 H

D. 6 H

Answer: A

Solution:

Flux linked with each turn of the solenoid is, $\phi = 2.8 \times 10^{-3}$ Wb

$\therefore$ Total magnetic flux of the solenoid is,

$\phi_{\text{net}} = \mathrm{N}\phi$
$\phi_{\text{net}} = 1500 \times 2.8 \times 10^{-3} = 4.2\ \mathrm{Wb}$

Flux can also be given by, $\phi = \mathrm{LI}$

$\therefore$ The self-inductance of the solenoid is,

$$L = \frac{\phi}{I} = \frac{4.2}{3.5}$$
$$L = 1.2\ \mathrm{H}$$

Question 107

A metal rod cools at the rate of $4^\circ\mathrm{C}/\mathrm{min}$ whon its temperature is $90^\circ\mathrm{C}$ and the rate of $1^\circ\mathrm{C}/\mathrm{min}$ when its temperature is $30^\circ\mathrm{C}$. The temperature of the surrounding is

Options:

A. $20^\circ\mathrm{C}$

B. $15^\circ\mathrm{C}$

C. $10^\circ\mathrm{C}$

D. $5^\circ\mathrm{C}$

Answer: C

Solution:

According to Newton's law of cooling,

$$\frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$$
$$\therefore \quad \frac{4}{1} = \frac{90 - \theta_0}{30 - \theta_0}$$
$$\therefore \quad 120 - 90 = 4\theta_0 - \theta_0$$
$$\therefore \quad 3\theta_0 = 30$$

$\therefore$ Temperature of the surroundings is,

$\theta_0 = 10^\circ\mathrm{C}$

Question 108

On replacing a thin film of mica of thickness 12×10^{-5} cm in the path of one of the interfering beams in Young's double slit experiment using monochromatic light, the fringe pattern shifts through a distance equal to the width of bright fringe. If $\lambda = 6 \times 10^{-5}$ cm, the refractive index of mica is

Options:

A. 1.1

B. 1.3

C. 1.5

D. 1.4

Answer: C

Solution:

The equation for the shift in the fringe pattern is given as,

$\mathrm{n}\lambda = (\mu - 1)\mathrm{t}$

$\therefore$ Refractive index of mica is:

$$\mu = \frac{\mathrm{n}\lambda}{\mathrm{t}} + 1$$

$$\therefore \quad \mu = \frac{1 \times 6 \times 10^{-5}}{12 \times 10^{-5}} + 1 = 0.5 + 1$$

$$\therefore \quad \mu = 1.5$$

Question 109

In the hysteresis curve the value of magnetization (B) which will be present in a substance when value of magnetizing force (H) is made zero $(\mathrm{H} = 0)$ is called as

Options:

A. coercivity

B. retentivity

C. domain

D. saturation

Answer: B

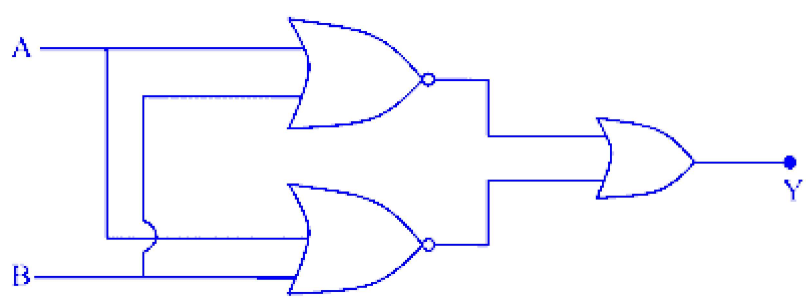
Solution:

Retentivity or remanence is the ability of a magnetic substance to retain magnetism even in the absence of a magnetising field.

∴ Retentivity corresponds to the value of B when value of H is zero.

Question 110

The output of following combination is same as that of



Options:

A. AND gate

B. OR gate

C. NAND gate

D. NOR gate

Answer: D

Solution:

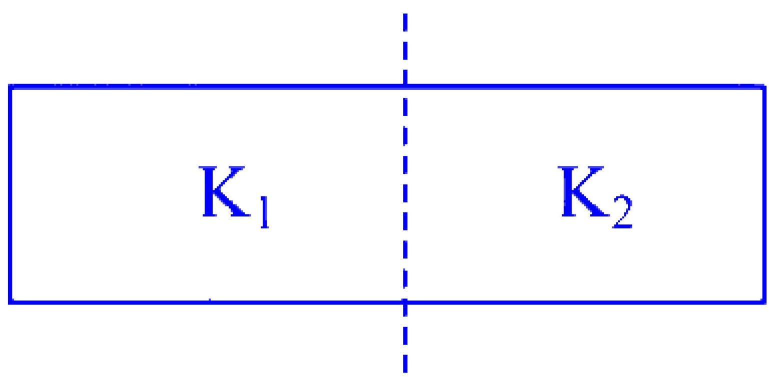
Considering the truth table for the given circuit,

A	B	$C = A + B$	$D = A + B$	$Y = C + D$
0	0	1	1	1
0	1	0	0	0
1	0	0	0	0
1	1	0	0	0

$\therefore$ The output of the given combination is same as that of a NOR gate.

Question 111

A parallel plate capacitor with air medium between the plates has a capacitance of $10 \mu F$. The area of capacitor is divided into two equal halves and filled with two media (as shown in figure) having dielectric constant $K_1 = 2$ and $K_2 = 4$. The capacitance of the system will be



Options:

A. 10 μF

B. 20 μF

C. 30 μF

D. 40 μF

Answer: C

Solution:

$C = \frac{\varepsilon_0 A}{d} = 10\mu F$

After dividing the area into two equal halves, the resultant capacitance is calculated as,

$$
\begin{aligned}
C_{eq} &= C_1 + C_2 \\
&= \frac{K_1\varepsilon_0 A_1}{d} + \frac{K_1\varepsilon_0 A_1}{d} = \frac{\frac{2\varepsilon_0 A}{2}}{d} + \frac{\frac{4\varepsilon_0 A}{2}}{d} \\
\therefore \quad C_{eq} &= \frac{\varepsilon_0 A}{d} + \frac{2\varepsilon_0 A}{d} = \frac{3\varepsilon_0 A}{d} = 3 \times 10 = 30\mu F
\end{aligned}
$$

Question 112

Refractive index of a glass convex lens is 1.5. The radius of curvature of each of the two surfaces of the lens is 20 cm. The ratio of the power of the lens when immersed in a liquid of refractive index 1.25 to that when placed in air is

Options:

A. 2 : 3

B. 2 : 5

C. 3 : 5

D. 5 : 2

Answer: B

Solution:

Given data: $\mu_g = 1.5, R_1 = R_2 = 20$ cm$, \mu_l = 1.25$

$$
\begin{aligned}
P_1 &= \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\
\therefore \quad P_2 &= \left(\frac{\mu_g}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1.5}{1.25} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\
&= (1.2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)
\end{aligned}
$$

Taking ratio,

$\frac{P_2}{P_1} = \frac{1.2-1}{1.5-1} = \frac{2}{5}$

Question 113

Earth is assumed to be a sphere of radius R. If 'g_ϕ' is value of effective acceleration due to gravity at latitude 30° and 'g' is the value at equator, then the value of $|g - g_\phi|$ is (ω is angular velocity of rotation of earth, $\cos 30^\circ = \frac{\sqrt{3}}{2}$)

Options:

A. $\frac{1}{4}\omega^2\mathrm{R}$

B. $\frac{3}{4}\omega^2\mathrm{R}$

C. $\omega^2\mathrm{R}$

D. $\frac{1}{2}\omega^2\mathrm{R}$

Answer: A

Solution:

The acceleration due to gravity on the surface of the Earth varies with latitude due to the centrifugal force resulting from the Earth's rotation. The effective acceleration due to gravity at a latitude ϕ, denoted as g_ϕ, takes into account the centrifugal force and is less than the acceleration due to gravity that would be experienced if the Earth were not rotating (denoted as g_0). At the equator ($\phi = 0^\circ$), this effect is maximal because the velocity due to Earth's rotation is maximal. As we go to the poles, the effect of rotation becomes minimal.

The effective acceleration due to gravity at latitude ϕ is given by:

$g_\phi = g_0 - \omega^2\mathrm{R}\cos^2\phi$

Where:

- g_0 is the acceleration due to gravity without the Earth's rotation, at the equator
- ω is the angular velocity of the Earth's rotation
- R is the radius of the Earth
- ϕ is the latitude

At the equator, the effective acceleration due to gravity, g, is:

$g=g_0-\omega^2\mathrm{R}$

because $\cos 0^\circ=1$.

At latitude 30°, the effective acceleration due to gravity, g_ϕ, using $\cos 30^\circ=\frac{\sqrt{3}}{2}$, is:

$g_\phi=g_0-\omega^2\mathrm{R}\left(\frac{\sqrt{3}}{2}\right)^2$

$g_\phi=g_0-\omega^2\mathrm{R}\times\frac{3}{4}$

To find $|g-g_\phi|$, we subtract the above two equations:

$|g-g_\phi|=\left|\left(g_0-\omega^2\mathrm{R}\right)-\left(g_0-\omega^2\mathrm{R}\times\frac{3}{4}\right)\right|$

$|g-g_\phi|=\omega^2\mathrm{R}-\omega^2\mathrm{R}\times\frac{3}{4}$

$|g-g_\phi|=\omega^2\mathrm{R}\left(1-\frac{3}{4}\right)$

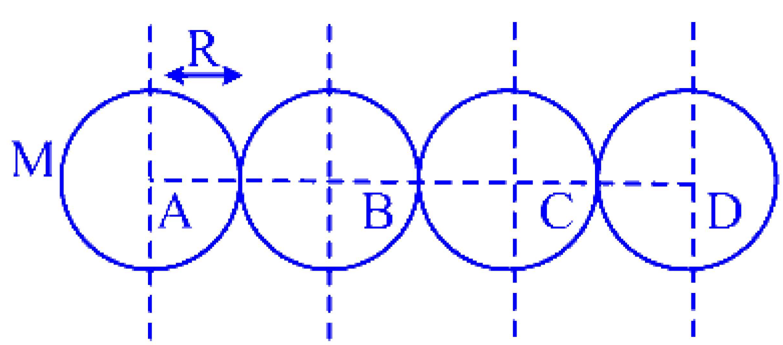
$|g-g_\phi|=\omega^2\mathrm{R}\times\frac{1}{4}$

So the value of $|g-g_\phi|$ is given by the option A, which is:

$\frac{1}{4}\omega^2\mathrm{R}$

Question 114

Four identical uniform solid spheres each of same mass 'M' and radius 'R' are placed touching each other as shown in figure, with centres A, B, C, D. $\mathrm{I_A}, \mathrm{I_B}, \mathrm{I_C}$ and $\mathrm{I_D}$ are the moment of inertia of these spheres respectively about an axis passing through centre and perpendicular to the plane. The difference in $\mathrm{I_A}$, and $\mathrm{I_B}$ is



Options:

A. 24 MR^2

B. 32 MR^2

C. 56 MR^2

D. 80 MR^2

Answer: B

Solution:

Using the parallel axes theorem, the M.I. of the system about the axis passing through the centre of the sphere A is

$$I_A = I_A' + I_B' + I_{C'} + I_D'$$
$$I_A = \frac{2}{5}MR^2 + \left(\frac{2}{5}MR^2 + 4MR^2\right) + \left(\frac{2}{5}MR^2 + 16MR^2\right) + \left(\frac{2}{5}M^2 + 36MR^2\right)$$

$$\therefore \quad I_A = 57.6MR^2$$

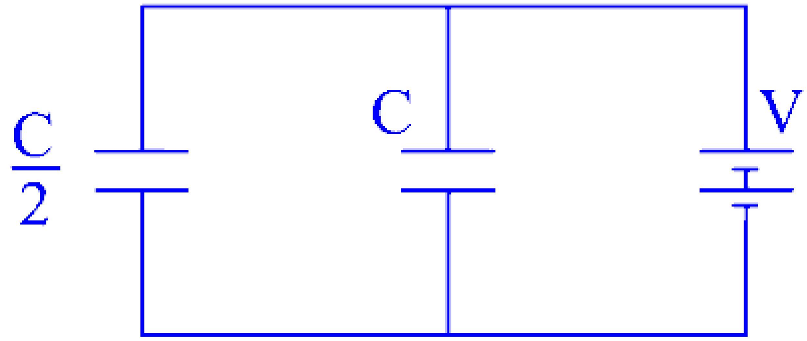
The M.I. about sphere B is,

$$I_B = \frac{2}{5}MR^2 + \left(\frac{2}{5}MR^2 + 4MR^2\right) + \left(\frac{2}{5}MR^2 + 16MR^2\right) + \left(\frac{2}{5}MR^2 + 4MR^2\right)$$

$$I_B = 25.6MR^2$$
$$\therefore \quad I_A - I_B = 57.6MR^2 - 25.6MR^2 = 32MR^2$$

Question 115

Two condensers one of capacity $\frac{C}{2}$ and other capacity C are connected to a battery of voltage V as shown. The work done in charging fully both the condensers is



Options:

A. $\frac{1}{2}$ CV^2

B. $\frac{3}{4}$ CV^2

C. $\frac{3}{2}$ CV^2

D. 2 CV^2

Answer: B

Solution:

The two condensers are in parallel. Their equivalent capacitance is $C_c = C + \frac{C}{2} = \frac{3C}{2}$

The formula for work done is

$$W = \frac{1}{2} C_e V^2$$
$$W = \frac{1}{2}\left(\frac{3C}{2}\right) V^2$$
$$W = \frac{3}{4} CV^2$$

Question 116

When two light waves each of amplitude 'A' and having a phase difference of $\frac{\pi}{2}$ superimposed then the amplitude of resultant wave is

Options:

A. $\frac{A}{\sqrt{2}}$

B. 2 A

C. $\sqrt{2}$ A

D. $\frac{A}{2}$

Answer: C

Solution:

The formula for resultant amplitude is,

$R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos\phi}$

Here, $A_1=A_2=A$ and $\phi=90^\circ$

$\therefore \quad R=\sqrt{A^2+A^2+2A^2\cos 90^\circ}=\sqrt{2A^2}=\sqrt{2}A$

Question 117

In an n-p-n transistor, the collector current is $28\ \mathrm{mA}$. If 80% of electrons reach the collector, its base current in mA is

Options:

A. 35

B. 24

C. 14

D. 7

Answer: D

Solution:

In an n-p-n transistor, the collector current (Ic) is primarily due to the flow of electrons from the emitter to the collector. However, not all the electrons emitted by the emitter reach the collector; a small percentage of them recombine in the base. This percentage is given by the common base current gain (α), which in this case is 80% or 0.80.

The base current (I_b) can be calculated by knowing that the emitter current (I_e) is the sum of the collector current (I_c) and the base current (I_b). The relationship is given by the equation:

$I_e = I_c + I_b$

Given that α, which is the common base current gain, is the ratio of the collector current to the emitter current:

$\alpha = \frac{I_c}{I_e}$

Let's solve for I_e:

$I_e = \frac{I_c}{\alpha} = \frac{28 \text{ mA}}{0.80} = 35 \text{ mA}$

Now, we can find the base current using the initial relation between I_e, I_c, and I_b:

$I_b = I_e - I_c = 35 \text{ mA} - 28 \text{ mA}$

$I_b = 7 \text{ mA}$

Hence, the base current $\mathbf{I_b}$ is **7** mA, which corresponds to Option D.

Question 118

A light spring is suspended with mass m_1 at its lower end and its upper end fixed to a rigid support. The mass is pulled down a short distance and then released. The period of oscillation is T second. When a mass m_2 is added to m_1 and the system is made to oscillate, the period is found to be $\frac{3}{2}T$. The ratio $m_1 : m_2$ is

Options:

A. 2 : 3

B. 3 : 4

C. 4 : 5

D. 5 : 6

Answer: C

Solution:

The time period is $T \propto \sqrt{\mathrm{m}}$

$$\therefore \quad \frac{T}{\frac{3}{2}T} = \frac{\sqrt{m_1}}{\sqrt{m_1+m_2}}$$
$$\therefore \quad \frac{2}{3} = \frac{\sqrt{m_1}}{\sqrt{m_1+m_2}}$$
$$\therefore \quad \frac{4}{9} = \frac{m_1}{m_1+m_2}$$
$$\therefore \quad 4m_1 + 4m_2 = 9m_1$$
$$\therefore \quad 5m_1 = 4m_2$$
$$\therefore \quad \frac{m_1}{m_2} = \frac{4}{5}$$

Question 119

The molecular mass of a gas having r.m.s. speed four times as that of another gas having molecular mass 32 is

Options:

A. 2

B. 4

C. 16

D. 32

Answer: A

Solution:

$$\mathrm{v_{rms}} \propto \frac{1}{\sqrt{\mathrm{M}}}$$
$$\therefore \quad \frac{(\mathrm{v_{rms}})_2}{(\mathrm{v_{rms}})_1} = \sqrt{\frac{\mathrm{M_1}}{\mathrm{M_2}}}$$
Given that $(\mathrm{v_{rms}})_2 = 4(\mathrm{v_{rms}})_1$
$$\therefore \quad \sqrt{\frac{32}{\mathrm{M_2}}} = 4$$
$$\therefore \quad \frac{32}{\mathrm{M_2}} = 16$$
$$\therefore \quad \mathrm{M_2} = 2$$

Question 120

The position 'x' of a particle varies with a time as $x = at^2 - bt^3$ where 'a' and 'b' are constants. The acceleration of the particle will be zero at

Options:

A. $\frac{2\mathrm{a}}{3\ \mathrm{b}}$

B. $\frac{\mathrm{a}}{\mathrm{b}}$

C. $\frac{a}{3b}$

D. zero

Answer: C

Solution:

To find when the acceleration of the particle is zero, we first need to understand how acceleration relates to position mathematically. Acceleration is the second derivative of position with respect to time. Given the position function

$x = at^2 - bt^3$,

let's first find the velocity of the particle, which is the first derivative of position with respect to time ($\frac{dx}{dt}$), and then the acceleration, which is the second derivative of position with respect to time ($\frac{d^2x}{dt^2}$).

The first derivative (velocity) is:

$v = \frac{dx}{dt} = \frac{d}{dt}(at^2 - bt^3) = 2at - 3bt^2$.

Now, let's take the second derivative (acceleration):

$a(t) = \frac{d^2x}{dt^2} = \frac{d}{dt}(2at - 3bt^2) = 2a - 6bt$.

To find when the acceleration is zero, set the acceleration function to zero and solve for t:

$2a - 6bt = 0$.

We can solve for t as follows:

$2a = 6bt$

$t = \frac{2a}{6b}$

$t = \frac{a}{3b}$.

Therefore, the acceleration of the particle will be zero at $t = \frac{a}{3b}$.

This corresponds to option C, which is the correct answer.

Question 121

When the conductivity of a semiconductor is only due to the breaking of the covalent bonds, the semiconductor is called

Options:

A. donor

B. intrinsic

C. acceptor

D. extrinsic

Answer: B

Solution:

The correct answer is Option B, intrinsic.

Intrinsic semiconductors are materials where the electrical conductivity is derived purely from the semiconductor itself, without any external doping. The conductivity in intrinsic semiconductors occurs because of the breaking of covalent bonds at higher temperatures, which creates electron-hole pairs. These electron-hole pairs contribute to the electrical conductivity.

Here's a brief explanation of each term to clarify the concepts:

Intrinsic Semiconductors (Option B): This is the pure form of a semiconductor, such as silicon or germanium, with no impurities added. The charge carriers (electrons and holes) are generated by thermal energy breaking the covalent bonds. Electrons are excited from the valence band to the conduction band, leaving behind holes in the valence band. Both the excited electrons and the holes contribute to the charge transport in intrinsic semiconductors.

Extrinsic Semiconductors (Option D): These are impure semiconductors intentionally 'doped' with impurities to alter their electrical properties. The addition of a small amount of dopant to a semiconductor can greatly increase its conductivity. There are two types of extrinsic semiconductors:

- **Donor (n-type) semiconductors (Option A)**: These have been doped with elements that have more valence electrons than the semiconductor, typically five-valence electrons such as phosphorus or arsenic when doping silicon. These extra electrons can easily move to the conduction band and significantly increase the conductivity of the material.

- **Acceptor (p-type) semiconductors (Option C)**: These are doped with elements that have fewer valence electrons (typically three valence electrons) than the semiconductor, such as boron or aluminum when doping silicon. These create holes (positive charge carriers) in the valence band by accepting electrons that jump from the valence band, thus increasing the conductivity.

To summarize, the answer is Option B, intrinsic, because this is the term used to describe a pure semiconductor where conductivity is due solely to its own lattice structure and not influenced by impurities or added elements.

Question 122

A coil having effective area A, is held with its plane normal to magnetic field of induction B. The magnetic induction is quickly reduced by 25% of its initial value in 2 second. Then the e.m.f. induced across the coil will be

Options:

A. $\frac{\mathrm{AB}}{8}$

B. $\frac{\mathrm{AB}}{2}$

C. $\frac{3\mathrm{AB}}{4}$

D. $\frac{3\mathrm{AB}}{8}$

Answer: A

Solution:

The induced emf is given as

$\mathrm{e}=\frac{\Delta \phi}{\Delta \mathrm{t}}=\frac{\mathrm{A} \triangle \mathrm{B}}{\Delta \mathrm{t}}$

The magnetic field is reduced by 25% of its initial value.

The final magnetic field is,

$$\mathrm{B}_2=\mathrm{B}-\frac{1}{4} \mathrm{~B}=\frac{3}{4} \mathrm{~B}$$

$$\therefore \quad \mathrm{e}=\frac{\mathrm{A}\left(\mathrm{B}-\frac{3}{4} \mathrm{~B}\right)}{2}=\frac{\mathrm{AB}}{8}$$

Question 123

At constant temperature, increasing the pressure of a gas by 5% its volume will decrease by

Options:

A. 5%

B. 5.26%

C. 4.20%

D. 4.70%

Answer: D

Solution:

According to ideal gas law at constant temperature, $\mathrm{P} \propto \frac{1}{\mathrm{V}}$

$$\therefore \quad \mathrm{P_1 V_1 = P_2 V_2}$$
$$\mathrm{P_1 = P}$$
$$\mathrm{P_2 = P + \frac{5}{100} P = 1.05P}$$

Substituting the values

$$\mathrm{PV_1 = 1.05PV_2}$$
$$\therefore \quad \frac{\mathrm{V_2}}{\mathrm{V_1}} = \frac{1}{1.05}$$
$$\therefore \quad \frac{\mathrm{V_2 - V_1}}{\mathrm{V_1}} \times 100 = \frac{1 - 1.05}{1.05} \times 100$$
$$\therefore \quad \frac{\Delta \mathrm{V}}{\mathrm{V_1}} = -4.76\% \approx -4.7\%$$

The negative sign indicates that the volume is decreasing.

Question 124

Half life of radio-active element is 1600 years. The fraction of sample remains undecayed after 6400 years will be

Options:

A. $\frac{1}{16}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{24}$

Answer: A

Solution:

The half-life of a radioactive substance is the time it takes for half of the substance to decay. The remaining fraction of the original substance after a certain number of half-lives can be calculated using the formula:

$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

Where:

N is the remaining quantity of the substance,

N_0 is the initial quantity of the substance,

t is the elapsed time,

$T_{1/2}$ is the half-life of the substance.

Given that the half-life $T_{1/2}$ is 1600 years, and we want to find the amount of substance left after 6400 years, we can use the above formula:

$\frac{t}{T_{1/2}} = \frac{6400}{1600} = 4$ half-lives

Therefore,

$N = N_0\left(\frac{1}{2}\right)^4$

$N = N_0\left(\frac{1}{2^4}\right)$

$N = N_0\left(\frac{1}{16}\right)$

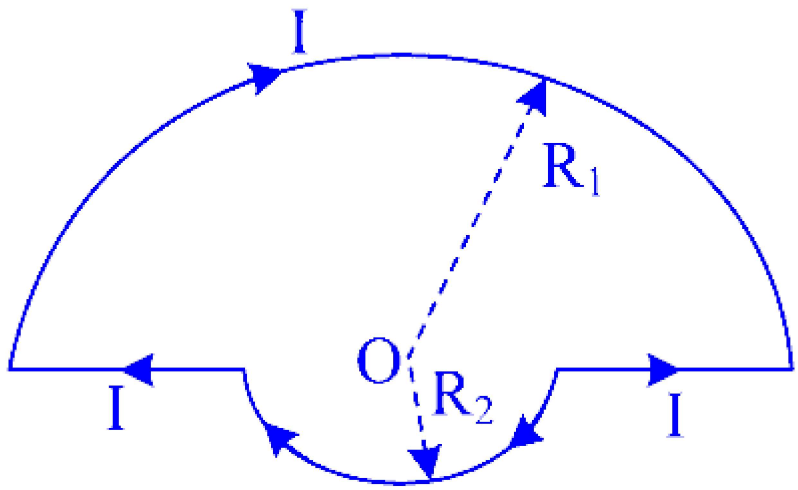
The fraction of the original sample remaining undecayed after 6400 years is $\frac{1}{16}$.

Thus, the correct answer is:

Option A $\frac{1}{16}$

Question 125

Figure shows two semicircular loops of radii R_1 and R_2 carrying current I. The magnetic field at the common centre 'O' is



Options:

A. $\frac{\mu_0 \mathrm{I}}{4}\left(\frac{1}{\mathrm{R}_1}+\frac{1}{\mathrm{R}_2}\right)$

B. $\frac{\mu_0 \mathrm{I}}{4}\left(\frac{1}{\mathrm{R}_1}-\frac{1}{\mathrm{R}_2}\right)$

C. $\frac{\mu_0 \mathrm{I}}{2\pi}\left(\frac{1}{\mathrm{R}_1}+\frac{1}{\mathrm{R}_2}\right)$

D. $\frac{\mu_0 I}{2\pi}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Answer: A

Solution:

For semicircular arc the magnetic field is given as $B=\frac{\mu_0 i}{4R}$

The equivalent magnetic field at the centre is

$$\begin{aligned} B_{\text{eq}} &= \frac{\mu_0 I}{4R_1}+\frac{\mu_0 I}{4R_2} \\ B_{\text{eq}} &= \frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \end{aligned}$$

Question 126

When radiation of wavelength 'λ' is incident on a metallic surface, the stopping potential is 4.8 V. If the surface is illuminated with radiation of double the wavelength then the stopping potential becomes 1.6 V. The threshold wavelength for the surface is

Options:

A. 2λ

B. 4λ

C. 6λ

D. 8λ

Answer: B

Solution:

Stopping potential:

$$\mathrm{eV_0} = \mathrm{h}\nu - \phi_0$$
$$\mathrm{eV_0} = \frac{\mathrm{hc}}{\lambda} - \phi_0$$

For first case: $e(4.8) = \frac{hc}{\lambda} - \phi_0$ ..... (i)

For Second case: $e(1.6) = \frac{hc}{2\lambda} - \phi_0$ ..... (ii)

Dividing equation (i) by equation (ii),

$3\left(\frac{\mathrm{hc}}{2\lambda} - \phi_0\right) = \frac{\mathrm{hc}}{\lambda} - \phi_0$

$$\therefore \quad \frac{3\mathrm{hc}}{2\lambda} - 3\phi_0 = \frac{\mathrm{hc}}{\lambda} - \phi_0$$
$$\frac{\mathrm{hc}}{2\lambda} = 2\phi_0$$
$$\frac{\mathrm{hc}}{4\lambda} = \phi_0$$

$\therefore$ Threshold wavelength is 4λ.

Question 127

A long wire is bent into a circular coil of one turn and then into a circular coil of smaller radius having n turns. If the same current passes in both the cases, the ratio of magnetic fields produced at the centre for one turn to that of n turns is

Options:

A. $1 : \mathrm{n}$

B. $\mathrm{n} : 1$

C. $1 : \mathrm{n}^2$

D. $\mathrm{n}^2 : 1$

Answer: C

Solution:

Magnetic field at the centre of coil is,

$\mathrm{B}_1 = \frac{\mu_0 \mathrm{I}}{2\mathrm{r}_1}$

$\therefore$ Magnetic field of n turns coil at the centre:

$$B_2 = \frac{\mu_0 n I}{2 r_2}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2 r_1}}{\frac{\mu_0 n I}{2 r_2}} = \frac{r_2}{\mathrm{n r}_1}$$

But, radius: $r_1 = \frac{1}{2\pi}$ and $r_2 = \frac{1}{2\pi n}$

$$\therefore \quad \frac{\mathrm{r}_2}{\mathrm{r}_1} = \frac{1}{\mathrm{n}}$$

$$\therefore \quad \frac{\mathrm{B}_1}{\mathrm{B}_2} = \frac{1}{\mathrm{n}^2}$$

Question 128

When moving coil galvanometer (MCG) is converted into a voltmeter, the series resistance is 'n' times the resistance of galvanometer. How many times that of MCG the voltmeter is now capable of measuring voltage?

Options:

A. n

B. $\frac{n+1}{n}$

C. $n+1$

D. $n-1$

Answer: C

Solution:

Series resistance is n times of galvanometer resistance: $R_s = nR_G$

Relation between voltage and current:

$$\begin{aligned} \therefore \quad & V = I(R_s + R_G) \\ \therefore \quad & V = I(nR_G + R_G) \\ \therefore \quad & V = IR_G(n+1) \end{aligned}$$

Question 129

The self induction (L) produced by solenoid of length 'l' having 'N' number of turns and cross sectional area 'A' is given by the formula ($\phi =$ magnetic flux, $\mu_0 =$ permeability of vacuum)

Options:

A. $L = N\phi$

B. $L = \mu_0 NAl$

C. $L = \frac{\mu_0 N^2 A}{l}$

D. $L = \frac{\mu_0 NA}{l}$

Answer: C

Solution:

Magnetic field inside the solenoid is, $B = \frac{\mu_0 NI}{l}$

Flux inside the coil, $\phi = N(BA)$

$\therefore \quad \phi = \frac{\mu_0 N^2 IA}{l}$

Also, self inductance, $L = \frac{\phi}{I} = \frac{\frac{\mu_0 N^2 IA}{l}}{I}$

$\therefore \quad L = \frac{\mu_0 N^2 A}{l}$

Question 130

A wire PQ has length $4.8 \mathrm{~m}$ and mass $0.06 \mathrm{~kg}$. Another wire QR has length $2.56 \mathrm{~m}$ and mass $0.2 \mathrm{~kg}$. Both wires have same radii and are joined as a single wire. This wire is under tension of $80 \mathrm{~N}$. A wave pulse of amplitude $3.5 \mathrm{~cm}$ is sent along the wire PQ from end P. the time taken by the wave pulse to travel along the wire from point P to R is ?

Options:

A. 0.1 s

B. 0.12 s

C. 0.14 s

D. 0.16 s

Answer: C

Solution:

To determine the time taken by the wave pulse to travel along the wire from point P to R, we first need to calculate the speed of the wave in each segment of the wire (PQ and QR) and then find the time taken to travel through each segment.

We have two wires PQ and QR with different lengths and masses but the same radius and therefore the same cross-sectional area A. Both wires are under the same tension T. Given that the tension and cross-sectional area are the same for both wires, we can assume that the linear density (mass per unit length) is different because they have different masses and lengths.

To find the linear density (μ) for each wire, we use the formula:

$\mu = \frac{m}{L}$

where

m is the mass of the wire, and

L is the length of the wire.

For wire PQ:

$$\mu_{PQ} = \frac{m_{PQ}}{L_{PQ}} = \frac{0.06\ kg}{4.8\ m} = 0.0125\ kg/m$$

For wire QR:

$$\mu_{QR} = \frac{m_{QR}}{L_{QR}} = \frac{0.2\ kg}{2.56\ m} = 0.078125\ kg/m$$

The speed of a wave in a stretched string or wire is given by the formula:

$v = \sqrt{\frac{T}{\mu}}$

where

v is the speed of the wave, and

T is the tension in the wire.

Using this formula, we can calculate the speed of the wave in each section of the wire.

For wire PQ:

$$v_{PQ} = \sqrt{\frac{T}{\mu_{PQ}}} = \sqrt{\frac{80\ N}{0.0125\ kg/m}} = \sqrt{6400\ m^2/s^2} = 80\ m/s$$

For wire QR:

$$v_{QR} = \sqrt{\frac{T}{\mu_{QR}}} = \sqrt{\frac{80\ N}{0.078125\ kg/m}} = \sqrt{1024\ m^2/s^2} = 32\ m/s$$

Now we will find the time taken for the wave to travel through each section.

Time is distance over speed, so for wire PQ:

$$t_{PQ} = \frac{L_{PQ}}{v_{PQ}} = \frac{4.8\ m}{80\ m/s} = 0.06\ s$$

And for wire QR:

$t_{QR} = \frac{L_{QR}}{v_{QR}} = \frac{2.56\ m}{32\ m/s} = 0.08\ s$

To find the total time taken by the wave pulse to travel from P to R, we add the times for PQ and QR:

$t_{total} = t_{PQ} + t_{QR} = 0.06\ s + 0.08\ s = 0.14\ s$

The correct answer is:

Option C: 0.14 s

Question 131

When a monochromatic ray of light is passed through an equilateral glass prism, it is found that the refracted ray in glass is parallel to the base of the prism. If 'i' and 'e' denote the angles of incidence and emergence respectively, then

Options:

A. i $>$ e

B. i $<$ e

C. i $=$ e

D. i $+$ e $= 90^\circ$

Answer: C

Solution:

When light travels from one medium to another, it undergoes refraction twice in the case of a prism: first when entering the prism and second when exiting it. The path of light inside the prism is parallel to the base, which means that the angle of incidence at the first surface is equal to the angle of emergence at the second surface.

Since we are dealing with an equilateral prism, the angles at each corner are 60°. Let's denote the angle of the prism as A. When the ray inside the prism is parallel to the base, the angle of refraction at the first surface (inside the prism) will be equal to the angle of incidence at the second surface. Let's denote this angle as r (which is also the angle of refraction at the second surface), then according to Snell's Law, where the refractive index of glass is denoted by n:

$n \cdot \sin(i) = \sin(r)$

and

$n \cdot \sin(e) = \sin(r)$

Since the refracted ray in glass is parallel to the base of the prism, the deviation of the ray inside the prism is minimal, and this occurs when r equals $A/2$ for an equilateral prism, so

$r = 60^\circ/2 = 30^\circ$

Since the values of $\sin(i)$ and $\sin(e)$ are both obtained by multiplying $\sin(r)$ with the refractive index n, we have

$\sin(i) = \sin(e)$

Because the sine function is increasing for angles between 0° and 90°, if $\sin(i)$ equals $\sin(e)$, the corresponding angles must also be equal, thus

$i = e$

Hence, the correct option is:

Option C $\mathrm{i} = \mathrm{e}$

Question 132

A solid cylinder and a solid sphere having same mass and same radius roll down on the same inclined plane. The ratio of the acceleration of the cylinder 'a_c' to that of sphere 'a_s' is

Options:

A. $\frac{11}{15}$

B. $\frac{13}{14}$

C. $\frac{15}{14}$

D. $\frac{14}{15}$

Answer: D

Solution:

For sphere: M.I. $\mathrm{I_S} = \frac{2}{5}\mathrm{MR}^2$

For cylinder: M.I. $I_C = \frac{1}{2}MR^2$

Acceleration: $a = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$

$$\therefore \quad \frac{a_c}{a_s} = \frac{1+\frac{I_s}{MR^2}}{1+\frac{I_C}{MR^2}} = \frac{MR^2+I_s}{MR^2+I_c} = \frac{MR^2+\frac{2}{5}MR^2}{MR^2+\frac{1}{2}MR^2}$$
$$\therefore \quad \frac{a_c}{a_s} = \frac{7}{5}\times\frac{2}{3} = \frac{14}{15}$$

Question 133

An alternating voltage $E = 200\sqrt{2}\sin(100t)$ volt is connected to a 1μf capacitor through an a.c. ammeter. The reading of the ammeter shall

Options:

A. 10 mA

B. 20 mA

C. 40 mA

D. 80 ma

Answer: B

Solution:

$$V_{rms} = \frac{I_{ms}}{X_C}$$
$$\therefore \quad I_{rms} = \frac{V_0\omega C}{\sqrt{2}} = \frac{200\sqrt{2}\times 100\times 1\times 10^{-6}}{\sqrt{2}} = 20\text{ mA}$$

Ammeters always read r.m.s. value of the current.

Question 134

A sonometer wire 49 cm long is in unison with a tuning fork of frequency 'n'. If the length of the wire is decreased by 1 cm and it is

vibrated with the same tuning fork, 6 beats are heard per second. The value of 'n' is

Options:

A. 256 Hz

B. 288 Hz

C. 320 Hz

D. 384 Hz

Answer: B

Solution:

$$\begin{aligned} & \mathrm{n}_1 \mathrm{~L}_1=\mathrm{n}_2 \mathrm{~L}_2 \\ \therefore \quad & \mathrm{n}_1 \times 49=\mathrm{n}_2 \times 48 \\ \therefore \quad & \mathrm{n}_2=\tfrac{49}{48} \mathrm{n}_1 \end{aligned}$$

Given that, beat frequency, $|\mathrm{n}_1-\mathrm{n}_2|=6$

$$\begin{aligned} \therefore \quad & \left|\mathrm{n}_1-\frac{49}{48} \mathrm{n}_1\right|=6 \\ \therefore \quad & \frac{\mathrm{n}_1}{48}=6 \\ \therefore \quad & \mathrm{n}_1=288 \mathrm{~Hz} \end{aligned}$$

Question 135

Two spherical soap bubbles of radii 'a' and 'b' in vacuum coalesce under isothermal conditions. The resulting bubble has a radius equal to

Options:

A. $a+b$

B. $\frac{a+b}{2}$

C. $\sqrt{a^2+b^2}$

D. $\frac{a+b}{ab}$

Answer: C

Solution:

Number of moles is conserved, so

$\mathrm{P}_1 \mathrm{~V}_1+\mathrm{P}_2 \mathrm{~V}_2=\mathrm{P}_3 \mathrm{~V}$

But, $P=\frac{4T}{r}$ where, r is the radius of the bubble

$$\begin{aligned} \therefore \quad & \frac{4 \mathrm{~T}}{\mathrm{a}}\left(\frac{4}{3} \pi \mathrm{a}^3\right)+\frac{4 \mathrm{~T}}{\mathrm{b}}\left(\frac{4}{3} \pi \mathrm{b}^3\right)=\frac{4 \mathrm{~T}}{\mathrm{c}}\left(\frac{4}{3} \pi \mathrm{c}^3\right) \\ & \mathrm{a}^2+\mathrm{b}^2=\mathrm{c}^2 \\ & \mathrm{c}=\sqrt{\mathrm{a}^2+\mathrm{b}^2} \end{aligned}$$

Question 136

A mass 'M' is moving with constant velocity parallel to X-axis. Its angular momentum with respect to the origin is

Options:

A. constant

B. zero

C. decreasing

D. increasing

Answer: A

Solution:

The angular momentum $(\vec{L})$ of a particle with respect to a point (in this case, the origin) is given by the cross product of the position vector $(\vec{r})$ and the linear momentum of the particle $(\vec{p})$, which is the product of the mass M and its velocity $\vec{v}$:

$\vec{L}=\vec{r} \times \vec{p}$

If a mass M is moving with constant velocity parallel to the X-axis, then its momentum $\vec{p}$ is also constant both in magnitude and direction, given by:

$$\vec{p} = M\vec{v}$$

where $\vec{v}$ is the constant velocity vector of the mass parallel to the X-axis.

The position vector $\vec{r}$ is a vector from the origin to the location of the mass M. Since the mass is moving with a constant velocity and not approaching or moving away from the origin, the perpendicular distance from the origin to the line of motion (essentially, the "arm" of the moment arm) remains constant.

The cross product of two vectors yields a vector that is perpendicular to the plane formed by the two original vectors and its magnitude is given by:

$$|\vec{L}| = |\vec{r}||\vec{p}| \sin \theta$$

where θ is the angle between $\vec{r}$ and $\vec{p}$.

Since the mass is moving parallel to the X-axis, the angle θ between the position vector from the origin and the momentum vector is constant, and so is the sine of that angle. Thus, the product $|\vec{r}||\vec{p}|$ is also constant.

Therefore, the magnitude of the angular momentum $|\vec{L}|$ is constant, and because the mass M is not changing its direction of motion or speed, the direction of the angular momentum vector is also constant.

The correct answer to the question is:

Option A: constant

since the angular momentum does not change with time when the velocity is constant and the direction of motion is a straight line, and because there are no external forces or torques acting on the mass to change its state of motion or angular momentum.

Question 137

A block of mass 'M' rests on a piston executing S.H.M. of period one second. The amplitude of oscillations, so that the mass is separated from the piston, is (acceleration due to gravity, $\mathrm{g} = 10 \mathrm{~ms}^{-2}, \pi^2 = 10$)

Options:

A. 0.25 m

B. 0.5 m

C. 1 m

D. ∞

Answer: A

Solution:

Given that, $T = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k}}} = 1\ \mathrm{s}$

$\therefore \quad \frac{\mathrm{m}}{\mathrm{k}} = \frac{1}{4\pi^2}$

The block will separate, when the restoring force exceeds the force due to gravity.

i.e., $\mathrm{kx} \geq \mathrm{Mg}$

$\therefore \quad x \geq \frac{Mg}{k} = \frac{g}{4\pi^2} = \frac{10}{4\times10} = 0.25\ \mathrm{m}$

Question 138

A machine gun fires bullets of mass $30\ \mathrm{g}$ with velocity of $1000\ \mathrm{m/s}$. The man holding the gun can exert a maximum force of $300\ \mathrm{N}$ on it. How many bullets can he fire per second at most?

Options:

A. 3

B. 6

C. 10

D. 9

Answer: C

Solution:

To determine how many bullets the man can fire per second without exceeding the maximum force he can exert, we need to compute the force applied due to the momentum change of the bullets being fired.

The momentum p of a single bullet is given by:

$p = m \cdot v$

where m is the mass of the bullet and v is the velocity of the bullet. Here, $m = 30\ \mathrm{g} = 0.03\ \mathrm{kg}$ (since $1\ \mathrm{g} = 0.001\ \mathrm{kg}$) and $v = 1000\ \mathrm{m/s}$.

Now we calculate the momentum of a single bullet:

$p = 0.03\ \mathrm{kg} \cdot 1000\ \mathrm{m/s} = 30\ \mathrm{kg} \cdot \mathrm{m/s}$

The rate of change of momentum gives us the force exerted by the bullets being fired. If F is the force and n is the number of bullets fired per second, the force is equal to the rate of change of momentum:

$F = n \cdot \frac{\Delta p}{\Delta t}$

Since Δp is the change in momentum per bullet and $\Delta t = 1$ second (because n is measured in bullets per second), the force exerted by n bullets is:

$F = n \cdot \frac{30\ \mathrm{kg}\ \cdot \mathrm{m/s}}{1\ \mathrm{s}} = n \cdot 30\ \mathrm{N}$

We know the man can exert a maximum force of $F_{\max} = 300\ \mathrm{N}$. Therefore, the maximum number of bullets ($n_{\max}$) that can be fired per second without exceeding the maximum force is obtained by equating the two:

$300\ \mathrm{N} = n_{\max} \cdot 30\ \mathrm{N}$

Solving for $n_{\max}$:

$n_{\max} = \frac{300\ \mathrm{N}}{30\ \mathrm{N/bullet}} = 10\ \mathrm{bullets/s}$

Thus, the man can fire at most 10 bullets per second. Hence, the correct answer is:

Option C: 10

Question 139

The temperature of a gas is measure of

Options:

A. the average kinetic energy of gas molecules.

B. the average potential energy of gas molecules.

C. the average distance between the molecules of a gas

D. the size of the molecules of a gas

Answer: A

Solution:

The temperature of a gas is indeed a measure of the average kinetic energy of its molecules. This is a principle that comes from kinetic molecular theory, which helps us understand the behavior of gases. According to this theory, temperature is directly proportional to the average kinetic energy of the gas molecules. This means that as the temperature increases, the average kinetic energy of the gas molecules also increases, and vice versa. The correct answer is therefore:

Option A: the average kinetic energy of gas molecules.

Here's why the other options are incorrect:

Option B: The temperature of a gas does not measure the average potential energy of gas molecules. Potential energy in the context of gas molecules often refers to the energy associated with the positions of the molecules relative to each other (e.g., due to intermolecular forces), which is not what temperature measures.

Option C: The temperature of a gas is not a measure of the average distance between the molecules of a gas. While temperature can affect the volume of gas (and thus indirectly influence average distances between molecules in some contexts), the temperature itself is specifically a measure of kinetic energy, not spatial distribution or distances between molecules.

Option D: The temperature of a gas does not measure the size of the molecules of a gas. Molecular size is related to the type of gas and its molecular structure, not to the temperature of the gas.

Question 140

A body (mass m) starts its motion from rest from a point distant $R_0\ (R_0 > R)$ from the centre of the earth. The velocity acquired by the body when it reaches the surface of earth will be ($\mathrm{G}=$ universal constant of gravitation, $\mathrm{M}=$ mass of earth, R = radius of earth)

Options:

A. $2\mathrm{GM}\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{R}_0}\right)$

B. $\left[2\mathrm{GM}\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{R}_0}\right)\right]^{\frac{1}{2}}$

C. $\mathrm{GM}\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{R}_0}\right)$

D. $2\mathrm{GM}\left[\left(\frac{1}{\mathrm{R}}-\frac{1}{\mathrm{R}_0}\right)\right]^{\frac{1}{2}}$

Answer: B

Solution:

According to law of conservation of energy,

$$\frac{1}{2}mv^2 = -\frac{GMm}{R_0} - \left(\frac{GMm}{R}\right) = GMm\left(\frac{1}{R} - \frac{1}{R_0}\right)$$

$\therefore \quad v^2 = 2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)$

$\therefore$ The velocity acquired by the body when it reaches the surface of earth is:

$V = \left[2GM\left(\frac{1}{R} - \frac{1}{R_0}\right)\right]^{\frac{1}{2}}$

Amongst the given options, only option (B) dimensionally equates to velocity.

Question 141

An ideal refrigerator has freezer at a temperature of $-13^\circ\mathrm{C}$. The coefficient of performance of the engine is 5. The temperature of the air (to which heat is rejected) is

Options:

A. $320^\circ\mathrm{C}$

B. $39^\circ\mathrm{C}$

C. $325\ \mathrm{K}$

D. $325^\circ\mathrm{C}$

Answer: B

Solution:

Coefficient of performance, $\beta = \frac{T_2}{T_1 - T_2}$

Given that, $\beta = 5$ and $\mathrm{T}_2 = -13^\circ\mathrm{C} = 260\ \mathrm{K}$

$\therefore \quad \frac{260}{\mathrm{T}_1 - 260} = 5$

$\therefore \quad 5\ \mathrm{T}_1 - 1300 = 260$

$\therefore \quad \mathrm{T}_1 = \frac{1560}{5} = 312\ \mathrm{K}$

$\therefore \quad \mathrm{T}_1 = 39^\circ\mathrm{C}$

Question 142

1000 small water drops of equal size combine to form a big drop. The ratio of final surface energy to the total initial surface energy is

Options:

A. 10 : 1

B. 1 : 10

C. 1000 : 1

D. 1 : 1000

Answer: B

Solution:

To solve this problem, we need to understand the relationship between the surface energy of a drop and its radius. The surface energy E of a drop is proportional to its surface area A. The surface area of a sphere (which is the shape of a water drop) can be calculated using the formula:

$A = 4\pi r^2$

where r is the radius of the sphere. The surface energy is then given by:

$E = \sigma A = \sigma 4\pi r^2$

where σ is the surface tension of the liquid. When 1000 small drops combine to form one large drop, the volume of the large drop is equal to the sum of the volumes of the small drops. The volume of a sphere is given by

$V = \frac{4}{3}\pi r^3$

If the radius of each small drop is r, the total volume of the small drops is $1000 \times \dfrac{4}{3}\pi r^3$. Let the radius of the large drop be R. Then,

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

Canceling common terms and taking the cube root of both sides, we get

$1000^{1/3} \times r = R$

Since $1000 = 10^3$, the cube root of 1000 is 10.

$10 \times r = R$

So the radius of the large drop is ten times the radius of each small drop.

Now we can compare the initial surface energy of all small drops with the final surface energy of the large drop. Let's denote the initial surface energy as E_{initial} and the final surface energy as E_{final}.

$E_{\text{initial}} = 1000 \times \sigma 4\pi r^2$ $E_{\text{final}} = \sigma 4\pi R^2$

Substituting $R = 10r$ into the final energy equation, we get:

$E_{\text{final}} = \sigma 4\pi (10r)^2$ $E_{\text{final}} = \sigma 4\pi (100r^2)$ $E_{\text{final}} = 100\sigma 4\pi r^2$

Now, we compare the final energy to the initial energy:

$\frac{E_{\text{final}}}{E_{\text{initial}}} = \frac{100\sigma 4\pi r^2}{1000\sigma 4\pi r^2}$ $\frac{E_{\text{final}}}{E_{\text{initial}}} = \frac{100}{1000}$ $\frac{E_{\text{final}}}{E_{\text{initial}}} = \frac{1}{10}$

Hence, the ratio of the final surface energy to the total initial surface energy is 1 : 10. Therefore, the correct answer is:

Option B 1 : 10

Question 143

It is easier to spray water to which soap is added because addition of soap to water

Options:

A. decreases surface tension of water

B. increases surface tension of water.

C. makes surface tension of water zero.

D. increases its density.

Answer: A

Solution:

The correct answer is **A) decreases surface tension of water**

Surface Tension

- Water molecules are attracted to each other due to cohesive forces.
- At the surface of the water, there's an imbalance of force because there are no water molecules above to create attraction.

- This results in a net inward force, causing the surface to behave like a stretched membrane – this is surface tension.

Effect of Soap

- Soap molecules have a hydrophilic (water-loving) head and a hydrophobic (water-hating) tail.
- When soap is added to water, the hydrophobic tails disrupt the cohesive forces between water molecules at the surface.
- This reduces the net inward force, decreasing the surface tension.

Why it's easier to spray

- Spraying creates tiny droplets of water, which increases the surface area significantly.
- High surface tension would require extra energy to overcome the cohesive forces and create those droplets.
- Soap, by reducing surface tension, makes this process easier, requiring less energy for spraying.

The other options are incorrect:

- **B) increases surface tension of water:** This would make it harder to spray.
- **C) makes surface tension of water zero:** While surfactants reduce surface tension significantly, they don't reduce it to zero.
- **D) increases its density:** Density isn't directly related to ease of spraying in this context.

Question 144

What will be the phase difference between virtual voltage and virtual current when current in the circuit is wattless?

Options:

A. 60°

B. 45°

C. 90°

D. 180°

Answer: C

Solution:

When the current in an AC circuit is described as "wattless," it means that the current does not do any real work. This happens when the current and the voltage are out of phase by 90°. In such a situation, the power factor of the circuit is zero, as power factor is defined as the cosine of the phase angle between the voltage and the current, given by:

$\text{Power Factor} = \cos(\phi)$

where ϕ is the phase difference. If the power is wattless, then:

$\text{Power Factor} = \cos(90^\circ) = 0$

The phase difference that makes the cosine equal to zero is 90° since:

$\cos(90^\circ) = 0$

Therefore, the correct answer is Option C: 90°. This phase difference implies that when the voltage reaches its peak value, the current is zero, and vice versa, meaning the voltage and current waves are out of phase by a quarter cycle.

Question 145

Two wavelengths of sodium light $590\ \text{nm}$ and $596\ \text{nm}$ are used one after another to study diffraction due to single slit of aperture $2 \times 10^{-6}\ \text{m}$. The distance between the slit and the screen is $1.5\ \text{m}$. The separation between the positions of first maximum of the diffraction pattern obtained in the two cases is

Options:

A. 5.5 mm

B. 5.75 mm

C. 6.25 mm

D. 6.75 mm

Answer: D

Solution:

First maximum in single slit diffraction pattern is obtained at, $x = \frac{3\lambda D}{2d}$

$\therefore \quad \Delta \mathrm{x} = \frac{3\mathrm{D} \times \Delta\lambda}{2\ \mathrm{d}} = \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 - 590)$

$\therefore \quad \Delta \mathrm{x} = 6.75\ \mathrm{mm}$

Question 146

A coil having an inductance of $\frac{1}{\pi}\mathrm{H}$ is connected in series with a resistance of 300Ω. If $20\ \mathrm{V}$ from a $200\ \mathrm{Hz}$ source are impressed across the combination, the value of the phase angle between the voltage and the current is

Options:

A. $\tan^{-1}\left(\frac{5}{4}\right)$

B. $\tan_{-1}\left(\frac{4}{5}\right)$

C. $\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: D

Solution:

$$\mathrm{X_L} = \mathrm{L}\omega = \mathrm{L} \times 2\pi\mathrm{f}$$

$$\therefore \quad \mathrm{X_L} = \frac{1}{\pi} \times 2\pi \times 200$$

$$\therefore \quad \mathrm{X_L} = 400\Omega$$

Now, the phase angle between voltage and current is given by, $\tan\phi = \frac{X_L}{R} = \frac{400}{300}$

$\therefore \quad \phi = \tan^{-1}\left(\frac{4}{3}\right)$

Question 147

The pressure and density of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes adiabatically from (P, ρ) to (P', ρ'). If $\frac{\rho'}{\rho} = 32$ then $\frac{\mathrm{P}'}{\mathrm{P}}$ should be

Options:

A. $\frac{1}{128}$

B. 128

C. 32

D. 64

Answer: B

Solution:

For adiabatic process, $\mathrm{PV}^{\gamma} = \text{a constant}$

$\therefore \quad \frac{\mathrm{P}'}{\mathrm{P}} = \left(\frac{\mathrm{V}'}{\mathrm{V}}\right)^{\gamma} = \left(\frac{\rho'}{\rho}\right)^{\gamma}$

Given that, $\frac{\rho'}{\rho} = 32$ and $\gamma = \frac{7}{5}$

$\therefore \quad \frac{\mathrm{P}'}{\mathrm{P}} = (32)^{\frac{7}{5}} = 128$

Question 148

A source of sound is moving towards a stationary observer with $\left(\frac{1}{10}\right)^{\text{th}}$ the of the speed of sound. The ratio of apparent to real frequency is

Options:

A. 10 : 9

B. 11 : 10

C. $(11)^2 : (10)^2$

D. $(9)^2 : (10)^2$

Answer: A

Solution:

The frequency of the sound as perceived by an observer when the source of sound is moving towards the observer is given by the Doppler Effect. According to the Doppler Effect, if a source of sound with the frequency f_0 (real frequency) moves towards a stationary observer with velocity v_s and the sound travels with a velocity v, then the apparent frequency f' heard by the observer is given by:

$f' = f_0 \left(\frac{v+v_0}{v-v_s}\right)$

Here, v_0 is the velocity of the observer which is zero and $v_s = \frac{v}{10}$ since the source of sound is moving at one tenth the speed of sound. Substituting these values back into the equation, we get:

$f' = f_0 \left(\frac{v}{v-\frac{v}{10}}\right)$

$f' = f_0 \left(\frac{v}{\frac{9v}{10}}\right)$

$f' = f_0 \left(\frac{10}{9}\right)$

Hence, the ratio of the apparent frequency to the real frequency is 10 : 9, which corresponds to Option A.

Question 149

If $\mathrm{E_a}$ and $\mathrm{E_q}$ represent the electric field intensity due to a short dipole at a point on its axial line and on the equatorial line at the same distance 'r' from the centre of the dipole, then

Options:

A. $\mathrm{E_a} = \mathrm{E_q}$

B. $\mathrm{E_a} = \frac{1}{2}\mathrm{E_q}$

C. $\mathrm{E_a} = \frac{1}{\sqrt{2}}\mathrm{E_q}$

D. $\mathrm{E_a} = 2\mathrm{E_q}$

Answer: D

Question 150

In potentiometer experiment, the balancing length is $8 \mathrm{~m}$ when two cells E_1 and E_2 are joined in series. When two cells are connected in opposition the balancing length is $4 \mathrm{~m}$. The ratio of the e.m.f. of the two cells $\left(\frac{E_1}{E_2}\right)$ is

Options:

A. 1 : 2

B. 2 : 1

C. 1 : 3

D. 3 : 1

Answer: D

Solution:

$$\frac{\mathrm{E}_1}{\mathrm{E}_2}=\frac{l_1+l_2}{l_1-l_2}=\frac{8+4}{8-4}$$
$$\frac{\mathrm{E}_1}{\mathrm{E}_2}=\frac{12}{4}$$
$$\therefore \quad \frac{\mathrm{E}_1}{\mathrm{E}_2}=3$$
