MHT CET 2023 : 9th May Evening Shift

Mathematics

Question 1

If the slope of one of the lines represented by $ax^2 + (2a+1)xy + 2y^2 = 0$ is reciprocal of the slope of the other, then the sum of squares of slopes is

Options:

A. $\frac{17}{4}$

B. $\frac{82}{9}$

C. $\frac{97}{36}$

D. 2

Answer: A

Solution:

Given equation of pair of lines is

$ax^2 + (2\mathrm{a}+1)xy + 2y^2 = 0$

$\mathrm{A} = \mathrm{a}, \mathrm{H} = \frac{2\mathrm{a}+1}{2}, \ \mathrm{B} = 2$

Given condition,

$\mathrm{m}_1 = \frac{1}{\mathrm{m}_2}$

$\therefore \quad \mathrm{m}_1 \cdot \mathrm{m}_2 = 1$

Product of slopes $= \frac{\mathrm{A}}{\mathrm{B}} = \frac{\mathrm{a}}{2}$

$\therefore \quad \mathrm{m}_1 \cdot \mathrm{m}_2 = 1 = \frac{\mathrm{a}}{2}$

$\therefore \quad \mathrm{a} = 2$

Also, sum of slopes $= \frac{-2\mathrm{H}}{\mathrm{B}} = -\left(\frac{2\mathrm{a}+1}{2}\right) = \frac{-5}{2}$

Using $(\mathrm{m}_1 + \mathrm{m}_2)^2 = \mathrm{m}_1^2 + \mathrm{m}_2^2 + 2\,\mathrm{m}_1\,\mathrm{m}_2$

$\left(\frac{-5}{2}\right)^2 = \mathrm{m}_1^2 + \mathrm{m}_2^2 + 2 \times 1$

$\therefore \quad \mathrm{m}_1^2 + \mathrm{m}_2^2 = \frac{25}{4} - 2$

$\therefore \quad \mathrm{m}_1^2 + \mathrm{m}_2^2 = \frac{17}{4}$

Question 2

In $\triangle PQR$, $\sin P$, $\sin Q$ and $\sin R$ are in A.P., then

Options:

A. its altitudes are in A.P.

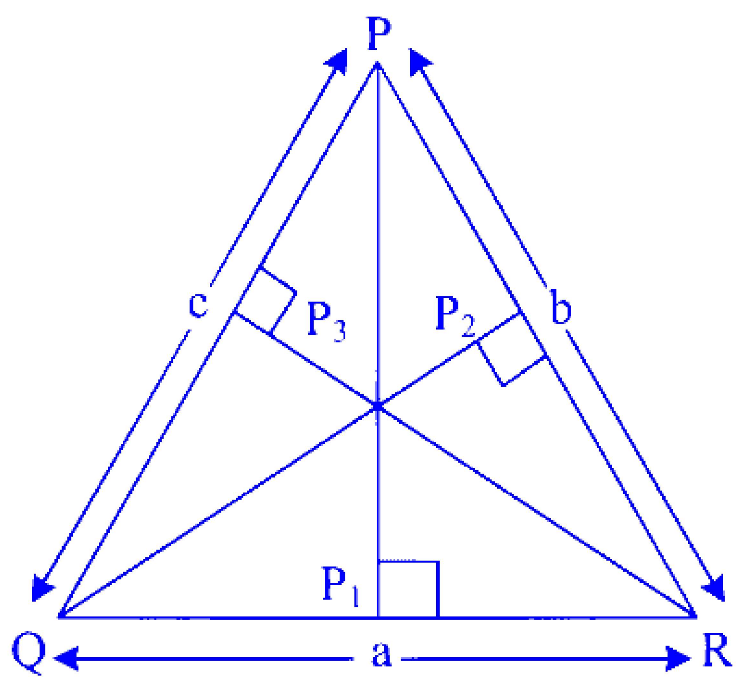
B. its altitudes are in H.P.

C. its medians are in G.P.

D. its medians are in A.P.

Answer: B

Solution:



Let p_1, p_2, p_3 be the altitudes of $\triangle PQR$ Area of $\triangle PQR$

$$= \frac{1}{2} \times \text{ base } \times \text{ height}$$
$$= \frac{1}{2} \times p_1 \times a$$

Area $= \frac{1}{2} p_1 a$

$\therefore \quad p_1 = 2 \times \frac{\text{Area}}{a}$ ..... (i)

$\therefore \quad$ similarly,

$p_2 = \frac{2\times \text{Area}}{b}$ ..... (ii)

$p_3 = \frac{2\times \text{Area}}{c}$ ..... (iii)

$\therefore$ By sine Rule,

$\frac{a}{\sin P} = \frac{b}{\sin Q} = \frac{c}{\sin R}$

Let $\frac{a}{\sin P} = \frac{b}{\sin Q} = \frac{c}{\sin R} = k$

$\therefore$ $\sin P = \frac{a}{k}, \sin Q = \frac{b}{k}, \sin R = \frac{c}{k}$

$\sin P, \sin Q$ and $\sin R$ are in A.P.

$\therefore$ a, b, c are in A.P.

$\therefore$ $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P. ..... (iv)

From equations (i), (ii), (iii) and (iv), we get p_1, p_2 and p_3 are in H.P.

Question 3

The value of $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right), |x| < \frac{1}{2}, x \neq 0$

Options:

A. $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

B. $\frac{\pi}{4} + \cos^{-1} x^2$

C. $\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2$

D. $\frac{\pi}{4} - \cos^{-1} x^2$

Answer: A

Solution:

Let $T=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Put $x^2=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1} x^2$

$\therefore \quad \mathrm{T}=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right)$

$=\tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

$=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$

$=\frac{\pi}{4}+\theta$

$=\frac{\pi}{4}+\frac{1}{2}\cos^{-1} x^2$

Question 4

If slope of a tangent to the curve $xy+ax+by=0$ at the point $(1,1)$ on it is 2, then a - b is

Options:

A. 3

B. 1

C. 2

D. -1

Answer: A

Solution:

Given curve is

$$xy+ax+by=0$$
$$\therefore \quad \text{Slope } =2=\frac{dy}{dx}$$
$$\therefore \quad xy+ax+by=0$$

Differentiating w.r.t. x, we get

$$x \frac{\mathrm{d}y}{\mathrm{d}x} + y + \mathrm{a} + \mathrm{b} \frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

$$\therefore \quad (x + \mathrm{b}) \frac{\mathrm{d}y}{\mathrm{d}x} = -(y + \mathrm{a})$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-(y+\mathrm{a})}{x+\mathrm{b}}$$

$$\therefore \quad \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(1,1)} = 2$$

$$\therefore \quad 2 = \frac{-(1+\mathrm{a})}{1+\mathrm{b}}$$

$$\therefore \quad \mathrm{a} + 2\,\mathrm{b} = -3 \quad \text{..... (i)}$$

Since $(1, 1)$ lies on $xy + ax + by = 0$, we get
$a + b = -1$ .... (ii)

Solving (i), (ii), we get

$$a = 1, b = -2$$

$$\therefore \quad a - b = 1 - (-2) = 3$$

Question 5

The equation of a line, whose perpendicular distance from the origin is 7 units and the angle, which the perpendicular to the line from the origin makes, is 120° with positive X-axis, is

Options:

A. $x + \sqrt{3}y - 14 = 0$

B. $x + \sqrt{3}y + 14 = 0$

C. $x - \sqrt{3}y + 14 = 0$

D. $x - \sqrt{3}y - 14 = 0$

Answer: C

Solution:

Normal form of the equation of line is

$x \cos \alpha + y \sin \alpha = \mathrm{p}$

Here, $\alpha = 120^\circ$ and $\mathrm{p} = 7$

$\therefore \quad x \cos 120^\circ + y \sin 120^\circ = 7$

$\therefore \quad x\left(\frac{-1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 7$

$\therefore \quad \frac{-x+\sqrt{3}y}{2} = 7$

$\therefore \quad -x + \sqrt{3}y = 14$

$\therefore \quad -x + \sqrt{3}y - 14 = 0$

$\therefore \quad x - \sqrt{3}y + 14 = 0$

Question 6

Let a, b, c be the lengths of sides of triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9} = k$. Then $\frac{(\mathrm{A}(\triangle \mathrm{ABC}))^2}{\mathrm{k}^4} =$

Options:

A. 36

B. 32

C. 38

D. 40

Answer: A

Solution:

In $\triangle ABC$

$$\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9} = k \quad \text{... [Given]}$$

$\therefore \quad a + b = 7k \quad$ .... (i)

$b + c = 8k \quad$ .... (ii)

$c + a = 9k \quad$ .... (iii)

Adding above equations,

$2\mathrm{a} + 2\,\mathrm{b} + 2\mathrm{c} = 24\mathrm{k}$

$\mathrm{a} + \mathrm{b} + \mathrm{c} = 12\mathrm{k} \quad$ .... (iv)

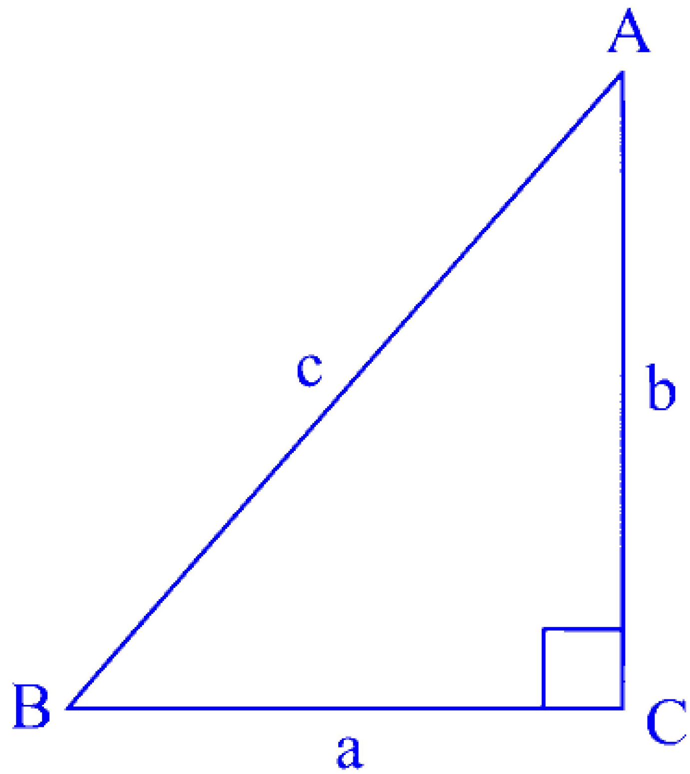
Solving equations (i), (ii), (iii), (iv) We get,

$\mathrm{c} = 5\mathrm{k}, \mathrm{a} = 4\mathrm{k}, \mathrm{b} = 3\mathrm{k}$

$\therefore \quad \mathrm{c}^2 = \mathrm{a}^2 + \mathrm{b}^2$

$\therefore \quad \triangle \mathrm{ABC}$ is right angled triangle

$\therefore \quad \angle C = 90^\circ$



$$\text{Area of } \triangle ABC = \frac{1}{2} ab \sin C$$
$$= \frac{1}{2} ab \sin 90$$
$$= \frac{1}{2} \times 4k \times 3k = 6k^2$$

$$\therefore \quad \text{Now, } \frac{[A(\triangle ABC)]^2}{k^4} = \frac{(6k^2)^2}{k^4} = \frac{36k^4}{k^4} = 36$$

Question 7

If the mean and S.D. of the data $3, 5, 7, a, b$ are 5 and 2 respectively, then a and b are the roots of the equation

Options:

A. $x^2 - 10x + 18 = 0$

B. $2x^2 - 20x + 19 = 0$

C. $x^2 - 10x + 19 = 0$

D. $x^2 - 20x + 18 = 0$

Answer: C

Solution:

$$\text{Mean } = 5 \quad \text{.... [Given]}$$

$$\therefore \quad \text{Mean } = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\Rightarrow 5 = \frac{3+5+7+a+b}{5}$$

$$\Rightarrow a+b = 10 \quad \text{... (i)}$$

$$\text{S.D. } = 2 \quad \text{... [Given]}$$

$$\text{S.D. } = \sqrt{\frac{\sum x_i^2}{n} - (x)^2}$$

$$\Rightarrow (2)^2 = \frac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2$$

$$\Rightarrow 4 = \frac{83+a^2+b^2}{5} - 25$$

$$\Rightarrow a^2+b^2 = 62 \quad \text{.... (ii)}$$

Now, (i) $\Rightarrow a+b=10$

Squaring both sides, we get

$$(a+b)^2 = 100$$

$$a^2+2ab+b^2 = 100$$

$$38 = 2ab \quad \text{.... [From (ii)]}$$

$$\therefore \quad \text{ab } = 19$$

Note that the required quadratic equation is expressed as

$$x^2-(a+b)x+ab=0$$

$$\therefore \quad x^2-10x+19=0$$

Question 8

A man takes a step forward with probability 0.4 and backwards with probability 0.6 . The probability that at the end of eleven steps, he is one step away from the starting point is

Options:

A. ${}^{11}\mathrm{C}_6(0.24)^6$

B. ${}^{11}\mathrm{C}_6(0.24)^5$

C. ${}^{11}\mathrm{C}_6(0.4)^6(0.6)^5$

D. ${}^{11}\mathrm{C}_6(0.4)^5(0.6)^6$

Answer: B

Solution:

Let a step forward be a success and the step backward be a failure.

$\therefore$ Probability of success $= p = 0.4$, and Probability of failure $= \mathrm{q} = 0.6$

Now, in 11 steps

number of successes $= 6$, number of failure $= 5$

OR

number of successes $= 5$, number of failures $= 6$

$\therefore$ Required probability $= {}^{11}\mathrm{C}_6 \cdot \mathrm{p}^6\mathrm{q}^5 + {}^{11}\mathrm{C}_5\mathrm{p}^5\mathrm{q}^6$.

$$
\begin{aligned}
&= \frac{11!}{6!5!}p^6q^5 + \frac{11!}{5!6!}p^5q^6 \\
&= {}^{11}C_6p^5q^5(p+q) \\
&= {}^{11}C_6(0\cdot 4)^5(0\cdot 6)^5(1) \\
&= {}^{11}C_6(0\cdot 4)^5(0\cdot 6)^5 \\
&= {}^{11}C_6(0\cdot 24)^5
\end{aligned}
$$

Question 9

If $\mathrm{f}'(x) = x - \frac{5}{x^5}$ and $\mathrm{f}(1) = 4$, then $\mathrm{f}(x)$ is

Options:

A. $\frac{x^2}{2} + \frac{9}{4}\frac{1}{x^4} + \frac{5}{4}$

B. $\frac{x^2}{2} - \frac{5}{4}\frac{1}{x^4} + \frac{9}{4}$

C. $\frac{x^2}{2} + \frac{5}{4}\frac{1}{x^4} + \frac{9}{4}$

D. $\frac{x^2}{2} - \frac{9}{4}\frac{1}{x^4} + \frac{5}{4}$

Answer: C

Solution:

Given $\mathrm{f}'(x) = x - \frac{5}{x^5}$

$\therefore$ Integrating both sides, we get

$$f(x) = \int\left(x - \frac{5}{x^5}\right)dx$$
$$f(x) = \frac{x^2}{2} + \frac{5}{4} \times \frac{1}{x^4} + c$$
$\therefore$ But $f(1) = 4$

$\therefore$ $\frac{1}{2} + \frac{5}{4} + c = 4$

$\therefore$ $c = \frac{9}{4}$

$\therefore$ $f(x) = \frac{x^2}{2} + \frac{5}{4}\frac{1}{x^4} + \frac{9}{4}$

Question 10

A linguistic club of a certain Institute consists of 6 girls and 4 boys. A team of 4 members to be selected from this group including the selection of a Captain (from among these 4 members) for the team. If the team has to include atmost one boy, the number of ways of selecting the team is

Options:

A. 95

B. 260

C. 320

D. 380

Answer: D

Solution:

Case I: No boy is included.

Selecting 4 girls from 6 girls $= {}^6C_4$

Selecting 1 captain from selected members $= {}^4C_1$

Total number of ways $= {}^6C_4 \times {}^4C_1 = 60$

Case II: One boy is included.

Selecting 3 girls and 1 boy from given members $= {}^6C_3 \times {}^4C_1$.

Selecting 1 captain from the selected members $= {}^4C_1$.

Total Number of ways $= {}^6C_3 \times {}^4C_1 \times {}^4C_1 = 320$.

$\therefore$ Total Number of ways $= 320 + 60 = 380$.

Question 11

Negation of the statement

"The payment will be made if and only if the work is finished in time." Is

Options:

A. The work is finished in time and the payment is not made.

B. The payment is made and the work is not finished in time.

C. The work is finished in time and the payment is not made, or the payment is made and the work is finished in time.

D. Either the work is finished in time and the payment is not made, or the payment is made and the work is not finished in time.

Answer: D

Solution:

Let p : Payment will be made

q : Work is finished in time.

Given statement is

$(\mathrm{p} \rightarrow \mathrm{q}) \wedge (\mathrm{q} \rightarrow \mathrm{p})$

Negation of above statement is

$(p \wedge \sim q) \vee (q \wedge \sim p)$

$\therefore$ Option (D) is correct.

Question 12

The equation $x^3 + x - 1 = 0$ has

Options:

A. no real root.

B. exactly two real roots.

C. exactly one real root.

D. all three real roots.

Answer: C

Solution:

$$\begin{aligned}
& \text{Given equation } x^3+x-1=0 \\
& \text{Let } \mathrm{f}(x)=x^3+x-1 \\
\therefore\quad & \mathrm{f}'(x)=3x^2+1 \\
& \Rightarrow \mathrm{f}'(x)>0 \forall x \in \mathrm{R} \\
& \Rightarrow \mathrm{f}(x) \text{ is increasing} \\
\therefore\quad & \text{for } x_2>x_1, \mathrm{f}\left(x_2\right)>\mathrm{f}\left(x_1\right) \\
& \text{Now, } \mathrm{f}(0)=-1 \text{ and } \mathrm{f}(1)=1 \\
\therefore\quad & \mathrm{f}(x)=0 \text{ for some } x \in(0,1) \\
\therefore\quad & \text{Equation has one real root.}
\end{aligned}$$

Question 13

If a line L is the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$. If line L makes an angle α with the positive X-axis, then the value of $\sec\alpha$ is

Options:

A. $\sqrt{3}$

B. 2

C. 1

D. $\sqrt{2}$

Answer: A

Solution:

Given equations:

$$\begin{aligned}
& 2x+3y+z=1 \\
& 2x+3y=1-z \quad \text{...... (i)} \\
& x+3y+2z=2 \\
& x+3y=2-2z \quad \text{...... (ii)}
\end{aligned}$$

Subtracting (ii) from (i), we get

$$2x + 3y - x - 3y = 1 - z - 2 + 2z$$
$$x = -1 + z$$
$$z = \frac{x+1}{1} \quad \text{...... (iii)}$$

Putting value of x in equation (ii), we get

$$-1 + z + 3y = 2 - 2z$$
$$3z = 3 - 3y$$
$$z = \frac{y-1}{-1} \quad \text{...... (iv)}$$

From (iii), (iv)

$\frac{x+1}{1} = \frac{y-1}{-1} = \frac{z}{1}$

Thus, angle between above line and X-axis having Direction Ratio's $(1, 0, 0)$ is given as

$$\cos\alpha = \left|\frac{1\cdot(1)+0+0}{\sqrt{1+1+1}\cdot\sqrt{1}}\right| = \frac{1}{\sqrt{3}}$$
$$\therefore \quad \sec\alpha = \sqrt{3}$$

Question 14

The range of values of x for which $f(x) = x^3 + 6x^2 - 36x + 7$ is increasing in

Options:

A. $(-\infty, -6) \cup (2, \infty)$

B. $(-6, 2)$

C. $(-\infty, -2) \cup (6, \infty)$

D. $(-6, 2]$

Answer: A

Solution:

$$f(x) = x^3 + 6x^2 - 36x + 7$$
$$f'(x) = 3x^2 + 12x - 36$$
$$= 3\left(x^2 + 4x - 12\right)$$

For f(x) to be increasing,

$\mathrm{f}'(x) > 0$
$\Rightarrow 3\left(x^2 + 4x - 12\right) > 0$
$\Rightarrow x^2 + 4x - 12 > 0$
$\Rightarrow (x+6)(x-2) > 0$
$\Rightarrow x \in (-\infty, -6) \cup (2, \infty)$

Question 15

The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $\mathrm{S} = \left\{x \in \mathbb{R} / x^2 + 30 \leq 11x\right\}$ is

Options:

A. 122

B. 132

C. 112

D. 222

Answer: A

Solution:

$\mathrm{S} = \left\{x \in \mathbb{R} / x^2 + 30 \leq 11x\right\}$
$x^2 + 30 \leq 11x$
$x^2 - 11x + 30 \leq 0$
$(x-5)(x-6) \leq 0$
$x \in [5, 6]$
Now, $\mathrm{f}(x) = 3x^3 - 18x^2 + 27x - 40$
$\mathrm{f}'(x) = 9x^2 - 36x + 27$
$\mathrm{f}'(x) = 9\left(x^2 - 4x + 3\right)$
$= 9\left[\left(x^2 - 4x + 4\right) - 1\right]$
$= 9(x-2)^2 - 9$
$\therefore \quad \mathrm{f}'(x) > 0 \forall x \in [5, 6]$

$\therefore \quad \mathrm{f}(x)$ is strictly increasing in the interval $[5, 6]$

$\therefore \quad$ Maximum value of $\mathrm{f}(x)$ when $x \in [5, 6]$ is $f(6) = 122$

Question 16

Let p, q, r be three statements, then $[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$ is

Options:

A. equivalent to p $\leftrightarrow$ q.

B. contingency.

C. tautology.

D. contradiction.

Answer: C

Solution:

Given statement,

$$\begin{aligned} &[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r] \\ &p \to (q \to r) \equiv \sim p \vee (q \to r) \\ &\equiv \sim p \vee (\sim q \vee r) \\ &\equiv [(\sim p) \vee (-q)] \vee r \quad \ldots [\text{ Associative law}] \\ &\equiv \sim (p \wedge q) \vee r \quad \ldots [\text{ De'morgans law}] \\ &\equiv p \wedge q \to r \end{aligned}$$

$\therefore$ Given statement is tautology.

Question 17

$\int_1^2 \frac{\mathrm{d}x}{(x^2-2x+4)^{\frac{3}{2}}} = \frac{\mathrm{k}}{\mathrm{k}+5}$, then k has the value

Options:

A. 1

B. 2

C. -1

D. -2

Answer: A

Solution:

$$\text{Let I} = \int_1^2 \frac{dx}{(x^2-2x+4)^{\frac{3}{2}}}$$

$$= \int_1^2 \frac{dx}{[(x-1)^2+3]^{\frac{3}{2}}}$$

Put $x - 1 = \sqrt{3}\tan\theta$

$dx = \sqrt{3}\sec^2\theta d\theta$

When $x = 1, \theta = 0$

When $x = 2, \theta = \frac{\pi}{6}$

$$\therefore \quad I = \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{\left[3\tan^2\theta+3\right]^{\frac{3}{2}}} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{\left[3\left(1+\tan^2\theta\right)\right]^{\frac{3}{2}}}$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sqrt{3}\sec^2\theta}{3\cdot\sqrt{3}(\sec^2\theta)^{\frac{3}{2}}}$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{3}\cdot\frac{\sec{}^2\theta}{\sec^3\theta}$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{6}} \cos\theta$$

$$= \frac{1}{3}[\sin\theta]_0^{\frac{\pi}{6}}$$

$$I = \frac{1}{3}\left[\sin\frac{\pi}{6} - \sin 0\right]$$

$$I = \frac{1}{6}$$

$$\therefore \quad \frac{k}{k+5} = \frac{1}{6}$$

$$6k = k + 5$$

$$\therefore \quad k = 1$$

Question 18

The sides of a rectangle are given by the equations $x=-2, x=4, y=-2$ and $y=5$

Then the equation of the circle, whose centre is the point of intersection of the diagonals, lying within the rectangle and touching only two opposite

sides, is

Options:

A. $x^2+y^2+2x+3y+9=0$

B. $x^2+y^2-2x+3y+9=0$

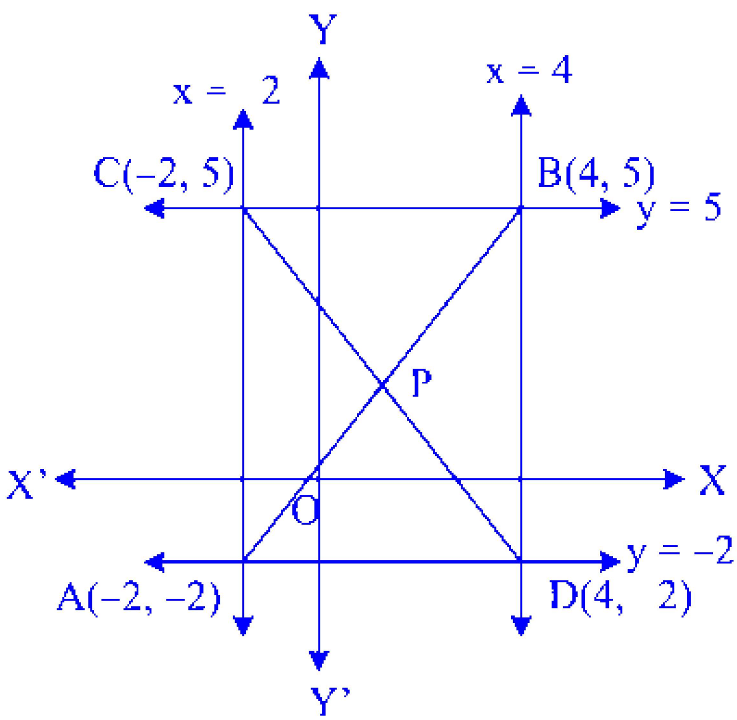
C. $x^2+y^2+2x-3y-9=0$

D. $x^2+y^2-2x-3y-9=0$

Answer: D

Solution:

The given equations of the sides are $x=-2, x=4, y=-2, y=5$



According to the given condition, centre of the required circle required circle is P.

$\therefore$ The co-ordinates of P are $\left(1, \frac{3}{2}\right)$.

As circle touches only 2 opposite sides, its radius is either 3.5 units or 3 units.

$\therefore$ Equation of the required circle is

$$x^2+y^2-2x-3y-\frac{23}{4}=0 \text{ or}$$
$$x^2+y^2-2x-3y-9=0$$

$\therefore$ Option (D) is correct.

Question 19

$\overline{a} = \hat{i} + \hat{j} + \hat{k}, \overline{b} = \hat{j} - \hat{k}$, then vector $\overline{r}$ satisfying $\overline{a} \times \overline{r} = \overline{b}$ and $\overline{a} \cdot \overline{r} = 3$ is

Options:

A. $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

B. $-\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

C. $\frac{5}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

D. $-\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$

Answer: A

Solution:

Given $\overline{a} \cdot \overline{r} = 3$

$\overline{a} \times \overline{r} = \overline{b}$

Let $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\overline{a} \times \overline{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}$$
$$= (z - y)\hat{i} - \hat{j}(z - x) + \hat{k}(y - x)$$

Given $\overline{a} \times \overline{r} = \overline{b}$

$\therefore \quad (z - y)\hat{i} - (z - x)\hat{j} + (y - x)\hat{k} = \hat{j} - \hat{k}$

Comparing

$z - y = 0 \quad$ ..... (i)
$z - x = -1 \quad$ ..... (ii)
$y - x = -1 \quad$ ..... (iii)

Also, $\overline{a} \cdot \overline{r} = 3$

$(\hat{i} + \hat{j} + \hat{k})(x\hat{i} + y\hat{j} + zk) = 3$
$x + y + z = 3 \quad$ ..... (iv)

Solving equations (i), (ii), (iii) and (iv), we get

$$x=\frac{5}{3}, y=\frac{2}{3}, z=\frac{2}{3}$$

$$\therefore \quad \bar{r}=\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$$

Question 20

The magnitude of the projection of the vector $2\hat{i}+\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ is

Options:

A. $\frac{2}{\sqrt{6}}$

B. $\frac{1}{\sqrt{6}}$

C. $\frac{5}{\sqrt{6}}$

D. $\frac{7}{\sqrt{6}}$

Answer: B

Solution:

The vector perpendicular to both vectors containing $(\hat{i}+\hat{j}+\hat{k})$ and $(\hat{i}+2\hat{j}+3\hat{k})$ is

$$\begin{aligned}
&=(\hat{i}+\hat{j}+\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})\\
&=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}\\
&=\hat{i}-2\hat{j}+\hat{k}
\end{aligned}$$

Therefore, the magnitude of the projection of vector $(2\hat{i}+\hat{j}+\hat{k})$ on $(\hat{i}-2\hat{j}+\hat{k})$ is

$$\begin{aligned}
&=\left|\frac{(2\hat{i}+\hat{j}+\hat{k})(\hat{i}-2\hat{j}+\hat{k})}{\sqrt{1^2+(-2)^2+(1)^2}}\right|\\
&=\left|\frac{2-2+1}{\sqrt{6}}\right|\\
&=\frac{1}{\sqrt{6}}
\end{aligned}$$

Question 21

The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is

Options:

A. $\frac{1}{\sqrt{14}}$ units.

B. $\frac{1}{\sqrt{5}}$ units.

C. $\frac{1}{\sqrt{11}}$ units.

D. $\frac{1}{\sqrt{6}}$ units.

Answer: D

Solution:

The lines arĕ

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

Comparing equations with

$\frac{x-x_1}{\mathrm{a}_1}=\frac{y-y_1}{\mathrm{b}_1}=\frac{\mathrm{z}-\mathrm{z}_1}{\mathrm{c}_1}$ and $\frac{x-x_2}{\mathrm{a}_2}=\frac{y-y_2}{\mathrm{b}_2}=\frac{\mathrm{z}-\mathrm{z}_2}{\mathrm{c}_2}$,

we get

$$x_1=1, y_1=2, \mathrm{z}_1=3 \quad x_2=2, y_2=4, \mathrm{z}_2=5$$
$$\mathrm{a}_1=2,\ \mathrm{b}_1=3, \mathrm{c}_1=4 \quad \mathrm{a}_2=3,\ \mathrm{b}_2=4, \mathrm{c}_2=5$$
$$\begin{vmatrix} x_2-x_1\cdot & y_2-y_1 & \mathrm{z}_2-\mathrm{z}_1 \\ \mathrm{a}_1 & \mathrm{b}_1 & \mathrm{c}_1 \\ \mathrm{a}_2 & \mathrm{b}_2 & \mathrm{c}_2 \end{vmatrix}=\begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$
$$=1(15-16)-2(10-12)+2(8-9)$$
$$=1$$
$$\therefore \quad \sqrt{\left(a_1 b_2-a_2 b_1\right)^2+\left(b_1 c_2-b_2 c_1\right)^2+\left(c_1 a_2-c_2 a_1\right)^2}$$
$$=\sqrt{(2\times 4-3\times 3)^2+(3\times 5-4\times 4)^2+(4\times 3-5\times 2)^2}$$
$$=\sqrt{1+1+4}$$
$$=\sqrt{6}$$

Shortest distance between line is d

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - b_2c_1)^2 + \sqrt{(c_1a_2 - c_2a_1)^2}}}$$
$$= \frac{1}{\sqrt{6}}$$

Question 22

If $\bar{a}, \bar{b}$ and $\bar{c}$ are any three non-zero vectors, then $(\bar{a} + 2\bar{b} + \bar{c}) \cdot [(\bar{a} - \bar{b}) \times (\bar{a} - \bar{b} - \bar{c})] =$

Options:

A. $\begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix}$

B. $2\begin{bmatrix} \overline{\mathrm{a}} & \overline{\mathrm{b}} & \overline{\mathrm{c}} \end{bmatrix}$

C. $3\begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix}$

D. $4\begin{bmatrix} \overline{\mathrm{a}} & \overline{\mathrm{b}} & \overline{\mathrm{c}} \end{bmatrix}$

Answer: C

Solution:

$(\bar{a} + 2\bar{b} + \bar{c}) \cdot [(\bar{a} - \bar{b}) \times (\bar{a} - \bar{b} - \bar{c})]$
$= (\bar{a} + 2\bar{b} + \bar{c}) \cdot (\bar{a} \times \bar{a} - \bar{a} \times \bar{b} - \bar{a} \times \bar{c} - \bar{b} \times \bar{a} + \bar{b} \times \bar{b} + \bar{b} \times \bar{c})$
$= (\bar{a} + 2\bar{b} + \bar{c}) \cdot (\bar{0} - \bar{a} \times \bar{b} - \bar{a} \times \bar{c} + \bar{a} \times \bar{b} + \bar{0} + \bar{b} \times \bar{c})$
$= (\bar{a} + 2\bar{b} + \bar{c}) \cdot (\bar{c} \times \bar{a} + \bar{b} \times \bar{c})$
$= \bar{a} \cdot (\bar{c} \times \bar{a}) + \bar{a} \cdot (\bar{b} \times \bar{c}) + 2\bar{b} \cdot (\bar{c} \times \bar{a}) + 2\bar{b} \cdot (\bar{b} \times \bar{c})$
$+ \bar{c} \cdot (\bar{c} \times \bar{a}) + \bar{c} \cdot (\bar{b} \times \bar{c})$

$= \bar{0} + \bar{a} \cdot (\bar{b} \times \bar{c}) + 2\bar{b} \cdot (\bar{c} \times \bar{a}) + 2 \times 0 - 0 - 0$
$= \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} + 2\begin{bmatrix} \bar{b} & \bar{c} & \bar{a} \end{bmatrix}$
$= \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} + 2\begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix}$
$= 3\begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix}$

Question 23

In $\triangle \mathrm{ABC}$, with usual notations, $\mathrm{m}\angle \mathrm{C}=\frac{\pi}{2}$, if $\tan\left(\frac{A}{2}\right)$ and $\tan\left(\frac{B}{2}\right)$ are the roots of the equation $a_1 x^2+b_1 x+c_1=0\left(a_1 \neq 0\right)$, then

Options:

A. $a_1+b_1=c_1$

B. $b_1+c_1=a_1$

C. $a_1+c_1=b_1$

D. $b_1=c_1$

Answer: A

Solution:

$$\begin{aligned}
& \text{In } \triangle \mathrm{ABC}, \\
& \angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=180^{\circ} \\
\therefore \quad & \angle \mathrm{A}+\tfrac{\pi}{2}+\angle \mathrm{B}=180^{\circ} \\
\therefore \quad & \angle \mathrm{A}+\angle \mathrm{B}=\tfrac{\pi}{2} \\
\therefore \quad & \tfrac{\angle \mathrm{A}}{2}+\tfrac{\angle \mathrm{B}}{2}=\tfrac{\pi}{4} \\
& \tan\left(\tfrac{\mathrm{A}}{2}\right) \text{ and } \tan\left(\tfrac{\mathrm{B}}{2}\right) \text{ are roots of equation} \\
& \mathrm{a}_1 x^2+\mathrm{b}_1 x+\mathrm{c}_1=0 \quad \ldots [\text{Given}] \\
\therefore \quad & \text{Sum of roots }=\tfrac{-\mathrm{b}_1}{\mathrm{a}_1} \\
& \tan\left(\tfrac{\mathrm{A}}{2}\right)+\tan\left(\tfrac{\mathrm{B}}{2}\right)=\tfrac{-\mathrm{b}_1}{\mathrm{a}_1}
\end{aligned}$$

Also, $\tan\left(\frac{\mathrm{A}}{2}\right) \cdot \tan\left(\frac{\mathrm{B}}{2}\right)=\frac{\mathrm{c}_1}{\mathrm{a}_1}$

Using $\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}$, we get

$\tan\left(\frac{\pi}{4}\right)=\frac{\frac{-b_1}{a_1}}{1-\frac{c_1}{a_1}}$

$1=\frac{-b_1}{a_1-c_1}$

$a_1-c_1=-b_1$

$a_1+b_1=c_1$

Question 24

If $\int \frac{\sin x}{3+4\cos^2 x}\,\mathrm{d}x=\mathrm{A}\tan^{-1}(\mathrm{~B}\cos x)+\mathrm{c}$, (where c is a constant of integration), then the value of $\mathrm{A}+\mathrm{B}$ is

Options:

A. $\frac{5}{2\sqrt{3}}$

B. $\frac{-1}{2\sqrt{3}}$

C. $\frac{-2}{\sqrt{3}}$

D. $\frac{\sqrt{3}}{2}$

Answer: D

Solution:

$\text{Let I} = \int \frac{\sin x}{3+4\cos^2 x}\, dx$
$\text{Put } \cos x = t$
$-\sin x\, dx = dt$
$\therefore \quad \sin x\, dx = -dt$
$\therefore \quad I = \int \frac{-dt}{3+4t^2}$
$= -1 \int \frac{1}{4t^2+3}$
$= -1 \int \frac{1}{(2t)^2+(\sqrt{3})^2}$
$= -1 \times \frac{1}{2\times\sqrt{3}} \tan^{-1}\left(\frac{2t}{\sqrt{3}}\right) + c$
$I = \frac{-1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + c$
$\text{But } \int \frac{\sin x}{3+4\cos^2 x}\, dx = A\tan^{-1}(B\cos x) + c$
Comparing above equations, we get
$A = \frac{-1}{2\sqrt{3}},\ B = \frac{2}{\sqrt{3}}$
$\therefore A + B = \frac{-1}{2\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

Question 25

Vectors $\overline{a}$ and $\overline{b}$ are such that $|\overline{a}| = 1; |\overline{b}| = 4$ and $\bar{a} \cdot \bar{b} = 2$. If $\bar{c} = 2\bar{a} \times \bar{b} - 3\bar{b}$, then the angle between $\bar{b}$ and $\bar{c}$ is

Options:

A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer: B

Solution:

Given: $|\overline{\mathrm{a}}| = 1, |\overline{\mathrm{b}}| = 4$ and $\overline{\mathrm{a}} \cdot \overline{\mathrm{b}} = 2, \overline{\mathrm{c}} = 2\overline{\mathrm{a}} \times \overline{\mathrm{b}} - 3\overline{\mathrm{b}}$

Now that, $|\overline{\mathrm{a}} \times \overline{\mathrm{b}}|^2 = |\overline{\mathrm{a}}|^2|\overline{\mathrm{b}}|^2 - |\overline{\mathrm{a}} \cdot \overline{\mathrm{b}}|^2$

$\therefore \quad |\overline{\mathrm{a}} \times \overline{\mathrm{b}}|^2 = 16 - 4 = 12$

Given that $\bar{c} = 2\bar{a} \times \bar{b} - 3\bar{b}$

$|\bar{c}|^2 = (2\bar{a} \times \bar{b} - 3\bar{b})^2$
$|\bar{c}|^2 = 4|\bar{a} \times \bar{b}|^2 + 9|\bar{b}|^2$

$|\bar{c}|^2 = 4(12) + 9(16)$
$|\bar{c}|^2 = 192$
$|\bar{c}| = 8\sqrt{3}$

Now, $\bar{b} \cdot \bar{c} = b \cdot (2\bar{a} \times \bar{b} - 3\bar{b})$

$\therefore \quad \bar{b} \cdot \bar{c} = -3|\bar{b}|^2 = -48$

Angle between b and c is given by

$$\cos\theta = \frac{(\bar{b} \cdot \bar{c})}{(|b||c\ |)} = \frac{-48}{(4 \times 8\sqrt{3})}$$
$$\cos\theta = \frac{-\sqrt{3}}{2}$$
$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$
$$\theta = \frac{5\pi}{6}$$

Question 26

If y is a function of x and $\log(x+y) = 2xy$, then the value of $y'(0)$ is

Options:

A. 1

B. -1

C. 2

D. 0

Answer: A

Solution:

$\log(x+y)=2xy$

Differentiating w.r.t. x, we get

$$\frac{1}{x+y}\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)=2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$$
$$\frac{1}{x+y}+\frac{1}{(x+y)}\frac{\mathrm{d}y}{\mathrm{d}x}=2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$$
$$\left(\frac{1}{x+y}-2x\right)\frac{\mathrm{d}y}{\mathrm{d}x}=2y-\frac{1}{x+y}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{1}{x+y}-2x\right)=2y-\frac{1}{x+y}$$
$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(2y-\frac{1}{x+y}\right)}{\left(\frac{1}{x+y}-2x\right)}$$
$$\text{For } x=0, \log(y)=0$$
$$\Rightarrow y=1$$
$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(0,1)}=\frac{\left(2-\frac{1}{0+1}\right)}{\left(\frac{1}{0+1}-0\right)}=1$$

Question 27

If $\cos 2B=\frac{\cos(A+C)}{\cos(A-C)}$. Then $\tan A, \tan B, \tan C$ are in

Options:

A. Geometric Progression.

B. Arithmetic Progression.

C. Harmonic Progression.

D. Arithmetico-Geometric Progression.

Answer: A

Solution:

$\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$

$\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{\cos A \cos C - \sin A \sin C}{\cos A \cos C + \sin A \sin C}$

$\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{\cos A \cos C(1-\tan A \tan C)}{\cos A \cos C(1+\tan A \tan C)}$

$\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{(1-\tan A \tan C)}{(1+\tan A \tan C)}$

$$\begin{aligned}
&\left(1-\tan^2 \mathrm{~B}\right)(1+\tan \mathrm{A} \tan \mathrm{C}) \\
&=(1-\tan \mathrm{A} \tan \mathrm{C})\left(1+\tan^2 \mathrm{~B}\right) \\
&1+\tan \mathrm{A} \tan \mathrm{C}-\tan^2 \mathrm{~B}-\tan^2 \mathrm{~B} \tan \mathrm{A} \tan \mathrm{C} \\
&=1+\tan^2 \mathrm{~B}-\tan \mathrm{A} \tan \mathrm{C}-\tan \mathrm{A} \tan \mathrm{C} \tan^2 \mathrm{~B} \\
&2 \tan \mathrm{A} \tan \mathrm{C}=2 \tan^2 \mathrm{~B} \\
&\tan^2 B=\tan A \cdot \tan C \\
\therefore\ &\tan A, \tan B, \tan C \text{ are in G.P.}
\end{aligned}$$

Question 28

If $\log_2 x + \log_4 x + \log_8 x + \log_{16} x = \frac{25}{36}$ and $x = 2^{\mathrm{k}}$ then k is

Options:

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{8}$

Answer: C

Solution:

$$\log_2 x+\log_4 x+\log_8 x+\log_{16} x=\frac{25}{36}$$
$$\frac{\log x}{\log 2}+\frac{\log x}{\log 4}+\frac{\log x}{\log 8}+\frac{\log x}{\log 16}=\frac{25}{36}$$
$$\frac{\log x}{\log 2}+\frac{\log x}{2\log 2}+\frac{\log x}{3\log 2}+\frac{\log x}{4\log 2}=\frac{25}{36}$$
$$\frac{\log x}{\log 2}\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right]=\frac{25}{36}$$
$$\frac{\log x}{\log 2}\left[\frac{25}{12}\right]=\frac{25}{36}$$
$$\frac{25}{12}\log_2 x=\frac{25}{36}$$
$$\log_2 x=\frac{1}{3}$$
$$\therefore \quad x=2^{\frac{1}{3}}$$
But $x=2^{\mathrm{k}}$ .... [Given]
$$\therefore \quad \mathrm{k}=\frac{1}{3}$$

Question 29

If $\begin{vmatrix}\cos(A+B) & -\sin(A+B) & \cos(2B)\\ \sin A & \cos A & \sin B\\ -\cos A & \sin A & \cos B\end{vmatrix}=0$**, then the value of B is**

Options:

A. $\mathrm{n}\pi, \mathrm{n}\in\mathbb{Z}$

B. $(2\mathrm{n}+1)\frac{\pi}{2}, \mathrm{n}\in\mathbb{Z}$

C. $(2\mathrm{n}+1)\frac{\pi}{4}, \mathrm{n}\in\mathbb{Z}$

D. $2\mathrm{n}\frac{\pi}{3}, \mathrm{n}\in\mathbb{Z}$

Answer: B

Solution:

$$\begin{vmatrix}\cos(A+B) & -\sin(A+B) & \cos(2B)\\ \sin A & \cos A & \sin B\\ -\cos A & \sin A & \cos B\end{vmatrix}=0$$

$$\therefore \quad \cos(\mathrm{A}+\mathrm{B})[(\cos\mathrm{A}\cos\mathrm{B}-\sin\mathrm{A}\sin\mathrm{B})]+\sin(A+B)[\sin A\cos B+\sin B\cos A]+\cos 2B\left[\sin^2 A+\cos^2 A\right]=0$$

$\therefore \quad \cos(A+B) \cdot \cos(A+B)$
$+\sin(A+B) \cdot \sin(A+B)+\cos 2B=0$
$\cos^2(A+B)+\sin^2(A+B)+\cos 2B=0$
$1+\cos 2B=0$

$2\cos^2 B=0$
$\therefore \quad \cos B=0$

$\therefore \quad B=(2n+1)\frac{\pi}{2}$ for $(n \in Z)$

Question 30

General solution of the differential equation $\log\left(\frac{dy}{dx}\right)=ax+by$ is

Options:

A. $ae^{by}+be^{ax}=c_1$, where c_1 is a constant.

B. $ae^{-by}+b^{-ax}=c_1$, where c_1 is a constant.

C. $ae^{-by}+be^{ax}=c_1$, where c_1 is a constant.

D. $ae^{by}+be^{-ax}=c_1$, where c_1 is a constant.

Answer: C

Solution:

Given differential equation is

$$\log\left(\frac{dy}{dx}\right)=ax+by$$

$$\therefore \quad \frac{dy}{dx}=e^{ax+by}$$

$$\therefore \quad \frac{dy}{dx}=e^{ax} \cdot e^{by}$$

$$\therefore \quad \frac{dy}{e^{by}}=e^{ax} \cdot dx$$

$$e^{-by}\,dy-e^{ax}\,dx=0$$

Integrating both sides, we get

$$\int e^{-by}dy - \int e^{ax}dx = 0$$

$$\frac{e^{-by}}{-b} - \frac{e^{ax}}{a} + c = 0$$

$$\text{i.e., } \frac{e^{-by}}{b} + \frac{e^{ax}}{a} = c$$

$$ae^{-by} + be^{ax} = abc$$

$$ae^{-by} + be^{ax} = c_1, \text{ where } c_1 = abc$$

Question 31

$\int(\sqrt{\tan x} + \sqrt{\cot x})dx =$

Options:

A. $\sqrt{2}\sin^{-1}(\sin x - \cos x) + \text{c}$, where c is a constant of integration.

B. $\frac{1}{\sqrt{2}}\sin^{-1}(\sin x - \cos x) + \text{c}$, where c is a constant of integration.

C. $\sin^{-1}(\sin x - \cos x) + c$, where c is a constant of integration.

D. $2\sin^{-1}(\sin x - \cos x) + \text{c}$, where c is a constant of integration.

Answer: A

Solution:

Let $\mathrm{I} = \int (\sqrt{\tan x} + \sqrt{\cot x}) \mathrm{d}x$

$$= \int \left(\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}} \right) dx$$

$$\mathrm{I} = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} \mathrm{~d}x$$

Let $\sin x - \cos x = \mathrm{t}$

$\therefore \quad (\sin x + \cos x) \mathrm{d}x = \mathrm{dt}$

Consider, $\sin x - \cos x = \mathrm{t}$

Squaring on both sides, we get

$1 - 2 \sin t \cdot \cos t = t^2$

$1 - t^2 = 2 \sin t \cdot \cos t$

$$\therefore \quad \sin t \cdot \cos t = \frac{1 - t^2}{2}$$

$$\therefore \quad \mathrm{I} = \int \frac{\mathrm{dt}}{\frac{\sqrt{1-\mathrm{t}^2}}{\sqrt{2}}}$$

$$\therefore \quad \mathrm{I} = \sqrt{2} \int \frac{1}{\sqrt{1 - \mathrm{t}^2}} \mathrm{dt}$$

$\therefore \quad \mathrm{I} = \sqrt{2} \sin^{-1}(\mathrm{t}) + \mathrm{c}$

$\therefore \quad \mathrm{I} = \sqrt{2} \sin^{-1}(\sin x - \cos x) + \mathrm{c}$

Question 32

If $Z_1 = 2 + i$ and $Z_2 = 3 - 4i$ and $\frac{\overline{Z_1}}{\overline{Z_2}} = a + bi$, then the value of $-7a + b$ is (where $i = \sqrt{-1}$ and $a, b \in R$)

Options:

A. 1

B. -1

C. $\frac{-3}{25}$

D. $\frac{-9}{25}$

Answer: B

Solution:

$Z_1 = 2 + i$
$Z_2 = 3 - 4i$
$\overline{Z_1} = 2 - i$
$\overline{Z_2} = 3 + 4i$

$$\frac{\overline{Z_1}}{\bar{Z}_2} = \frac{2 - i}{3 + 4i}$$

$$= \frac{(2 - i)(3 - 4i)}{(3 + 4i)(3 - 4i)}$$

$$= \frac{6 - 8i - 3i + 4i^2}{(3)^2 - (4i)^2}$$

$$= \frac{6 - 11i - 4}{9 + 16} \quad \ldots [i^2 = -1]$$

$$a + bi = \frac{2 - 1li}{25}$$

$$\therefore \quad a = \frac{2}{25}, b = \frac{-11}{25}$$

$$\text{Now } -7a + b$$

$$= -7\left(\frac{2}{25}\right) - \frac{11}{25}$$

$$= \frac{-14 - 11}{25}$$

$$= \frac{-25}{25} = -1$$

Question 33

Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x) \sin 2\alpha + c$, (where c is a constant of integration), then functions $A(x)$ and $B(x)$ are respectively

Options:

A. $x + \alpha$ and $\log|\sin(x + \alpha)|$.

B. $x - \alpha$ and $\log|\sin(x - \alpha)|$.

C. $x - \alpha$ and $\log|\cos(x - \alpha)|$.

D. $x + \alpha$ and $\log|\sin(x - \alpha)|$.

Answer: B

Solution:

Let $\mathrm{I}=\int \frac{\tan x+\tan \alpha}{\tan x-\tan \alpha} \mathrm{d} x$

$=\int \frac{\frac{\sin x}{\cos x}+\frac{\sin \alpha}{\cos \alpha}}{\frac{\sin x}{\cos x}-\frac{\sin \alpha}{\cos \alpha}} d x$

$=\int \frac{\sin x \cos \alpha+\sin \alpha \cos x}{\sin x \cos \alpha-\sin \alpha \cos x} \mathrm{~d} x$

$=\int \frac{\sin (x+\alpha)}{\sin (x-\alpha)} d x$

Let $x-\alpha=\mathrm{t}$

$\therefore \quad I=\frac{\sin (t+2 \alpha)}{\sin t}$

$=\int \frac{\sin (t) \cos 2 \alpha+\cos (t) \sin 2 \alpha}{\sin (t)} d x$

$=\cos 2 \alpha \int 1 d t+\sin 2 \alpha \int \cot (\mathrm{t}) \mathrm{dt}$

$=\cos 2 \alpha \cdot \mathrm{t}+\sin 2 \alpha \cdot \log |\sin (\mathrm{t})|+\mathrm{c}$

$\therefore \quad \mathrm{I}=(x-\alpha) \cos 2 \alpha+\log |\sin (x-\alpha)| \sin 2 \alpha+\mathrm{c}$

But $\int \frac{\tan x+\tan \alpha}{\tan x-\tan \alpha} \mathrm{d} x=\mathrm{A}(x) \cos 2 \alpha+\mathrm{B}(x) \sin 2 \alpha+\mathrm{c}$ .... [Given]

$\Rightarrow \mathrm{A}(x)=x-\alpha, \mathrm{B}(x)=\log |\sin (x-\alpha)|+\mathrm{c}$

Question 34

Two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$, then the unit vector parallel to its diagonal is

Options:

A. $\frac{3}{7}\hat{\mathrm{i}}-\frac{6}{7}\hat{\mathrm{j}}+\frac{2}{7}\hat{\mathrm{k}}$

B. $\frac{2}{7}\hat{\mathrm{i}}-\frac{6}{7}\hat{\mathrm{j}}+\frac{3}{7}\hat{\mathrm{k}}$

C. $\frac{6}{7}\hat{\mathrm{i}}-\frac{2}{7}\hat{\mathrm{j}}+\frac{3}{7}\hat{\mathrm{k}}$

D. $\frac{1}{7}\hat{\mathrm{i}}+\frac{1}{7}\hat{\mathrm{j}}-\frac{3}{7}\hat{\mathrm{k}}$

Answer: A

Solution:

Let $\vec{a}$ and $\vec{b}$ be the adjacent sides of a parallelogram, where

$\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$
$\vec{b}=\hat{i}-2\hat{j}-3\hat{k}$

Let diagonal be $\overrightarrow{c}$

$$\vec{c} = \vec{a} + \vec{b}$$
$$\vec{c} = 2\hat{i} - 4\hat{j} + 5\hat{k} + \hat{i} - 2\hat{j} - 3\hat{k}$$
$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Magnitude of $\overrightarrow{c} = \sqrt{3^2 + (-6)^2 + (2)^2}$
$= \sqrt{49} = 7$

$\therefore$ Unit vector in direction of diagonal $\overrightarrow{c}$ is

$$= \frac{\vec{c}}{|\vec{c}|}$$
$$= \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$
$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

Question 35

A water tank has a shape of inverted right circular cone whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at constant rate of 5 cubic meter/minute. The rate in meter/ minute at which level of water is rising, at the instant when depth of water in the tank is 10 m is

Options:

A. $\frac{1}{5\pi}$

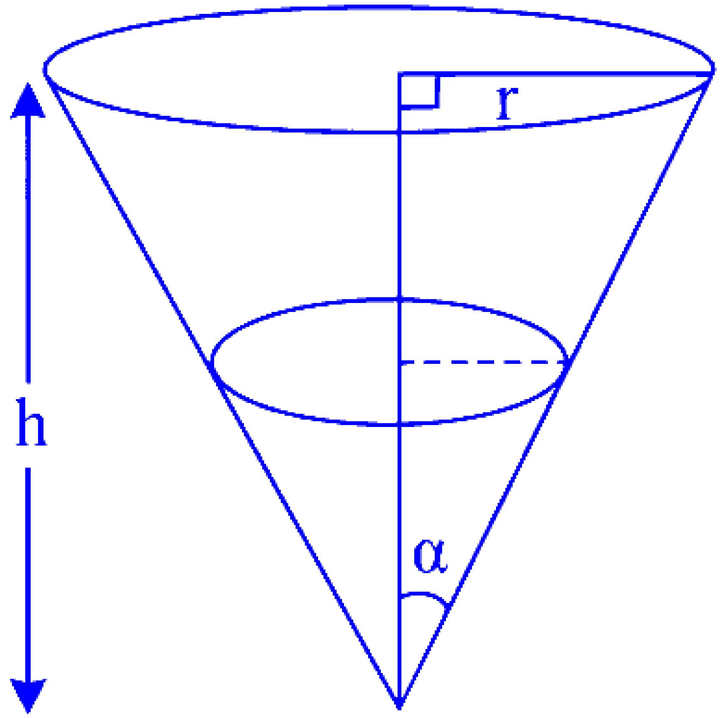
B. $\frac{1}{15\pi}$

C. $\frac{2}{\pi}$

D. $\frac{1}{10\pi}$

Answer: A

Solution:



Semi-vertical angle $= \tan^{-1}\left(\frac{1}{2}\right)$

Let $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$$\tan\alpha = \frac{1}{2}$$
$$\therefore \quad \frac{\mathrm{r}}{\mathrm{h}} = \frac{1}{2}$$
$$\mathrm{r} = \frac{\mathrm{h}}{2}$$

Given, $\frac{\mathrm{dV}}{\mathrm{dt}} = 5\ \mathrm{m}^3/\mathrm{min}.$

V = Volume of cone

Volume of cone $= \frac{1}{3}\pi \mathrm{r}^2\ \mathrm{h}$

$$\mathrm{V} = \frac{1}{3}\pi\left(\frac{\mathrm{h}}{2}\right)^2 \times \mathrm{h}$$
$$\mathrm{V} = \frac{1}{12}\pi \mathrm{h}^3$$

Differentiating w. r. t. t, we get $\frac{\mathrm{dV}}{\mathrm{dt}} = \frac{1}{12} \times \pi \times 3\ \mathrm{h}^2 \times \frac{\mathrm{dh}}{\mathrm{dt}}$

$5 = \frac{1}{4}\pi \mathrm{h}^2 \frac{\mathrm{dh}}{\mathrm{dt}}$

$\frac{\mathrm{dh}}{\mathrm{dt}} = \frac{20}{\pi \mathrm{h}^2}$

Now, $\mathrm{h} = 10$ .... [Given]

$$\therefore \quad \frac{\mathrm{dh}}{\mathrm{dt}} = \frac{20}{\pi \times (10)^2}$$
$$\frac{\mathrm{dh}}{\mathrm{dt}} = \frac{1}{5\pi}$$

$\therefore$ Rate of change of water level is $\frac{1}{5\pi}\mathrm{m/min}$.

Question 36

The differential equation of all circles which pass through the origin and whose centres lie on Y-axis is

Options:

A. $\left(x^2-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}-2xy=0$

B. $\left(x^2-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=0$

C. $\left(x^2-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}+xy=0$

D. $\left(x^2-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}-xy=0$

Answer: A

Solution:

Circle passes through origin and centre lie on Y-axis.

Let $(0, k)$ be centre and 'k' be radius

$\therefore$ Equation of circle is

$$\begin{aligned}
&(x-0)^2+(y-\mathrm{k})^2=\mathrm{k}^2\\
&x^2+y^2-2y\mathrm{k}+\mathrm{k}^2=\mathrm{k}^2\\
&x^2+y^2-2\mathrm{k}y=0\\
&x^2+y^2=2\mathrm{k}y \quad \text{.... (i)}\\
&\frac{x^2+y^2}{2y}=\mathrm{k} \quad \text{.... (ii)}
\end{aligned}$$

Differentiating equation (i) with respect to x, we get

$2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{k}\frac{\mathrm{d}y}{\mathrm{d}x}$

$2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}-2\mathrm{k}\frac{\mathrm{d}y}{\mathrm{d}x}=0$

$2x+2(y-\mathrm{k})\frac{\mathrm{d}y}{\mathrm{d}x}=0$

$2x+2\left[y-\left(\frac{x^2+y^2}{2y}\right)\right]\frac{\mathrm{d}y}{\mathrm{d}x}=0$ ...[From (ii)]

$2x+2\left[\frac{2y^2-x^2-y^2}{2y}\right]\frac{\mathrm{d}y}{\mathrm{d}x}=0$

$2x+\left(\frac{y^2-x^2}{y}\right)\frac{\mathrm{d}y}{\mathrm{d}x}=0$

$2xy+\left(y^2-x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}=0$

i.e. $\left(x^2-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}-2xy=0$

Question 37

If $x^{\mathrm{k}}+y^{\mathrm{k}}=\mathrm{a}^{\mathrm{k}}(\mathrm{a},\mathrm{k}>0)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{y}{x}\right)^{\frac{1}{3}}=0$, then k has the value

Options:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{4}$

D. $\frac{2}{7}$

Answer: B

Solution:

$x^k+y^k=a^k$

Differentiating w.r.t. x, we get

$$\mathrm{kx}^{\mathrm{k}-1}+\mathrm{k}y^{\mathrm{k}-1}\frac{\mathrm{d}y}{\mathrm{d}x}=0$$

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\mathrm{k}x^{k-1}}{\mathrm{k}y^{k-1}}$$

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-\left(\frac{x}{y}\right)^{\mathrm{k}-1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{y}{x}\right)^{1-k}=0$$

But $\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{y}{x}\right)^{\frac{1}{3}}=0$ ... [Given]

Comparing above equations, we get

$$1-\mathrm{k}=\frac{1}{3}$$

$$1-\frac{1}{3}=\mathrm{k}$$

$$\therefore \quad \mathrm{k}=\frac{2}{3}$$

Question 38

The area (in sq. units) bounded by the curve $y=x|x|$, X-axis and the lines $x=-1$ and $x=1$ is

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D. $\frac{4}{3}$

Answer: A

Solution:

$y = x|x| \quad \ldots$ [Given]
Required area

$$= \int_{-1}^{1} x|x| \mathrm{d}x$$

$$= 2\int_{0}^{1} x^2 \, \mathrm{d}x \quad \text{...[}\because \text{ Area is always positive]}$$

$$= 2 \times \left[\frac{x^3}{3}\right]_0^1$$

$$= 2 \times \left(\frac{1}{3} - 0\right) = \frac{2}{3} \text{ sq.units}$$

Question 39

The co-ordinates of the point, where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+5}{4}$ meets the plane $2x + 4y - \text{z} = 3$, are

Options:

A. $(3, -1, -1)$

B. $(3, 1, -1)$

C. $(3, -1, 1)$

D. $(-3, -1, -1)$

Answer: A

Solution:

Given line is

$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+5}{4}$

Let $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+5}{4} = \lambda$

$\therefore \quad x - 1 = 2\lambda, y - 2 = -3\lambda, z + 5 = 4\lambda$

$x = 2\lambda + 1, y = -3\lambda + 2, z = 4\lambda - 5$

$\therefore \quad 2x + 4y - z = 3$

$\Rightarrow 2(2\lambda + 1) + 4(-3\lambda + 2) - (4\lambda - 5) = 3$

$\Rightarrow 4\lambda + 2 - 12\lambda + 8 - 4\lambda + 5 = 3$

$\Rightarrow -12\lambda = 3 - 15$

$\therefore \quad x = 3, y = -1, z = -1,$

$\therefore$ Required co-ordinates are: $(3, -1, -1)$

Question 40

$\lim\limits_{x \to 0} \frac{(1-\cos 2x)\cdot\sin 5x}{x^2 \sin 3x}$ **is**

Options:

A. $\frac{10}{3}$

B. $\frac{5}{3}$

C. $\frac{5}{6}$

D. $\frac{2}{3}$

Answer: A

Solution:

$$\lim_{x \to 0} \frac{(1-\cos 2x)\sin 5x}{x^2 \sin 3x}$$
$$\lim_{x \to 0} \frac{(1-\cos 2x)\sin 5x \cdot x}{x^3 \cdot \sin 3x}$$
$$\lim_{x \to 0} \frac{2\sin^2 x}{x^2} \cdot \frac{\sin 5x}{x} \cdot \frac{x}{\sin 3x}$$
$$= 2\left(\lim_{x \to 0} \frac{\sin x}{x}\right)^2 \cdot 5\left(\lim_{x \to 0} \frac{\sin 5x}{5x}\right) \cdot \frac{1}{3}\left(\frac{1}{\lim\limits_{x \to 0} \frac{\sin 3x}{3x}}\right)$$
$$= 2 \times 1 \times 5 \times 1 \times \frac{1}{3} \times 1 = \frac{10}{3}$$

Question 41

If g is the inverse of f and $f'(x) = \frac{1}{1+x^3}$, then $g'(x)$ is

Options:

A. $\frac{1}{1+(g(x))^3}$

B. $1 + (g(x))^3$

C. $\frac{g(x)}{1+(g(x))^3}$

D. $\frac{(\mathrm{g}(x))^3}{1+(\mathrm{g}(x))^3}$

Answer: B

Solution:

$$\begin{aligned}
& \mathrm{g}(x) \text{ is inverse of function } \mathrm{f}(x) \\
& \text{ i.e., } \mathrm{g}(x)=\mathrm{f}^{-1}(x) \\
& \mathrm{f}(\mathrm{g}(x))=x \\
& \text{ differentiating w.r.t. } x, \text{ we get} \\
& \mathrm{f}'(\mathrm{g}(x)) \times \mathrm{g}'(x)=1 \\
\therefore \quad & \mathrm{g}'(x)=\frac{1}{\mathrm{f}'(\mathrm{g}(x))} \quad \text{.... (i)} \\
& \text{ Now, } \mathrm{f}'(x)=\frac{1}{1+x^3} \quad \text{....[Given]} \\
\therefore \quad & \mathrm{f}'(\mathrm{g}(x))=\frac{1}{1+(\mathrm{g}(x))^3} \quad \ldots \text{(ii)} \\
\therefore \quad & \text{ From (ii) and (ii), we get} \\
\therefore \quad & \mathrm{g}'(x)=1+(\mathrm{g}(x))^3
\end{aligned}$$

Question 42

A problem in statistics is given to three students A, B and C. Their probabilities of solving the problem are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If all of them try independently, then the probability, that problem is solved, is

Options:

A. $\frac{2}{3}$

B. $\frac{3}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: B

Solution:

$$P(A)=\frac{1}{2}$$

$$\therefore \quad P\left(A'\right)=1-\frac{1}{2}=\frac{1}{2}$$

$$P(B)=\frac{1}{3}$$

$\therefore \quad \mathrm{P}\left(\mathrm{B}'\right)=1-\frac{1}{3}=\frac{2}{3}$

$\mathrm{P}(\mathrm{C})=\frac{1}{4}$

$\therefore \quad \mathrm{P}\left(\mathrm{C}'\right)=1-\frac{1}{4}=\frac{3}{4}$

$\therefore \quad \mathrm{P}\left(\text{Problem}' \text{ is not solve }\right)$

$=\mathrm{P}\left(\mathrm{A}' \cap \mathrm{B}' \cap \mathrm{C}'\right)$

$=\mathrm{P}\left(\mathrm{A}'\right) \cdot \mathrm{P}\left(\mathrm{B}'\right) \cdot \mathrm{P}\left(\mathrm{C}'\right) \quad \cdots\left[\because \mathrm{A}', \mathrm{B}', \mathrm{C}' \text{ are independent }\right]$

$=\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$=\frac{1}{4}$

$\therefore \quad \mathrm{P}(\text{ the problem will be solve })$

$= \quad 1-\mathrm{P}(\text{ Problem is not solved })$

$= \quad 1-\frac{1}{4}$

$= \quad \frac{3}{4}$

Question 43

Let $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 3\\ 1 & -2 & 1\end{bmatrix}, B=\begin{bmatrix}6\\ 11\\ 0\end{bmatrix}$ and $X=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$, if $\mathrm{AX}=\mathrm{B}$, then the value of $2\mathrm{a}+\mathrm{b}+2\mathrm{c}$ is

Options:

A. 10

B. 8

C. 6

D. 12

Answer: A

Solution:

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$\therefore \quad a + b + c = 6 \quad .... (i)$

$b + 3c = 11 \quad .... (ii)$

$a - 2b + c = 0$

i.e., $a + c = 2b \quad .... (iii)$

From (i) and (ii), we get $b = 2$

From (ii), $c = 3$

From (i), $a = 1$

$\therefore \quad 2a + b + 2c = 2(1) + 2 + 2(3) = 10$

Question 44

$f(x) = \begin{cases} \frac{1-\cos kx}{x^2}, & \text{if } x \leq 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & \text{if } x > 0 \end{cases}$ is continuous at $x = 0$, **then the value of** k **is**

Options:

A. 4

B. 2

C. -1

D. -3

Answer: A

Solution:

$\mathrm{f}(x)$ is continuous at $x = 0$

$$\therefore \quad \text{L.H.L.} = \lim_{x \to 0} \frac{1 - \cos kx}{x^2}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 \frac{kx}{2}}{x^2}$$

$$= 2 \lim_{x \to 0} \frac{\sin^2 \frac{kx}{2}}{\frac{k^2 x^2}{4} \times 4} \cdot k^2$$

$$= \frac{1}{2} k^2 \lim_{x \to 0} \left(\frac{\sin \frac{kx}{2}}{\frac{kx}{2}} \right)^2$$

$$\therefore \quad \text{L.H.L.} = \frac{1}{2} k^2$$

$\therefore \quad \text{R.H.L.} = \lim_{x\to0} \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$

$= \lim_{x\to0} \frac{(\sqrt{x})(\sqrt{16+\sqrt{x}}+4)}{(\sqrt{16+\sqrt{x}}-4)(\sqrt{16+\sqrt{x}}+4)}$

$= \lim_{x\to0} \frac{\sqrt{x}(\sqrt{16+\sqrt{x}}+4)}{16+\sqrt{x}-16}$

$= \sqrt{16+\sqrt{0}}+4$

$\therefore \quad \text{R.H.L.} = 8$

$\mathrm{f}(x)$ is continuous at $x=0$ ....[Given]

$\therefore \quad \text{L.H.L.} = \text{R.H.L.}$

$\therefore \quad \frac{1}{2}\mathrm{k}^2 = 8$

$\therefore \quad \mathrm{k}^2 = 16$

$\therefore \quad \mathrm{k} = \pm 4$

Question 45

If D, E and F are the mid-points of the sides BC, CA and AB of triangle ABC respectively, then $\overline{\mathrm{AD}} + \frac{2}{3}\overline{\mathrm{BE}} + \frac{1}{3}\overline{\mathrm{CF}} =$

Options:

A. $\frac{1}{2}\overline{\mathrm{AB}}$

B. $\frac{1}{2}\overline{\mathrm{AC}}$

C. $\frac{1}{2}\overline{\mathrm{BC}}$

D. $\frac{2}{3}\overline{\mathrm{AC}}$

Answer: B

Solution:

Let the position vector of A, B, C, D, E, F be $\overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}, \overline{\mathrm{d}}, \overline{\mathrm{e}}, \overline{\mathrm{f}}$ respectively.

$\therefore \quad \overline{\mathrm{d}} = \frac{\overline{\mathrm{b}}+\overline{\mathrm{c}}}{2}, \overline{\mathrm{e}} = \frac{\overline{\mathrm{c}}+\overline{\mathrm{a}}}{2}, \overline{\mathrm{f}} = \frac{\overline{\mathrm{a}}+\overline{\mathrm{b}}}{2}$

Now, $\overline{\mathrm{AD}} + \frac{2}{3}\overline{\mathrm{BE}} + \frac{1}{3}\overline{\mathrm{CF}}$

$$= \overline{d} - \overline{a} + \frac{2}{3}(\overline{e} - \overline{b}) + \frac{1}{3}(\overline{f} - \overline{c})$$
$$= \frac{\overline{b} + \overline{c}}{2} - \overline{a} + \frac{2}{3}\left(\frac{\overline{c} + \overline{a}}{2} - \overline{b}\right) + \frac{1}{3}\left(\frac{\overline{a} + \overline{b}}{2} - \overline{c}\right)$$
$$= \frac{\overline{b} + \overline{c} - 2\overline{a}}{2} + \frac{\overline{c} + \overline{a} - 2\overline{b}}{3} + \frac{\overline{a} + \overline{b} - 2\overline{c}}{6}$$
$$= \frac{3\overline{c} - 3\overline{a}}{6}$$
$$= \frac{3}{6}(\overline{c} - \overline{a})$$
$$= \frac{1}{2}\overline{AC}$$

Question 46

Two cards are drawn successively with replacement from a well shuffled pack of 52 cards, then mean of number of queens is

Options:

A. $\frac{1}{13}$

B. $\frac{1}{169}$

C. $\frac{2}{13}$

D. $\frac{4}{169}$

Answer: C

Solution:

Total number of cards $= 52$

Total number of queens $= 4$

Probability of getting a queen

P(queen) $= \frac{4}{52} = \frac{1}{13}$

Probability of not getting a queen

P(non queen) $= \frac{48}{52} = \frac{12}{13}$

Let X be a random variable such that $X =$ number of queens in 2 draws

Case I: No queens are drawn $(X = 0)$

$\mathrm{P}(\mathrm{X}=0)=\mathrm{P}(\text{non queen}) \times \mathrm{P}(\text{non queen})$

$=\frac{12}{13} \times \frac{12}{13}=\frac{144}{169}$

Case II: One queen is drawn $(X=1)$

$P(X=1)=P(\text{ non queen and queen }) \text{ or } \mathrm{P}(\text{ queen and non queen})$

$$=\frac{12}{13} \times \frac{1}{13}+\frac{1}{13} \times \frac{12}{13}$$
$$=\frac{24}{169}$$

Case III: Two queens are drawn $(\mathrm{X}=2)$

$$\begin{aligned} \mathrm{P}(\mathrm{X}=2)=\mathrm{P}(\text{ queen }) \times \mathrm{P}(\text{ queen }) \\ =\frac{1}{13} \times \frac{1}{13} \\ =\frac{1}{169} \end{aligned}$$

$\therefore$ Required Mean is

$$\begin{aligned} \mathrm{E}(\mathrm{X}) & =\Sigma x \cdot \mathrm{P}(x) \\ & =0 \times \frac{144}{169}+1 \times \frac{24}{169}+2 \times \frac{1}{169} \\ & =\frac{26}{169} \\ \mathrm{E}(\mathrm{X}) & =\frac{2}{13} \end{aligned}$$

Question 47

The equation of a plane, containing the line of intersection of the planes $2x-y-4=0$ and $y+2z-4=0$ and passing through the point $(2,1,0)$, is

Options:

A. $3x-2y+z=4$

B. $3x+2y+z=4$

C. $3x-2y-z=4$

D. $3x+2y-z=-4$

Answer: C

Solution:

Equation of plane passing through the intersection of given planes is

$2x - y - 4 + \lambda(y + 2z - 4) = 0$ .... (i)

Since, the plane passes through $(2, 1, 0)$

$$2(2) - 1 - 4 + \lambda(1 + 2(0) - 4) = 0$$
$$4 - 1 - 4 - 3\lambda = 0$$
$$-1 - 3\lambda = 0$$
$$\lambda = \frac{-1}{3}$$

Substituting $\lambda = \frac{-1}{3}$ in equation (i), we get

$$2x - y - 4 - \frac{1}{3}(y + 2z - 4) = 0$$
$$6x - 3y - 12 - y - 2z + 4 = 0$$
$$6x - 4y - 2z - 8 = 0$$
$$3x - 2y - z = 4$$

Question 48

A random variable X assumes values 1, 2, 3, ..., n with equal probabilities, if $\operatorname{var}(X) = E(X)$, then n is

Options:

A. 4

B. 5

C. 7

D. 9

Answer: C

Solution:

$$\mathrm{X} = 1, 2, 3, \ldots.\mathrm{n}$$
$$\mathrm{P(X)} = \frac{1}{\mathrm{n}}$$
$$\mathrm{E(X)} = \sum_{\mathrm{i}=1}^{\mathrm{n}} x_{\mathrm{i}} \mathrm{p}_{\mathrm{i}}$$
$$= \frac{(1 + 2 + 3 + \ldots + \mathrm{n})}{\mathrm{n}}$$
$$= \frac{\mathrm{n(n+1)}}{2\mathrm{n}}$$
$$\mathrm{E(X)} = \frac{\mathrm{n}+1}{2}$$

$$\begin{aligned}
\operatorname{Var}(X) &= \sum_{i=1}^{n} x_i^2 p_i - [E(X)]^2 \\
&= \frac{1^2+2^2+3^2+\ldots+n^2}{n} - \left(\frac{n+1}{2}\right)^2 \\
&= \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2 \\
&= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2
\end{aligned}$$

$\operatorname{Var}(X) = E(X)$ .... [Given]

$$\frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n+1}{2}$$

$$\frac{2n^2+n+2n+1}{6} - \left(\frac{n^2+2n+1}{4}\right) = \frac{n+1}{2}$$

$$\frac{4n^2+6n+2-3n^2-6n-3}{12} = \frac{n+1}{2}$$

$$n^2 - 1 = 6(n+1)$$

$$n^2 - 1 = 6n + 6$$

$$n^2 - 6n - 7 = 0$$

$\therefore$ $n = -1$ or $n = 7$

But $n \neq -1$

$\therefore$ $n = 7$

Question 49

The graphical solution set for the system of inequations $x - 2y \leq 2, 5x + 2y \geq 10, 4x + 5y \leq 20, x \geq 0, y \geq 0$ is given by

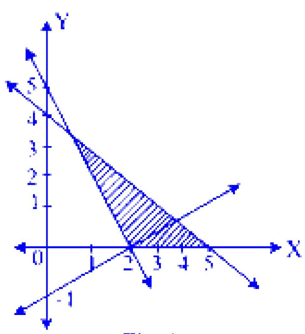


Fig. 1

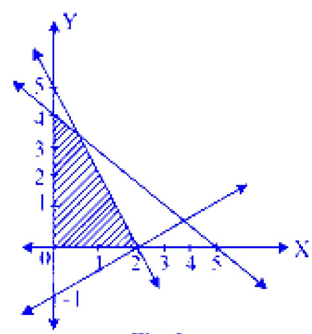


Fig. 2

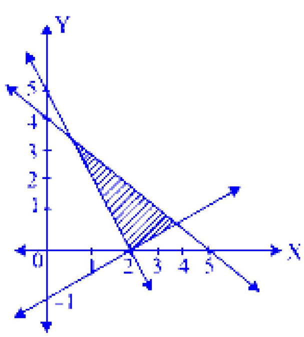


Fig. 3

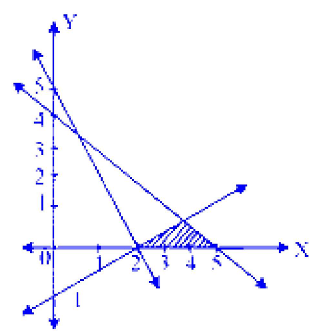


Fig. 4

Options:

A. Fig. 2

B. Fig. 4

C. Fig. 1

D. Fig. 3

Answer: D

Solution:

Feasible region lies on origin side of $x - 2y = 2, 4x + 5y = 20$ and on non-origin side of $5x + 2y = 10$, in 1^{st} quadrant.

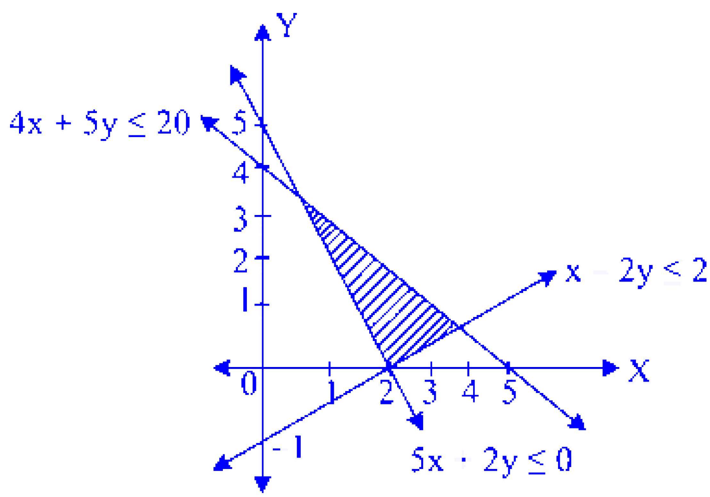


Fig. 3

Question 50

Let $\mathrm{f}(0) = -3$ and $\mathrm{f}'(x) \leq 5$ for all real values of x. The $\mathrm{f}(2)$ can have possible maximum value as

Options:

A. 10

B. 5

C. 7

D. 13

Answer: C

Solution:

Applying Lagrange's mean value theorem on interval $[0, 2]$, we get

there exist atleast one 'c' $\in (0, 2)$ such that

$$\frac{f(2)-f(0)}{2-0} = f'(c)$$

$\therefore \quad f(2) - f(0) = 2f'(c)$

$\therefore \quad f(2) = f(0) + 2f'(c)$

$\therefore \quad f(2) = -3 + 2f'(c)$

Given that $f'(x) \leq 5$ for all x

$\therefore \quad f(2) \leq -3 + 10$

$\therefore \quad f(2) \leq 7$

$\therefore$ Largest possible value of $f(2)$ is 7.

Chemistry

Question 51

Identify monodentate ligand from the following.

Options:

A. Cyanide ion

B. Ethylenediamine

C. Oxalate ion

D. Ethylendiaminetetraacetate

Answer: A

Solution:

Ligand	Type of ligand
CN^-	Monodentate

Ligand	Type of ligand
Ethylenediamine	Bidentate
Oxalate ion	Bidentate
Ethylendiaminetetraacetate	Hexadentate

Question 52

Identify linear polymer from the following.

Options:

A. High density polythene

B. Low density polythene

C. Bakelite

D. Melamine

Answer: A

Solution:

Option A, High-density polyethylene (HDPE), is the linear polymer among the options listed.

HDPE is known for its linear structure, which contributes to its high-density characteristics. This linearity is due to the lack of branching in its polymer chains, leading to a more closely packed structure and higher strength compared to low-density polyethylene (LDPE).

The other options are not linear polymers :

- Low-density polyethylene (LDPE) (Option B) has a branched structure, which results in its lower density and less rigid nature.
- Bakelite (Option C) is a thermosetting phenol-formaldehyde resin, known for its network structure.
- Melamine (Option D) is another type of thermosetting plastic that forms a cross-linked, three-dimensional network rather than a linear structure.

Question 53

What is the conductivity of $0.05 \mathrm{~M} \mathrm{BaCl}_2$ solution if its molar conductivity is $220 \Omega^{-1} \mathrm{~cm}^2 \mathrm{~mol}^{-1}$?

Options:

A. $0.011 \Omega^{-1} \mathrm{~cm}^{-1}$

B. $0.022 \Omega^{-1} \mathrm{~cm}^{-1}$

C. $0.033 \Omega^{-1} \mathrm{~cm}^{-1}$

D. $0.044 \Omega^{-1} \mathrm{~cm}^{-1}$

Answer: A

Solution:

$$
\begin{aligned}
& \Lambda=\frac{1000 \mathrm{k}}{\mathrm{c}} \\
& \mathrm{k}=\frac{\Lambda \mathrm{c}}{1000} \\
& \mathrm{k}=\frac{220 \Omega^{-1} \mathrm{~cm}^2 \mathrm{~mol}^{-1} \times 0.05 \mathrm{~mol} \mathrm{~L}^{-1}}{1000 \mathrm{~cm}^3 \mathrm{~L}^{-1}} \\
& \quad=0.011 \Omega^{-1} \mathrm{~cm}^{-1}
\end{aligned}
$$

Question 54

Which from following polymers is grouped in the category of elastomers?

Options:

A. Nylon 6,6

B. Buna-S

C. Terylene

D. Polythene

Answer: B

Solution:

Option B, Buna-S, is grouped in the category of elastomers.

Elastomers are polymers that exhibit elastic properties, meaning they can stretch significantly and return to their original shape. Buna-S, also known as Styrene-Butadiene Rubber (SBR), is a synthetic rubber that demonstrates these elastic properties, making it an elastomer.

The other options are not elastomers :

- Nylon 6,6 (Option A) is a type of polyamide, known for its strength and thermal resistance.
- Terylene (Option C) is a polyester, valued for its durability and resistance to various chemicals and environmental conditions.
- Polythene (Option D), also known as polyethylene, is a common plastic used in a wide range of products, noted for its toughness and flexibility, but it is not an elastomer.

Question 55

Which element from following exhibits diagonal relationship with beryllium?

Options:

A. B

B. Na

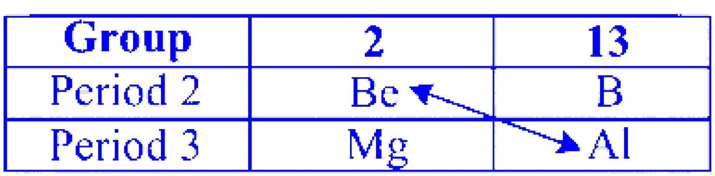
C. Mg

D. Al

Answer: D

Solution:

Diagonal relationship



Question 56

What is the stock notation of Manganese dioxide?

Options:

A. $Mn(I)O_2$

B. $Mn(II)O_2$

C. $Mn(III)O_2$

D. $Mn(IV)O_2$

Answer: D

Solution:

The oxidation state of Mn in manganese dioxide (MnO_2) is +4 . Hence, the stock notation is $Mn(IV)O_2$

Question 57

A reaction, $Ni_{(s)} + Cu^{+}_{((M))} \rightarrow Ni^{+}_{(IM)} + Cu_{(s)}$ occurs in a cell. Calculate E°_{cell} if $E^{\circ}_{Cu} = 0.337\ V$ and $E^{\circ}_{Ni} = -0.257\ V$

Options:

A. $0.594\ V$

B. $-0.594\ V$

C. $-0.08\ V$

D. $0.08\ V$

Answer: A

Solution:

The standard cell potential is given by

$$
\begin{aligned}
E^{\circ}_{cell} &= E^{\circ}_{cathode} - E^{\circ}_{anode} \\
E^{\circ}_{cell} &= E^{\circ}_{Cu} - E^{\circ}_{Ni} \\
&= (0.337\ V) - (-0.257\ V) \\
&= 0.337\ V + 0.257\ V \\
&= 0.594\ V
\end{aligned}
$$

Question 58

What volume of ammonia is formed when $10\ \mathrm{dm}^3$ dinitrogen reacts with $30\ \mathrm{dm}^3$ dihydrogen at same temperature and pressure?

Options:

A. $30\ \mathrm{dm}^3$

B. $20\ \mathrm{dm}^3$

C. $15\ \mathrm{dm}^3$

D. $10\ \mathrm{dm}^3$

Answer: B

Solution:

$\text{Nitrogen}_{(g)}$	+	$\text{Hydrogen}_{(g)}$	$\longrightarrow$	$\text{Ammonia}_{(g)}$
[1 L]		[3 L]		[2 L]
[10 dm^3]		[30 dm^3]		[20 dm^3]

Question 59

What is the number of moles of sp^2 hybrid carbon atoms present in n moles of isopentane?

Options:

A. zero

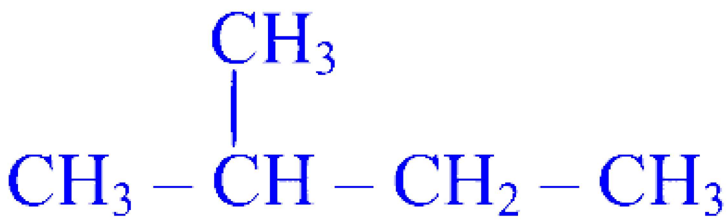
B. one

C. two

D. three

Answer: A

Solution:



Isopentane

There are no sp^2 hybridized carbon atoms in isopentane.

Question 60

Find solubility of $\mathrm{PbI_2}$ if its solubility product is 7.0×10^{-9}.

Options:

A. $1.21 \times 10^{-3}\ \mathrm{mol\ L^{-1}}$

B. $3.228 \times 10^{-3}\ \mathrm{mol\ L^{-1}}$

C. $2.831 \times 10^{-3}\ \mathrm{mol\ L^{-1}}$

D. $1.811 \times 10^{-3}\ \mathrm{mol\ L^{-1}}$

Answer: A

Solution:

For $\mathrm{PbI_2}$,

$$\mathrm{PbI_{2(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2I^{-}_{(aq)}}$$

$$x = 1, \mathrm{y} = 2$$

$$\therefore \quad \mathrm{K_{sp}} = x^{x}\mathrm{y}^{\mathrm{y}}\mathrm{S}^{x+y} = (1)^1(2)^2\,\mathrm{S}^{1+2} = 4\,\mathrm{S}^3$$

$$\therefore \quad \mathrm{S} = \sqrt[3]{\frac{\mathrm{Ksp}}{4}} = \sqrt[3]{\frac{7.0\times10^{-9}}{4}} = 1.21 \times 10^{-3}\ \mathrm{mol\ L^{-1}}$$

Question 61

Identify the product formed when vapours of 2-methylpropan-2-ol are passed over hot copper.

Options:

A. Propanone

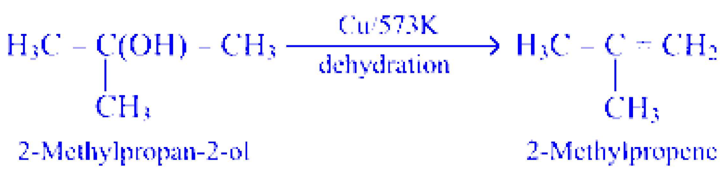
B. 2-Methylpropene

C. 2-Methylpropanoic acid

D. Propanal

Answer: B

Solution:



Question 62

Calculate the rate constant of first order reaction if the concentration of the reactant decreases by 90% in 30 minutes.

Options:

A. 7.7×10^{-2} minute $^{-1}$

B. 4.2×10^{-2} minute $^{-1}$

C. 2.1×10^{-2} minute $^{-1}$

D. 3.5×10^{-2} minute $^{-1}$

Answer: A

Solution:

Concentration decreases by 90%. Hence, 10% reactant is left after 30 minutes.

$$
\begin{aligned}
& \mathrm{k}=\frac{2.303}{\mathrm{t}} \log_{10} \frac{[\mathrm{A}]_0}{[\mathrm{A}]_{\mathrm{t}}} \\
& \mathrm{k}=\frac{2.303}{30} \log_{10} \frac{100}{10} \\
& \quad=\frac{2.303}{30 \text{ min}} \\
& \quad=7.7 \times 10^{-2} \text{ min}^{-1}
\end{aligned}
$$

Question 63

What different elements are found in baryte?

Options:

A. Ca, S, O

B. Zn, S, O

C. Ba, S, O

D. Mg, S, H, O

Answer: C

Solution:

Baryte, a mineral composed of barium sulfate ($BaSO_4$), falls under the study of s-block elements because barium is an s-block element. This chapter would typically cover the properties, compounds, and uses of s-block elements, including barium, and how they form various compounds like barium sulfate.

Question 64

Identify the product 'B' in the following reaction.

$$\textbf{Cumene} \xrightarrow[\Delta]{KMnO_4\ .\ KOH} \textbf{A} \xrightarrow{H_3O^+} \textbf{B}$$

Options:

A. Phenol

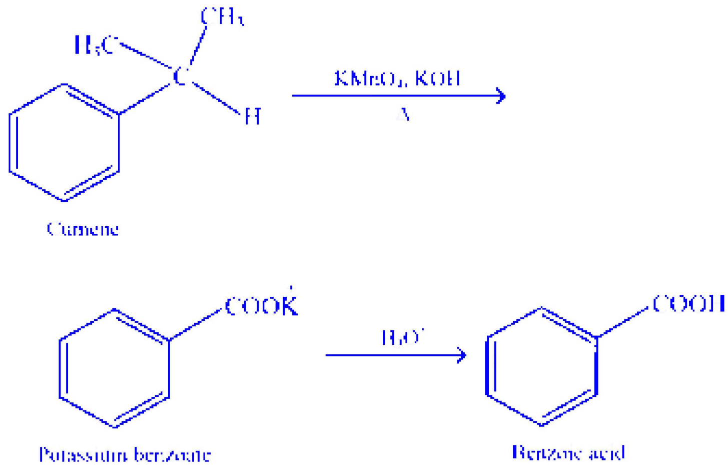
B. Potassium benzoate

C. Benzoic acid

D. Aniline

Answer: C

Solution:



Note: The entire alkyl chain on ring, regardless of its length, is oxidized to a carboxyl group.

Question 65

Which of the following on reaction with ammoniacal silver nitrate forms silver precipitate?

Options:

A. Ethanol

B. Ethanal

C. Ethoxyethane

D. Ethanoic acid

Answer: B

Solution:

Tollens' reagent oxidises aldehyde to the corresponding carboxylate anion. Silver gets precipitated as greyish black precipitate.

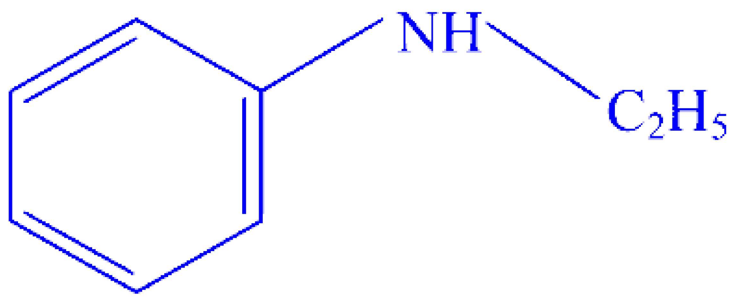
Tollens' reagent is a mild oxidizing agent. Thus, simple hydrocarbons, ethers, ketones and alcohols do not get oxidized by Tollens' reagent.

Question 66

Identify an aromatic, mixed, 3° amine from following.

Options:

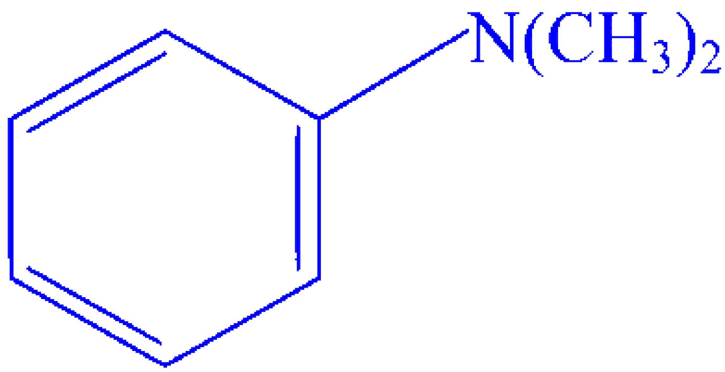
A.



B. $(C_2H_5)_3N$

C. $(CH_3)_3C - NH_2$

D.



Answer: D

Solution:

Tertiary amine is formed by removing 3 hydrogen atoms from NH_3. Hence, tertiary amine should not contain any $N - H$ bond. Only option (B) satisfies this condition.

Question 67

For $NaCl_{(s)}$ enthalpy of solution is 4 kJ mol^{-1} and lattice enthalpy is 790 kJ mol^{-1}. What is hydration enthalpy of $NaCl$?

Options:

A. 786 kJ

B. 794 kJ

C. $-786 \mathrm{~kJ}$

D. $-794 \mathrm{~kJ}$

Answer: C

Solution:

$$
\begin{aligned}
& \Delta_{\text {soln }} \mathrm{H}=\Delta_{\mathrm{L}} \mathrm{H}+\Delta_{\text {hyd }} \mathrm{H} \\
\therefore \quad & \Delta_{\text {hyd }} \mathrm{H}=\Delta_{\text {soln }} \mathrm{H}-\Delta_{\mathrm{L}} \mathrm{H} \\
& \quad=4 \mathrm{~kJ} \mathrm{~mol}^{-1}-790 \mathrm{~kJ} \mathrm{~mol}^{-1} \\
& \quad=-786 \mathrm{~kJ} \mathrm{~mol}^{-1}
\end{aligned}
$$

Question 68

Which from following statements regarding transition elements is NOT CORRECT?

Options:

A. These exhibit properties between 's' and 'p' block elements.

B. These form cations with incomplete d-subshell.

C. 5d series consists of all elements from lanthanum to mercury of periodic table.

D. These are arranged in four different 'd' series.

Answer: C

Solution:

The statement that is NOT CORRECT regarding transition elements is :

Option C : "5d series consists of all elements from lanthanum to mercury of periodic table."

This statement is incorrect because the 5d series of the transition elements does not start with lanthanum. The first element of the 5d series is hafnium (Hf), not lanthanum (La). Lanthanum is actually the first element in the lanthanide series.

Question 69

A solution of $8 \mathrm{~g}$ of certain organic compound in $2 \mathrm{dm}^3$ water develops osmotic pressure $0.6 \mathrm{~atm}$ at $300 \mathrm{~K}$. Calculate the molar mass of compound.

[R = 0.082 atm dm^3 K^{-1} mol^{-1}]

Options:

A. $148 \mathrm{~g} \mathrm{~mol}^{-1}$

B. $164 \mathrm{~g} \mathrm{~mol}^{-1}$

C. $172 \mathrm{~g} \mathrm{~mol}^{-1}$

D. $180 \mathrm{~g} \mathrm{~mol}^{-1}$

Answer: B

Solution:

$$
\begin{aligned}
& \pi=\frac{\mathrm{W}_2 \mathrm{RT}}{\mathrm{M}_2 \mathrm{~V}} \\
\therefore \quad & \mathrm{M}_2=\frac{\mathrm{W}_2 \mathrm{RT}}{\pi \mathrm{V}} \\
\therefore \quad & \mathrm{M}_2=\frac{8 \mathrm{~g} \times 0.082 \mathrm{dm}^3 \mathrm{~atm} \mathrm{~K}^{-1} \mathrm{~mol}^{-1} \times 300 \mathrm{~K}}{0.6 \mathrm{~atm} \times 2 \mathrm{dm}^3} \\
& =164 \mathrm{~g} \mathrm{~mol}^{-1}
\end{aligned}
$$

Question 70

What is the value of $\Delta H-\Delta U$ for the following reaction?

$2 \mathrm{C}_{(\mathrm{s})}+3 \mathrm{H}_{2(\mathrm{~g})} \rightarrow \mathrm{C}_2 \mathrm{H}_{6(\mathrm{~g})}$

Options:

A. 4RT

B. $-5 \mathrm{RT}$

C. RT

D. $-2 \mathrm{RT}$

Answer: D

Solution:

$\Delta n_g =$ (moles of product gases) $-$ (moles of reactant gases)

$\Delta n_g = 1 - 3 = -2$ mol

Now, using formula,

$\Delta H = \Delta U + \Delta n_g RT$

$\therefore \quad \Delta H = \Delta U - 2RT$

$\therefore \quad \Delta H - \Delta U = -2RT$

Question 71

Which from the following compound solutions in water of equal concentration has electrical conductivity nearly same as distilled water?

Options:

A. Urea

B. Sodium chloride

C. Sodium hydroxide

D. Acetic acid

Answer: A

Solution:

Urea is nonelectrolyte and hence, it has electrical conductivity nearly same as distilled water. Sodium chloride and sodium hydroxide are strong electrolytes while acetic acid is weak electrolyte.

Question 72

What is the pH of a solution containing $2.2 \times 10^{-6}\mathrm{M}$ hydrogen ions?

Options:

A. 6.34

B. 5.66

C. 4.34

D. 3.80

Answer: B

Solution:

$$\begin{aligned} \mathrm{pH} = -\log_{10}\left[\mathrm{H_3O^+}\right] &= -\log_{10}\left[2.2 \times 10^{-6}\right] \\ &= 6 - \log_{10}(2.2) = 6 - 0.34 \\ &= 5.66 \end{aligned}$$

Question 73

Which of the following statements is NOT true about Rutherford atomic model?

Options:

A. Each atom consists of massive, +vely charged centre.

B. The electrons are revolving continuously around the nucleus.

C. This model does not describe the distribution of electrons around the nucleus.

D. This model describes the energies of electrons.

Answer: D

Solution:

Bohr's atomic model describes the energies of electrons.

Question 74

What is IUPAC name of propylene glycerol?

Options:

A. Propane-1,2-diol

B. Propane-1,3-diol

C. Propane-1,2,3-triol

D. Propene-1,2,3-triol

Answer: C

Solution:

Common Name	Structural formula	IUPAC Name
Propylene glycerol	$\underset{\mathrm{OH}}{\underset{\vert}{H_2C}} - \underset{\mathrm{OH}}{\underset{\vert}{CH}} - \underset{\mathrm{OH}}{\underset{\vert}{CH_2}}$	Propane-1,2,3-triol

----

Question 75

Identify the CORRECT decreasing order of melting point of cluster of sodium atoms depending on size.

Options:

A. Cluster of 10^3 atoms > cluster of 10^4 atoms > Bulk sodium

B. Bulk sodium > cluster of 10^4 atoms > Cluster of 10^3 atoms

C. Cluster of 10^4 atoms > Cluster of 10^3 atoms > Bulk sodium

D. Bulk sodium > cluster of 10^3 atoms > Cluster of 10^4 atoms

Answer: B

Solution:

Sodium clusters (Na_n) of 1000 atoms melts at 288 K while cluster of 10,000 atoms melts at 303 K and bulk sodium melts at 371 K.

Question 76

Which from the following coordinate complexes contains anionic and neutral ligands in it?

Options:

A. Potassium trioxalatoaluminate(III)

B. Hexacyanoferrate (II)

C. Pentaamminecarbonatocobalt (III) chloride

D. Tetraamminecopper(II) ion

Answer: C

Solution:

Pentaamminecarbonatocobalt(III) chloride: $[Co(NH_3)_5(CO_3)]Cl$

It contains anionic ligand $\left(CO_3^{2-}\right)$ and neutral ligand (NH_3).

NH_3 (named as ammine) is a neutral ligand. Both (C) and (D) contain NH_3 ligand. However, (D) contains only NH_3. Hence, option (C) should be valid.

Question 77

Which among the following is allylic halide?

Options:

A. $CH_3 - CH_2 - CH_2 - X$

B. $C_6H_5 - CH_2 - X$

C. $CH_2 = CH - CH_2X$

D. $CH_3 - CH = CH - X$

Answer: C

Solution:

In allylic halides, halogen atom is bonded to a sp^3 hybridized carbon atom next to a carboncarbon double bond $(CH_2 = CH - CH_2 - X)$.

Question 78

Which of the following compounds has the highest boiling point?

Options:

A. Chloromethane

B. Fluoromethane

C. Iodomethane

D. Bromomethane

Answer: C

Solution:

Molecules with higher molecular mass generally have higher boiling points. So, for given alkyl group (here $-CH_3$) boiling point decreases as, $RI > RBr > RCl > RF$.

Question 79

A solution of nonvolatile solute is obtained by dissolving 1 g in 100 g solvent, decreases its freezing point by 0.3 K. Calculate cryoscopic constant of solvent if molar mass of solute is 60 g mol^{-1}.

Options:

A. $1.0 \text{ K kg mol}^{-1}$

B. $1.4 \text{ K kg mol}^{-1}$

C. $2.4 \text{ K kg mol}^{-1}$

D. $1.8 \text{ K kg mol}^{-1}$

Answer: D

Solution:

$$\begin{aligned} \Delta T_f &= \frac{1000 \, K_f W_2}{M_2 \, W_1} \\ \therefore \quad K_f &= \frac{\Delta T_f M_2 \, W_1}{1000 \, W_2} \\ &= \frac{0.3 \text{ K} \times 60 \text{ g mol}^{-1} \times 100 \text{ g}}{1000 \times 1 \text{ g}} \\ &= 1.8 \text{ K kg mol}^{-1} \end{aligned}$$

Question 80

What is the IUPAC name of allylamine?

Options:

A. Propan-1-amine

B. Prop-2-en-1-amine

C. Prop-1-en-2-amine

D. Propan-2-amine

Answer: B

Solution:

$CH_2 = CH - CH_2 - NH_2$
Prop-2-en-1-amine
(Allylamine)

Question 81

What is the wave number of photon emitted during transition from orbit, $\mathrm{n} = 4$ to $\mathrm{n} = 2$ in hydrogen atom $\left[R_H = 109677\ \mathrm{cm}^{-1}\right]$

Options:

A. $20564.44\ \mathrm{cm}^{-1}$

B. $23032.17\ \mathrm{cm}^{-1}$

C. $15354.78\ \mathrm{cm}^{-1}$

D. $25225.7\ \mathrm{cm}^{-1}$

Answer: A

Solution:

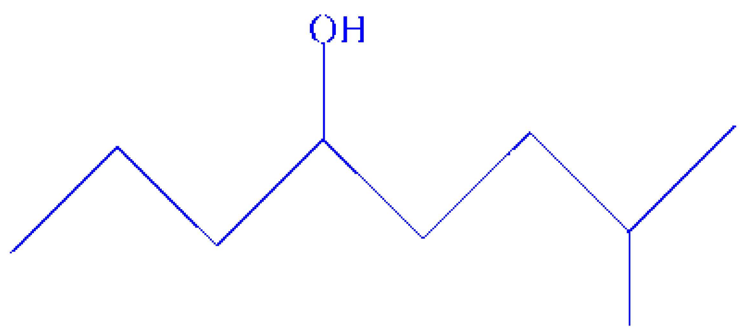
$$\bar{v} = 109677\left[\frac{1}{2^2} - \frac{1}{4^2}\right]\text{cm}^{-1}$$
$$= 109677\left[\frac{1}{4} - \frac{1}{16}\right]\text{cm}^{-1}$$
$$= 109677\left[\frac{12}{64}\right]\text{cm}^{-1}$$
$$= 20564.44\ \text{cm}^{-1}$$

Question 82

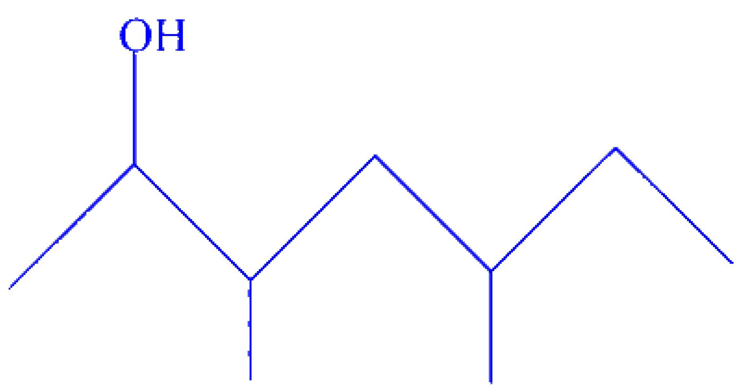
Which from following is a CORRECT bond line formula of $HO(CH_2)_3CH\,(CH_3)CH(CH_3)_2$

Options:

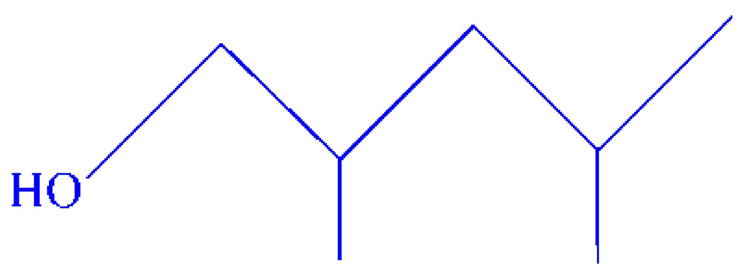
A.



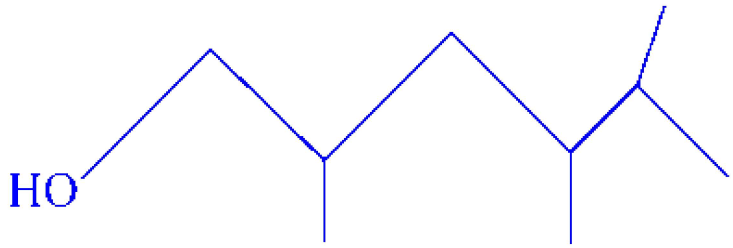
B.



C.



D.



Answer: D

Question 83

Identify the carbon atoms of α-glucose and β-fructose forming glycosidic linkage in sucrose.

Options:

A. C - 1 of β-fructose and C - 2 of α-glucose

B. C-1 of α-glucose and $\mathrm{C}-2$ of β-fructose

C. $\mathrm{C}-1$ of α-glucose and $\mathrm{C}-1$ of β-fructose

D. $\mathrm{C}-2$ of α-glucose and C-2 of β-fructose

Answer: B

Solution:

Structure of sucrose contains glycosidic linkage between $\mathrm{C}-1$ of α-glucose and $\mathrm{C}-2$ of β-fructose. Aldehyde and ketone groups of both the monosaccharide units are involved in formation of the glycosidic bond.

Question 84

Calculate the molar mass of an element having density $21 \mathrm{~g} \mathrm{~cm}^{-3}$ that forms fcc unit cell $\left[\mathrm{a}^3 \cdot \mathrm{N}_{\mathrm{A}}=36 \mathrm{~cm}^3 \mathrm{~mol}^{-1}\right]$

Options:

A. $292.00 \mathrm{~g} \mathrm{~mol}^{-1}$

B. $189.00 \text{ g mol}^{-1}$

C. $140.00 \text{ g mol}^{-1}$

D. $108.00 \text{ g mol}^{-1}$

Answer: B

Solution:

$$\begin{aligned} & \text{Density } (\rho) = \frac{\text{Mn}}{\text{a}^3 \text{ N}_\text{A}} \\ \therefore \quad & 21 \text{ g cm}^{-3} = \frac{\text{M} \times 4}{36 \text{ cm}^3 \text{ mol}^{-1}} \\ \therefore \quad & M = \frac{21 \text{ g cm}^{-3} \times 36 \text{ cm}^3 \text{ mol}^{-1}}{4} \\ & = 189.00 \text{ g mol}^{-1} \end{aligned}$$

Question 85

Which of the following is the SI unit of coefficient of viscosity?

Options:

A. $\text{N s}^{-1} \text{ m}^{-2}$

B. N s m^{-2}

C. N s m^2

D. $\text{N s}^{-1} \text{ m}^{-1}$

Answer: B

Question 86

Which from following metal has ccp crystal structure?

Options:

A. Cu

B. Zn

C. Mg

D. Po

Answer: A

Solution:

Among the given options, Copper (Cu) has a cubic close-packed (ccp) crystal structure.

In a ccp structure, atoms are arranged in a specific manner that allows them to occupy the maximum available space in the crystal lattice. Copper, represented here as Option A, is well-known for having this type of arrangement.

The other metals listed have different crystal structures :

- Zinc (Zn) has a hexagonal close-packed (hcp) structure.
- Magnesium (Mg) also crystallizes in an hcp structure.
- Polonium (Po) has a simple cubic structure at room temperature, but under different conditions, it can exhibit other structures.

Question 87

Identify the reagent 'A' used in the following conversion.

Ethyl bromide $\xrightarrow{\text{A}}$ Ethylpropanoate

Options:

A. Sodium propoxide

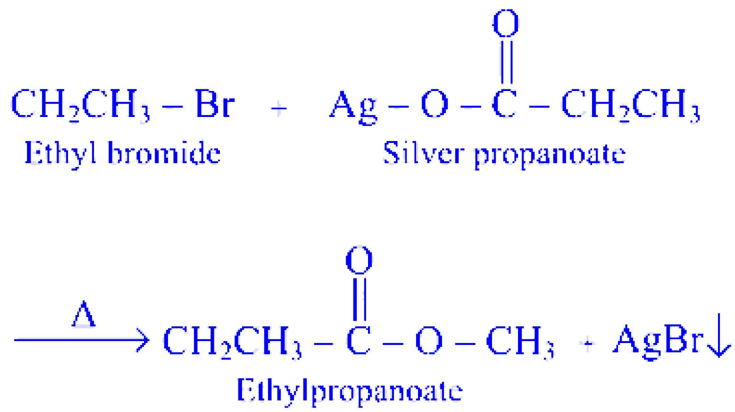
B. Ethoxy propane

C. Silver propanoate

D. Silver ethanoate

Answer: C

Solution:



Question 88

What is the concentration of $[H_3O^+]$ ion in $\mathrm{mol\ L^{-1}}$ of 0.001 M acetic acid $(\alpha = 0.134)$?

Options:

A. 1.34×10^{-4}

B. 1.54×10^{-4}

C. 1.80×10^{-4}

D. 1.70×10^{-4}

Answer: A

Solution:

$$[H_3O^+] = \alpha \times c = 0.134 \times 0.001$$
$$= 1.34 \times 10^{-4}\ \mathrm{mol\ dm^{-3}}$$

Question 89

Which group elements from following are called as chalcogens?

Options:

A. group 13

B. group 15

C. group 16

D. group 17

Answer: C

Solution:

Chalcogens are the elements in group 16 of the periodic table. This group includes oxygen (O), sulfur (S), selenium (Se), tellurium (Te), and polonium (Po). Therefore, the correct answer is :

Option C : group 16

Question 90

What is the number of electrons around sulfur in H_2SO_4 molecule?

Options:

A. 4

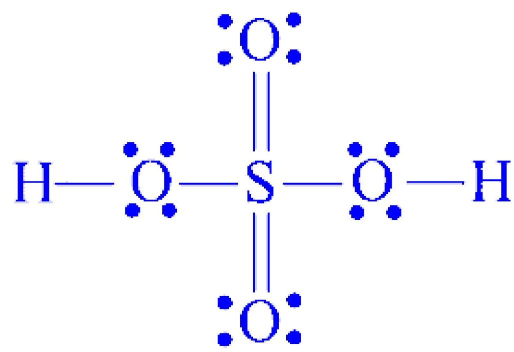
B. 6

C. 10

D. 12

Answer: D

Solution:



H_2SO_4;

12 electrons around sulphur

Question 91

An ideal gas expands by $1.5\ \mathrm{L}$ against a constant external pressure of $2\ \mathrm{atm}$ at $298\ \mathrm{K}$. Calculate the work done?

Options:

A. $-75\ \mathrm{J}$

B. $-303.9\ \mathrm{J}$

C. $13.3\ \mathrm{J}$

D. $-30\ \mathrm{J}$

Answer: B

Solution:

$$\begin{aligned} W &= -P_{\text{ext}}\ \ \Delta V \\ &= -2\ \text{atm} \times (1.5\ \text{L}) \\ &= -3\ \text{atm L} \times 1.01325 = -3.0398\ \text{dm}^3\text{bar} \end{aligned}$$

Now, $1\ \text{dm}^3\text{bar} = 100\ \text{J}$

Hence, $-3.0398\ \text{dm}^3\ \text{bar} \times \frac{100\ \text{J}}{1\ \text{dm}^3\ \text{bar}} = -303.98\ \text{J} \cong -303.9\ \text{J}$

Question 92

The rate law for the reaction $\mathrm{A}+\mathrm{B} \rightarrow$ product is given by rate $= k[A][B]$ Calculate $[A]$ if rate of reaction and rate constant are $0.25 \mathrm{~moldm}^{-3} \mathrm{~s}^{-1}$ and $6.25 \mathrm{~mol}^{-1} \mathrm{dm}^{3} \mathrm{~s}^{-1}$ respectively $\left[[\mathrm{B}]=0.25 \mathrm{~moldm}^{-3}\right]$

Options:

A. $0.22 \mathrm{~mol} \mathrm{~dm}^{3}$

B. $0.16 \mathrm{~mol} \mathrm{~dm}^{3}$

C. $0.30 \mathrm{~mol} \mathrm{~dm}^{3}$

D. $0.25 \mathrm{~mol} \mathrm{~dm}^{3}$

Answer: B

Solution:

$$
\begin{aligned}
& \text{rate } = \mathrm{k}[\mathrm{A}][\mathrm{B}] \\
\therefore \quad & [\mathrm{A}]=\frac{\text{rate}}{\mathrm{k}[\mathrm{B}]}=\frac{0.25 \mathrm{~moldm}^{-3} \mathrm{~s}^{-1}}{6.25 \mathrm{~mol}^{-1} \mathrm{dm}^{3} \cdot \mathrm{s}^{-1} \times 0.25 \mathrm{~moldm}^{-3}} \\
& \qquad =0.16 \mathrm{~mol} \mathrm{~dm}^{3}
\end{aligned}
$$

Question 93

Which among the following has the lowest boiling point?

Options:

A. Butyric acid

B. Valeric acid

C. Acetic acid

D. Formic acid

Answer: D

Solution:

Boiling point increases as the molar mass increases. Formic has the lowest molar mass among given and hence, it has the lowest boiling point.

Question 94

Find the average rate of formation $O_{2(g)}$ in the following reaction.

$$2NO_{2(g)} \rightarrow 2NO_{(g)} + O_{2(g)}$$
$$\left[-\frac{\Delta[NO]}{\Delta t}\right] = x \text{ moldm}^{-3}\text{ s}^{-1}$$

Options:

A. $\frac{x}{2}$ mol dm^{-3} s^{-1}

B. x mol dm^{-3} s^{-1}

C. $2x$ mol dm^{-3} s^{-1}

D. $4x$ mol dm^{-3} s^{-1}

Answer: A

Solution:

Average rate of reaction

$= -\frac{1}{2}\frac{\Delta[NO_2]}{\Delta t} = \frac{1}{2}\frac{\Delta[NO_2]}{\Delta t} = \frac{\Delta[O_2]}{\Delta t}$

Rate of formation of $O_2 = \frac{\Delta[O_2]}{\Delta t}$

$$= \frac{1}{2}\frac{-\Delta[NO_2]}{\Delta t}$$
$$= \frac{1}{2} \times x = \frac{x}{2} \text{ mol dm}^{-3}\text{ s}^{-1}$$

Question 95

Which among the following reactions occurs by breaking of $C-O$ bond in alcohol?

Options:

A. Reaction with propionic acid.

B. Reaction with acetic anhydride

C. Reaction with phosphorus trichloride

D. Reaction with acetyl chloride

Answer: C

Solution:

$3R - OH + PCl_3 \longrightarrow 3R - Cl + H_3PO_3$

Question 96

Which among the following gases exhibits very low solubility in water at room temperature?

Options:

A. O_2

B. CO_2

C. NH_3

D. HCl

Answer: A

Solution:

Option A, O_2 (Oxygen), is the gas that exhibits very low solubility in water at room temperature among the options provided.

Oxygen is a non-polar molecule and has a very low solubility in water, which is a polar solvent. This low solubility is due to the lack of strong interactions between the oxygen molecules and the water molecules.

Regarding the other options :

- CO_2 (Carbon Dioxide) is more soluble than oxygen due to its ability to react with water to form carbonic acid.
- NH_3 (Ammonia) is highly soluble in water and forms ammonium hydroxide.
- HCl (Hydrochloric Acid) is extremely soluble in water and dissociates completely to form hydrochloric acid.

Question 97

Identify the trisaccharide from following.

Options:

A. Maltose

B. Lactose

C. Raffinose

D. Stachyose

Answer: C

Solution:

Maltose, lactose - Disaccharides

Raffinose - Trisaccharides

Stachyose - Tetrasaccharides

Question 98

Identify the element from following having six unpaired electrons in observed electronic configuration?

Options:

A. Cu

B. Zn

C. Cr

D. Ti

Answer: C

Solution:

Chromium (Cr):

Expected electronic configuration: $[Ar]3\,d^4 4\,s^2$

Observed electronic configuration: $[Ar]3\,d^5 4\,s^1$

Hence, number of unpaired electrons in observed electronic configuration is 6.

Question 99

Find the radius of metal atom in simple cubic unit cell having edge length 334.7 pm?

Options:

A. 167.35 pm

B. 334.70 pm

C. 144.93 pm

D. 118.32 pm

Answer: A

Solution:

For simple cubic unit cell $r = \frac{a}{2} = \frac{334.7pm}{2} = 167.35$ pm

For simple cubic unit cell, radius is half of the edge length.

Question 100

Which of the following colloids is NOT a gel?

Options:

A. Cheese

B. Milk

C. Butter

D. Jellies

Answer: B

Solution:

Milk is an emulsion.

Physics

Question 101

Light of wavelength ',λ' is incident on a slit of width 'd'. The resulting diffraction pattern is observed on a screen at a distance 'D'. The linear width of the principal maximum is then equal to the width of the slit if D equals

Options:

A. $\frac{d}{\lambda}$

B. $\frac{\mathrm{d}^2}{2\lambda}$

C. $\frac{2\lambda}{\mathrm{d}}$

D. $\frac{2\lambda^2}{d}$

Answer: B

Solution:

In diffraction of light by single slit, the width of central maximum is given as

$\mathrm{W_c} = \frac{2\lambda \mathrm{D}}{\mathrm{d}}$

Given: $\mathrm{W_c} = \mathrm{d}$

$$\therefore \quad d = \frac{2\lambda D}{d}$$

$$\Rightarrow D = \frac{d^2}{2\lambda}$$

Question 102

Two S.H.Ms. are represented by equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos(100\pi t)$ The phase difference between the speeds of the two particles is

Options:

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{6}$

C. $+\frac{\pi}{6}$

D. $-\frac{\pi}{3}$

Answer: B

Solution:

Given: $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and

$y_2 = 0.1 \cos(100\pi t)$

$$\begin{aligned} \therefore \quad v_1 &= \frac{dy_1}{dt} = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right) \\ v_2 &= \frac{dy_2}{dt} = -0.1 \times 100\pi \sin(100\pi t) \\ &= (0.1 \times 100\pi) \cos\left(100\pi t + \frac{\pi}{2}\right) \end{aligned}$$

Phase difference of velocity of first particle with respect to the velocity of second particle at $t = 0$ is

$\therefore \quad \Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{-\pi}{6}$

Question 103

A film of soap solution is formed between two straight parallel wires of length 10 cm each separated by 0.5 cm. If their separation is increased by 1 mm while still maintaining their parallelism. How much work will have to be done?

(surface tension of solution $= 65 \times 10^{-2} \text{ N/m}$)

Options:

A. $7.22 \times 10^{-6} \text{ J}$

B. 13.0×10^{-5} J

C. 2.88×10^{-5} J

D. 5.76×10^{-5} J

Answer: B

Solution:

The increase in surface area of the film is,

$$\Delta A = A_2 - A_1$$
$$A_1 = 2 \times l \times b = 2 \times 10 \times 10^{-2} \times 0.5 \times 10^{-2}$$
$$A_2 = 2 \times l \times (b+1) = 2 \times 10 \times 10^{-2} \times (0.5 + 0.1) \times 10^{-2}$$
$$\begin{aligned} A_2 - A_1 = & \left[2 \times 10 \times 10^{-2} \times (0.5+0.1) \times 10^{-2}\right] \\ & - \left[2 \times 10 \times 10^{-2} \times 0.5 \times 10^{-2}\right] \\ & = 2 \times 10^{-4}\ \mathrm{m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \quad \text{Work done } & = \text{ Increase in surface energy} \\ & = \mathrm{TdA} \\ & = \left(65 \times 10^{-2}\right) \times \left(2 \times 10^{-4}\right) \\ & = 1.3 \times 10^{-4} \\ & = 13 \times 10^{-5}\ \mathrm{J} \end{aligned}$$

A factor of 2 appears in the calculation of area of the film because the soap film has two surfaces which are in the contact with the wire.

Question 104

An ideal gas in a container of volume 500 c.c. is at a pressure of $2 \times 10^{+5}\ \mathrm{N/m^2}$. The average kinetic energy of each molecule is 6×10^{-21} J. The number of gas molecules in the container is

Options:

A. 5×10^{25}

B. 5×10^{23}

C. 25×10^{23}

D. 2.5×10^{22}

Answer: D

Solution:

$\mathrm{PV}=\mathrm{Nk_B T}$

and, K.E./molecule $=\frac{3}{2}\mathrm{k_B T}=\frac{3}{2}\frac{\mathrm{PV}}{\mathrm{N}}$

$$\therefore \quad \mathrm{N}=\frac{3}{2}\times\frac{\mathrm{PV}}{(\text{ K.E./molecule })}$$
$$=\frac{3}{2}\times\frac{2\times10^{5}\times500\times10^{-6}}{6\times10^{-21}}=2.5\times10^{22}$$

Question 105

A gas at N.T.P. is suddenly compressed to onefourth of its original volume. If $\gamma=1.5$, then the final pressure is

Options:

A. 4 times

B. 1.5 times

C. 8 times

D. $\frac{1}{4}$ times

Answer: C

Solution:

As the change is sudden, the process is adiabatic

$\therefore \quad \mathrm{P_1 V_1^{\gamma}=P_2 V_2^{\gamma}}$

$\therefore \quad \frac{\mathrm{P_2}}{\mathrm{P_1}}=\left[\frac{\mathrm{V_1}}{\mathrm{V_2}}\right]^{\gamma}=\left[\frac{4}{1}\right]^{3/2}=\frac{8}{1}$

Question 106

A particle of mass 'm' moves along a circle of radius 'r' with constant tangential acceleration. If K.E. of the particle is 'E' by the end of third

revolution after beginning of the motion, then magnitude of tangential acceleration is

Options:

A. $\frac{\mathrm{E}}{2\pi\mathrm{rm}}$

B. $\frac{\mathrm{E}}{6\pi\mathrm{rm}}$

C. $\frac{\mathrm{E}}{8\pi\mathrm{rm}}$

D. $\frac{\mathrm{E}}{4\pi\mathrm{rm}}$

Answer: B

Solution:

Using 3^{rd} equation of motion,

$$v^2 = u^2 + 2as$$

$$\therefore \quad v^2 = 2a_t s \quad \text{.... (For u = 0)}$$

$$\therefore \quad a_t = \frac{v^2}{2s}$$

By the end of 3^{rd} revolution, distance covered, $\mathrm{s} = 3(2\pi\mathrm{r}) = 6\pi\mathrm{r}$

$\therefore \quad a_{\mathrm{t}} = \frac{\mathrm{v}^2}{2\times6\pi\mathrm{r}}$ .... (i)

Also, $\frac{1}{2}\mathrm{mv}^2 = \mathrm{E}$

$\therefore \quad v^2 = \frac{2\mathrm{E}}{\mathrm{m}}$ ..... (ii)

Substituting equation (ii) in equation (i),

$a_{\mathrm{t}} = \frac{2\mathrm{E}}{2\times6\pi\mathrm{r}\times\mathrm{m}} = \frac{\mathrm{E}}{6\pi\mathrm{rm}}$

Question 107

An e.m.f. $E = 4\cos(1000t)$ volt is applied to an LR circuit of inductance $3\ \mathrm{mH}$ and resistance $4\ \Omega$. The maximum current in the circuit is

Options:

A. $\frac{4}{\sqrt{7}}$ A

B. 1.0 A

C. $\frac{4}{7}$ A

D. 0.8 A

Answer: D

Solution:

For LR circuit,

$Z=\sqrt{R^2+(\omega L)^2}$

Comparing equation $E=4\cos(1000t)$ with standard equation, $E=E_0\cos\omega t$, we get $\omega=1000$ units and $E_0=4$ V

$$\therefore \quad Z=\sqrt{16+\left(1000\times 3^2\times 10^{-6}\right)}$$
$$Z=\sqrt{16+9}=5\Omega$$
$$\therefore \quad I=\frac{E_0}{Z}=\frac{4}{5}=0.8A$$

Question 108

In the reverse biasing of a p-n junction diode :

Options:

A. the width of the depletion layer decreases.

B. the width of the depletion layer increases.

C. the number of minority charge carriers increase.

D. the number of majority charge carriers increase.

Answer: B

Solution:

In a p-n junction diode :

- **Reverse Biasing** : This occurs when the p-type material is connected to the negative terminal of the battery and the n-type material to the positive terminal. In reverse bias, the built-in potential barrier increases, which tends to widen the depletion region.

Given this, let's evaluate the options :

Option A : The width of the depletion layer decreases.

- This is incorrect. In reverse bias, the width of the depletion layer increases.

Option B : The width of the depletion layer increases.

- This is correct. In reverse bias, the depletion layer widens as the potential barrier across the junction increases.

Option C : The number of minority charge carriers increase.

- This is not necessarily true. While the flow of minority carriers constitutes the reverse current, reverse bias does not increase their number.

Option D : The number of majority charge carriers increase.

- This is incorrect. Reverse biasing does not increase the number of majority charge carriers.

The correct answer is Option B : the width of the depletion layer increases.

Question 109

When a charge of $3 \mathrm{~C}$ is placed in uniform electric field, it experiences a force of $3000 \mathrm{~N}$. Within this field, potential difference between two points separated by a distance of $1 \mathrm{~cm}$ is

Options:

A. 10 V

B. 90 V

C. 1000 V

D. 3000 V

Answer: A

Solution:

Electric force, $F=q E$ and potential difference, $\mathrm{V}=\mathrm{Ed}$

$\therefore \quad \mathrm{V}=\frac{\mathrm{Fd}}{\mathrm{q}}=\frac{3000 \times 10^{-2}}{3}=10 \mathrm{~V}$

[Note: Framing of question is modified to arrive at the correct answer.]

Question 110

A soap bubble of radius 'R' is blown. After heating a solution, a second bubble of radius '$2 \mathrm{R}$' is blown. The work required to blow the 2^{nd} bubble

in comparison to that required for the 1^{st} bubble is

Options:

A. exactly double.

B. slightly more than 4 times.

C. slightly less than 4 times.

D. slightly less than double.

Answer: C

Solution:

$$\begin{aligned} & W_1 = 8\pi R^2\ T_1 \\ & W_2 = 8\pi(2R)^2\ T_2 = 32\pi R^2\ T_2 \\ \therefore\quad & \frac{W_1}{W_2} = \frac{T_1}{4\ T_2} \\ & \text{When } T_1 = T_2,\ W_2 = 4\ W_1 \end{aligned}$$

But as work is done, temperature increases and surface tension decreases.

$\therefore\quad W_2 < 4\ W_1$

Question 111

In a transistor, in common emitter configuration, the ratio of power gain to voltage gain is

Options:

A. α

B. $\frac{\beta}{\alpha}$

C. $\beta\alpha$

D. β

Answer: D

Solution:

$$\text{Power gain} = \text{Voltage gain} \times \text{Current gain}$$

$$\therefore \quad \frac{\text{Power Gain}}{\text{Voltage Gain}} = \text{Current gain} = \beta$$

Question 112

A galvanometer of resistance $20\ \Omega$ gives a deflection of 5 divisions when $1\ \mathrm{mA}$ current flows through it. The galvanometer scale has 50 divisions. To convert the galvanometer into a voltmeter of range 25 volt, we should connect a resistance of

Options:

A. 1240 Ω in series.

B. 2480 Ω in series.

C. 2480 Ω in parallel.

D. 20 Ω in parallel.

Answer: B

Solution:

$$\mathrm{R} = \frac{\mathrm{V}}{\mathrm{I_g}} - \mathrm{G}$$

$$\text{Here, } \mathrm{I_g} = \frac{50}{5} = 10\ \mathrm{mA}$$

$$\therefore \quad \mathrm{R} = \frac{25}{10 \times 10^{-3}} - 20$$

$$= 2480\Omega \text{ in series}$$

Question 113

The equation of simple harmonic progressive wave is given by $y = a \sin 2\pi(bt - cx)$. The maximum particle velocity will be half the wave velocity, if $\mathrm{c} =$

Options:

A. $2\pi \mathrm{a}$

B. $\frac{1}{4\pi \mathrm{a}}$

C. $\frac{1}{2\pi \mathrm{a}}$

D. $4\pi \mathrm{a}$

Answer: B

Solution:

General equation of a simple harmonic progressive wave,

$y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$

Given: $\mathrm{y} = \mathrm{a} \sin 2\pi(bt - cx)$

$\Rightarrow \mathrm{A} = \mathrm{a}, \frac{1}{\mathrm{T}} = \mathrm{b}$ and $\frac{1}{\lambda} = \mathrm{cx}$

Also, $(\mathrm{v_p})_{\max} = \mathrm{a}\omega = \mathrm{a}(2\pi \mathrm{n}) = \frac{\mathrm{a}2\pi}{\mathrm{T}}$

$= \frac{\mathrm{A}2\pi}{\mathrm{T}} = \mathrm{a}2\pi \mathrm{b}$

From $\mathrm{v} = \frac{\lambda}{\mathrm{T}} = \frac{1/\mathrm{c}}{1/\mathrm{b}} = \frac{\mathrm{b}}{\mathrm{c}}$

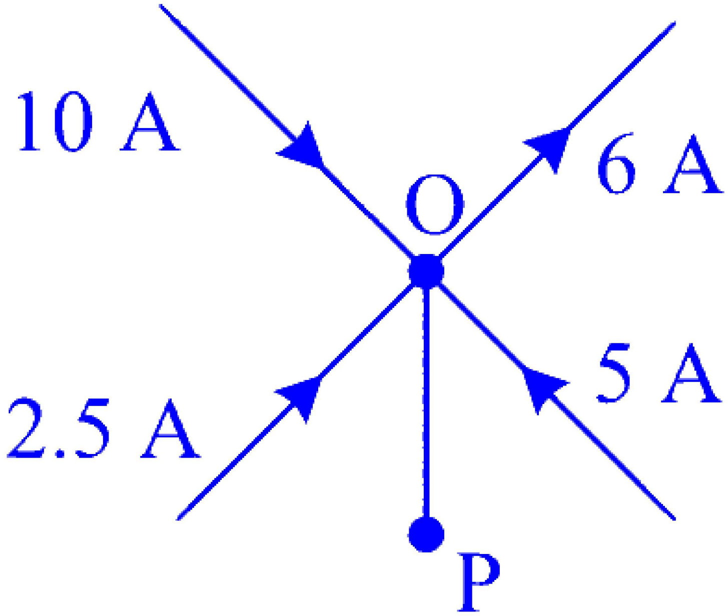
Given: $(\mathrm{v_p})_{\max} = \frac{1}{2}\mathrm{v}$

$$\Rightarrow 2\pi ab = \frac{1}{2} \times \frac{b}{c}$$

$$\therefore \quad c = \frac{1}{4\pi a}$$

Question 114

Five current carrying conductors meet at a point 'O' as shown in figure. The magnitude and direction of the current in conductor 'OP' is



Options:

A. 6.5 A from O to P.

B. 9 A from P to O.

C. 10.5 A from P to O.

D. 11.5 A from O to P.

Answer: D

Solution:

Using Kirchhoff's current Law,

Current flowing in = Current flowing out $10 + 2.5 + 5 = 6 + x$

$\therefore \quad x = 17.5 - 6$

$\therefore \quad \mathrm{x} = 17.5 - 6$
$\quad\quad = 11.5 \mathrm{~A}$

$\therefore$ Magnitude and direction of the current in 'OP' should be 11.5 A from O to P.

Question 115

An alternating voltage of frequency 'ω' is induced in electric circuit consisting of an inductance 'L' and capacitance 'C', connected in parallel. Then across the inductance coil

Options:

A. current is maximum when $\omega^2 = \frac{1}{LC}$

B. current is zero

C. voltage is minimum when $\omega^2 = \frac{1}{LC}$

D. voltage is maximum when $\omega^2 = \frac{1}{LC}$

Answer: D

Solution:

In a parallel LC circuit at $\omega^2 = \frac{1}{LC}$, the current is minimum.

As current and voltage are out of phase by 90°, the voltage would be maximum.

Question 116

A gas is compressed at a constant pressure of $50\ \mathrm{N/m^2}$ from a volume of $10\ \mathrm{m}^3$ to a volume of $4\ \mathrm{m}^3$. Energy of $100\ \mathrm{J}$ is then added to the gas by heating. Its internal energy is

Options:

A. increased by 400 J

B. increased by 200 J

C. increased by 100 J

D. decreased by 200 J

Answer: A

Solution:

From first law of thermodynamics,

$Q = \Delta U + \Delta W = \Delta U + P\Delta V$

Change in volume due to compression

$\Delta V = V_2 - V_1 = 4 - 10 = -6m^3$

Negative sign indicates gas is compressed.

$\therefore \quad \Delta U = Q - P\Delta V = 100 - [50 \times (-6)] = 400 \text{ J}$

As, ΔU is positive, the internal energy is increased.

Question 117

The reactance of capacitor at 50 Hz is 5Ω. If the frequency is increased to 100 Hz, the new reactance is

Options:

A. 5Ω

B. 10Ω

C. 2.5Ω

D. 125Ω

Answer: C

Solution:

$$X_C = \frac{1}{2\pi fc}$$

$$\therefore \quad C = \frac{1}{2\pi f(5)} \quad \ldots .(\because X_C = 5\Omega)$$

New reactance,

$X'_C = \frac{1}{2\pi f'C} = \frac{1}{2\pi(2f)C} = \frac{X_C}{2} = \frac{1}{2} \times 5 = 2.5\Omega$

Question 118

Stationary waves can be produced in

Options:

A. only solid and gaseous media

B. only liquid and gaseous media

C. only solid and liquid media

D. solid, liquid and gaseous media

Answer: D

Question 119

The pressure exerted by an ideal gas at a particular temperature is directly proportional to

Options:

A. the mean speed of the gas molecules.

B. mean of the square of the speed of the gas molecules.

C. the square of the mean speed of the gas molecules.

D. the root mean square speed of the gas molecules.

Answer: B

Question 120

If a ray of light in denser medium strikes a rarer medium at angle of incidence i, the angles of reflection and refraction are r and r' respectively. If the reflected and refracted rays are at right angles to each other, the critical angle for the given pair of media is

Options:

A. $\sin^{-1}(\tan r')$

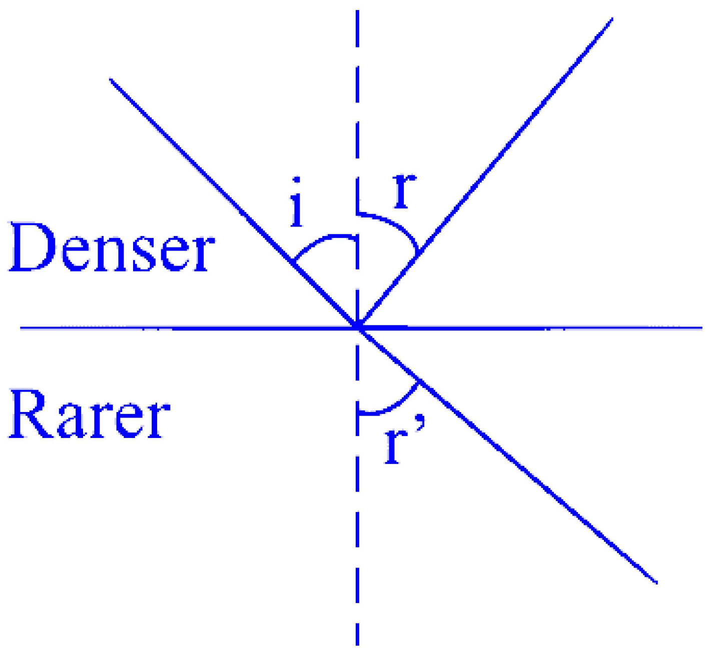
B. $\sin^{-1}(\tan r)$

C. $\tan^{-1}(\sin \mathrm{i})$

D. $\cot^{-1}(\tan \mathrm{i})$

Answer: B

Solution:



According to Snell's Law,

$\frac{\sin i}{\sin r} = \frac{1}{n}$

But $\mathrm{i} = \mathrm{r}$,

$\therefore \quad \frac{\sin \mathrm{r}}{\sin \mathrm{r}'} = \frac{1}{\mathrm{n}}$

From figure $r' + r + 90 = 180^{\circ}$

$\therefore \quad r' = 180^{\circ} - 90^{\circ} - r = 90^{\circ} - r$

$\therefore \quad \Rightarrow \frac{\sin \mathrm{r}}{\sin(90^{\circ} - r)} = \frac{1}{\mathrm{n}}$

$\frac{\sin r}{\cos r} = \frac{1}{n}$

$\therefore \quad \tan \mathrm{r} = \frac{1}{\mathrm{n}}$

Critical angle is given by $\sin \mathrm{i_c} = \frac{1}{\mathrm{n}}$

$\therefore \quad \tan \mathrm{r} = \sin \mathrm{i_c}$

$\therefore \quad \mathrm{i_c} = \sin^{-1}(\tan \mathrm{r})$

Question 121

A spring has a certain mass suspended from it and its period for vertical oscillations is 'T_1'. The spring is now cut in to two equal halves and the same mass is suspended from one of the halves. The period of vertical oscillations is now '$\mathrm{T_2}$'. The ratio $\mathrm{T_1}/\mathrm{T_2}$ is

Options:

A. 2

B. $\sqrt{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer: B

Solution:

$T_1 = 2\pi\sqrt{\frac{m}{k}}$.

Also, spring constant $(k) \propto \frac{1}{\text{Length} \quad (l)}$

When the spring is half in length, then k becomes twice.

$\therefore \quad T_2 = 2\pi\sqrt{\frac{m}{2k}}$
$\therefore \quad \frac{T_1}{T_2} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

Question 122

When photons of energies twice and thrice the work function of a metal are incident on the metal surface one after other, the maximum velocities of the photoelectrons emitted in the two cases are v_1 and v_2 respectively. The ratio $v_1 : v_2$ is

Options:

A. $\sqrt{2} : 1$

B. $\sqrt{3} : 1$

C. $\sqrt{3} : \sqrt{2}$

D. $1 : \sqrt{2}$

Answer: D

Solution:

$K.E_{\max} = hv - \phi_0$

Given,

$E_1 = 2\phi_0$ and $E_2 = 3\phi_0$

$\Rightarrow \quad K \cdot E_1 = 2\phi_0 - \phi_0 = \phi_0$

$\Rightarrow \quad K \cdot E_2 = 3\phi_0 - \phi_0 = 2\phi_0$

but, $K \cdot E_1 = \frac{1}{2} m v_1^2$ and $K \cdot E_2 = \frac{1}{2} m_2^2$

$\therefore \quad \frac{K \cdot E_1}{K \cdot E_2} = \frac{v_1^2}{v_2^2} = \frac{1}{2}$

$\therefore \quad \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$

Question 123

A simple pendulum of length $2 \mathrm{~m}$ is given a horizontal push through angular displacement of 60°. If the mass of bob is 200 gram, the angular velocity of the bob will be (Take Acceleration due to gravity $= 10 \mathrm{~m/s}^2$)

$\left(\sin 30^{\circ} = \cos 60^{\circ} = 0.5, \cos 30^{\circ} = \sin 60^{\circ} = \sqrt{3}/2\right)$

Options:

A. $2\sqrt{2} \mathrm{~rad/s}$

B. $3\sqrt{2} \mathrm{~rad/s}$

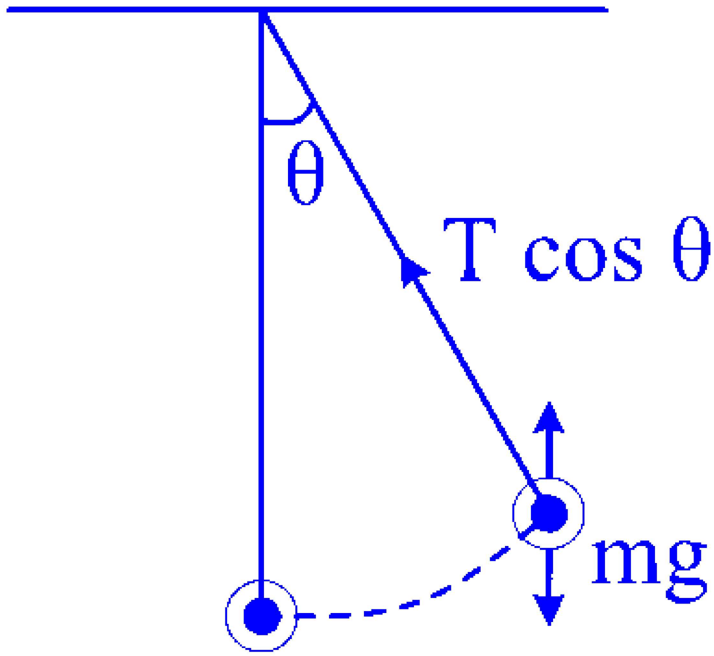
C. $2\sqrt{2.5} \mathrm{~rad/s}$

D. $3\sqrt{2.5} \mathrm{~rad/s}$

Answer: C

Solution:

Given : $l = 2$ m, $\theta = 60^{\circ}$, m = 200 g = 2g



From the figure,

$T = mr\omega^2$ ..... (i)

also $T\cos\theta = mg$ ..... (ii)

putting (i) into (ii)

$mr\omega^2 - \cos\theta = mg$ ..... (iii)

putting the given values into equation (iii)

$$2 \times 2 \times \omega^2 \frac{1}{2} = 2 \times 10 \quad \ldots \left(\because \cos 60 = \frac{1}{2}\right)$$

$\omega^2 = 10$

$$\Rightarrow \omega = \sqrt{10} \quad \ldots . (\because \sqrt{10} = \sqrt{2 \times 2 \times 2.5})$$

$$= 2\sqrt{2.5}\ \mathrm{rad/s}$$

Question 124

The force acting on the electron in hydrogen atom (Bohr' theory) is related to the principle quantum number 'n' as

Options:

A. n^4

B. n^{-4}

C. n^2

D. n^{-2}

Answer: B

Solution:

The centripetal force of the rotating electron is given by $F = \frac{mv^2}{r}$

But, according to Bohr,

$v \propto \frac{1}{n}$ and $r \propto n^2$

i.e. $F \propto \frac{v^2}{r^2}$

$\Rightarrow F \propto \frac{1}{n^2 n^2}$

$F \propto \frac{1}{n^4}$

Question 125

If the frequency of the input voltage is $50 \mathrm{~Hz}$, applied to a (a) half wave rectifier and (b) full wave rectifier. The output frequency in both cases is respectively

Options:

A. $50 \mathrm{~Hz}, 50 \mathrm{~Hz}$

B. $50 \mathrm{~Hz}, 100 \mathrm{~Hz}$

C. $100 \mathrm{~Hz}, 100 \mathrm{~Hz}$

D. $100 \mathrm{~Hz}, 50 \mathrm{~Hz}$

Answer: B

Solution:

The key aspect in understanding this question is to grasp how rectifiers work and how they affect the output frequency given an input frequency.

(a) Half-wave rectifier: A half-wave rectifier converts only one half of the AC input signal into DC output. Regardless of whether it's the positive or the negative half of the AC cycle that gets rectified, the result is that for every cycle of the AC input, there's a corresponding pulse in the output. Thus, if the input frequency is 50 Hz, the output frequency remains at 50 Hz as well. Each cycle of the input produces one pulse of output.

(b) Full-wave rectifier: A full-wave rectifier, on the other hand, converts both the positive and negative halves of the AC input signal into DC output. This means that for each cycle of the input AC signal, you get two pulses of the DC output. Consequently, if the input frequency is 50 Hz, the output frequency is doubled, resulting in a 100 Hz output frequency. Each cycle of the input results in two output pulses due to the rectification of both the positive and negative half-cycles.

Therefore, the correct answer to the question is:

Option B: $50 \text{ Hz}, 100 \text{ Hz}$

This means that for a half-wave rectifier, the output frequency is the same as the input frequency (50 Hz), and for a full-wave rectifier, the output frequency is double the input frequency (100 Hz).

Question 126

Periodic time of a satellite revolving above the earth's surface at a height equal to radius of the earth 'R' is [$g=$ acceleration due to gravity]

Options:

A. $2\pi\sqrt{\frac{2R}{g}}$

B. $4\pi\sqrt{\frac{2R}{g}}$

C. $2\pi\sqrt{\frac{R}{g}}$

D. $8\pi\sqrt{\frac{\mathrm{R}}{\mathrm{g}}}$

Answer: B

Solution:

$$\begin{aligned}
\mathrm{T}=2\pi\sqrt{\frac{(\mathrm{R}+\mathrm{h})^3}{\mathrm{gR}^2}} &= 2\pi\sqrt{\frac{(2\mathrm{R})^3}{\mathrm{gR}^2}} \quad \cdots .(\because \mathrm{h}=\mathrm{R}) \\
&= 4\pi\sqrt{\frac{2\mathrm{R}}{\mathrm{g}}}
\end{aligned}$$

Question 127

The wavelength of light for the least energetic photons emitted in the Lyman series of the hydrogen spectrum is nearly [Take $\mathrm{hc}=1240 \mathrm{~eV}$ - nm, change

in energy of the levels $= 10.2$ eV]

Options:

A. 150 nm

B. 122 nm

C. 102 nm

D. 82 nm

Answer: B

Solution:

For least energetic photon emitted in Lyman series, $E = E_2 - E_1 = 10.2$ eV

$$\lambda = \frac{hc}{E} = \frac{1240}{10.2} \text{ nm}$$
$$= 121.57 \text{ nm} \approx 122 \text{ nm}$$

Question 128

In Young's double slit experiment, the wavelength of light used is 'λ'. The intensity at a point is 'I' where path difference is $\left(\frac{\lambda}{4}\right)$. If I_0 denotes the maximum intensity, then the ratio $\left(\frac{I}{I_0}\right)$ is

$$\left(\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$$

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. $\frac{\sqrt{3}}{2}$

Answer: B

Solution:

Phase difference, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta l$

For path difference $\frac{\lambda}{4}$,

Phase difference $\Delta\phi = \frac{\pi}{2}$

Using, $\mathrm{I} = \mathrm{I}_0 \cos^2 \frac{\phi}{2}$

$\therefore \quad \frac{\mathrm{I}}{\mathrm{I}_0} = \cos^2 \frac{\phi}{2} = \cos^2\left(\frac{\pi}{4}\right)$

$\therefore \quad \frac{\mathrm{I}}{\mathrm{I}_0} = \frac{1}{2}$

Question 129

The side of a copper cube is $1 \mathrm{~m}$ at $0^{\circ}\mathrm{C}$. What will be the change in its volume, when it is heated to $100^{\circ}\mathrm{C}$? $\left[\alpha_{\text{copper}} \quad = 18 \times 10^{-6}/^{\circ}\mathrm{C}\right]$

Options:

A. $45 \times 10^{-4} \mathrm{~m}^3$

B. $54 \times 10^{-4} \mathrm{~m}^3$

C. $34 \times 10^{-4} \mathrm{~m}^3$

D. $64 \times 10^{-4} \mathrm{~m}^3$

Answer: B

Solution:

In volumetric expansion for a cube,

Change in volume $\Delta \mathrm{V} = \mathrm{V} 3 \alpha \Delta \mathrm{T}$

$\Delta \mathrm{V} = 3 \times 18 \times 10^{-6} \times 100$

$\Delta \mathrm{V} = 54 \times 10^{-4} \mathrm{~m}^3$

Question 130

If current 'I' is flowing in the closed circuit with collective resistance 'R', the rate of production of heat energy in the loop as we pull it along with a constant speed 'V' is ($\mathrm{L}=$ length of conductor, $\mathrm{B}=$ magnetic field)

Options:

A. $\frac{\mathrm{BLV}}{\mathrm{R}}$

B. $\frac{\mathrm{B}^2 \mathrm{~L}^2 \mathrm{~V}^2}{\mathrm{R}^2}$

C. $\frac{B L V}{R^2}$

D. $\frac{\mathrm{B}^2 \mathrm{~L}^2 \mathrm{~V}^2}{\mathrm{R}}$

Answer: D

Solution:

From motional emf,

$\mathrm{e}_{\max }=\mathrm{BLV}$

$$
\begin{aligned}
\therefore \quad & \text{Heat produced }=\frac{\mathrm{V}^2}{\mathrm{r}}=\frac{\mathrm{B}^2 \mathrm{~L}^2 \mathrm{~V}^2}{\mathrm{R}} \\
& \mathrm{i}=\frac{\mathrm{BLV}}{\mathrm{R}} \\
& |\mathrm{F}|=\mathrm{BiL} \text{ and } \mathrm{P}=\mathrm{F} \cdot \mathrm{V} \\
\therefore \quad & \mathrm{P}=\mathrm{B}\left(\frac{\mathrm{BLV}}{\mathrm{R}}\right) \mathrm{LV} \\
& =\frac{\mathrm{B}^2 \mathrm{~L}^2 \mathrm{~V}^2}{\mathrm{R}}
\end{aligned}
$$

Question 131

Two coils A and B have mutual inductance 0.008 H. The current changes in the coil A, according to the equation $\mathrm{I}=\mathrm{I}_{\mathrm{m}} \sin \omega \mathrm{t}$, where $\mathrm{I}_{\mathrm{m}}=5 \mathrm{~A}$ and $\omega=200 \pi \mathrm{~rad} \mathrm{~s}^{-1}$. The maximum value of the e.m.f. induced in the coil B in volt is

Options:

A. 4π

B. 8π

C. 10π

D. 16π

Answer: B

Solution:

$e = M\frac{dI}{dt}$

Given: $M = 0.008, I_m = 5A, \omega = 200\ \mathrm{rad/s}$

$\therefore \quad \mathrm{e} = 0.008 \times \mathrm{I_m}\omega\cos\omega\mathrm{t}$

For $\mathrm{e} = \mathrm{e_{max}}, \cos\omega\mathrm{t} = 1$

$$\begin{aligned}\therefore \quad \mathrm{e_{max}} &= 0.008 \times \mathrm{I_m} \times \omega \\ &= 0.008 \times 5 \times 200\pi \\ &= 8\pi\end{aligned}$$

Question 132

A thin rod of length L has magnetic moment M when magnetised. If rod is bent in a semicircular arc what is magnetic moment in new shape?

Options:

A. $\frac{M}{L}$

B. $\frac{M}{\pi}$

C. $\frac{\mathrm{M}}{2\pi}$

D. $\frac{2\mathrm{M}}{\pi}$

Answer: D

Solution:

Magnetic moment of rod $= \mathrm{M}$

Let r be the radius after bending it into a semicircular arc.

$\therefore$ The separation between the two ends is 2r. Here length $=$ circumference of the semicircle i.e., $\mathrm{L} = \pi\mathrm{r}$

$\therefore \quad r = \frac{L}{\pi}$

Also $M = m \times L$ and $m = \frac{M}{L}$

$$\therefore \quad M_{\text{new}} = m(2r) = \frac{M}{L} \times \frac{2L}{\pi} = \frac{2M}{\pi}$$

Question 133

A fluid of density 'ρ' and viscosity 'η' is flowing through a pipe of diameter 'd', with a velocity 'v'. Reynold number is

Options:

A. $\frac{2 \text{ d}\rho\text{v}}{\eta}$

B. $\frac{\text{d}\rho\text{v}}{\eta}$

C. $\frac{\text{d}\rho v}{\eta^2}$

D. $\frac{2\eta\text{dv}}{\rho}$

Answer: B

Solution:

The Reynolds number (R_e) is given by the formula :

$R_e = \frac{\rho v d}{\eta}$

where ρ is the density of the fluid, v is the velocity of the fluid, d is the characteristic linear dimension (diameter of the pipe in this case), and η is the dynamic viscosity of the fluid.

Based on this formula, the correct answer would be Option B : $\frac{d\rho v}{\eta}$.

Question 134

In Young's double slit experiment, the fringe width is 2 mm. The separation between the 13^{th} bright fringe and the 4^{th} dark fringe from the centre of the screen on same side will be

Options:

A. 13 mm.

B. 17 mm.

C. 19 mm.

D. 23 mm.

Answer: C

Solution:

Given: Fringe width $\mathrm{W}=2 \mathrm{~mm}$

The distance of the n^{th} bright fringe from centre of the screen $y_n=\frac{n \lambda D}{d}$ ...... (i)

The distance of the n^{th} dark fringe from centre of the screen $y_n^{\prime}=(2 n-1) \frac{\lambda D}{2 d}$ ..... (ii)

Substituting, $\mathrm{n}=13$ in (i) and $\mathrm{n}=4$ in (ii) we get $\mathrm{y}_{13}=\frac{13 \lambda \mathrm{D}}{\mathrm{d}}$ and $\mathrm{y}_4^{\prime}=\frac{7}{2} \frac{\lambda \mathrm{D}}{\mathrm{d}}$

$\therefore$ The separation between the 13^{th} bright fringe and the 7^{th} dark fringe is

$$
\begin{aligned}
& \mathrm{y}_{13}-\mathrm{y}_4^{\prime} \\
& =\frac{13 \lambda \mathrm{D}}{\mathrm{d}}-\frac{7}{2} \frac{\lambda \mathrm{D}}{\mathrm{d}} \\
& =\left(13-\frac{7}{2}\right) \frac{\lambda \mathrm{D}}{\mathrm{d}} \\
& =\frac{19}{2} \frac{\lambda \mathrm{D}}{\mathrm{d}}=\frac{19}{2} \mathrm{~W} \\
& \text{but } \mathrm{W}=2 \mathrm{~mm} \\
\therefore \quad & \mathrm{y}_{13}-\mathrm{y}_4^{\prime}=\frac{19}{2} \times 2=19 \mathrm{~mm}
\end{aligned}
$$

Question 135

10 A current is flowing in two straight parallel wires in the same direction. Force of attraction between them is 1×10^{-3} N. If the current is doubled in both the wires the force will be

Options:

A. 1×10^{-3} N

B. 2×10^{-3} N

C. 4×10^{-3} N

D. 0.25×10^{-3} N

Answer: C

Solution:

$F = \frac{\mu_0}{4\pi} \frac{2I_1I_2}{r} l = 10^{-3}$ N

When current in both the wires is doubled, then

$F' = \frac{\mu_0}{4\pi} \frac{2(2I_1 \times 2I_2)}{r} l = 4 \times 10^{-3}$ N

Question 136

Consider a planet whose density is same as that of the earth but whose radius is three times the radius 'R' of the earth. The acceleration due to gravity 'g_n' on the surface of planet is $g_n = x.\ g$ where g is acceleration due to gravity on surface of earth. The value of 'x' is

Options:

A. 9

B. 3

C. $\frac{1}{3}$

D. $\frac{1}{9}$

Answer: B

Solution:

As, $g = \frac{GM}{R^2}$ and $M = \rho V$

$\therefore \quad g = \frac{G\rho V}{R^2} = \frac{G\rho \frac{4}{3}\pi R^3}{R^2}$

$\therefore \quad g \propto R$

For the planet: Radius $R = 3R$

$\therefore \quad g_{\text{planet}} = \frac{G\rho V_{\text{planet}}}{(3R)^2}$

where $V_{\text{planet}} = \frac{4}{3}\pi(3R)^3$

$\therefore \quad g_{\text{planet}} = \frac{G\rho \frac{4}{3}\pi(3R)^3}{(3R)^2}$

$\therefore \quad g_{\text{planet}} \propto 3R$

$\therefore \quad \frac{g}{g_{\text{planet}}} = \frac{R}{3R}$ ....(since ρ is constant)

$\therefore \quad g_{\text{planet}} = 3g$

$\therefore \quad x = 3$

Question 137

When a certain metal surface is illuminated with light of frequency v, the stopping potential for photoelectric current is V_0. When the same surface is illuminated by light of frequency $\frac{v}{2}$, the stopping potential is $\frac{\mathrm{V}_0}{4}$, the threshold frequency of photoelectric emission is

Options:

A. $\frac{v}{6}$

B. $\frac{v}{3}$

C. $\frac{2v}{3}$

D. $\frac{4v}{3}$

Answer: B

Solution:

$\mathrm{eV}_0 = \mathrm{h}v - \mathrm{h}v_0$ ..... (i)

$\frac{\mathrm{eV}_0}{4} = \frac{\mathrm{h}v}{2} - \mathrm{h}v v_0$ ..... (ii)

Dividing equation (i) by equation (ii),

$4=\frac{v-v_0}{\frac{v}{2}-v_0}$

$\therefore \quad 2v-4v_0=v-v_0$

$\therefore \quad 3v_0=v$

$\therefore \quad v_0=\frac{v}{3}$

Question 138

If the length of an open organ pipe is $33.3 \mathrm{~cm}$, then the frequency of fifth overtone is [Neglect end correction, velocity of sound $=333 \mathrm{~m/s}$]

Options:

A. 3500 Hz

B. 3000 Hz

C. 2500 Hz

D. 2000 Hz

Answer: B

Solution:

For a pipe open at both ends,

$\mathrm{n}=\frac{\mathrm{v}}{2 \mathrm{l}}=\frac{333}{2 \times 33.3 \times 10^{-2}}=500 \mathrm{~Hz}$

$\therefore$ Frequency of 5^{th} overtone,

$\mathrm{n}=6 \mathrm{n}=6 \times 500=3000 \mathrm{~Hz}$

Question 139

A transparent glass cube of length $24 \mathrm{~cm}$ has a small air bubble trapped inside. When seen normally through one surface from air outside, its apparent distance is $10 \mathrm{~cm}$ from the surface. When seen normally from opposite surface, its apparent distance is $6 \mathrm{~cm}$. The distance of the air bubble from first surface is

Options:

A. 15 cm

B. 14 cm

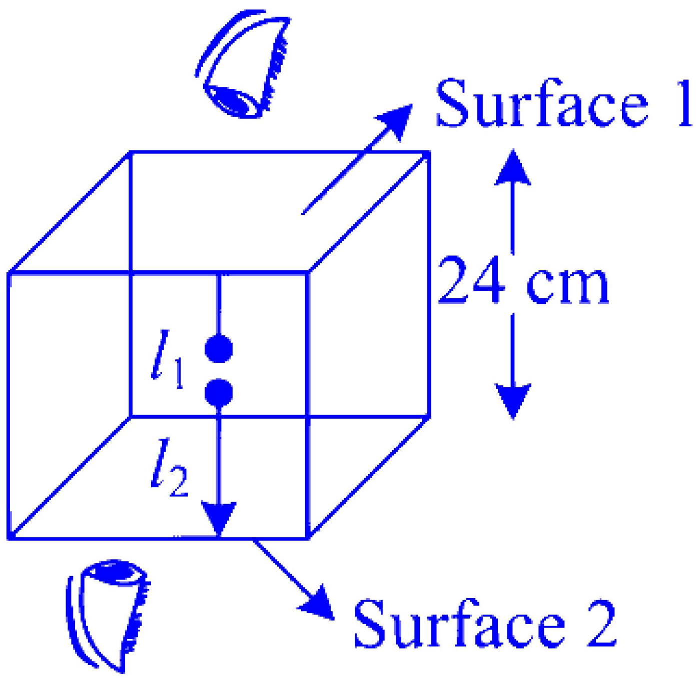
C. 12 cm

D. 8 cm

Answer: A

Solution:

Given: Length of cube = 12 cm



$\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{l_1}{h_1} = \frac{24-l_1}{h_2}$

putting $h_1 = 10$ cm and $h_2 = 6$ cm into (i), we get $\frac{l_1}{10} = \frac{24-l_1}{6}$

$$6l_1 = 240 - 10l_1$$
$$16l_1 = 240$$
$$\therefore \quad l_1 = 15 \text{ cm}$$

Question 140

The temperature of an ideal gas is increased from $27^\circ C$ to $927^\circ C$. The r.m.s. speed of its molecules becomes

Options:

A. twice

B. four times.

C. half.

D. one-fourth.

Answer: A

Solution:

We know $v_{\mathrm{rms}} \quad = \sqrt{\frac{3RT}{M}}$

$\Rightarrow \mathrm{v}_{\mathrm{rms}} \quad \propto \sqrt{\mathrm{T}}$

$\therefore \quad \frac{\mathrm{v}_{\mathrm{mm}}}{\mathrm{v}_{\mathrm{mm}_2}} = \sqrt{\frac{\mathrm{T}_1}{\mathrm{T}_2}} = \sqrt{\frac{300}{1200}} = \sqrt{\frac{1}{4}}$

$\frac{\mathrm{v}_{\mathrm{ms}}}{\mathrm{v}_{\mathrm{ms}_2}} - \frac{1}{2}$

$\therefore \quad \mathrm{v}_{\mathrm{mss}_2} = 2 \cdot \mathrm{v}_{\mathrm{rms1}}$

Question 141

A disc of radius R and thickness $\frac{R}{6}$ has moment of inertia I about an axis passing through its centre and perpendicular to its plane. Dise is melted and recast into a solid sphere. The moment of inertia of a sphere about its diameter is

Options:

A. $\frac{\mathrm{I}}{5}$

B. $\frac{\mathrm{I}}{6}$

C. $\frac{\mathrm{I}}{32}$

D. $\frac{\mathrm{I}}{64}$

Answer: A

Solution:

M.I. of disc, $I = \frac{1}{2}\ \mathrm{MR_d^2}$ ..... (i)

M.I. of sphere, $\mathrm{I_{sphere}} \quad = \frac{2}{5}\ \mathrm{MR_s^2}$ .... (ii)

$\because$ volume of disc = volume of sphere

$\therefore \quad \pi \mathrm{R_d^2}\left(\frac{\mathrm{R_d}}{6}\right) = \frac{4}{3}\pi \mathrm{R_s^3}$

$\therefore \quad \mathrm{R_d^3} = 8\mathrm{R_s^3}$

$\therefore \quad \mathrm{R_S} = \frac{\mathrm{R_d}}{2}$ .... (iii)

Substitute equation (iii) in equation (ii)

$$\therefore \quad I_{\text{sphere}} = \frac{2}{5}\mathrm{M}\left(\frac{\mathrm{R_d}}{2}\right)^2 = \frac{2}{5} \times \frac{1}{4}\mathrm{MR_d^2}$$
$$= \frac{1}{5}\left(\frac{1}{2}\mathrm{MR_d^2}\right) = \frac{1}{5} \quad \text{.... [from (i)]}$$

Question 142

A charge 'q' moves with velocity 'v' through electric (E) as well as magnetic field (B). Then the force acting on it is

Options:

A. $\mathrm{q}(\overrightarrow{\mathrm{v}} \times \overrightarrow{\mathrm{B}})$

B. $\mathrm{q}(\overrightarrow{\mathrm{B}} \times \overrightarrow{\mathrm{v}})$

C. $\mathrm{q}(\overrightarrow{\mathrm{E}} \times \overrightarrow{\mathrm{v}})$

D. $\overrightarrow{\mathrm{qE}} + \mathrm{q}(\overrightarrow{\mathrm{v}} \times \overrightarrow{\mathrm{B}})$

Answer: D

Question 143

A parallel combination of two capacitors of capacities '$2\ \mathrm{C}$' and 'C' is connected across $5\ \mathrm{V}$ battery. When they are fully charged, the charges and energies stored in them be 'Q_1', 'Q_2' and 'E_1', 'E_2' respectively. Then $\frac{E_1 - E_2}{Q_1 - Q_2}$ in $\mathrm{J/C}$ is (capacity is in Farad, charge in Coulomb and energy in J)

Options:

A. $\frac{5}{4}$

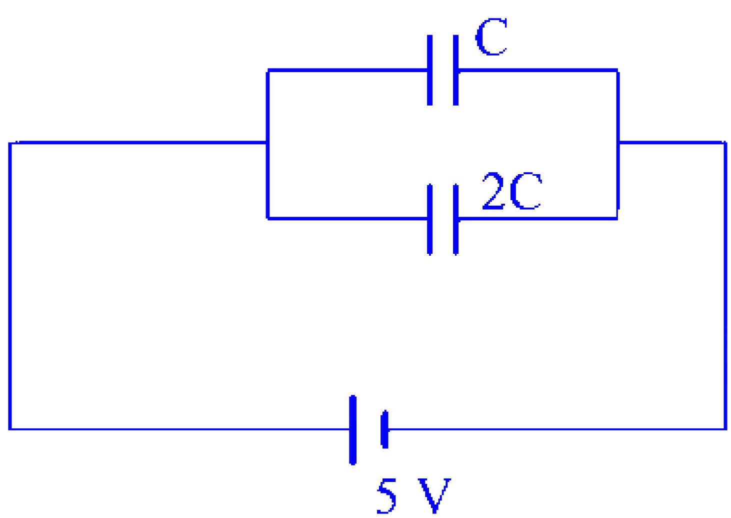
B. $\frac{4}{5}$

C. $\frac{5}{2}$

D. $\frac{2}{5}$

Answer: C

Solution:



We know, $\mathrm{Q} = \mathrm{C}.\mathrm{V}$

$\therefore \quad Q_1 = 10\mathrm{C}$ and $\mathrm{Q}_2 = 5\mathrm{C}$

Energy stored, $\mathrm{E} = \frac{1}{2}\mathrm{CV}^2$

$\therefore \quad \mathrm{E}_1 = \frac{1}{2}\mathrm{C}_1\,\mathrm{V}^2 = \frac{1}{2} \times 2\mathrm{C} \times 25 = 25\ \mathrm{J}$

Similarly, $\mathrm{E}_2 = \frac{1}{2}\mathrm{C}_2\,\mathrm{V}^2 = \frac{1}{2} \times \mathrm{C} \times 25 = 12.5\ \mathrm{J}$

$\therefore \quad \frac{\mathrm{E}_1 - \mathrm{E}_2}{\mathrm{Q}_1 - \mathrm{Q}_2} = \frac{12.5}{5} = \frac{5}{2}$

Question 144

Consider the following statements A and B. Identify the correct choice in the given answers.

A. In an inelastic collision, there is no loss in kinetic energy during collision.

B. During a collision, the linear momentum of the entire system of particles is conserved if there is no external force acting on the system.

Options:

A. Both A and B are wrong.

B. Both A and B are correct.

C. A is wrong and B is correct.

D. A is correct and B is wrong.

Answer: C

Solution:

In elastic collision, there is a loss in kinetic energy. However, momentum is conserved if there is no external force acting on the system.

Question 145

The charges $2q, -q, -q$ are located at the vertices of an equilateral triangle. At the circumcentre of the triangle

Options:

A. the field is zero but potential is not zero.

B. the field is non-zero but the potential is zero.

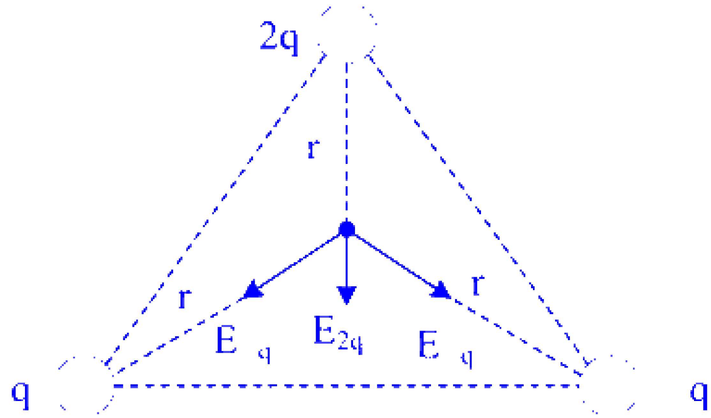
C. both, field and potential are zero.

D. both. field and potential are non-zero.

Answer: B

Solution:

From charge configuration, at the centre electric field is non-zero.



Potential at the centre due to 2q charge $V_{2q} = \frac{2q}{r}$ and potential due to $-q$ charge

$V_{-q} = -\frac{q}{r}$ (r = distance of centre point)

$\therefore$ total potential $V = V_{2q} + V_{-q} + V_{-q} = 0$

Question 146

The magnetic field at a point P situated at perpendicular distance 'R' from a long straight wire carrying a current of $12\ \mathrm{A}$ is $3 \times 10^{-5}\ \mathrm{Wb/m^2}$. The value of 'R' in mm is $\left[\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb/Am}\right]$

Options:

A. 0.08

B. 0.8

C. 8

D. 80

Answer: D

Solution:

Using Biot-Savart's Law,

$$B = \frac{\mu_0 I}{2\pi R}$$

$$\therefore \quad R = \frac{\mu_0 I}{2\pi B}$$

$$= \frac{4\pi \times 10^{-7} \times 12}{2\pi \times 3 \times 10^{-5}}$$

$$= 8 \times 10^{-2}\ \mathrm{m}$$

$$= 80\ \mathrm{mm}$$

Question 147

A particle at rest starts moving with constant angular acceleration $4\ \mathrm{rad/s^2}$ in circular path. At what time the magnitudes of its tangential acceleration and centrifugal acceleration will be equal?

Options:

A. 0.4 s

B. 0.5 s

C. 0.8 s

D. 1.0 s

Answer: B

Solution:

In rotational motion,

$\omega = \omega_0 + \alpha t$
$\omega = \alpha t$

...... ($\because \omega_0 = 0$; particle at rest.)

$\therefore$ Centrifugal acceleration $a = \omega^2 r$

$\therefore \quad a = \alpha^2 t^2 r$

Tangential acceleration $a_t = \alpha \times r$

Given: $a = a_t$

$\Rightarrow \alpha^2 t^2 r = \alpha r$

$t^2 = \frac{1}{\alpha} = \frac{1}{4}$

$\therefore \quad t = \frac{1}{2} = 0.5\ \mathrm{s}$

Question 148

If the end correction of an open pipe is 0.8 cm, then the inner radius of that pipe is

Options:

A. $\frac{1}{3}$ cm

B. $\frac{2}{3}$ cm

C. $\frac{3}{2}$ cm

D. 0.2 cm

Answer: B

Solution:

For an open pipe, $\mathrm{e} = 0.6 \mathrm{~d}$

$\therefore \quad \mathrm{d} = \frac{\mathrm{e}}{0.6}$
$\therefore \quad 2\mathrm{r} = \frac{\mathrm{e}}{0.6}$
$\therefore \quad \mathrm{r} = \frac{0.8}{1.2} = \frac{2}{3} \text{ cm}$

Question 149

The mutual inductance (M) of the two coils is 3 H. The self inductances of the coils are 4 H and 9 H respectively. The coefficient of coupling between the coils is

Options:

A. 0.3

B. 0.4

C. 0.5

D. 0.6

Answer: C

Solution:

Coefficient of coupling,

$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{3}{\sqrt{36}} = 0.5$

Question 150

A particle is vibrating in S.H.M. with an amplitude of $4 \mathrm{~cm}$. At what displacement from the equilibrium position is its energy half potential and half kinetic?

Options:

A. $1 \mathrm{~cm}$

B. $\sqrt{2} \mathrm{~cm}$

C. $2 \mathrm{~cm}$

D. $2\sqrt{2} \mathrm{~cm}$

Answer: D

Solution:

$$\begin{aligned} \text{K.E } &= \frac{1}{2} m\omega^2 \left(A^2 - x^2\right) \text{ and} \\ \text{P.E } &= \frac{1}{2} m\omega^2 x^2 \\ \therefore \quad \text{T.E } &= \text{K.E } + \text{ P.E} \\ &= \frac{1}{2} m\omega^2 \left(A^2 - x^2\right) + \frac{1}{2} m\omega^2 x^2 \\ &= \frac{1}{2} m\omega^2 A^2 \\ &\text{Given: K.E } = \text{ P.E} \end{aligned}$$

$$\therefore \quad A^2 - x^2 = x^2$$

$$\therefore \quad A^2 = 2x^2$$

$$\therefore \quad x = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$$

$$\therefore \quad x = \frac{4}{\sqrt{2}} = 2\sqrt{2} \text{ cm}$$
