# MHT CET 2023 : 9th May Morning Shift

# Mathematics

## Question 1

**If two vertices of a triangle are $\mathrm{A}(3,1,4)$ and $\mathrm{B}(-4,5,-3)$ and the centroid of the triangle is $G(-1,2,1)$, then the third vertex $C$ of the triangle is**

**Options:**

A. $(2,0,2)$

B. $(-2,0,2)$

C. $(0,-2,2)$

D. $(2,-2,0)$

**Answer: B**

## Solution:

Let $\overline{\mathrm{a}}, \overline{\mathrm{b}}, \overline{\mathrm{c}}$ and $\overline{\mathrm{g}}$ be the position vectors of $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and $\mathrm{G}$ respectively.

$$\bar{a}=3\hat{i}+1\hat{j}+4\hat{k},$$
$$\bar{b}=-4\hat{i}+5\hat{j}-3\hat{k},$$
$$\bar{g}=-\hat{i}+2\hat{j}+\hat{k},$$

G is centroid of $\triangle\mathrm{ABC}$.

$$\therefore \quad \bar{g}=\frac{\bar{a}+\bar{b}+\bar{c}}{3}$$
$$3\bar{g}=\bar{a}+\bar{b}+\bar{c}$$
$$3(-\hat{i}+2\hat{j}+\hat{k})=3\hat{i}+\hat{j}+4\hat{k}-4\hat{i}+5\hat{j}-3\hat{k}+\bar{c}$$
$$\therefore \quad \bar{c}=-3\hat{i}+6\hat{j}+3\hat{k}-3\hat{i}-\hat{j}-4\hat{k}+4\hat{i}-5\hat{j}+3\hat{k}$$
$$=-2\hat{i}+0\hat{j}+2\hat{k}$$

$\therefore \quad$ Third vertex $C \equiv(-2,0,2)$

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# Question 2

## In a Binomial distribution with $n = 4$, if $2P(X = 3) = 3P(X = 2)$, then the variance is

**Options:**

A. $\frac{36}{169}$

B. $\frac{144}{169}$

C. $\frac{9}{169}$

D. $\frac{16}{169}$

**Answer: B**

## Solution:

$$P(X = 3) = {}^4C_3 p^3(1 - p) = 4p^3(1 - p)$$
$$P(X = 2) = {}^4C_2 p^2(1 - p)^2 = 6p^2(1 - p)^2$$
$$\text{Given } 2P(X = 3) = 3P(X = 2)$$
$$\therefore \quad 8p^3(1 - p) = 18p^2(1 - p)^2$$
$$\therefore \quad 8p = 18(1 - p)$$
$$\therefore \quad p = \frac{9}{13}$$
$$\begin{aligned} \text{Variance } &= np(1 - p) \\ &= 4 \times \frac{9}{13}\left(1 - \frac{9}{13}\right) \\ &= \frac{144}{169} \end{aligned}$$

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# Question 3

## Let two non-collinear vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point $P$ moves, so that at any time $t$ the position vector $\overline{OP}$, where $O$ is origin,

**is given by $\hat{a}\sin t + \hat{b}\cos t$, when $P$ is farthest from origin $O$, let $M$ be the length of $\overline{\mathrm{OP}}$ and $\hat{\mathrm{u}}$ be the unit vector along $\overline{\mathrm{OP}}$, then**

**Options:**

A. $\hat{\mathrm{u}} = \frac{\hat{\mathrm{a}}+\hat{\mathrm{b}}}{|\hat{\mathrm{a}}+\hat{\mathrm{b}}|}$ and $\mathrm{M} = (1 + \hat{\mathrm{a}} \cdot \hat{\mathrm{b}})^{\frac{1}{2}}$

B. $\hat{\mathrm{u}} = \frac{\hat{\mathrm{a}}-\hat{\mathrm{b}}}{|\hat{\mathrm{a}}-\hat{\mathrm{b}}|}$ and $\mathrm{M} = (1 + \hat{\mathrm{a}} \cdot \hat{\mathrm{b}})^{\frac{1}{2}}$

C. $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$

D. $\hat{u} = \frac{\hat{a}-\hat{b}}{|\hat{a}-\hat{b}|}$ and $M = (1 - 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$

**Answer: A**

## Solution:

$\mathrm{M} = |\overrightarrow{\mathrm{OP}}|$

$M = \sqrt{(\hat{a}\sin t + \hat{b}\cos t)^2}$

$= \sqrt{(\hat{a}\sin t)^2 + (\hat{b}\cos t)^2 + 2(\hat{a}\sin t) \cdot (\hat{b}\cos t)}$

$= \sqrt{\sin^2 \mathrm{t} + \cos^2 \mathrm{t} + \hat{\mathrm{a}} \cdot \hat{\mathrm{b}}(2\sin \mathrm{t}\cos \mathrm{t})}$

$= \sqrt{1 + \hat{a} \cdot \hat{b}(\sin 2t)}$

Maximum value of $\sin 2t = 1$

$\therefore \quad 2\mathrm{t} = \sin^{-1}(1)$

$\therefore \quad \mathrm{t} = \frac{\pi}{4}$

$\therefore \quad \mathrm{M} = \sqrt{1 + \hat{\mathrm{a}} \cdot \hat{\mathrm{b}}(1)}$

$= (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$

Now, $\hat{u} = \frac{\overline{\mathrm{OP}}}{|\overline{\mathrm{OP}}|}$

$= \frac{\hat{a}\sin t + \hat{b}\cos t}{|\hat{a}\sin t + \hat{b}\cos t|}$

$= \frac{\hat{a}\left(\frac{1}{\sqrt{2}}\right) + \hat{b}\left(\frac{1}{\sqrt{2}}\right)}{\left|\hat{a}\left(\frac{1}{\sqrt{2}}\right) + \hat{b}\left(\frac{1}{\sqrt{2}}\right)\right|}$

Unit vector of OP is

$\hat{u} = \frac{\hat{a}+\hat{b}}{|(\hat{a}+\hat{b})|}$

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# Question 4

## The number of solutions in $[0, 2\pi]$ of the equation $16^{\sin^2 x} + 16^{\cos^2 x} = 10$ is

**Options:**

A. 2

B. 4

C. 6

D. 8

**Answer: D**

## Solution:

$$16^{\sin^2 x} + 16^{\cos^2 x} = 10$$
$$16^{\sin^2 x} + 16^{1-\sin^2 x} = 10$$
$$16^{\sin^2 x} + \frac{16}{16^{\sin^2 x}} = 10$$

Let $16^{\sin^2 x} = t$

$\therefore \quad \mathrm{t} + \frac{16}{\mathrm{t}} = 10$
$\therefore \quad \mathrm{t}^2 - 10\mathrm{t} + 16 = 0$
$\Rightarrow \mathrm{t} = 2 \text{ and } \mathrm{t} = 8$

Now, $16^{\sin^2 x} = 2$ and $16^{\sin^2 x} = 8$

$2^{4\sin^2 x} = 2^1$ and $2^{4\sin^2 x} = 2^3$

$\therefore \quad 4\sin^2 x = 1$ and $4\sin^2 x = 3$

$\therefore \quad \sin^2 x = \frac{1}{4}$ and $\sin^2 x = \frac{3}{4}$

$\quad \sin x = \pm\frac{1}{2}$ and $\sin x = \pm\frac{\sqrt{3}}{2}$

$\therefore \quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ and $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\therefore$ number of solutions $= 8$.

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# Question 5

## The differential equation of all parabolas, whose axes are parallel to $\mathrm{Y}$-axis, is

### Options:

A. $y_3 = 1$

B. $y_3 = 0$

C. $y_3 = -1$

D. $yy_3 + y_1 = 0$

**Answer: B**

## Solution:

Parabola whose axes are parallel to Y-axis. Vertex is not $(0, 0)$

Equation becomes

$(x - \mathrm{h})^2 = 4\,\mathrm{b}(y - \mathrm{k})$

Differentiating w.r.t. $x$, we get

$2(x - h) = 4b\left(\frac{dy}{dx}\right)$

Again differentiating w.r.t. $x$, we get

$2 = 4b\left(\frac{d^2y}{dx^2}\right)$

Again differentiating w.r.t. $x$, we get

$$0 = 4b\left(\frac{d^3y}{dx^3}\right)$$
$$\therefore \quad \frac{d^3y}{dx^3} = 0$$
$$\text{i.e., } y_3 = 0$$

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# Question 6

## The value of $c$ of Lagrange's mean value theorem for $f(x) = \sqrt{25 - x^2}$ on $[1, 5]$ is

**Options:**

A. $\sqrt{15}$

B. 5

C. $\sqrt{10}$

D. 1

**Answer: A**

## Solution:

$$\mathrm{f}(x) = \sqrt{25 - x^2}$$
$$\therefore \quad \mathrm{f}'(x = \mathrm{c}) = \frac{-2\mathrm{c}}{2\sqrt{25 - \mathrm{c}^2}}$$
$$= \frac{-\mathrm{c}}{\sqrt{25 - \mathrm{c}^2}}$$

Applying Lagrange's mean value theorem, we get

$$\mathrm{f}'(\mathrm{c}) = \frac{\mathrm{f}(1) - \mathrm{f}(5)}{1 - 5}$$
$$\therefore \quad \frac{\mathrm{c}}{\sqrt{25 - c^2}} = \frac{\sqrt{25 - 1} - \sqrt{25 - 5^2}}{1 - 5}$$
$$\therefore \quad \frac{-c}{\sqrt{25 - c^2}} = \frac{-\sqrt{24}}{4}$$

$\therefore \quad 4c = \sqrt{24} \cdot \sqrt{25 - c^2}$

$\therefore \quad 16c^2 = 24\left(25 - c^2\right)$

$\therefore \quad c^2 = 15$

$\therefore \quad c = \pm\sqrt{15}$

Since $c = -\sqrt{15}$ does not lie in $[1, 5]$

$c = \sqrt{15}$

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# Question 7

$$\int \frac{x+1}{x\left(1+x\mathrm{e}^x\right)^2}\,\mathrm{d}x =$$

**Options:**

A. $\log\left|\frac{x\mathrm{e}^x}{1+x\mathrm{e}^x}\right| + c$, where c is a constant of integration.

B. $\log\left|\frac{x\mathrm{e}^x}{1+x\mathrm{e}^x}\right| - \frac{1}{1+x\mathrm{e}^x} + \mathrm{c}$, where c is a constant of integration.

C. $\log\left|1 + x\mathrm{e}^x\right| + \frac{1}{1+x\mathrm{e}^x} + \mathrm{c}$, where c is constant of integration.

D. $\log\left|\frac{x\mathrm{e}^x}{1+x\mathrm{e}^x}\right| + \frac{1}{1+x\mathrm{e}^x} + \mathrm{c}$, where c is constant of integration.

**Answer: D**

## Solution:

Let $\mathrm{I}=\int \frac{x+1}{x\left(1+x \mathrm{e}^{x}\right)^{2}} \mathrm{~d} x=\int \frac{\mathrm{e}^{\prime}(x+1)}{\mathrm{e}^{x} \cdot x\left(1+x \mathrm{e}^{x}\right)^{2}} \mathrm{~d} x$

Let $x \cdot \mathrm{e}^{x}=\mathrm{t}$

$\Rightarrow(x+1) \mathrm{e}^{x} \mathrm{~d} x=\mathrm{dt}$

$\therefore \quad I=\int \frac{d t}{t(1+t)^{2}}$

$=\int \frac{1+t-t}{t(1+t)^{2}} d t$

$=\int \frac{1 d t}{t(1+t)}-\int \frac{1}{(1+t)^{2}} d t$

$=\int \frac{1+t-t}{t(1+t)}-\int \frac{1}{(1+t)^{2}} d t$

$=\int \frac{1}{t} d t-\int \frac{1}{(t+1)} d t-\int \frac{1}{(1+t)^{2}} d t$

$=\log \mathrm{t}-\int \frac{\mathrm{dt}}{(\mathrm{t}+1)}-\int \frac{1}{(1+\mathrm{t})^{2}} d \mathrm{dt}+\mathrm{c}$

Let $y=1+\mathrm{t}$

$\Rightarrow \mathrm{d} y=\mathrm{dt}$

$\therefore \quad \mathrm{I}=\log \mathrm{t}-\int \frac{1}{y} \mathrm{~d} y-\int \frac{1}{y^{2}} \mathrm{~d} y+\mathrm{c}$

$=\log \mathrm{t}-\log y+\frac{1}{y}+\mathrm{c}$

$=\log t-\log (1+t)+\frac{1}{(1+t)}+c$

$=\log x \mathrm{e}^{x}-\log \left(1+x \mathrm{e}^{x}\right)+\frac{1}{\left(1+x \mathrm{e}^{x}\right)}+c$

$\therefore \quad \mathrm{I}=\log \left|\frac{x \mathrm{e}^{x}}{1+x \mathrm{e}^{x}}\right|+\frac{1}{1+x \mathrm{e}^{x}}+\mathrm{c}$

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# Question 8

## If $Z_1=4 i^{40}-5 i^{35}+6 i^{17}+2, Z_2=-1+i$, where $i=\sqrt{-1}$, then $\left|Z_1+Z_2\right|=$

**Options:**

A. 5

B. 13

C. 12

D. 15

**Answer: B**

## Solution:

$$
\begin{aligned}
&Z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2 \\
&Z_1 = 6 + 11i \\
&Z_2 = -1 + i \\
\therefore \quad &Z_1 + Z_2 = 6 + 11i - 1 + i \\
&\ = 5 + 12i \\
\therefore \quad &|Z_1 + Z_2| = \sqrt{(5)^2 + (12)^2} = \sqrt{169} = 13
\end{aligned}
$$

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# Question 9

## The approximate value of $\log_{10} 998$ is (given that $\log_{10} \mathrm{e} = 0.4343$ )

**Options:**

A. 3.0008686

B. 1.9991314

C. 2.0008686

D. 2.9991314

**Answer: D**

## Solution:

$$
\begin{aligned}
\text{Let } f(x) = \log_{10} x = \frac{\log_e x}{\log_e 10} &= (\log_{10} e)(\log_e x) \\
&= 0.4343 (\log_e x)
\end{aligned}
$$

On differentiating w.r.t. $x$, we get

$\mathrm{f}'(x) = \dfrac{0.4343}{x}$

Let $x = 998$

$= 1000 - 2 = \mathrm{a} + \mathrm{h}$

$\therefore \quad \mathrm{a} = 1000,\ \mathrm{h} = -2$

$f(a) = f(1000)$

$= \log_{10}(1000)$

$= 3\log_{10} 10$

$\therefore \quad \mathrm{f(a)} = 3$

Also, $\mathrm{f}'(\mathrm{a}) = \mathrm{f}'(1000) = \dfrac{0.4343}{1000} = 0.0004343$

$\mathrm{f(a+h)} \approx \mathrm{f(a)} + \mathrm{hf}'(\mathrm{a})$

$\therefore \quad \log_{10}(998) \approx 3 - 2(0.0004343)$

$\approx 2.9991314$

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# Question 10

## If $\sin^{-1} x + \cos^{-1} y = \frac{3\pi}{10}$, then the value of $\cos^{-1} x + \sin^{-1} y$ is

**Options:**

A. $\frac{\pi}{10}$

B. $\frac{7\pi}{10}$

C. $\frac{9\pi}{10}$

D. $\frac{3\pi}{10}$

**Answer: B**

## Solution:

$$\begin{aligned}
& \sin^{-1} x + \cos^{-1} y = \tfrac{3\pi}{10} \\
\therefore \quad & \tfrac{\pi}{2} - \cos^{-1} x + \tfrac{\pi}{2} - \sin^{-1} y = \tfrac{3\pi}{10} \\
\therefore \quad & \pi - \cos^{-1} x - \sin^{-1} y = \tfrac{3\pi}{10} \\
\therefore \quad & \cos^{-1} x + \sin^{-1} y = \pi - \tfrac{3\pi}{10} = \tfrac{7\pi}{10}
\end{aligned}$$

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# Question 11

**The area (in sq. units) of the region** $\mathrm{A}=\left\{(x, y) / \frac{y^2}{2} \leq x \leq y+4\right\}$ **is**

**Options:**

A. 30

B. $\frac{53}{3}$

C. 16

D. 18

**Answer: D**

## Solution:

Given that $\frac{y^2}{2} \leq x \leq y+4$

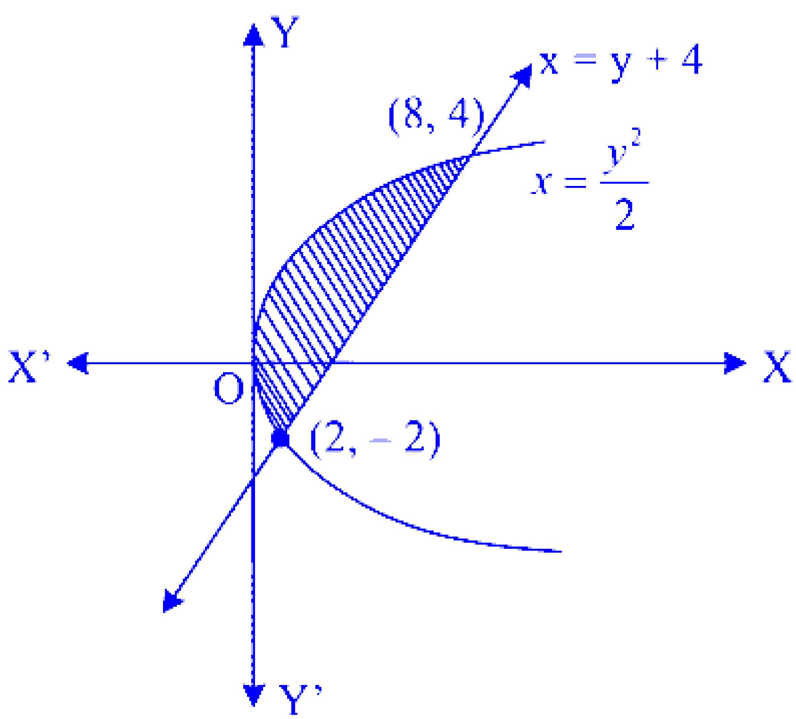
$\therefore \quad x=\frac{y^2}{2}$ and $x=y+4$

$\frac{y^2}{2}=y+4^2$

$\therefore \quad y^2-2 y-8=0$

$\therefore \quad y=4$ or $-2$

$\Rightarrow x=8$ or $2$



$\therefore \quad \mathrm{A}=\int_{-2}^{4}\left(y+4-\frac{y^2}{2}\right) \mathrm{d} y$

$\therefore \quad \mathrm{A}=\left[\frac{y^2}{2}+4 y-\frac{y^3}{6}\right]_{-2}^4$

$\therefore \quad A=\left(8+16-\frac{64}{6}\right)-\left(2-8+\frac{8}{6}\right)$

$\therefore \quad A=18$

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# Question 12

## $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are three events, one of which must and only one can happen. The odds in favor of $\mathrm{A}$ are $4: 6$, the odds against $B$ are $7: 3$. Thus, odds against $\mathrm{C}$ are

**Options:**

A. $7: 3$

B. $4: 6$

C. $6: 4$

D. $3: 7$

**Answer: A**

## Solution:

Odd in favor of A is $4: 6$.

$\therefore \quad \mathrm{P}(\mathrm{A})=\frac{4}{10}$

Odd against B is $7: 3$

$\therefore \quad \mathrm{P}(\mathrm{B})=\frac{3}{10}$

Since only one of the events A, B and C can happen, we get

$$
\begin{aligned}
& \mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})+\mathrm{P}(\mathrm{C})=1 \\
\therefore \quad & \frac{4}{10}+\frac{3}{10}+\mathrm{P}(\mathrm{C})=1 \\
\therefore \quad & \mathrm{P}(\mathrm{C})=\frac{3}{10} \\
\therefore \quad & \mathrm{P}\left(\mathrm{C}^{\prime}\right)=\frac{7}{10}
\end{aligned}
$$

$\therefore$ odds against the event C are $\mathrm{P}\left(\mathrm{C}'\right) : \mathrm{P}(\mathrm{C})$

$$= \frac{7}{10} : \frac{3}{10}$$
$$= 7 : 3$$

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# Question 13

## The value of $\alpha$, so that the volume of the parallelopiped formed by $\hat{i}+\alpha\hat{j}+\hat{k}, \hat{j}+\alpha\hat{k}$ and $\alpha\hat{i}+\hat{k}$ becomes maximum, is

**Options:**

A. $\frac{-1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

**Answer: A**

## Solution:

Volume of parallelopiped is $[\vec{a}\vec{b}\vec{c}]$

$$\therefore \quad V = \begin{vmatrix} 1 & \alpha & 1 \\ 0 & 1 & \alpha \\ \alpha & 0 & 1 \end{vmatrix}$$
$$= 1 - \alpha\left(-\alpha^2\right) - \alpha$$
$$= 1 + \alpha^3 - \alpha$$

Differentiating w.r.t. $\alpha$, we get

$\frac{dV}{d\alpha} = 3\alpha^2 - 1$

$\therefore \quad \frac{d^2V}{d\alpha^2} = 6\alpha$

Let $\frac{dV}{d\alpha} = 0$

$\therefore \quad 3\alpha^2 - 1 = 0$

$\therefore \quad \alpha = \pm\frac{1}{\sqrt{3}}$

at $\alpha = \frac{-1}{\sqrt{3}}$,

$\frac{d^2V}{d\alpha^2} = \frac{-6}{\sqrt{3}} < 0$

$\therefore$ V is maximum at $\alpha = \frac{-1}{\sqrt{3}}$

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# Question 14

## Two sides of a triangle are $\sqrt{3}+1$ and $\sqrt{3}-1$ and the included angle is $60^\circ$, then the difference of the remaining angles is

**Options:**

A. $30^\circ$

B. $45^\circ$

C. $60^\circ$

D. $90^\circ$

**Answer: D**

## Solution:

Let $a = \sqrt{3}+1, b = \sqrt{3}-1, C = 60^\circ$

Using cosine Rule,

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - 2(\sqrt{3}+1)(\sqrt{3}-1)\frac{1}{2}$

$c^2 = 6$

$\therefore \quad c^2 = 6$

$c = \sqrt{6}$

Using sine Rule,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Consider

$$\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$\therefore \quad \frac{\sqrt{3}-1}{\sin B}=\frac{\sqrt{6}}{\frac{\sqrt{3}}{2}}$$
$$\therefore \quad \sin B=\frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$\therefore \quad \angle B=15^{\circ}$$
$$\therefore \quad \angle A=105^{\circ}$$

.... [Remaining angle of a Triangle]

$\therefore \quad$ Difference $=90^{\circ}$

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# Question 15

## The standard deviation of the following distribution

| C.I. | 0 - 6 | 6 - 12 | 12 - 18 |
|---|---|---|---|
| $f_i$ | 2 | 4 | 6 |

## is

**Options:**

A. $5\sqrt{2}$

B. $\sqrt{5}$

C. $2\sqrt{5}$

D. 20

**Answer: C**

## Solution:

| C. I. | $f_i$ | $x_i$ | $x_i^2$ | $f_i x_i$ | $f_i x_i^2$ |
|---|---|---|---|---|---|
| 0 - 6 | 2 | 3 | 9 | 6 | 18 |
| 6 - 12 | 4 | 9 | 81 | 36 | 324 |
| 12 - 18 | 6 | 15 | 225 | 90 | 1350 |
| **Total** | **12** | | | **132** | **1692** |

Here $\sum f_i = 12, \sum f_i x_i = 132, \sum f_i x_i^2 = 1692$

$$\therefore \quad V(X) = \frac{1692}{12} - \left(\frac{132}{12}\right)^2 = 141 - 121 = 20$$

$\therefore$ Standard deviation $= \sqrt{20} = 2\sqrt{5}$

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# Question 16

## The maximum value of xy when x + 2y = 8 is

**Options:**

A. 20

B. 16

C. 24

D. 8

**Answer: D**

## Solution:

$$x + 2y = 8$$
$$\therefore \quad 2y = 8 - x$$
$$\therefore \quad y = \frac{8 - x}{2}$$

Let $f(x) = xy$

$\therefore \quad \mathrm{f}(x) = x \cdot \frac{(8-x)}{2}$

Differentiating w.r.t $x$, we get

$$f'(x) = \frac{(8-x)-x}{2}$$
$$f'(x) = 4 - x$$

To find critical points,

$$\mathrm{f}'(x) = 0$$
$$\therefore \quad 4 - x = 0$$
$$\therefore \quad x = 4$$

critical point at $x = 4$

$\therefore \quad \mathrm{f}(4) = \frac{4(8-4)}{2} = 8$

$\therefore$ Maximum value of the given function is 8.

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# Question 17

## If truth values of statements $\mathrm{p}, \mathrm{q}$ are true, and $\mathrm{r}$, $s$ are false, then the truth values of the following statement patterns are respectively

$$\mathrm{a} : \sim (\mathrm{p} \wedge \sim \mathrm{r}) \vee (\sim \mathrm{q} \vee \mathrm{s})$$
$$\mathrm{b} : (\sim \mathrm{q} \wedge \sim \mathrm{r}) \leftrightarrow (\mathrm{p} \vee \mathrm{s})$$
$$\mathrm{c} : (\sim \mathrm{p} \vee \mathrm{q}) \rightarrow (\mathrm{r} \wedge \sim \mathrm{s})$$

**Options:**

A. T, F, F

B. F, F, F

C. F, T, T

D. T, F, T

**Answer: B**

## Solution:

a. $\sim(p\wedge\sim \mathrm{r})\vee(\sim \mathrm{q}\vee \mathrm{s})$
$\equiv\sim(T\wedge\sim \mathrm{F})\vee(\sim T\vee F)$
$\equiv(F\vee F)\vee(F\vee F)$
$\equiv F\vee F$
$\equiv F$

b. $(\sim \mathrm{q}\wedge\sim \mathrm{r})\leftrightarrow(\mathrm{p}\vee \mathrm{s})$
$\equiv(\sim \mathrm{T}\wedge\sim \mathrm{F})\leftrightarrow(\mathrm{T}\vee \mathrm{F})$
$\equiv \mathrm{F}\leftrightarrow \mathrm{T}$
$\equiv \mathrm{F}$

c. $(\sim p\vee q)\rightarrow(r\wedge\sim s)$
$\equiv(\sim T\vee T)\rightarrow(F\wedge\sim F)$
$\equiv(F\vee T)\rightarrow(F\wedge T)$
$\equiv T\rightarrow F$
$\equiv F$

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# Question 18

## The rate of change of $\sqrt{x^2+16}$ with respect to $\frac{x}{x-1}$ at $x=5$ is

**Options:**

A. $\frac{-80}{\sqrt{41}}$

B. $\frac{80}{\sqrt{41}}$

C. $\frac{12}{5}$

D. $\frac{-12}{5}$

**Answer: A**

## Solution:

$y=\sqrt{x^2+16}$

$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x}{2\sqrt{x^2+16}}$

$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{\sqrt{x^2+16}}$ ..... (i)

Let z $=\frac{x}{x-1}$

$\therefore \quad \frac{\mathrm{dz}}{\mathrm{d}x}=\frac{(x-1)-x}{(x-1)^2}$

$\therefore \quad \frac{\mathrm{dz}}{\mathrm{d}x}=\frac{-1}{(x-1)^2}$ ..... (ii)

$\therefore \quad \left(\frac{\mathrm{d}y}{\mathrm{dz}}\right)_{x=5}=\frac{\frac{x}{\sqrt{x^2+16}}}{\frac{-1}{(x-1)^2}}$

$=\frac{-5}{\sqrt{25+16}}\times 16=\frac{-80}{\sqrt{41}}$

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# Question 19

## If $x^2+y^2=\mathrm{t}+\frac{1}{\mathrm{t}}$ and $x^4+y^4=\mathrm{t}^2+\frac{1}{\mathrm{t}^2}$, then $\frac{\mathrm{d}y}{\mathrm{d}x}$ is equal to

**Options:**

A. $\frac{y}{x}$

B. $\frac{-y}{x}$

C. $\frac{x}{y}$

D. $\frac{-x}{y}$

**Answer: B**

## Solution:

$x^2+y^2=t+\frac{1}{t}$

Squaring on both sides, we get

$x^4+y^4+2x^2y^2=t^2+\frac{1}{t^2}+2$

$\therefore \quad t^2+\frac{1}{t^2}+2x^2y^2=t^2+\frac{1}{t^2}+2 \quad \ldots[\because x^4+y^4=t^2+\frac{1}{t^2}, \text{given}]$

$2x^2y^2=2$
$x^2y^2=1$
differentiating w.r.t. $x$, we get
$x^2 2y\frac{dy}{dx}+2xy^2=0$
$x^2 2y\frac{dy}{dx}=-2xy^2$
$\frac{dy}{dx}=\frac{-2xy^2}{2x^2y}=\frac{-y}{x}$

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# Question 20

## Two tangents to the circle $x^2+y^2=4$ at the points $\mathrm{A}$ and $\mathrm{B}$ meet at the point $\mathrm{P}(-4,0)$. Then the area of the quadrilateral $\mathrm{PAOB}$, $\mathrm{O}$ being the origin, is

**Options:**

A. $2\sqrt{3}$ sq. units

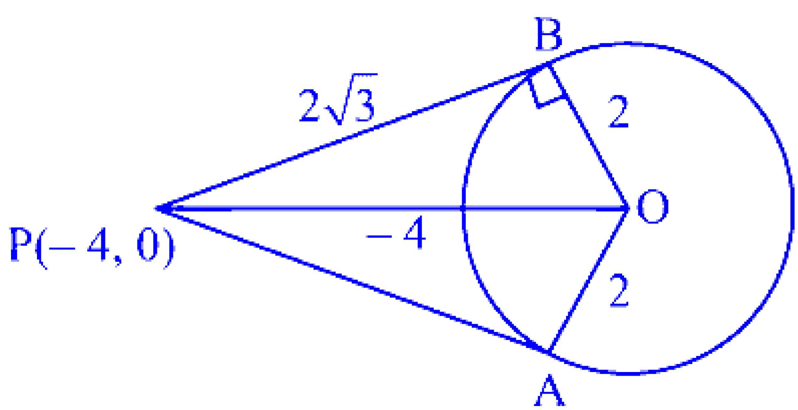
B. $8\sqrt{3}$ sq. units

C. $4\sqrt{3}$ sq. units

D. $6\sqrt{3}$ sq. units

**Answer: C**

## Solution:



$$\begin{aligned}\text{Required area } &= 2\times \text{ Area of }\triangle PBO\\ &= 2\times\frac{1}{2}\times 2\times 2\sqrt{3}\\ &= 4\sqrt{3}\text{ sq. units}\end{aligned}$$

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# Question 21

**f : $\mathbb{R}-\left(-\frac{3}{5}\right)\rightarrow\mathbb{R}$ is defined by $f(x)=\frac{3x-2}{5x+3}$, then $f\circ f(1)$ is**

**Options:**

A. 1

B. $\frac{-13}{29}$

C. $\frac{13}{29}$

D. $-1$

**Answer: B**

## Solution:

$$\begin{aligned}f(x) &= \frac{3x-2}{5x+3}\\ f(f(x)) &= \frac{3\left(\frac{3x-2}{5x+3}\right)-2}{5\left(\frac{3x-2}{5x+3}\right)+3}\\ &= \frac{3(3x-2)-2(5x+3)}{5(3x-2)+3(5x+3)}\\ &= \frac{9x-6-10x-6}{15x-10+15x+9}\\ &= \frac{-x-12}{30x-1}\\ \text{fof }(1) &= \frac{-1-12}{30-1}=\frac{-13}{29}\end{aligned}$$

---

# Question 22

**The value of** $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ **is**

**Options:**

A. $\frac{23}{25}$

B. $\frac{25}{23}$

C. $\frac{23}{24}$

D. $\frac{24}{23}$

**Answer: B**

## Solution:

$$\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$$
$$=\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+2\times\frac{n(n+1)}{2}\right)\right)$$
$$=\cot\left(\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))\right)$$
$$=\cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)\right)$$
$$=\cot\left(\sum_{n=1}^{23}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right)$$
$$=\cot\left(\sum_{n=1}^{23}\tan^{-1}(n+1)-\sum_{n=1}^{23}\tan^{-1}n\right)$$
$$=\cot\left[\left(\tan^{-1}(2)+\tan^{-1}(3)+\ldots+\tan^{-1}(24)\right)-\left(\tan^{-1}(1)+\tan^{-1}(2)+\ldots.+\tan^{-1}(23)\right)\right]$$
$$=\cot\left(\tan^{-1}(24)-\tan^{-1}(1)\right)$$
$$=\cot\left(\tan^{-1}\left(\frac{24-1}{1+24(1)}\right)\right)$$
$$=\cot\left(\tan^{-1}\left(\frac{23}{25}\right)\right)$$
$$=\cot\left(\cot^{-1}\left(\frac{25}{23}\right)\right)$$
$$=\frac{25}{23}$$

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# Question 23

## An object is moving in the clockwise direction around the unit circle $x^2 + y^2 = 1$. As it passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, its $y$-co-ordinate is decreasing at the rate of 3 units per sec. The rate at which the $x$-co-ordinate changes at this point is

**Options:**

A. 2 units/sec

B. $3\sqrt{3}$ units/sec

C. $\sqrt{3}$ units /sec

D. $2\sqrt{3}$ units /sec

**Answer: B**

## Solution:

Given equation is $x^2 + y^2 = 1$

Differentiating w.r.t. t, we get

$$2x\frac{\mathrm{d}x}{\mathrm{d}t} + 2y\frac{\mathrm{d}y}{\mathrm{d}t} = 0$$
$$2x\frac{\mathrm{d}x}{\mathrm{d}t} + 2y(-3) = 0$$
$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{6y}{2x}$$
$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3y}{x}$$
$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)} = \frac{3 \times \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$
$$= 3\sqrt{3} \text{ units /sec}$$

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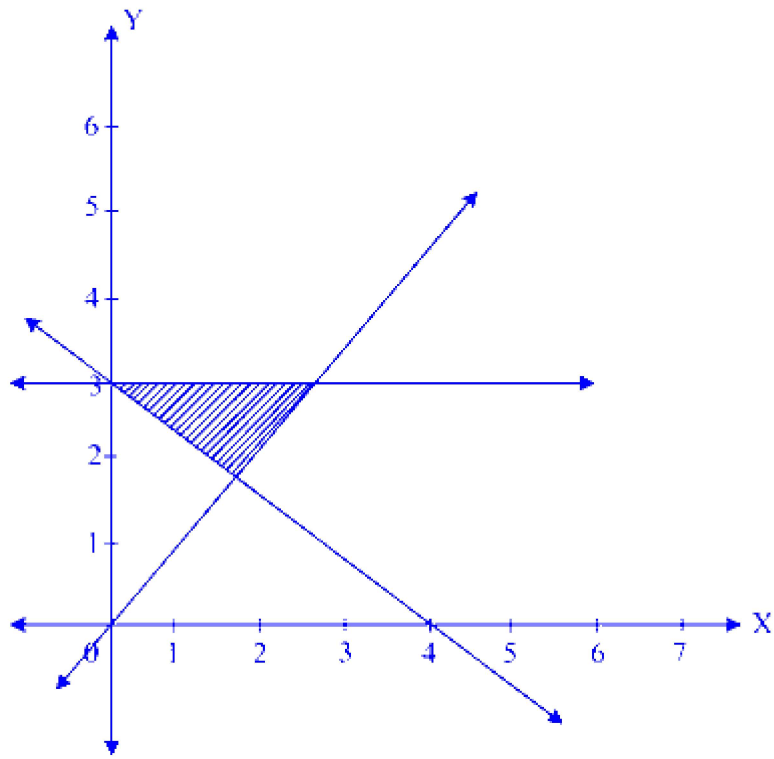
# Question 24

## If feasible region is as shown in the figure, then the related inequalities are




**Options:**

A. $3x + 4y \geq 12, y - x \geq 0, y \leq 3, x, y \geq 0$

B. $3x + 4y \leq 12, y - x \leq 0, y \geq 3, x, y \geq 0$

C. $3x + 4y \leq 12, y - x \geq 0, y \leq 3, x, y \geq 0$

D. $3x + 4y \geq 12, y - x \leq 0, y \geq 3, x, y \geq 0$

**Answer: A**

# Solution:

The shaded region lies:

on non-origin side of line $3x + 4y = 12$ i.e., $3x + 4y \geq 12$,

on the side of the line $y - x = 0$, where $y \geq x$ i.e., $y - x \geq 0$,

on origin side of line $y = 3$ i.e., $y \leq 3$,

and in first quadrant i.e., $x \geq 0, y \geq 0$.

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# Question 25

$$\int \frac{e^{\tan^{-1} x}}{1+x^2}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] \mathrm{d}x, x>0=$$

**Options:**

A. $\left(\tan^{-1} x\right)^2 \mathrm{e}^{\tan^{-1} x}+\mathrm{c}$, where c is a constant of integration.

B. $\left(\tan^{-1} x\right) \mathrm{e}^{\tan^{-1} x}+\mathrm{c}$, where c is a constant of integration.

C. $\left(\tan^{-1} x\right) \mathrm{e}^{2 \tan^{-1} x}+\mathrm{c}$, where c is a constant of integration.

D. $\left(\tan^{-1} x\right)^2 \mathrm{e}^{2 \tan^{-1} x}+c$, where c is a constant of integration.

**Answer: A**

## Solution:

$\int \frac{e^{\tan^{-1} x}}{1+x^2}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] \mathrm{d}x,$

Put $x=\tan \mathrm{t}$

$\therefore \quad \mathrm{d}x=\sec^2 \mathrm{t} d\mathrm{t}$

$$\begin{aligned}
\therefore \quad I &= \int \frac{e^{\operatorname{sen}^{-1}(\tan t)}}{1+\tan^2 t}\left[\left(\sec^{-1}\sqrt{1+\tan^2 t}\right)^2+\cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\right] \sec^2 t\, dt \\
&= \int \frac{e^t}{\sec^2 t}\left[\left(\sec^{-1}(\sec t)\right)^2+\cos^{-1}(\cos 2t)\right] \sec^2 t dt \\
&= \int e^t\left[t^2+2t\right] dt \\
&= e^t \cdot t^2+c \quad \ldots\left[\int e^x f(x) \cdot f'(x)=e^x f(x)+c\right] \\
&= t^2 \cdot e^t+c \\
&= \left(\tan^{-1} x\right)^2 e^{\tan^{-1} x}+c
\end{aligned}$$

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# Question 26

**If $\mathrm{f}(x)$ is a function satisfying $\mathrm{f}'(x)=\mathrm{f}(x)$ with $\mathrm{f}(0)=1$ and $\mathrm{g}(x)$ is a function that satisfies $\mathrm{f}(x)+\mathrm{g}(x)=x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is**

**Options:**

A. $e-\frac{e^2}{2}-\frac{5}{2}$

B. $\mathrm{e}+\frac{\mathrm{e}^2}{2}-\frac{3}{2}$

C. $\mathrm{e}-\frac{\mathrm{e}^2}{2}-\frac{3}{2}$

D. $\mathrm{e}+\frac{\mathrm{e}^2}{2}+\frac{5}{2}$

**Answer: C**

## Solution:

As $\mathrm{f}'(x)=\mathrm{f}(x)$

$\frac{\mathrm{f}'(x)}{\mathrm{f}(x)}=1$

Integrating on both sides, we get

$\log \mathrm{f}(x)=x+\mathrm{c}$ ..... (i)

As $\mathrm{f}(0)=1$

$\therefore \quad$ (i) $\Rightarrow \mathrm{c}=0$
$\therefore \quad \log \mathrm{f}(x)=x$
$\therefore \quad \mathrm{f}(x)=\mathrm{e}^x$

As $\mathrm{f}(x)+\mathrm{g}(x)=x^2$

$\mathrm{g}(x)=x^2-\mathrm{e}^x$

$$\begin{aligned}\therefore \quad \mathrm{f}(x)\mathrm{g}(x) &= \mathrm{e}^x\left(x^2-\mathrm{e}^x\right)\\ &= \int_0^1\left(\mathrm{e}^x x^2-\mathrm{e}^{2x}\right)\mathrm{d}x\\ &= \left[\left(x^2-2x+2\right)\mathrm{e}^x\right]_0^1-\frac{1}{2}\mathrm{e}^2+\frac{1}{2}\\ &= \mathrm{e}-\frac{1}{2}\mathrm{e}^2-\frac{3}{2}\end{aligned}$$

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# Question 27

## The foot of the perpendicular drawn from the origin to the plane is $(4, -2, 5)$, then the Cartesian equation of the plane is

**Options:**

A. $4x - 2y + 5z = 45$

B. $-4x + 2y + 5z = 45$

C. $4x - 2y + 5z + 45 = 0$

D. $4x + 2y - 5z + 45 = 0$

**Answer: A**

## Solution:

Substitute $x = 4, y = -2, \mathrm{z} = 5$ in all options.

$\therefore$ For option A

$$\begin{aligned} 4x - 2y + 5z &= 4(4) - 2(-2) + 5(5) \\ &= 16 + 4 + 25 \\ &= 45 \end{aligned}$$

$\therefore$ Only option A is satisfied by $(4, -2, 5)$

---

# Question 28

## $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

**Options:**

A. $\frac{1}{24}$

B. $\frac{1}{16}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

**Answer: B**

## Solution:

$$\text{Let I} = \lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$
$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x(1 - \sin x)}{\sin x(\pi - 2x)^3}$$
$$\text{Put } x = \frac{\pi}{2} - \text{h}$$
$$\therefore \quad \pi - 2x = 2\text{ h}$$
$$\text{As } x \to \frac{\pi}{2},\ \text{h} \to 0$$

$$\therefore \quad I = \lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2} - h\right)\left(1 - \sin\left(\frac{\pi}{2} - h\right)\right)}{\sin\left(\frac{\pi}{2} - h\right)(2h)^3}$$
$$= \lim_{h \to 0} \frac{\text{sinh} \cdot 2\sin^2\left(\frac{h}{2}\right)}{\text{cosh} \cdot 8h^3}$$
$$= \frac{2}{8} \lim_{h \to 0} \frac{\text{sinh}}{h} \times \lim_{h \to 0} \frac{\sin^2 \frac{h}{2}}{\frac{h^2}{4} \cdot 4}$$
$$= \frac{2}{8}(1) \times (1) \times \frac{1}{4}$$
$$= \frac{1}{16}$$

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# Question 29

### If the distance between the parallel lines given by the equation $x^2 + 4xy + py^2 + 3x + qy - 4 = 0$ is $\lambda$, then $\lambda^2 =$

**Options:**

A. 5

B. $\sqrt{5}$

C. 25

D. $\frac{9}{5}$

**Answer: A**

## Solution:

Given equation is

$x^2 + 4xy + 4y^2 + 3x + 6y - 4 = 0$
$(x + 2y)^2 + 3(x + 2y) - 4 = 0$
$(x + 2y + 4)(x + 2y - 1) = 0$

$\therefore$ The lines are: $x + 2y + 4 = 0$ and $x + 2y - 1 = 0$

$$\therefore \quad \text{Required distance } = \frac{|4 - (-1)|}{\sqrt{1+4}} = \frac{5}{\sqrt{5}} = \sqrt{5} \text{ units}$$

$\lambda = \sqrt{5}$ units
$\lambda^2 = 5$

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# Question 30

## The particular solution of the differential equation $\left(1 + y^2\right)\mathrm{d}x - xy\,\mathrm{d}y = 0$ at $x = 1, y = 0$, represents

**Options:**

A. circle

B. pair of straight lines

C. hyperbola

D. ellipse

**Answer: C**

## Solution:

$$\left(1+y^2\right)\mathrm{d}x - xy\,\mathrm{d}y = 0$$

$\therefore \quad \left(1+y^2\right)\mathrm{d}x = xy\,\mathrm{d}y$

$\therefore \quad \frac{1}{x}\,\mathrm{d}x = \frac{y\,\mathrm{d}y}{1+y^2}$

Integrating both sides, we get

$$\int \frac{1}{x}\,\mathrm{d}x = \int \frac{y}{1+y^2}\,\mathrm{d}y$$

$$\log x = \frac{1}{2}\log\left(1+y^2\right) + \mathrm{c}$$

At $x = 1, y = 0$ .... [Given]

$\therefore \quad 0 = \frac{1}{2}\log(1+0) + \mathrm{c}$

$\therefore \quad \mathrm{c} = 0$

$\therefore \quad \log x = \frac{1}{2}\log\left(1+y^2\right)$

$\therefore \quad x^2 = 1 + y^2$

$\therefore \quad x^2 - y^2 = 1,$

Which is a rectangular hyperbola.

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# Question 31

## If at the end of certain meeting, everyone had shaken hands with everyone else, it was found that 45 handshakes were exchanged, then the number of members present at the meeting, are

**Options:**

A. 10

B. 15

C. 20

D. 21

**Answer: A**

## Solution:

Let 'n' be the number of members in the meeting

$\therefore$ Total number of handshakes $= {}^{\mathrm{n}}\mathrm{C}_2$

$\therefore \quad {}^{n}C_2 = 45$

$$\frac{n!}{2!(n-2)!} = 45$$
$$\frac{n(n-1)(n-2)!}{2\times(n-2)!} = 45$$
$$n(n-1) = 90$$
$$\therefore \quad n^2 - n - 90 = 0$$
$$n = 10 \text{ or } n = -9 \text{ (not possible)}$$
$$\therefore \quad n = 10$$

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# Question 32

## If $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, then $\cos^2 48^\circ - \sin^2 12^\circ$ has the value

**Options:**

A. $\frac{-\sqrt{5}+1}{8}$

B. $\frac{\sqrt{5}-1}{8}$

C. $\frac{\sqrt{5}+1}{8}$

D. $\frac{-1-\sqrt{5}}{8}$

**Answer: C**

## Solution:

$$\cos^2 \mathrm{A} - \sin^2 \mathrm{B} = \cos(\mathrm{A}+\mathrm{B}) \cdot \cos(\mathrm{A}-\mathrm{B})$$
$$\therefore \quad \cos^2 48^\circ - \sin^2 12^\circ = \cos(60^\circ) \cdot \cos(36^\circ)$$
$$= \frac{1}{2} \cdot \left(1 - 2\sin^2 \frac{36}{2}\right)$$
$$= \frac{1}{2}\left(1 - 2\sin^2 18^\circ\right)$$
$$= \frac{1}{2}\left[1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2\right]$$
$$= \frac{\sqrt{5}+1}{8}$$

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# Question 33

**If $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$, then the inverse of $\left(2A^2 + 5A\right)$ is**

**Options:**

A. $\frac{1}{95}\begin{bmatrix} 7 & 3 \\ 3 & 4 \end{bmatrix}$

B. $\frac{1}{95}\begin{bmatrix} -7 & 3 \\ 3 & -4 \end{bmatrix}$

C. $\frac{1}{95}\begin{bmatrix} -7 & -3 \\ 3 & 4 \end{bmatrix}$

D. $\frac{1}{95}\begin{bmatrix} 4 & 3 \\ 3 & 7 \end{bmatrix}$

**Answer: A**

## Solution:

$A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix}$

Now, $2A^2 + 5A = 2\begin{bmatrix} 5 & -5 \\ -5 & 10 \end{bmatrix} + 5\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 20 & -15 \\ -15 & 35 \end{bmatrix}$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\therefore \quad \left(2\text{ A}^2 + 5\text{ A}\right)^{-1} = \frac{1}{475}\begin{bmatrix} 35 & 15 \\ 15 & 20 \end{bmatrix}$

$= \frac{1}{95}\begin{bmatrix} 7 & 3 \\ 3 & 4 \end{bmatrix}$

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# Question 34

**If $I=\int \frac{\sin x+\sin^3 x}{\cos 2x} dx = P\cos x + Q\log\left|\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}\right|$ (where $c$ is a constant of integration), then values of $\mathrm{P}$ and $\mathrm{Q}$ are respectively**

**Options:**

A. $\frac{1}{2}, \frac{3}{4\sqrt{2}}$

B. $\frac{1}{2}, \frac{-3}{4\sqrt{2}}$

C. $\frac{1}{2}, \frac{3}{2\sqrt{2}}$

D. $\frac{1}{2}, \frac{-3}{2\sqrt{2}}$

**Answer: B**

## Solution:

$$\begin{aligned} I &= \int \frac{\sin x+\sin^3 x}{\cos 2x} dx \\ &= \int \frac{\sin x\left(1+\sin^2 x\right)}{2\cos^2 x-1} dx \\ &= \int \frac{\sin x\left(2-\cos^2 x\right)}{2\cos^2 x-1} dx \end{aligned}$$

$$\begin{aligned} &\text{Put } \cos x = \mathrm{t} \\ &-\sin x \, \mathrm{d}x = \mathrm{dt} \end{aligned}$$

$$\begin{aligned} I &= \int \frac{t^2-2}{2t^2-1} dt \\ &= \frac{1}{2}\int \frac{2t^2-4}{2t^2-1} dt \\ &= \frac{1}{2}\left[\int \frac{2t^2-1}{2t^2-1} dt - \int \frac{3}{2t^2-1} dt\right] \\ I &= \frac{1}{2}t - \frac{1}{2} \times \frac{3}{2\sqrt{2}} \log\left|\frac{\sqrt{2}t-1}{\sqrt{2}t+1}\right| + c \\ &= \frac{1}{2}\cos x - \frac{3}{4\sqrt{2}} \log\left|\frac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}\right| + c \\ \therefore \quad & P = \frac{1}{2} \text{ and } Q = \frac{-3}{4\sqrt{2}} \end{aligned}$$

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# Question 35

## The negation of the statement $(p \wedge q) \rightarrow (\sim p \vee r)$ is

**Options:**

A. $p \vee q \vee \sim r$

B. p ∧ q∧ ∼ r

C. $\sim p \vee q \wedge r$

D. $\sim p \vee \sim q \vee \sim r$

**Answer: B**

## Solution:

$\sim [(p \wedge q) \rightarrow (\sim p \vee r)]$
$\equiv (p \wedge q) \wedge \sim (\sim p \vee r) \ldots [\because \sim (p \rightarrow q) \equiv p \wedge \sim q]$
$\equiv p \wedge q \wedge p \wedge \sim r \quad \ldots [\text{ Associative Law }]$
$\equiv p \wedge q \wedge \sim r$ ...[Idempotent Law]

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# Question 36

## The probability mass function of random variable X is given by

$$P[X=r] = \begin{cases} \frac{{}^nC_r}{32}, & n, r \in \mathbb{N} \\ 0, & \text{otherwise} \end{cases} \text{, then } P[X \leq 2] =$$

**Options:**

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{5}$

**Answer: B**

## Solution:

$$\text{Since } \sum_{x=0}^{n} P(X=n)=1$$

$$\frac{{}^{n}C_0+{}^{n}C_1+{}^{n}C_2\ldots+\ldots+{}^{n}C_n}{32}=1$$

$$2^n=32$$

$$\therefore \quad n=5$$

$$P(X\le 2)=P(X=0)+P(X=1)+P(X=2)$$

$$=\frac{{}^{5}C_0}{32}+\frac{{}^{5}C_1}{32}+\frac{{}^{5}C_2}{32}=\frac{1}{2}$$

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# Question 37

**The distance of a point $(2,5)$ from the line $3x+y+4=0$ measured along the line $L_1$ and $L_1$ are same. If slope of line $L_1$ is $\frac{3}{4}$, then slope of the line $\mathrm{L}_2$ is**

**Options:**

A. $-\frac{3}{4}$

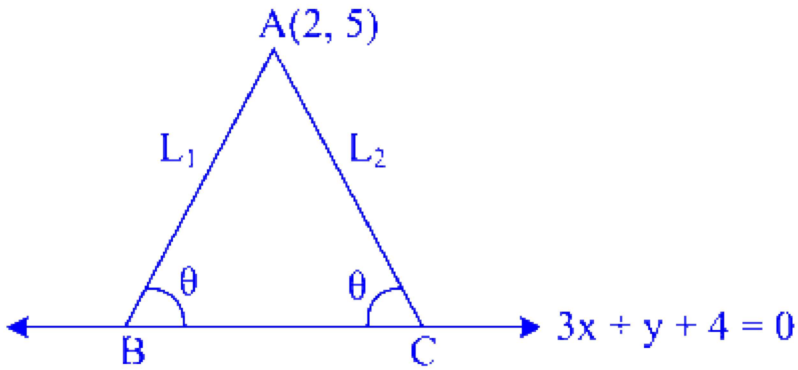
B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. 0

**Answer: D**

## Solution:




According to the given condition, in $\triangle ABC$, $AB = AC$.

$\therefore \quad \triangle ABC$ is an isosceles triangle.

Let $m, m_1,\ m_2$ be the slopes of given line, $L_1$ and $L_2$ respectively.

$\therefore \quad m = -3,\ m_1 = \frac{3}{4}$

$\therefore \quad \left|\frac{m-m_1}{1+mm_1}\right| = \left|\frac{m-m_2}{1+mm_2}\right|$

$\therefore \quad 3 = \left|\frac{-3-m_2}{1+3\ m_2}\right|$

$\Rightarrow m_2 = 0$

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# Question 38

### The distance of the point having position vector $\hat{i} - 2\hat{j} - 6\hat{k}$, from the straight line passing through the point $(2, -3, -4)$ and parallel to the vector $6\hat{i} + 3\hat{j} - 4\hat{k}$ is units.

**Options:**

A. $\sqrt{\frac{340}{61}}$

B. $\frac{341}{61}$

C. $\frac{\sqrt{341}}{61}$

D. $\sqrt{\frac{341}{61}}$

**Answer: D**

## Solution:

Given equation of the line is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

$$\therefore \quad \bar{r} = 2\hat{i} - 3\hat{j} - 4\hat{k} + \lambda(6\hat{i} + 3\hat{j} - 4\hat{k})$$

To find: Its distance from point $\bar{\alpha} = \hat{i} - 2\hat{j} - 6\hat{k}$,

$$\therefore \quad \text{Required distance } = \sqrt{|\bar{\alpha} - \bar{a}|^2 - \left[\frac{(\bar{\alpha} - \bar{a}) \cdot \bar{b}}{|\bar{b}|}\right]^2}$$

Here, $|\bar{\alpha} - \bar{a}|^2 = 6$ and $\left[\frac{(\bar{\alpha} - \bar{a}) \cdot \bar{b}}{|\bar{b}|}\right]^2 = \frac{25}{61}$

$$\therefore \quad \text{Required distance } = \sqrt{\frac{341}{61}}$$

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# Question 39

## A vector $\overrightarrow{n}$ is inclined to X-axis at $45^\circ$, Y-axis at $60^\circ$ and at an acute angle to Z-axis If $\overrightarrow{n}$ is normal to a plane passing through the point $(-\sqrt{2}, 1, 1)$, then equation of the plane is

**Options:**

A. $\sqrt{2}x + y + z = 0$

B. $x + \sqrt{2}y + z = 1$

C. $-\sqrt{2}x + y + 2z = 5$

D. $x + y + \sqrt{2}z = 1$

**Answer: A**

## Solution:

Let $\overrightarrow{n}$ be inclined at angles $\alpha, \beta, \gamma$ to X, Y, Z axes respectively.

$\alpha = 45^\circ, \beta = 60^\circ, \gamma = ?$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \quad \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$

$\therefore \quad \cos^2\gamma = \frac{1}{4}$

$\therefore \quad \gamma = 60^\circ$

$\vec{n} = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}$

$\therefore \quad \vec{n} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$

$\therefore$ Equation of the required plane is

$\frac{1}{\sqrt{2}}(x + \sqrt{2}) + \frac{1}{2}(y - 1) + \frac{1}{2}(z - 1) = 0$

i.e., $\sqrt{2}x + y + z = 0$

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# Question 40

**If $\cos^{-1} x - \cos^{-1} \frac{y}{3} = \alpha$, where $-1 \leq x \leq 1, -3 \leq y \leq 3, x \leq \frac{y}{3}$, then for all $x, y$ $9x^2 - 6xy\cos\alpha + y^2$ is equal to**

**Options:**

A. $\sin^2\alpha$

B. $3\sin^2\alpha$

C. $9\sin^2\alpha$

D. $\frac{4}{9}\sin^2\alpha$

**Answer: C**

## Solution:

$$\cos^{-1} a - \cos^{-1} b = \cos^{-1}\left(ab + \sqrt{1-a^2} \cdot \sqrt{1-b^2}\right)$$

$$\therefore \quad \cos^{-1} x - \cos^{-1} \frac{y}{3}$$

$$= \cos^{-1}\left(\frac{xy}{3} + \sqrt{1-x^2} \cdot \sqrt{1-\frac{y^2}{9}}\right) = a$$

$$\therefore \quad \frac{xy}{3} + \frac{\sqrt{1-x^2} \cdot \sqrt{9-y^2}}{3} = \cos\alpha$$

$$xy + \sqrt{1-x^2} \cdot \sqrt{9-y^2} = 3\cos\alpha$$

$$xy - 3\cos\alpha = -\sqrt{1-x^2} \cdot \sqrt{9-y^2}$$

squaring on both sides, we get

$$x^2y^2 - 6xy\cos\alpha + 9\cos^2\alpha = \left(1-x^2\right)\left(9-y^2\right)$$

$$x^2y^2 - 6xy\cos\alpha + 9\cos^2\alpha = 9 - y^2 - 9x^2 + x^2y^2$$

$$\text{i.e., } 9x^2 - 6xy\cos\alpha + y^2 = 9\sin^2\alpha$$

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# Question 41

## If $f(1) = 1, f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

**Options:**

A. 12

B. 19

C. 23

D. 33

**Answer: D**

## Solution:

$$\text{Let } y = \mathrm{f}(\mathrm{f}(\mathrm{f}(x))) + (\mathrm{f}(x))^2$$
$$\therefore \quad \frac{dy}{dx} = f'(f(f(x))) \cdot \mathrm{f}'(\mathrm{f}(x)) \cdot \mathrm{f}'(x) + 2\mathrm{f}(x)\mathrm{f}'(x)$$
$$\left.\frac{dy}{dx}\right|_{x=1} = f'(f(f(1))) \cdot f'(f(1)) \cdot f'(1) + 2f(1)f'(1)$$
$$= 3 \cdot 3 \cdot 3 + 2 \cdot 1 \cdot 3$$
$$= 33$$

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# Question 42

## Three fair coins numbered 1 and 0 are tossed simultaneously. Then variance Var (X) of the probability distribution of random variable $\mathrm{X}$, where $\mathrm{X}$ is the sum of numbers on the uppermost faces, is

**Options:**

A. 0.7

B. 0.75

C. 0.65

D. 0.62

**Answer: B**

## Solution:

Three fair coins numbered 1,0 are tossed.⁄p>

$$\therefore \quad \text{Sample space } = \{111, 110, 101, 011, 100, 010, 001, 000\}$$
$$\therefore \quad n(S) = 8$$

X represents the sum of numbers on upper most face

$\mathrm{P}(\mathrm{X} = 0) = \frac{1}{8}$
$\mathrm{P}(\mathrm{X} = 1) = \frac{3}{8}$
$\mathrm{P}(\mathrm{X} = 2) = \frac{3}{8}$,
$\mathrm{P}(\mathrm{X} = 3) = \frac{1}{8}$

$\therefore$ Probability distribution of X is

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

$$\mathrm{E}(\mathrm{X})=\sum_{x=0}^{3} x_{\mathrm{i}} \mathrm{P}\left(x_{\mathrm{i}}\right)$$

$$\therefore \quad \mathrm{E}(\mathrm{X})=0 \times \frac{1}{8}+1 \times \frac{3}{8}+2 \times \frac{3}{8}+3 \times \frac{1}{8}$$

$$=\frac{12}{8}=\frac{3}{2}$$

$$\mathrm{E}\left(\mathrm{X}^{2}\right)=\sum_{x=0}^{3} x_{\mathrm{i}}^{2} \mathrm{P}\left(x_{\mathrm{i}}\right)=3$$

Variance of $\mathrm{X}=\mathrm{E}\left(\mathrm{X}^{2}\right)-[\mathrm{E}(\mathrm{X})]^{2}$

$$=3-\left(\frac{3}{2}\right)^{2}$$

$$=3-\frac{9}{4}$$

$$=\frac{3}{4}=0.75$$

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# Question 43

$$\int \frac{1}{\sin(x-a)\sin x} dx =$$

**Options:**

A. $\sin \mathrm{a}(\log (\sin (x-\mathrm{a}) \cdot \operatorname{cosec} x))+\mathrm{c}$, where c is a constant of integration.

B. $\operatorname{cosec} a(\log (\sin (x-a) \cdot \operatorname{cosec} x))+c$, where c is a constant of integration.

C. $-\sin \mathrm{a}(\log (\sin (x-\mathrm{a}) \cdot \sin x))+\mathrm{c}$, where c is a constant of integration.

D. $-\operatorname{cosec} \mathrm{a}(\log (\sin (x-\mathrm{a}) \cdot \sin x))+\mathrm{c}$, where c is a constant of integration.

**Answer: B**

## Solution:

$$\text{Let I} = \int \frac{1}{\sin(x-\mathrm{a})\sin x}\,\mathrm{d}x$$
$$\text{Put } x-\mathrm{a}=\mathrm{t} \rightarrow x=\mathrm{a}+\mathrm{t}$$
$$\mathrm{d}x=\mathrm{dt}$$

$$\begin{aligned}
\therefore \quad I &= \int \frac{1}{\sin t\cdot\sin(a+t)}dt \\
&= \frac{1}{\sin a}\int \frac{\sin a}{\sin t\cdot\sin(a+t)}dt \\
&= \frac{1}{\sin a}\int \frac{\sin((a+t)-t)}{\sin(a+t)\cdot\sin t}dt \\
&= \frac{1}{\sin a}\left[\int \frac{\sin(a+t)\cos t}{\sin(a+t)\sin t}dt - \int \frac{\sin t\cos(a+t)}{\sin(a+t)\sin t}dt\right] \\
&= \frac{1}{\sin a}\left[\int \cot t dt - \int \cot(a+t)dt\right] \\
&= \operatorname{cosec} a[\log|\sin t| - \log|\sin(a+t)|] + c \\
&= \operatorname{cosec} a\left[\log\left|\frac{\sin t}{\sin(a+t)}\right|\right] + c \\
&= \operatorname{cosec} a[\log(\sin(x-a)\cdot\operatorname{cosec} x)] + c
\end{aligned}$$

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# Question 44

## The derivative of $\mathrm{f}(\sec x)$ with respect to $g(\tan x)$ at $x=\frac{\pi}{4}$, where $f'(\sqrt{2})=4$ and $g'(1)=2$, is

**Options:**

A. 2

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. $\frac{1}{2\sqrt{2}}$

**Answer: C**

## Solution:

Let $y = \mathrm{f}(\sec x)$ and $\mathrm{z} = \mathrm{g}(\tan x)$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(\sec x) \cdot \sec x \tan x$

$\frac{dz}{dx} = g'(\tan x) \cdot \sec^2 x$

Now, $\frac{\mathrm{d}y}{\mathrm{dz}} = \frac{\mathrm{f}'(\sec x) \sec x \tan x}{\mathrm{g}'(\tan x) \sec^2 x}$

$\frac{dy}{dz} = \frac{f'(\sec x) \tan x}{g'(\tan x) \cdot \sec x}$

$\left.\frac{\mathrm{dy}}{\mathrm{dz}}\right|_{x=\frac{\pi}{4}} = \frac{\mathrm{f}'\left(\sec \frac{\pi}{4}\right) \tan \frac{\pi}{4}}{\mathrm{g}'\left(\tan \frac{\pi}{4}\right) \sec \frac{\pi}{4}}$

$= \frac{f'(\sqrt{2}) \cdot (1)}{g'(1) \cdot \sqrt{2}} \Rightarrow \frac{4 \times 1}{2\sqrt{2}}$

$= \sqrt{2}$

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# Question 45

**The scalar product of the vector $\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}$ with a unit vector along the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1 , then value of $\lambda$ is**

**Options:**

A. 1

B. 2

C. 3

D. 4

**Answer: A**

## Solution:

$(2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k}) = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

Unit vector along the above vector is

$\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$

Scalar product of $(\hat{i}+\hat{j}+\hat{k})$ with this unit vector is 1 .

$\therefore \quad (\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}) \cdot \frac{(2+\lambda)\hat{\mathrm{i}}+6\hat{\mathrm{j}}-2\hat{\mathrm{k}}}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$

$\sqrt{\lambda^2+4\lambda+44}=\lambda+6$

$\therefore \quad \lambda^2+4\lambda+44=(\lambda+6)^2$

$8\lambda=8$

$\lambda=1$

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# Question 46

## The number of discontinuities of the greatest integer function $\mathrm{f}(x)=[x], x \in\left(-\frac{7}{2}, 100\right)$

**Options:**

A. 104

B. 100

C. 102

D. 103

**Answer: D**

## Solution:

$\mathrm{f}(x)=[x]$

$x \in\left(\frac{-7}{2}, 100\right)$

$x \in(-3.5,100)$

As we know greatest integer is discontinuous on integer values in given interval, the integer values are $\{-3,-2,-1,0 \ldots 99\}$

$\therefore$ Total number of discontinuities are 103 .

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# Question 47

**If $[(\bar{a}+2\bar{b}+3\bar{c})\times(\bar{b}+2\bar{c}+3\bar{a})]\cdot(\bar{c}+2\bar{a}+3\bar{b})=54$ then the value of $\begin{bmatrix}\bar{a} & \bar{b} & \bar{c}\end{bmatrix}$ is**

**Options:**

A. 0

B. 1

C. 3

D. 2

**Answer: C**

## Solution:

R.H.S. of the given equality can be written as

$$
\begin{aligned}
& (2\bar{a}+3\bar{b}+\bar{c})\cdot[(\bar{a}+2\bar{b}+3\bar{c})\times(3\bar{a}+\bar{b}+2\bar{c})] \\
& =(2\bar{a}+3\bar{b}+\bar{c})\cdot[3(\bar{a}\times\bar{a})+(\bar{a}\times\bar{b})+2(\bar{a}\times\bar{c}) \\
& +6(\bar{b}\times\bar{a})+2(\bar{b}\times\bar{b})+4(\bar{b}\times\bar{c}) \\
& +9(\bar{c}\times\bar{a})+3(\bar{c}\times\bar{b})+6(\bar{c}\times\bar{c})] \\
& =(2\bar{a}+3\bar{b}+\bar{c})[0+(\bar{a}\times\bar{b})+2(\bar{a}\times\bar{c}) \\
& -6(\bar{a}\times\bar{b})+0+4(\bar{b}\times\bar{c}) \\
& -9(\bar{a}\times\bar{c})-3(\bar{b}\times\bar{c})+0] \\
& =(2\bar{a}+3\bar{b}+\bar{c})[-5(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})-7(\bar{a}\times\bar{c})] \\
& =-10[\bar{a}\cdot(\bar{a}\times\bar{b})]+2[\bar{a}\cdot(\bar{b}\times\bar{c})]-14[\bar{a}\cdot(\bar{a}\times\bar{c})] \\
& -15[\bar{b}\cdot(\bar{a}\times\bar{b})]+3[\bar{b}\cdot(\bar{b}\times\bar{c})]-21[\bar{b}\cdot(\bar{a}\times\bar{c})] \\
& -5[\bar{c}\cdot(\bar{a}\times\bar{b})]+[\bar{c}\cdot(\bar{b}\times\bar{c})]-7[\bar{c}\cdot(\bar{a}\times\bar{c})] \\
& =0+2[\bar{a}\bar{b}\bar{c}]+0 \\
& +0+0+21[\bar{a}\bar{b}\bar{c}] \\
& -5[\bar{a}\bar{b}\bar{c}]+0+0 \\
& =18[\bar{a}\bar{b}\bar{c}] \\
\therefore\quad & 18[\bar{a}\bar{b}\bar{c}]=54 \\
& \Rightarrow[\bar{a}\bar{b}\bar{c}]=3
\end{aligned}
$$

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# Question 48

**A spherical raindrop evaporates at a rate proportional to its surface area. If originally its radius is $3\ \mathrm{mm}$ and 1 hour later it reduces to $2\ \mathrm{mm}$, then the expression for the radius $R$ of the raindrop at any time $t$ is**

**Options:**

A. $6\mathrm{R} = \mathrm{t} + 2$

B. $\mathrm{R}(\mathrm{t} + 2) = 6$

C. $\mathrm{R} = 6(\mathrm{t} + 2)$

D. $6\mathrm{R} = \mathrm{t}$

**Answer: B**

## Solution:

According to the given conditions, when $\mathrm{t} = 0, \mathrm{R} = 3$ and when $\mathrm{t} = 1, \mathrm{R} = 2$

This condition is satisfied by only option (B)

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# Question 49

## If the Cartesian equation of a line is $6x - 2 = 3y + 1 = 2z - 2$, then the vector equation of the line is

**Options:**

A. $\overline{\mathrm{r}} = \left(\frac{1}{3}\hat{\mathrm{i}} - \frac{1}{3}\hat{\mathrm{j}} + \hat{\mathrm{k}}\right) + \lambda(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}})$

B. $\overline{\mathrm{r}} = (\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}})$

C. $\overline{\mathrm{r}} = \left(\frac{-1}{3}\hat{\mathrm{i}} + \frac{1}{3}\hat{\mathrm{j}} + \hat{\mathrm{k}}\right) + \lambda(\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}})$

D. $\overline{\mathrm{r}} = \left(\frac{1}{3}\hat{\mathrm{i}} - \frac{1}{3}\hat{\mathrm{j}} - \hat{\mathrm{k}}\right) + \lambda(\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}})$

**Answer: A**

## Solution:

Given Cartesian equation of the line is

$6x - 2 = 3y + 1 = 2z - 2$

$\Rightarrow 6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z - 1)$

$\Rightarrow \frac{x - \frac{1}{3}}{\frac{1}{6}} = \frac{y + \frac{1}{3}}{\frac{1}{3}} = \frac{z - 1}{\frac{1}{2}}$

$\Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z - 1}{3}$

$\therefore$ The given line passes through $\left(\frac{1}{3}, \frac{-1}{3}, 1\right)$ and has direction ratios proportional to $1, 2, 3$.

$\therefore$ Vector equation is

$\overline{\mathrm{r}} = \left(\frac{1}{3}\hat{\mathrm{i}} - \frac{1}{3}\hat{\mathrm{j}} + \hat{\mathrm{k}}\right) + \lambda(\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}})$

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# Question 50

## The volume of parallelopiped, whose coterminous edges are given by $\overline{\mathrm{u}} = \hat{\mathrm{i}} + \hat{\mathrm{j}} + \lambda\hat{\mathrm{k}}, \vec{v} = \hat{i} + \hat{j} + 3\hat{k}, \bar{w} = 2\hat{i} + \hat{j} + \hat{k}$ is 1 cu. units. If $\theta$ is the angle between $\bar{u}$ and $\bar{w}$, then the value of $\cos\theta$ is

**Options:**

A. $\frac{3}{4}$

B. $\frac{5}{6}$

C. $\frac{1}{5}$

D. $\frac{1}{6}$

**Answer: B**

## Solution:

Volume of parallelepiped $= [\vec{u} \ \vec{v} \ \vec{w}]$

$$\begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = 1$$

$\Rightarrow \lambda = 2$

$$\therefore \quad \cos\theta = \frac{2+1+2}{\sqrt{6}\cdot\sqrt{6}} = \frac{5}{6}$$

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# Chemistry

# Question 51

## Which of the following molecules has no lone pair of electrons on central atom?

**Options:**

A. $SO_2$

B. $SF_6$

C. $NH_3$

D. $SF_4$

**Answer: B**

## Solution:

| Molecule | No. of lone pairs on the central atom |
|---|---|
| $SO_2$ | 1 |
| $SF_6$ | 0 |
| $NH_3$ | 1 |
| $SF_4$ | 1 |

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# Question 52

## Calculate $\Delta G^{\circ}$ for the cell:

$$Sn_{(s)}\left|Sn^{2+}_{(1M)}\right|\left|Ag^{+}_{(1M)}\right|Ag_{(s)} \text{ at } 25^{\circ}C \left(E^{\circ}_{cell} \quad = 0.90\text{ V}\right)$$

**Options:**

A. $-173.7$ kJ

B. $-225.3$ kJ

C. $-100.2$ kJ

D. $-290.8$ kJ

**Answer: A**

## Solution:

$\Delta G^{\circ} = -nFE^{\circ}_{cell}$

$= -2 \times 96500 \times 0.90$

$= -173700$ J

$= -173.7$ kJ

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# Question 53

## Which from following statements is NOT correct?

**Options:**

A. All alkali metals are silvery white.

B. Density of potassium is less than sodium.

C. Compounds of group-1 elements are diamagnetic.

D. Melting point of group-1 elements increase down the group.

**Answer: D**

## Solution:

Melting point of group-1 elements decreases down the group.

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# Question 54

## A compound made of elements $\mathrm{A}$ and $\mathrm{B}$ form fcc structure. Atoms of A are at the corners and atoms of B are present at the centres of faces of cube. What is the formula of the compound?

**Options:**

A. $\mathrm{AB}$

B. $\mathrm{AB_2}$

C. $\mathrm{AB_3}$

D. $\mathrm{A_2B}$

**Answer: C**

## Solution:

| Atom | Location | Contribution to a unit cell |
|---|---|---|
| A | Corners of cube | $\frac{1}{8} \times 8 = 1$ |
| B | Centres of faces | $\frac{1}{2} \times 6 = 3$ |
| Ratio | $\mathrm{A : B = 1 : 3}$ | |
| Formula | $\mathrm{AB_3}$ | |

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# Question 55

## What are different possible oxidation states exhibited by scandium?

**Options:**

A. $+4$

B. $+5$

C. $+4, +5$

D. $+2, +3$

**Answer: D**

## Solution:

Scandium typically exhibits a $+3$ oxidation state. This is because it has an electronic configuration of [Ar] $3d^1\,4s^2$, and it typically loses the three outer electrons to achieve a noble gas configuration. While a $+2$ oxidation state is theoretically possible, it is not commonly observed. Scandium does not commonly exhibit $+4$ or $+5$ oxidation states.

Thus, the correct answer to the question is :

Option D : $+2, +3$

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# Question 56

## Which from following is the slope of the graph of rate versus concentration of the reactant for first order reaction?

**Options:**

A. $-\mathrm{k}$

B. $\mathrm{k}$

C. $\frac{\mathrm{k}}{2.303}$

D. $\frac{-\mathrm{k}}{2.303}$

**Answer: B**

## Solution:

For a first-order reaction, the rate of the reaction is directly proportional to the concentration of the reactant. The rate law for a first-order reaction can be written as :

$\text{Rate} = k[\text{Reactant}]$

where $k$ is the rate constant.

The graph of rate versus concentration of the reactant for a first-order reaction would be a straight line with a slope equal to the rate constant $k$. This is because the rate is directly proportional to the concentration, making the slope of the line the rate constant itself.

Therefore, the correct answer is :

Option B : $k$

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# Question 57

## What is the packing efficiency of silver metal in its unit cell?

**Options:**

A. $52.4\%$

B. $68.0\%$

C. $32.0\%$

D. $74.0\%$

**Answer: D**

## Solution:

Silver has fcc crystal structure. Packing efficiency of fcc unit cell is 74.0%.

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# Question 58

**Which from following polymers is obtained from $C_2 F_4$ ?**

**Options:**

A. PVC

B. Polyisobutylene

C. Polyacrylonitrile

D. Teflon

**Answer: D**

## Solution:

The monomer used in preparation of teflon is tetrafluoroethylene, $(CF_2 = CF_2)$.

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# Question 59

## Calculate current in ampere required to deposit $4.8 \mathrm{~g} \mathrm{Cu}$ from it's salt solution in 30 minutes. [Molar mass of $\mathrm{Cu} = 63.5 \mathrm{~g} \mathrm{~mol}^{-1}$ ]

**Options:**

A. 8.1 ampere

B. 6.4 ampere

C. 10.5 ampere

D. 12.3 ampere

**Answer: A**

## Solution:

$Cu^{2+}_{(s)} + 2e^- \longrightarrow Cu_{(s)}$

$$\text{Mole ratio } = \frac{1 \text{ mol}}{2 \text{ mole}^-}$$

$$W = \frac{I(A) \times t(s)}{96500\,(C/\text{mole}^-)} \times \text{mole ratio } \times \text{ molar mass}$$

$$4.8 \text{ g} = \frac{I(A) \times 30 \times 60}{96500\,(C/\text{mole}^-)} \times \frac{1 \text{ mol}}{2 \text{ mol e}} \times 63.5 \text{ g mol}^{-1}$$

$$I(A) = \frac{4.8 \times 96500 \times 2}{63.5 \times 30 \times 60} = 8.1 \text{ A}$$

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# Question 60

## Identify number of moles of donor atoms in $2n$ mole of $SCN^-$.

**Options:**

A. 3n

B. 6n

C. 4n

D. n

**Answer: C**

## Solution:

$SCN^-$ has two donor atoms nitrogen and sulfur either of which links to metal atom/ion when in forms a coordinate bond. Therefore, 1 mole of $SCN^-$ will have 2 moles of donor atoms. Hence, $2n$ moles of $SCN^-$ will have $4n$ moles of donor atoms.

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# Question 61

## $0.2$ M aqueous solution of glucose has osmotic pressure 4.9 atm at $300$ K. What is the concentration of glucose if it has osmotic pressure $1.5$ atm at same temperature?

**Options:**

A. 0.03 M

B. 0.04 M

C. 0.05 M

D. 0.06 M

**Answer: D**

## Solution:

$\pi = \mathrm{M} \times \mathrm{R} \times \mathrm{T}$

$\therefore \quad \mathrm{M} = \frac{\pi}{\mathrm{RT}} = \frac{1.5}{0.082 \times 300} = 0.06\ \mathrm{M}$

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# Question 62

## Identify the product when phenol is heated with zinc dust.

**Options:**

A. Benzoquinone

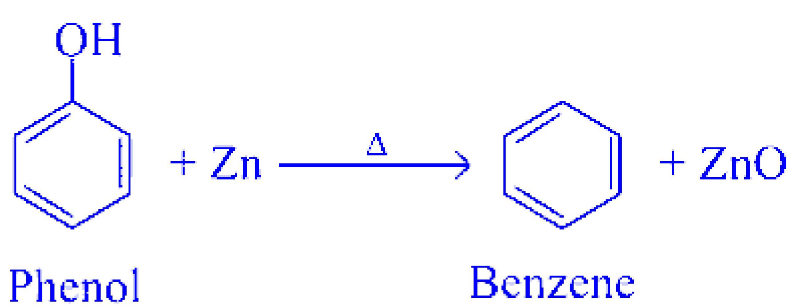
B. Cyclohexane

C. Benzene

D. Cyclohexanol

**Answer: C**

## Solution:



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# Question 63

## Identify the product 'B' in the following reaction.

$$\textbf{Dry ice} \xrightarrow[\text{Dry ether}]{CH_3MgBr} \textbf{A} \xrightarrow[\text{dil. HCl}]{H_2O} \textbf{B}$$

**Options:**

A. Methanoic acid

B. Ethanoic acid

C. Methanol

D. Ethanol

**Answer: B**

## Solution:

$$\underset{\text{Dry ice}}{CO_2} \xrightarrow[\text{Dry ether}]{CH_3MgBr} \underset{(A)}{CH_3-COOMgBr}$$

$$\xrightarrow[\text{dil. HCl}]{H_2O} \underset{\substack{\text{Ethanoic acid} \\ (B)}}{CH_3-COOH} + Mg(OH)Br$$

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# Question 64

## Which among the following reactions exhibits $\Delta H = \Delta U$ ?

**Options:**

A. $H_{2(g)} + Br_{2(g)} \rightarrow 2HBr_{(g)}$

B. $2CO_{(g)} + O_{2(g)} \rightarrow 2CO_{2(g)}$

C. $PCl_{5(g)} \rightarrow PCl_{3(g)} + Cl_{2(g)}$

D. $C_{(s)} + 2H_2O_{(g)} \rightarrow 2H_{2(g)} + CO_{2(g)}$

**Answer: A**

## Solution:

When $\Delta n_g = 0, \Delta H = \Delta U$.

For reaction $(A), \Delta n_g = 0$.

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# Question 65

## A buffer solution is prepared by mixing 0.2 M sodium acetate and 0.1 M acetic acid. If $pK_a$ for acetic acid is 4.7, find the pH.

**Options:**

A. 3.0

B. 4.0

C. 5.0

D. 2.0

**Answer: C**

## Solution:

Using Henderson's equation,

$$pH = pK_a + \log_{10} \frac{[Salt]}{[Acid]}$$
$$pH = 4.7 + \log_{10} \frac{0.2}{0.1}$$
$$pH = 4.7 + \log_{10} 2 = 4.7 + 0.3010 = 5.001$$

For acidic buffer, if [Salt] > [Acid], then $pH > pK_a$ of acid. Hence, only option (C) is valid.

---

# Question 66

**A solution of nonvolatile solute is obtained by dissolving $3.5 \text{ g}$ in $100 \text{ g}$ solvent has boiling point elevation $0.35 \text{ K}$. Calculate the molar mass of solute.**

**(Molal elevation constant $= 2.5 \text{ K kg mol}^{-1}$ )**

**Options:**

A. $270 \text{ g mol}^{-1}$

B. $260 \text{ g mol}^{-1}$

C. $250 \text{ g mol}^{-1}$

D. $240 \text{ g mol}^{-1}$

**Answer: C**

## Solution:

$M_2 = \frac{K_b \times W_2 \times 1000}{\Delta T_b \times W_1} = \frac{2.5 \times 3.5 \times 1000}{0.35 \times 100} = 250 \text{ g mol}^{-1}$

---

# Question 67

**At $0^{\circ}C$ a gas occupies 22.4 liters. What is the temperature in Kelvin to reach the volume of 224 liters?**

**Options:**

A. 546 K

B. 273 K

C. 2730 K

D. 5460 K

**Answer: C**

## Solution:

According to Charles' law,

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (at constant n and P )

$\therefore \quad T_2 = \frac{V_2 \times T_1}{V_1} = \frac{224 \times 273}{22.4} = 2730 \text{ K}$

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# Question 68

## Which from following rule / principle states that "No two electrons in an atom can have the same set of four quantum numbers"?

**Options:**

A. Pauli's exclusion principle

B. Hund's rule

C. Aufbau rule

D. Heisenberg uncertainty principle

**Answer: A**

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# Question 69

## Which from following molecules exhibits highest acidic nature?

**Options:**

A. $H_2O$

B. $H_2S$

C. $H_2Se$

D. $H_2Te$

**Answer: D**

## Solution:

The acidic character of the hydrides of group 16 increases from $H_2O$ to $H_2Te$.

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# Question 70

## What is the number of moles of $sp^2$ hybrid carbon atoms in one mole of hexa-1,4-diyne?

**Options:**

A. 5

B. 3

C. 4

D. Zero

**Answer: D**

## Solution:

$$\underset{\text{hexa-1,4-diyne}}{CH_3 - C \equiv C - CH_2 - C \equiv C - H}$$

It does not contain $sp^2$ hybridized carbon atom. Hence, the number of moles of $sp^2$ hybrid carbon atoms in one mole of hexa-1,4-diyne is zero.

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# Question 71

## Which of the following is NOT obtained when a mixture of methyl chloride and n-propyl chloride is treated with sodium metal in dry

## ether?

**Options:**

A. Ethane

B. Butane

C. Propane

D. Hexane

**Answer: C**

## Solution:

When a mixture of methyl chloride and n-propyl chloride is treated with sodium metal in dry ether, three possible alkanes formed are: ethane, butane and hexane.

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# Question 72

## Which carbon atoms (numbered from $C_1$ to $C_6$ ) are involved in the formation of ring structure of glucose?

**Options:**

A. $C_2$ and $C_5$

B. $C_1$ and $C_3$

C. $C_1$ and $C_4$

D. $C_1$ and $C_5$

**Answer: D**

## Solution:

The ring structure of glucose is formed by reaction between the formyl $(-CHO)$ group (C-1) and the alcoholic ( $-OH)$ group at C-5.

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# Question 73

## Calculate the amount of reactant in percent that remains after 60 minutes involved in first order reaction. $(\mathrm{k}=0.02303$ minute $^{-1})$

**Options:**

A. $25\%$

B. $75\%$

C. $50\%$

D. $12.5\%$

**Answer: A**

## Solution:

For a first order reaction,

$$k=\frac{0.693}{t_{1/2}}$$

$$t_{1/2}=\frac{0.693}{0.02303}=30 \text{ min}$$

Percent of reactant that remains after $\mathrm{t}_{1/2}=50\%$

$2 \times \mathrm{t}_{1/2}=60$ min

Therefore, percent of reactant that remains after $2 t_{1/2}=25\%$

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# Question 74

## Which salt from following forms aqueous solution having $\mathrm{pH}$ less than 7 ?

**Options:**

A. $CH_3COONa$

B. $Na_2SO_4$

C. $CuSO_4$

D. $Na_2CO_3$

**Answer: C**

## Solution:

A salt of strong acid and weak base forms aqueous solution having pH less than 7 (that is, it forms an acidic solution). $CuSO_4$ is salt of strong acid $H_2SO_4$ and weak base $Cu(OH)_2$.

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# Question 75

## Identify neutral ligand from following

**Options:**

A. Ammine

B. Nitrato

C. Cyano

D. Chloro

**Answer: A**

## Solution:

The neutral ligand from the options provided is :

Option A

Ammine

An ammine in coordination chemistry refers to an ammonia molecule acting as a ligand, attached to the central metal atom/ion through a lone pair of electrons. It is neutral because the ammonia molecule itself does not carry any net charge.

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# Question 76

**Identify the product 'B' in following reaction.**

**Ethyl phenyl ketone** $\xrightarrow{H_2N-NH_2}$ **A** $\xrightarrow[\Delta]{KOH, HO-(CH_2)_2-OH}$

**Options:**

A. Phenylhydrazone

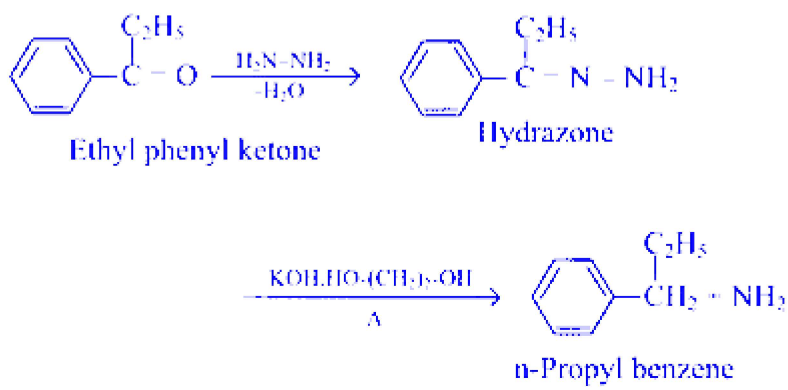
B. Ethyl benzene

C. n-Propyl benzene

D. Isopropyl benzene

**Answer: C**

## Solution:

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# Question 77

## Which of the following amines undergoes acylation reaction?

**Options:**

A. Ethyldimethylamine

B. N-Methylaniline

C. N,N-Dimethylmethanamine

D. N,N-Dimethylaniline

**Answer: B**

## Solution:

Aliphatic and aromatic primary and secondary amines undergo acylation reaction.

Among the given options, N-Methylaniline is a secondary amine and hence, it undergoes acylation reaction. All other amines given are tertiary amines.

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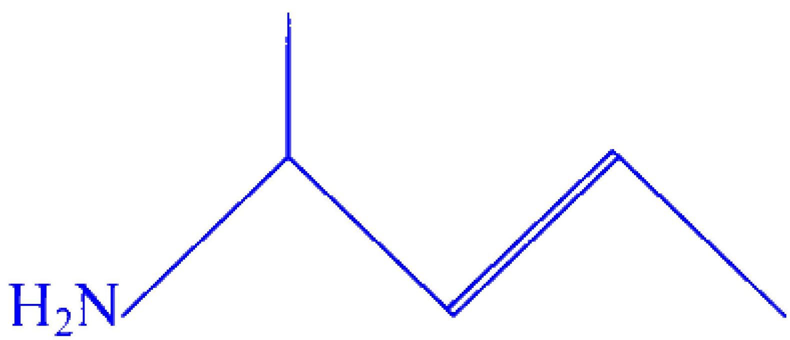
# Question 78

## What is the IUPAC name of following compound?




**Options:**

A. 1-Methylbut-2-en-1-amine

B. Pent-3-en-2-amine

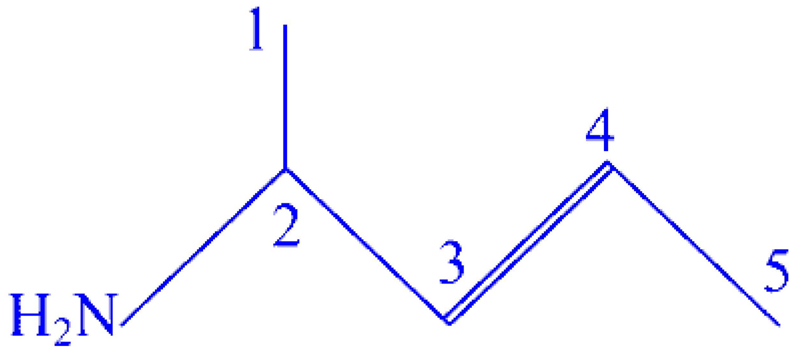
C. Pent-2-en-4-amine

D. 4-Methylbut-2-en-4-amine

**Answer: B**

## Solution:

In IUPAC nomenclature of organic compounds, $-NH_2$ group has a higher priority than a carbon-carbon double bond.




Pent-3-en-2-amine

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# Question 79

## What is the number of unit cells when one mole atom of a metal that forms simple cubic structure?

**Options:**

A. $6.022 \times 10^{23}$

B. $1.204 \times 10^{24}$

C. $9.033 \times 10^{23}$

D. $3.011 \times 10^{23}$

**Answer: A**

## Solution:

Number of atoms in one mole of atom of a metal $= 6.022 \times 10^{23}$

Number of atoms in a simple cubic unit cell $= 1$

$\therefore$ Number of unit cells $= 6.022 \times 10^{23}$

---

# Question 80

**Identify the gas produced due to reduction of $NH_4^+$ ions at cathode during working of dry cell.**

**Options:**

A. Ammonia

B. Hydrogen

C. Hydrogen chloride

D. Chlorine

**Answer: B**

## Solution:

In a dry cell, at cathode, $NH_4^+$ ions are reduced and hydrogen gas is produced.

$$2NH_{4(aq)}^{+} + 2e^{-} \longrightarrow 2NH_{3(aq)} + H_{2(g)}$$

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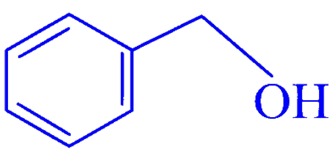
# Question 81

## Which of the following is secondary benzylic alcohol?

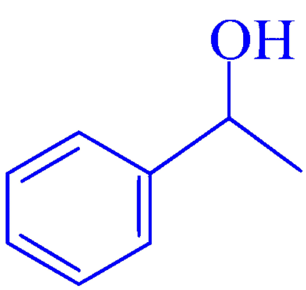
**Options:**

A.



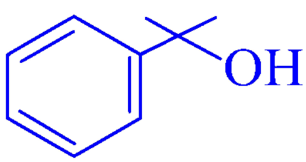


B.



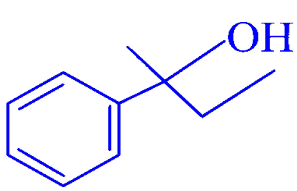


C.





D.



**Answer: B**

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# Question 82

## Find the value of spin only magnetic moment for chromium cation in +2 state.

**Options:**

A. 3.87 BM

B. 4.90 BM

C. 2.84 BM

D. 1.73 BM

**Answer: B**

## Solution:

$Cr^{2+}$ : $[Ar]3\,d^4 \Rightarrow n = 4$

$\therefore$ Spin-only magnetic moment,

$\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{4(4+2)} = \sqrt{24}$ BM

$= 4.90$ BM

Since the number of unpaired electrons (n) = 4, $\mu \approx 5$. Hence, option (B) is the correct answer.

---

# Question 83

## An ideal gas expands by performing 200 J of work, during this internal energy increases by 432 J. What is enthalpy change?

**Options:**

A. 200 J

B. 232 J

C. 432 J

D. 632 J

**Answer: D**

## Solution:

Assuming constant pressure,

$\Delta H = \Delta U + P_{ext}\Delta V = \Delta U - W$

Work done $(W) = -200$ J (for expansion of a gas)

$\Delta U = +432$ J
$\Delta H = \Delta U - W = +432 - (-200) = +632$ J

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# Question 84

## Calculate the wavenumber of the photon emitted during transition from the orbit of $n = 2$ to $n = 1$ in hydrogen atom. $\left[R_H = 109677 \text{ cm}^{-1}\right]$

**Options:**

A. $27419.3 \text{ cm}^{-}$

B. $109677.0 \text{ cm}^{-1}$

C. $12064.5 \text{ cm}^{-1}$

D. $82257.8 \text{ cm}^{-1}$

**Answer: D**

## Solution:

For hydrogen atom,

$\bar{v} = 109677\left[\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right]\text{cm}^{-1}$

Here, $n_i = 2, n_f = 1$

$$\therefore \quad \bar{v} = 109677\left[\frac{1}{1^2} - \frac{1}{2^2}\right]\text{cm}^{-1}$$
$$= 109677\left[\frac{1}{1} - \frac{1}{4}\right]\text{cm}^{-1}$$
$$= 109677\left[\frac{3}{4}\right]\text{cm}^{-1}$$
$$= 82257.8\ \text{cm}^{-1}$$

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# Question 85

### Calculate the value of $\Delta G$ for following reaction at $300\ \text{K}$.

$$H_2O_{(s)} \longrightarrow H_2O_{(l)}$$
$$\left(\Delta H = 7\ \text{kJ}, \Delta S = 24.8\ \text{J K}^{-1}\right)$$

**Options:**

A. $0.74\ \text{kJ mol}^{-1}$

B. $-0.82\ \text{kJ mol}^{-1}$

C. $0.21\ \text{kJ mol}^{-1}$

D. $-0.44\ \text{kJ mol}^{-1}$

**Answer: D**

## Solution:

$$\Delta H = 7\ \text{kJ}$$
$$\Delta S = 24.8\ \text{J K}^{-1} = 24.8 \times 10^{-3}\ \text{kJ K}^{-1}$$
$$T = 300\ \text{K}$$
$$\Delta G = \Delta H - T\Delta S$$
$$\therefore \quad \Delta G = 7\ \text{kJ} - \left(300\ \text{K} \times 24.8 \times 10^{-3}\ \text{kJ K}^{-1}\right)$$
$$= 7\ \text{kJ} - 7.44\ \text{kJ} = -0.44\ \text{kJ}$$

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# Question 86

**What is the oxidation state of carbon in $CaC_2$ and $K_2C_2O_4$ respectively?**

**Options:**

A. $-2$ and $+6$

B. $-1$ and $+3$

C. $+2$ and $+2$

D. $-2$ and $+3$

**Answer: B**

## Solution:

$$\begin{aligned} & CaC_2 \\ \therefore \quad & +2 + (x \times 2) = 0 \text{ or } x = -1 \\ & K_2C_2O_4 \\ \therefore \quad & (+1 \times 2) + (x \times 2) + (-2 \times 4) = 0 \text{ or } x = +3 \end{aligned}$$

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# Question 87

## What is the number of moles of water molecules required to prepare n moles of methane from n moles of methyl magnesium iodide?

**Options:**

A. n

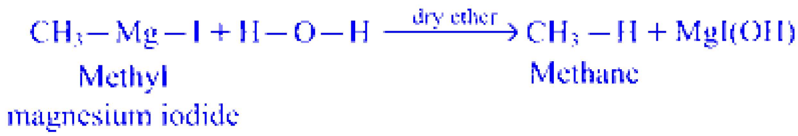
B. 2n

C. 0.5n

D. 0.1n

**Answer: A**

## Solution:




From the reaction, n moles water molecules are required to prepare n moles of methane from n moles of methyl magnesium iodide.

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# Question 88

## Which of the following metals is used as catalyst in manufacture of sulphuric acid by contact process?

**Options:**

A. Iron

B. Platinum

C. Nickel

D. Cobalt

**Answer: B**

## Solution:

In the contact process of industrial production of sulfuric acid; sulphur dioxide and oxygen from the air react reversibly over a solid catalyst of platinised asbestos.

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# Question 89

## The rate for reaction $\mathrm{A}+\mathrm{B} \rightarrow$ product, is $1.8 \times 10^{-2} \mathrm{~mol} \mathrm{dm}^{-3} \mathrm{~s}^{-1}$. Calculate the rate constant if the reaction is second order in $\mathrm{A}$ and first order in $\mathrm{B}$. $([\mathrm{A}]=0.2 \mathrm{M} ;[\mathrm{B}]=0.1 \mathrm{M})$

**Options:**

A. $9.0 \mathrm{~mol}^{-2} \mathrm{dm}^6 \mathrm{~s}^{-1}$

B. $18.0 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

C. $4.5 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

D. $16.0 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

**Answer: C**

## Solution:

$$\text{Rate } = k[A]^2[B]$$
$$\therefore \quad k = \frac{\text{rate}}{[A]^2[B]}$$
$$k = \frac{1.8 \times 10^{-2} \text{mol dm}^{-3} \text{ s}^{-1}}{\left(0.2 \text{ mol dm}^{-3}\right)^2 \times 0.1 \text{mol dm}^{-3}}$$
$$= 4.5 \text{ mol}^{-2} \text{dm}^6 \text{ s}^{-1}$$

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# Question 90

## Which of the following concentration terms depends on temperature?

**Options:**

A. Molality

B. Molarity

C. Mole fraction

D. Percent by mass

**Answer: B**

## Solution:

Among the given options, the concentration term that depends on temperature is :

- **Option B - Molarity**

Molarity is defined as the number of moles of solute per liter of solution. Because the volume of a solution changes with temperature due to thermal expansion or contraction, molarity is temperature-dependent. As temperature increases, the volume of the solution can increase, causing the molarity to decrease, and vice versa.

The other options are not temperature-dependent :

- **Option A - Molality :** Molality is the number of moles of solute per kilogram of solvent. Since it's based on mass, which does not change with temperature, molality is independent of temperature.

- **Option C - Mole Fraction :** Mole fraction is the ratio of the number of moles of a component to the total number of moles in the mixture. Since it is a ratio of moles and does not involve volume, it is not affected by temperature changes.

- **Option D - Percent by Mass :** Percent by mass (or weight percent) is the ratio of the mass of the solute to the total mass of the solution, multiplied by 100. Since mass is not affected by temperature, this concentration term is also temperature-independent.

Therefore, molarity is the term among the given options that is affected by changes in temperature.

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# Question 91

## What type of reaction is the formation of aldol from aldehyde?

### Options:

A. Condensation reaction

B. Addition reaction

C. Substitution reaction

D. Elimination reaction

**Answer: A**

## Solution:

The formation of aldol from an aldehyde is a :

Option A : Condensation reaction

This is because aldol formation involves the combination of two aldehyde molecules (or one aldehyde and one ketone) to form a β-hydroxyaldehyde (aldol) with the loss of a water molecule, which is characteristic of a condensation reaction.

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# Question 92

## Identify homopolymer from following.

**Options:**

A. Polyacrylonitrile

B. Glyptal

C. Polycarbonate

D. Buna-S

**Answer: A**

## Solution:

The homopolymer from the options given is :

Option A : Polyacrylonitrile

This polymer is a homopolymer because it is made up of only one type of monomer, acrylonitrile.

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# Question 93

## Identify the chiral molecule from following.

**Options:**

A. 2-Bromopropane

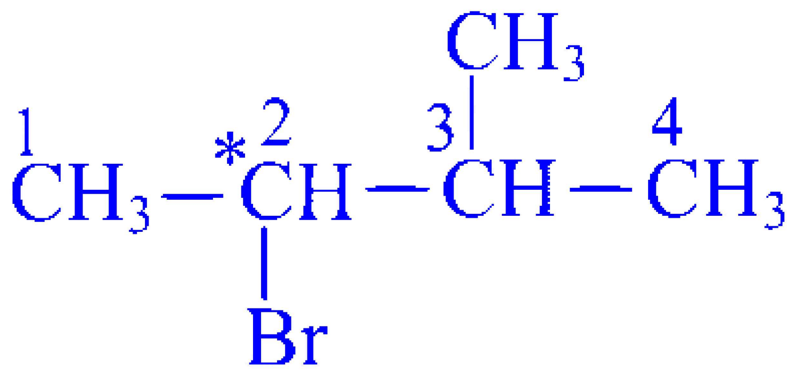
B. 2-Bromo-2-methylbutane

C. 2-Bromo-3-methylbutane

D. 3-Bromopentane

**Answer: C**

## Solution:



2-Bromo-3-methylbutane is a chiral molecule.

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# Question 94

## What is the mass in gram of 1 atom of an element if its atomic mass is 10 u ?

### Options:

A. $2.06056 \times 10^{-22}$ g

B. $1.66056 \times 10^{-23}$ g

C. $1.06056 \times 10^{-24}$ g

D. $3.66056 \times 10^{-25}$ g

**Answer: B**

## Solution:

Atomic mass is the mass of an atom of the element.

Mass of 1 atom of the element $= 10\text{u}$

Now, $1\text{u} = 1.66056 \times 10^{-24}$ g

Therefore, $10\text{u} = 1.66056 \times 10^{-23}$ g

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# Question 95

**Solubility of a salt $A_2B_3$ is $1 \times 10^{-3}$ mol dm$^{-3}$. What is the value of its solubility product?**

**Options:**

A. $1.08 \times 10^{-13}$

B. $8.1 \times 10^{-15}$

C. $2.7 \times 10^{-15}$

D. $2.0 \times 10^{-13}$

**Answer: A**

## Solution:

$A_2B_{3(s)} \rightleftharpoons 2A^{3+}_{(aq)} + 3B^{2-}_{(aq)}$

$$\begin{aligned} &\text{Here, } x = 2, y = 3 \\ &K_{sp} = x^x y^y S^{x+y} \\ &\quad = (2)^2 (3)^3 S^{2+3} \\ &\quad = 108\, S^5 \\ &K_{sp} = 108 \times \left(1 \times 10^{-3}\right)^5 \\ &\quad = 108 \times 10^{-15} \\ &\quad = 1.08 \times 10^{-13} \end{aligned}$$

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# Question 96

## Identify the element having general electronic configuration $ns^2np^4$ from following.

**Options:**

A. Se

B. Br

C. Xe

D. Kr

**Answer: A**

## Solution:

The general electronic configuration of the group 16 elements is $ns^2np^4$. Among the given options, Se is a group 16 element.

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# Question 97

## Identify the product A obtained in the following reaction.

**Phenol + Conc. Nitric acid** $\xrightarrow{\text{conc, } H_2SO_4}$ A

**Options:**

A. o-Nitrophenol

B. p-Nitrophenol

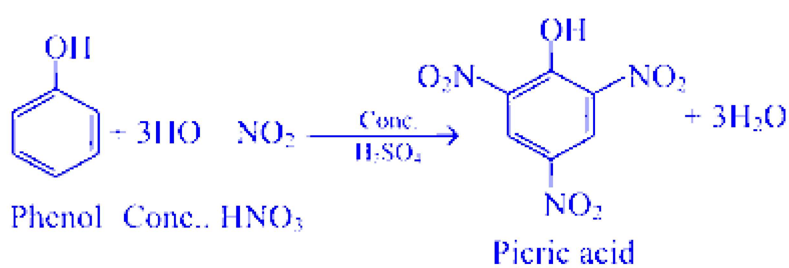
C. 2,4,6-Trinitrophenol

D. Mixture of ortho and para-nitrophenol

**Answer: C**

## Solution:

Phenol reacts with concentrated nitric acid in presence of concentrated $H_2SO_4$ to form 2,4,6-trinitrophenol (picric acid).

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# Question 98

## What is the value of specific rotation exhibited by glucose molecule?

**Options:**

A. $+52.7^{\circ}$

B. $-92.4^{\circ}$

C. $+66.5^{\circ}$

D. $+40.3^{\circ}$

**Answer: A**

## Solution:

The specific rotation of a glucose molecule is a measure of the degree to which it rotates the plane of polarized light. The specific rotation for glucose is :

Option A : $+52.7^{\circ}$

This value is typical for the D-form of glucose (dextrose) in solution at a standard temperature and wavelength.

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# Question 99

## What type of information is collected using scanning electron microscopy?

**Options:**

A. Structure of material surface

B. Crystal structure

C. Binding nature

D. Particle size

**Answer: A**

## Solution:

Scanning Electron Microscopy (SEM) is a powerful technique used in various scientific fields for analyzing the surface characteristics of materials.

- **Option A - Structure of Material Surface :** This is a primary use of SEM. It allows for high-resolution imaging of the surface topography and morphology of materials. SEM can provide detailed images of the surface structures, revealing features like cracks, pores, and the arrangement of particles on a surface.

- **Option B - Crystal Structure :** SEM itself is not typically used for determining crystal structure. This aspect is more directly studied using techniques like X-ray diffraction (XRD). However, SEM can be combined with diffraction techniques, such as electron backscatter diffraction (EBSD), to study crystallographic information in materials.

- **Option C - Binding Nature :** SEM does not directly provide information about the binding nature of atoms or molecules in a material. However, when combined with other techniques like Energy-dispersive X-ray spectroscopy (EDX or EDS), it can be used to analyze the elemental composition and infer binding information.

- **Option D - Particle Size :** SEM is highly effective for determining particle size and distribution. It is commonly used in materials science, nanotechnology, and other fields to measure and analyze the size of particles, including nanoparticles.

In summary, the most direct application of SEM among options is the study of the structure of material surfaces (Option A), though it can also contribute to understanding aspects of crystal structure and particle size when combined with other techniques.

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# Question 100

## Identify the reagent A in the following conversion.

## Alkyl halide $\xrightarrow{A}$ Alkyl nitrite

### Options:

A. $KNO_3$

B. $NaNO_3$

C. $AgNO_2$

D. $KNO_2$

**Answer: D**

## Solution:

Alkyl halide $(R - X)$ on treatment with $KNO_2$ forms alkyl nitrite $(R - O - N = O)$.

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# Physics

# Question 101

## A rubber ball filled with water, having a small hole is used as the bob of a simple pendulum. The time period of such a pendulum

**Options:**

A. is a constant.

B. decreases with time.

C. increases with time.

D. first increases and then decreases, finally having same value as at the beginning.

**Answer: D**

## Solution:

From $T = 2\pi\sqrt{\frac{L}{g}}$

The effective length increases due to this flow of water. Therefore, T increases. As the water flows out, the length decreases and becomes equal to the original length. Hence, the time period becomes equal to the value at the beginning.

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# Question 102

# The ratio of wavelengths for transition of electrons from $2^{\text{nd}}$ orbit to $1^{\text{st}}$ orbit of Helium $\left(\mathrm{He}^{++}\right)$ and Lithium $\left(\mathrm{Li}^{++1}\right)$ is (Atomic number of Helium $= 2$, Atomic number of Lithium $= 3$ )

**Options:**

A. $9 : 4$

B. $9 : 36$

C. $4 : 9$

D. $2 : 3$

**Answer: A**

## Solution:

To find the ratio of the wavelengths for the transition of electrons from the $2^{\text{nd}}$ orbit to the $1^{\text{st}}$ orbit for Helium ion $\mathrm{He}^{++}$ and Lithium ion $\mathrm{Li}^{+}$, we can use the formula derived from the Rydberg equation for wavelengths of emitted photons during electron transitions in hydrogen-like atoms.

The Rydberg formula for the wavelength $\lambda$ is given by:

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

where,

- $R$ is the Rydberg constant for hydrogen,
- $Z$ is the atomic number of the nucleus,
- $n_1$ is the lower energy level,
- $n_2$ is the higher energy level.

For a transition from the $2^{\text{nd}}$ orbit to the $1^{\text{st}}$ orbit ($n_2 = 2$ and $n_1 = 1$), the formula simplifies to:

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = RZ^2\left(\frac{3}{4}\right)$

Therefore, $\lambda = \frac{4}{3RZ^2}$.

**For Helium ion** $\mathrm{He}^{++}$ **($Z = 2$):**

$\lambda_{\text{He}} = \frac{4}{3R\cdot 2^2} = \frac{4}{12R} = \frac{1}{3R}$

**For Lithium ion** $\mathrm{Li}^{+}$ **($Z = 3$):**

$\lambda_{\mathrm{Li}} = \frac{4}{3R \cdot 3^2} = \frac{4}{27R}$

To find the ratio of the wavelengths ($\lambda_{\mathrm{He}} : \lambda_{\mathrm{Li}}$), we divide the expressions obtained for $\lambda_{\mathrm{He}}$ and $\lambda_{\mathrm{Li}}$ :

$\frac{\lambda_{\mathrm{He}}}{\lambda_{\mathrm{Li}}} = \frac{\frac{1}{3R}}{\frac{4}{27R}} = \frac{27}{12} = \frac{9}{4}$

Thus, the ratio of the wavelengths for the transition of electrons from the $2^{\text{nd}}$ orbit to the $1^{\text{st}}$ orbit of Helium ion ($\mathrm{He}^{++}$) to Lithium ion ($\mathrm{Li}^{+}$) is $9 : 4$.

Therefore, the correct answer is **Option A:** $9 : 4$.

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# Question 103

## For an intrinsic semiconductor ($\mathrm{n_h}$ and $\mathrm{n_e}$ are the number of holes per unit volume and number of electrons per unit volume respectively)

**Options:**

A. $\mathrm{n_h} < \mathrm{n_e}$

B. $\mathrm{n_h} = \mathrm{n_e}$

C. $\mathrm{n_h} = \frac{\mathrm{n_c}}{2}$

D. $\mathrm{n_h} > \mathrm{n_e}$

**Answer: B**

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# Question 104

## A $5.0 \mathrm{~V}$ stabilized power supply is required to be designed using a $12 \mathrm{~V}$ DC power supply as input source. The maximum power rating of zener diode is $2.0 \mathrm{~W}$. The minimum value of resistance $R_{\mathrm{s}}$ in $\Omega$ connected in series with zener diode will be

**Options:**

A. 16.5

B. 17.5

C. 18.5

D. 15.5

**Answer: B**

## Solution:

Using the series resistance formula for a zener diode

$$
\begin{aligned}
& R_S=\frac{\left(V_S-V_Z\right)}{I_{Z \max }} \\
& I_{Z \max }=\frac{P_Z}{V_Z}=\frac{2}{5}=400 \mathrm{~mA} \\
\therefore \quad & R_S=\frac{(12-5)}{400 \times 10^{-3}}=\frac{7}{400} \times 10^3 \\
& \quad=17.5 \Omega
\end{aligned}
$$

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# Question 105

## A jar 'P' is filled with gas having pressure, volume and temperature $\mathrm{P}, \mathrm{V}, \mathrm{T}$ respectively. Another gas jar $Q$ filled with a gas having pressure $2 \mathrm{P}$, volume $\frac{\mathrm{V}}{4}$ and temperature $2 \mathrm{~T}$. The ratio of the number of molecules in jar $\mathrm{P}$ to those in jar $Q$ is

**Options:**

A. 1 : 1

B. 1 : 2

C. 2 : 1

D. 4 : 1

**Answer: D**

## Solution:

According to the ideal gas equation $\mathrm{PV} = \mathrm{Nk_B T}$

For jar P, we have

$\mathrm{PV} = \mathrm{N_1 k_B T}$ ..... (i)

For jar Q, we have,

(2P) $\left(\frac{V}{4}\right) = \mathrm{N_2 k_B (2\ T)}$

$\Rightarrow$ PV $= 4\ \mathrm{N_2 k_B T}$ ..... (ii)

From equations (i) and (ii)

$$N_1 = 4N_2 \quad \Rightarrow \frac{N_1}{N_2} = 4$$
$$\therefore \quad N_1 : N_2 = 4 : 1$$

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# Question 106

## The magnetic flux through a loop of resistance $10\ \Omega$ varying according to the relation $\phi = 6\mathrm{t}^2 + 7\mathrm{t} + 1$, where $\phi$ is in milliweber, time is in second at time $\mathrm{t} = 1\ \mathrm{s}$ the induced e.m.f. is

**Options:**

A. 12 mV

B. 7 mV

C. 19 mV

D. 19 V

**Answer: C**

## Solution:

Given $\mathrm{R} = 10\ \Omega, \phi = 6\mathrm{t}^2 + 7\mathrm{t} + 1\ \mathrm{mWb}, \mathrm{t} = 1\ \mathrm{s}$

From $\mathrm{e} = \frac{\mathrm{d}\phi}{\mathrm{dt}}$,

$e = \frac{d}{dt}\left(6t^2 + 7t + 1\right) = 12t + 7$

Put $\mathrm{t} = 1$ in the above equation,

$\therefore \quad e = 19 \text{ mV}$

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# Question 107

## A thin rod of length '$L$' is bent in the form of a circle. Its mass is '$M$'. What force will act on mass '$m$' placed at the centre of this circle?

## ( $\mathrm{G}=$ constant of gravitation)

**Options:**

A. zero

B. $\frac{\mathrm{GMm}}{4 \mathrm{~L}^{2} \pi^{2}}$

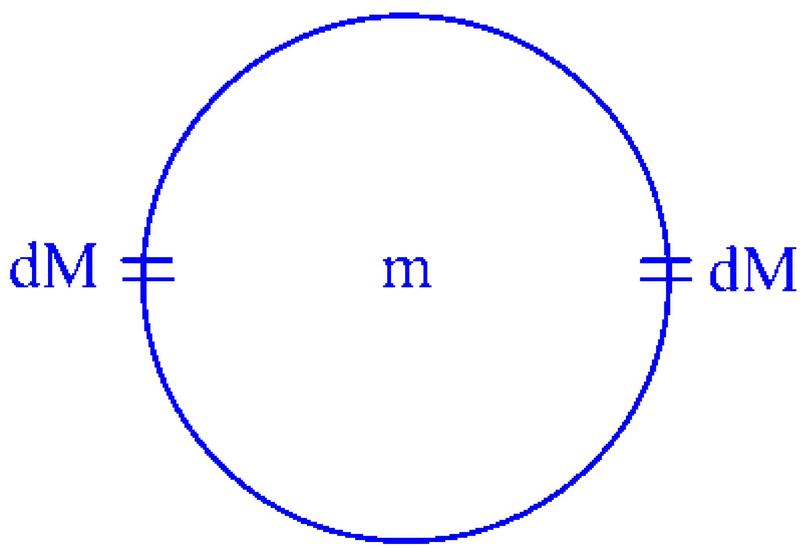
C. $\frac{4 \pi^{2} \mathrm{GMm}}{\mathrm{L}}$

D. $\frac{2 \mathrm{GMm}}{\mathrm{L}^{2}}$

**Answer: A**

## Solution:

Consider two diametrically and equal small and equal mass segments $dM_1$ and $dM_2$




$\therefore \quad$ Force at the centre due to $dM_1$ is

$F_1 = \frac{\mathrm{Gm} dM_1}{r^2}$

Similarly,

Force at the centre due to $dM_2$ is

$F_2 = \frac{GmdM_2}{r^2}$

But $F_1 = -F_2$

$\Rightarrow F_1 + F_2 = 0$ ($\because$ the forces cancel each other out as they are equal and opposite)

If this process is done for all such dM segments, we find the net force at the centre of the circle to be zero.

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# Question 108

## The coil of an a.c. generator has 100 turns, each of cross-sectional area $2 \text{ m}^2$. It is rotating at constant angular speed $30 \text{ rad/s}$, in a uniform magnetic field of $2 \times 10^{-2} \text{ T}$. If the total resistance of the circuit is $600 \ \Omega$ then maximum power dissipated in the circuit is

**Options:**

A. 6 W

B. 9 W

C. 12 W

D. 24 W

**Answer: C**

## Solution:

$$N = 100, A = 2 \text{ m}^2, \omega = 30 \text{rad/s}$$
$$B = 2 \times 10^{-2} \text{ T}, \text{R} = 600\Omega$$

Maximum power dissipated in the circuit

$$P_{\max} = E_{rms} \times I_{rms} = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}}$$
$$= \frac{E_0 I_0}{2} \quad ..... \text{(i)}$$

But $I_0 = \frac{E_0}{R}$ ..... (ii)

Putting (2) into (1) we get,

$I_0 = \frac{E_0{}^2}{2R}$

But $E_0 = NAB\omega$

$$\begin{aligned} E_0 &= 100 \times 2 \times 2 \times 10^{-2} \times 30 \\ &= 120 \text{ V} \end{aligned}$$

$$\therefore \quad P_{max} = \frac{120 \times 120}{2 \times 600} = 12 \text{ W}$$

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# Question 109

**A beam of unpolarized light passes through a tourmaline crystal A and then it passes through a second tourmaline crystal B oriented so that its principal plane is parallel to that of A. The intensity of emergent light is $I_0$. Now B is rotated by $45^\circ$ about the ray. The emergent light will have intensity $\left(\cos 45^\circ = \frac{1}{\sqrt{2}}\right)$**

**Options:**

A. $\frac{I_0}{2}$

B. $\frac{I_0}{\sqrt{2}}$

C. $\frac{\sqrt{2}}{I_0}$

D. $\frac{2}{I_0}$

**Answer: A**

## Solution:

Using law of Malus,

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= I_0 \left(\cos^2 45\right) \\ &= I_0 \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{I_0}{2} \end{aligned}$$

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# Question 110

## The materials suitable for making electromagnets should have

**Options:**

A. high retentivity and high coercivity

B. low retentivity and low coercivity

C. high retentivity and low coercivity

D. low retentivity and high coercivity

**Answer: B**

## Solution:

The correct option for materials suitable for making electromagnets is **Option B: low retentivity and low coercivity**.

To understand why, let's dive into the terms retentivity and coercivity in the context of magnetic materials:

- **Retentivity** (or remanence) is the ability of a magnetic material to retain a certain amount of residual magnetism after an external magnetizing field is removed. High retentivity means the material retains a significant amount of magnetization, while low retentivity means it loses most of the magnetization when the external field is removed.

- **Coercivity** is the measure of the coercive force required to reduce the magnetization of a magnetic material to zero after the magnetization of the sample has been driven to saturation. Essentially, it indicates how difficult it is to demagnetize the material. High coercivity means it is difficult to demagnetize, while low coercivity indicates it is easy to demagnetize.

Electromagnets are designed to be turned on and off (magnetized and demagnetized) easily with the application and removal of an electric current. Therefore, materials used for electromagnets should have **low retentivity** so they can easily lose their magnetism when the current is turned off and **low coercivity** so they can be magnetized and demagnetized easily with small changes in current. This ensures the electromagnet is effective in applications that require rapid switching between the magnetized and demagnetized states.

Materials with high retentivity and high coercivity are more suitable for permanent magnets, which are intended to maintain their magnetic field without a continuous external power source.

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# Question 111

## A body falls on a surface of coefficient of restitution 0.6 from a height of $1 \mathrm{~m}$. Then the body rebounds to a height of

**Options:**

A. 1 m

B. 0.36 m

C. 0.4 m

D. 0.6 m

**Answer: B**

## Solution:

As the body falls from a height

$$\begin{aligned} & \mathrm{u}=0 \\ & \mathrm{v}^2=\mathrm{u}^2+2 \mathrm{gh} \\ \therefore \quad & \mathrm{v}^2=2 \times 9.8 \times 1=19.6 \\ \therefore \quad & \mathrm{v}=\sqrt{19.6} \mathrm{~m} / \mathrm{s} \end{aligned}$$

Coefficient of restitution $\mathrm{e}=\frac{\mathrm{v}}{\mathrm{u}}$

$$\begin{aligned} & \mathrm{e}=\frac{\text { Velocity after collision }\left(\mathrm{v}_{\mathrm{f}}\right)}{\text { Velocity before collision }\left(\mathrm{v}_{\mathrm{b}}\right)} \\ \therefore \quad & \mathrm{v}_{\mathrm{f}}=\mathrm{e} \times \mathrm{v}_{\mathrm{b}} \\ & \mathrm{v}_{\mathrm{f}}=0.6 \times \sqrt{19.6} \mathrm{~m} / \mathrm{s} \end{aligned}$$

After the body rebounds,

$$\begin{aligned} & v^2=u^2-2 g h \\ & \Rightarrow u^2=2 g h \\ \therefore \quad & h=u^2 / 2 g \end{aligned}$$

Here, $\mathrm{u}=\mathrm{v}_{\mathrm{f}}$

$\therefore \quad h = \frac{(0.6 \times \sqrt{19.6})^2}{2 \times 9.8}$
$= 0.36 \text{ m}$

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# Question 112

## In a diffraction pattern due to single slit of width '$a$', the first minimum is observed at an angle of $30^\circ$ when the light of wavelength $5400\overset{o}{A}$ is incident on the slit. The first secondary maximum is observed at an angle of $\left(\sin 30^\circ = \frac{1}{2}\right)$

**Options:**

A. $\sin^{-1}\left(\frac{3}{4}\right)$

B. $\sin^{-1}\left(\frac{2}{3}\right)$

C. $\sin^{-1}\left(\frac{1}{2}\right)$

D. $\sin^{-1}\left(\frac{1}{4}\right)$

**Answer: A**

## Solution:

For $n^{th}$ secondary minimum,

path difference $= a \sin \theta_n = n\lambda$

For $n^{th}$ secondary maximum,

path difference $= a \sin \theta_n = (2n+1)\frac{\lambda}{2}$

$\therefore$ For $1^{st}$ minimum, a $\sin 30^\circ = \lambda$ ..... (i)

For $2^{nd}$ maximum, a $\sin \theta_n = (2+1)\frac{\lambda}{2} = \frac{3\lambda}{2}$ ..... (ii)

$\therefore$ Dividing equation (i) by equation (ii),

$\frac{\left(\frac{1}{2}\right)}{\sin \theta_n} = \frac{2}{3} \Rightarrow \theta_n = \sin^{-1}\left(\frac{3}{4}\right)$

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# Question 113

**A stone is projected vertically upwards with speed '$v$'. Another stone of same mass is projected at an angle of $60^\circ$ with the vertical with the same speed '$v$'. The ratio of their potential energies at the highest points of their journey is**

$$\left[\sin 30^\circ = \cos 60^\circ = 0.5, \cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$$

**Options:**

A. 4 : 1

B. 2 : 1

C. 3 : 2

D. 1 : 1

**Answer: A**

## Solution:

P.E. of the stone projected vertically is,

P.E. = mgh

But $h = \frac{v^2}{2g}$

$$\therefore \quad P.E_1 = mg\left(\frac{v^2}{2g}\right)$$

$$= \frac{\mathrm{mv}^2}{2} \quad \text{...... (i)}$$

For the second stone thrown at an angle $\theta_{\text{to the}}$ horizontal,

$$h = \frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \sin^2 30^\circ}{2g} = \frac{v^2}{8g}$$

$$\therefore \quad \text{P.E. } 2 = mg\left(\frac{v^2}{8g}\right) = \frac{mv^2}{8} \quad \text{...... (ii)}$$

Dividing equation (i) by equation (ii)

$$\frac{P.E_1}{P.E_2} = \frac{\left(\frac{mv^2}{2}\right)}{\left(\frac{m^2}{8}\right)} = 4 : 1$$

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# Question 114

## An electron (mass $\mathrm{m}$ ) is accelerated through a potential difference of '$V$' and then it enters in a magnetic field of induction '$B$' normal to the lines. The radius of the circular path is ($\mathrm{e}=$ electronic charge)

**Options:**

A. $\sqrt{\frac{2 \mathrm{eV}}{\mathrm{m}}}$

B. $\sqrt{\frac{2 \mathrm{Vm}}{\mathrm{eB}^2}}$

C. $\sqrt{\frac{2 V m}{e B}}$

D. $\sqrt{\frac{2 V m}{\mathrm{e}^2 \mathrm{~B}}}$

**Answer: B**

## Solution:

Radius of circular path in a cyclotron is given by

$\mathrm{R}=\frac{\mathrm{mV}}{\mathrm{qB}}$

Here $\mathrm{q}=\mathrm{e}$,

$\therefore \quad \mathrm{R}=\frac{\mathrm{mV}}{\mathrm{eB}}$ ..... (i)

On entering the field,

$\mathrm{KE}=\mathrm{eV}=\frac{1}{2} \mathrm{mv}^2$

$\therefore \quad \mathrm{v}=\sqrt{\frac{2 \mathrm{eV}}{\mathrm{m}}}$ .... (ii)

Putting (ii) into (i),

$\mathrm{R}=\frac{\mathrm{m} \sqrt{\frac{2 \mathrm{eV}}{\mathrm{m}}}}{\mathrm{eB}}=\sqrt{\frac{2 \mathrm{Vm}}{\mathrm{eB}^2}}$

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# Question 115

## A capacitor, an inductor and an electric bulb are connected in series to an a.c. supply of variable frequency. As the frequency of the supply is increased gradually, then the electric bulb is found to

**Options:**

A. increase in brightness.

B. decrease in brightness.

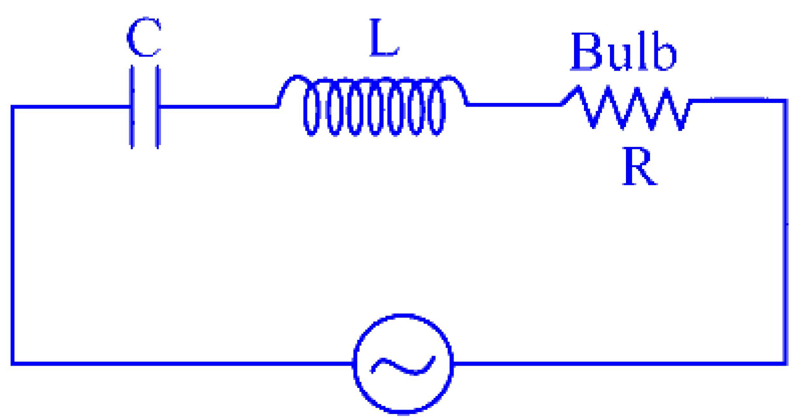
C. increase, reach a maximum and then decrease in brightness.

D. show no change in brightness.

**Answer: C**

### Solution:

The given circuit acts like a series LCR circuit.




$I = \frac{V}{Z} = \frac{V}{\sqrt{R_2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

From the above formula, when $\omega = 0$, I = maximum

When $\omega$ is increased, I starts to decrease and when $\omega = \frac{1}{\sqrt{LC}}$ the current will be zero. This cycle will repeat going forward. So brightness will increase, reach a maximum and then decrease.

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# Question 116

**When both source and listener are approaching each other the observed frequency of sound is given by $(V_L$ and $V_S$ is the velocity of listener and source respectively, $\mathrm{n}_0=$ radiated frequency)**

**Options:**

A. $\mathrm{n}=\mathrm{n}_0\left[\frac{\mathrm{V}+\mathrm{V}_\mathrm{L}}{\mathrm{V}-\mathrm{V}_\mathrm{s}}\right]$

B. $\mathrm{n}=\mathrm{n}_0\left[\frac{\mathrm{V}-\mathrm{V}_\mathrm{L}}{\mathrm{V}+\mathrm{V}_\mathrm{s}}\right]$

C. $\mathrm{n}=\mathrm{n}_0\left[\frac{\mathrm{V}-\mathrm{V}_\mathrm{L}}{\mathrm{V}-\mathrm{V}_\mathrm{s}}\right]$

D. $\mathrm{n}=\mathrm{n}_0\left[\frac{\mathrm{V}+\mathrm{V}_\mathrm{L}}{\mathrm{V}+\mathrm{V}_\mathrm{s}}\right]$

**Answer: A**

## Solution:

Using Dopper's effect formula for approaching frequency when both source and listener are approaching each other, the observed frequency of sound is given by,

$\mathrm{n}=\mathrm{n}_0\left[\frac{\mathrm{V}+\mathrm{V}_\mathrm{L}}{\mathrm{V}-\mathrm{V}_\mathrm{s}}\right]$

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# Question 117

## Water is flowing through a horizontal pipe in stream line flow. At the narrowest part of the pipe

**Options:**

A. velocity is maximum and pressure minimum.

B. pressure is maximum and velocity minimum.

C. both pressure and velocity are minimum.

D. both pressure and velocity are maximum.

**Answer: A**

## Solution:

Using equation of continuity,

$A_1 v_1 = A_2 v_2$

In streamlined flow, as the product of Av is a constant, the velocity at the narrowest part will be maximum.

By using Bernoulli's principle,

$\mathrm{P} + \frac{1}{2}\rho \mathrm{v}^2 + \rho \mathrm{gh} = \text{ Constant}$

We see that $\rho g h$ is constant because the pipe is horizontal.

As velocity increases, for the equation to remain constant, P has to decrease.

$\therefore$ Velocity will be maximum and pressure is minimum.

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# Question 118

## The angle of prism is $A$ and one of its refracting surface is silvered. Light rays falling at an angle of incidence '$2\mathrm{A}$' on the first surface return back through the same path after suffering reflection at the silvered surface. The refractive index of the material of the prism is

**Options:**

A. $2 \sin \left(\frac{\mathrm{A}}{2}\right)$

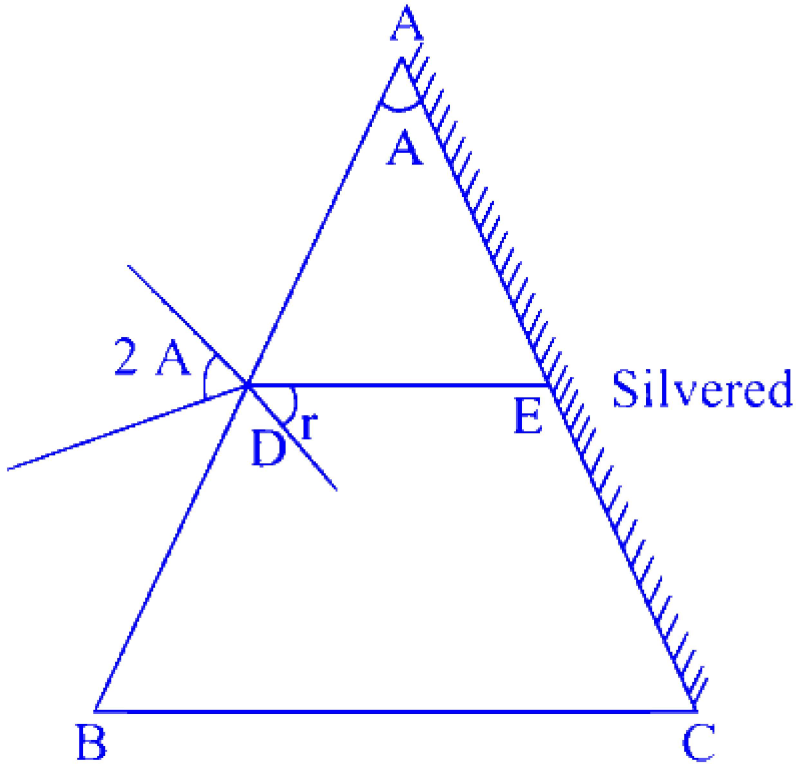
B. $2 \tan \mathrm{A}$

C. $2 \cos \mathrm{A}$

D. $2 \sin \mathrm{A}$

**Answer: C**

## Solution:




Given: Angle of Prism $= \mathrm{A}, \mathrm{I} = 2\ \mathrm{A}$

As DE is falling normally on the silvered side AC,

$\mathrm{r} = 90 - (90 - \mathrm{A})$

$\mathrm{r} = \mathrm{A}$

Using Snell's law,

$\frac{\sin \mathrm{i}}{\sin \mathrm{r}} = \frac{\sin 2\ \mathrm{A}}{\sin \mathrm{A}} = \frac{2 \sin \mathrm{A} \cos \mathrm{A}}{\sin \mathrm{A}} = 2 \cos \mathrm{A}$

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# Question 119

## The maximum velocity of a particle performing S.H.M. is 'V'. If the periodic time is made $\left(\frac{1}{3}\right)^{d}$ and the amplitude is doubled, then the new maximum velocity of the particle will be

**Options:**

A. $\frac{\mathrm{V}}{6}$

B. $\frac{3\ \mathrm{V}}{2}$

C. 3 V

D. 6 V

**Answer: D**

## Solution:

Given $\mathrm{T}' = \frac{1}{3}\mathrm{~T}$ and $\mathrm{A}' = 2\mathrm{~A}$

$\therefore \quad \omega' = \frac{2\pi}{T'} = \frac{2\pi}{\left(\frac{1}{3}T\right)} = \frac{6\pi}{T} = 3\omega$

$\therefore$ The new maximum velocity

$$\begin{aligned} \mathrm{V}' &= \mathrm{A}'\omega' \\ &= (2\mathrm{~A}) \times (3\omega) \\ &= 6\mathrm{~A}\omega \\ &= 6\mathrm{~V} \end{aligned}$$

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# Question 120

## A conducting wire of length $2500\mathrm{~m}$ is kept in east-west direction, at a height of $10\mathrm{~m}$ from the ground. If it falls freely on the ground then the current induced in the wire is (Resistance of wire $= 25\sqrt{2}\Omega$, acceleration due to gravity $\mathrm{g} = 10\mathrm{~m/s^2},\ \mathrm{B_H} = 2 \times 10^{-5}\mathrm{~T}$ )

**Options:**

A. 0.2 A

B. 0.02 A

C. 0.01 A

D. 2 A

**Answer: B**

## Solution:

We know, $e = B/v$

$v^2 = 2gh$

$\therefore \quad v = \sqrt{2gh}$

$\therefore \quad e = (2 \times 10^{-5}) \times 2500 \times \sqrt{2 \times 10 \times 10}$

$= (2 \times 10^{-5}) \times 25000 \times \sqrt{2}$

$= 2\sqrt{2} \times 25 \times 10^3 \times 10^{-5}$

$= 50\sqrt{2} \times 10^{-2}$ V

$\therefore \quad I = \frac{e}{R} = \frac{50\sqrt{2} \times 10^{-2}}{25\sqrt{2}} = 2 \times 10^{-2} = 0.02$ A

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# Question 121

## For an electron moving in the $n^{th}$ Bohr orbit the deBroglie wavelength of an electron is

**Options:**

A. $n\pi r$

B. $\frac{\pi r}{n}$

C. $\frac{nr}{2\pi}$

D. $\frac{2\pi r}{n}$

**Answer: D**

## Solution:

From de Broglie's hypothesis,

$\lambda = \frac{h}{p_n} = \frac{h}{mv}$ .... (i)

and from Bohr's atomic model,

$L = \frac{nh}{2\pi}$

Also,

$L = mvr_n$

Due to quantization of angular momentum, we can write,

$\frac{nh}{2\pi} = mvr_n$

$\therefore \quad v = \frac{nh}{2\pi mr} \quad$ .... (ii)

putting (ii) into (i), we get,

$\therefore \quad \lambda = \frac{h}{m(nh/2\pi mr)} = \frac{2\pi r}{n}$

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# Question 122

## A square lamina of side '$b$' has same mass as a disc of radius '$R$' the moment of inertia of the two objects about an axis perpendicular to the plane and passing through the centre is equal. The ratio $\frac{b}{R}$ is

**Options:**

A. 1 : 1

B. $\sqrt{3}$ : 1

C. $\sqrt{6}$ : 1

D. 1 : $\sqrt{3}$

**Answer: B**

## Solution:

$I_{lamina} \quad = \frac{Mb^2}{6}$

$I_{disc} \quad = \frac{MR^2}{2}$

Given $\frac{Mb^2}{6} = \frac{MR^2}{2}$

$\frac{b^2}{R^2} = 3$

$\therefore \quad \frac{b}{R} = \frac{\sqrt{3}}{1}$

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# Question 123

## A body weighs $300 \mathrm{~N}$ on the surface of the earth. How much will it weigh at a distance $\frac{R}{2}$ below the surface of earth? ( $R \rightarrow$ Radius of earth)

**Options:**

A. 300 N

B. 250 N

C. 200 N

D. 150 N

**Answer: D**

## Solution:

Acceleration due to gravity at depth d,

$$\begin{aligned} g_d &= g\left(1-\frac{d}{R}\right) \\ &= g\left(1-\frac{1}{2}\right) \quad \because d=\frac{R}{2} \\ &= \frac{g}{2} \end{aligned}$$

$\therefore$ Weight of the body at depth $d_1$

$W_d = mg_d = 300 \times \frac{1}{2} = 150 \mathrm{~N}$

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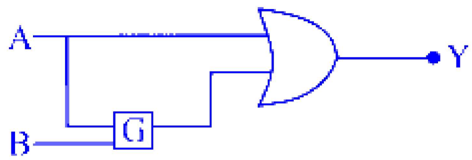
# Question 124

## To get the truth table shown, from the following logic circuit, the Gate G should be




| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**Options:**

A. OR

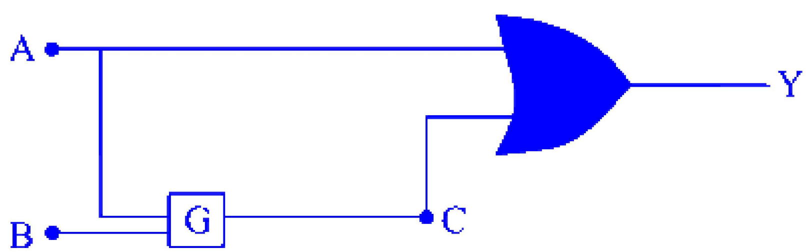
B. AND

C. NOR

D. NAND

**Answer: B**

# Solution:




Truth table for Y, with the possible values of C is,

| A | C | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 1 | 0, 1 | 1 |
| 1 | 0, 1 | 1 |

For gate G

| | A | B | C |
|---|---|---|---|
| (I) | 0 | 0 | 0 |

| | A | B | C |
|---|---|---|---|
| (II) | 0 | 1 | 0 |
| (III) | 1 | 0 | 0, 1 |
| (IV) | 1 | 1 | 0, 1 |

G is not a NOT gate as NOT gate takes only one input. (II) indicates G is not a OR gate as OR gate would give high output for the inputs in (II). Also, (II) indicates it is not a XOR gate as XOR would also give high output for inputs in (II). Hence, the given truth table is satisfied only by AND gate.

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# Question 125

## For a gas having 'X' degrees of freedom, '$\gamma$' is ($\gamma =$ ratio of specific heats $= \mathrm{C_P/C_V}$)

**Options:**

A. $\frac{1+X}{2}$

B. $1+\frac{X}{2}$

C. $1+\frac{2}{x}$

D. $1+\frac{1}{x}$

**Answer: C**

## Solution:

$\gamma$ and degrees of freedom is related by

$\gamma = \frac{\mathrm{f}+2}{\mathrm{f}}$

Where $f$ is the number of degrees of freedoms.

Given $\mathrm{f} = \mathrm{X}$,

$\therefore \quad \gamma = \frac{X+2}{X} = 1+\frac{2}{X}$

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# Question 126

## A galvanometer of resistance $\mathrm{G}$ is shunted with a resistance of $10\%$ of $\mathrm{G}$. The part of the total current that flows through the galvanometer is

**Options:**

A. $\frac{1}{11}\mathrm{I}$

B. $\frac{2}{11}\mathrm{I}$

C. $\frac{1}{10}\mathrm{I}$

D. $\frac{1}{5}\mathrm{I}$

**Answer: A**

## Solution:

$$\frac{I_g}{I} = \frac{S}{S+G} = \frac{0.1G}{0.1G+G} = \frac{1}{11}$$
$$\therefore \quad I_g = \frac{1}{11} I$$

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# Question 127

## If an electron in a hydrogen atom jumps from an orbit of level $n = 3$ to orbit of level $n = 2$, then the emitted radiation frequency is (where R = Rydberg's constant, C = Velocity of light)

**Options:**

A. $\frac{3\mathrm{RC}}{27}$

B. $\frac{\mathrm{RC}}{25}$

C. $\frac{8\mathrm{RC}}{9}$

D. $\frac{5\mathrm{RC}}{36}$

**Answer: D**

## Solution:

$$\frac{1}{\lambda} = \mathrm{R}\left[\frac{1}{\mathrm{n}_1^2} - \frac{1}{\mathrm{n}_2^2}\right]$$

$$\therefore \quad \frac{1}{\lambda} = \left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}\mathrm{R}$$

$$\therefore \quad \mathrm{f} = \frac{\mathrm{c}}{\lambda} = \frac{5}{36}\mathrm{Rc}$$

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# Question 128

### Self inductance of solenoid is

**Options:**

A. directly proportional to current flowing through the coil.

B. directly proportional to the length.

C. directly proportional to its area of cross-section.

D. inversely proportional to the area of cross-section.

**Answer: C**

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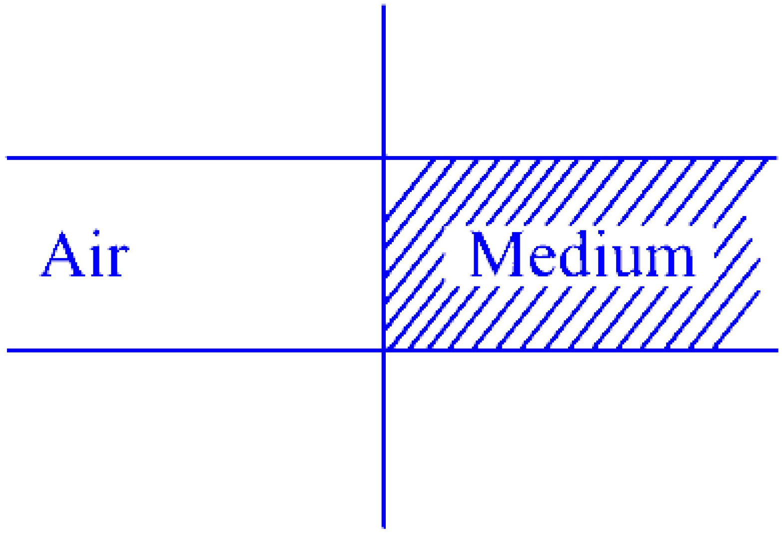
# Question 129

## The capacitance of a parallel plate capacitor is $2.5\ \mu\mathrm{F}$. When it is half filled with a dielectric as shown in figure, its capacitance becomes $5\ \mu\mathrm{F}$. The dielectric constant of the dielectric is




**Options:**

A. 7.5

B. 3

C. 4

D. 5

**Answer: B**

# Solution:

Given $C=\frac{\varepsilon_0 A}{d}=2.5\ \mu F$

When half filled with air,

$$\mathrm{C}_1=\frac{\varepsilon_0(\mathrm{~A}/2)}{\mathrm{d}}=\frac{\varepsilon_0 \mathrm{~A}}{2 \mathrm{~d}} \quad \ldots\ldots(\because \varepsilon_{\mathrm{r}}=1)$$

When half filled with a dielectric,

$$\mathrm{C}_2=\frac{\varepsilon_{\mathrm{r}} \varepsilon_0(\mathrm{~A}/2)}{\mathrm{d}}=\frac{\varepsilon_{\mathrm{r}} \varepsilon_0 \mathrm{~A}}{2 \mathrm{~d}}$$

From the figure, it can be seen that $C_1$ and $C_2$ are in parallel configuration.

$$\begin{aligned} \therefore \quad & \mathrm{C}_{\mathrm{eq}}=\mathrm{C}_1+\mathrm{C}_2 \\ & \text{Given, } \mathrm{C}_{\mathrm{eq}}=5 \mu \mathrm{F} \\ & \Rightarrow 5 \mu \mathrm{F}=\frac{\varepsilon_0 \mathrm{~A}}{2 \mathrm{~d}}+\frac{\varepsilon_{\mathrm{r}} \varepsilon_0 \mathrm{~A}}{2 \mathrm{~d}} \\ & 5=\frac{2.5}{2}+\varepsilon_{\mathrm{r}} \frac{2.5}{2} \\ & \frac{3.75}{1.25}=\varepsilon_{\mathrm{r}} \\ \therefore \quad & \varepsilon_{\mathrm{r}}=3 \end{aligned}$$

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# Question 130

## Equation of simple harmonic progressive wave is given by $y = \frac{1}{\sqrt{a}} \sin \omega t \pm \frac{1}{\sqrt{b}} \cos \omega t$ then the resultant amplitude of the wave is $(\cos 90^\circ = 0)$

**Options:**

A. $\frac{a \pm b}{ab}$

B. $\frac{\sqrt{\mathrm{a}} \pm \sqrt{\mathrm{b}}}{\mathrm{ab}}$

C. $\frac{\sqrt{\mathrm{a}} \pm \sqrt{\mathrm{b}}}{\sqrt{\mathrm{ab}}}$

D. $\sqrt{\frac{a+b}{ab}}$

**Answer: D**

## Solution:

$y = \frac{1}{\sqrt{a}} \sin \omega t \pm \frac{1}{\sqrt{b}} \sin \left(\omega t + \frac{\pi}{2}\right)$

Here phase difference $= \frac{\pi}{2}$

The resultant amplitude

$= \sqrt{\left(\frac{1}{\sqrt{a}}\right)^2 + \left(\frac{1}{\sqrt{b}}\right)^2} = \sqrt{\frac{1}{a} + \frac{1}{b}} = \sqrt{\frac{a+b}{ab}}$

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# Question 131

## Two uniform brass rods $A$ and $B$ of length '$l$' and '$2l$' and their radii '$2r$' and '$r$' respectively are heated to same temperature. The ratio of the increase in the volume of $\mathrm{rod\ A}$ to that of $\mathrm{rod\ B}$ is

**Options:**

A. 1 : 1

B. 1 : 2

C. 2 : 1

D. 1 : 4

**Answer: C**

## Solution:

To determine the ratio of the increase in the volume of rod $A$ to that of rod $B$, we need to consider the formula for the volumetric expansion due to heating. The change in volume ($\Delta V$) of a rod due to a change in temperature can be given by:

$\Delta V = \beta V \Delta T$

where:

$\beta$ = coefficient of volumetric expansion (which is the same for both rods since they are made of brass),

$V$ = initial volume of the rod,

$\Delta T$ = change in temperature (which is the same for both rods).

First, let's calculate the volume of each rod. The volume $V$ of a cylindrical rod is:

$V = \pi r^2 l$

For rod $A$:

Length = $l$,

Radius = $2r$.

Therefore, the initial volume of rod $A$ ($V_A$) is:

$V_A = \pi(2r)^2 l = \pi \cdot 4r^2 \cdot l = 4\pi r^2 l$

For rod $B$:

Length = $2l$,

Radius = $r$.

Therefore, the initial volume of rod $B$ ($V_B$) is:

$V_B = \pi r^2(2l) = 2\pi r^2 l$

Now, the increase in volume ($\Delta V$) of each rod due to heating is given by:

For rod $A$:

$\Delta V_A = \beta V_A \Delta T = \beta(4\pi r^2 l)\Delta T = 4\beta\pi r^2 l\Delta T$

For rod $B$:

$\Delta V_B = \beta V_B \Delta T = \beta(2\pi r^2 l)\Delta T = 2\beta\pi r^2 l\Delta T$

To find the ratio of the increase in the volume of rod $A$ to that of rod $B$, we take:

$\frac{\Delta V_A}{\Delta V_B} = \frac{4\beta\pi r^2 l\Delta T}{2\beta\pi r^2 l\Delta T} = \frac{4}{2} = 2$

Thus, the ratio of the increase in volume of rod $A$ to that of rod $B$ is: **2:1**.

Therefore, the correct option is **Option C: 2:1**.

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# Question 132

## In a single slit experiment, the width of the slit is doubled. Which one of the following statements is correct?

**Options:**

A. The intensity and width of the central maximum are unaffected.

B. The intensity remains same and angular width becomes half.

C. The intensity and angular width both are doubled.

D. The intensity increases by a factor 4 and the angular width decreases by a factor of $\frac{1}{2}$.

**Answer: D**

## Solution:

For single slit diffraction,

Fringe width $\mathrm{W} = \frac{\lambda \mathrm{D}}{\mathrm{d}}$

Also, $\mathrm{I} \propto \mathrm{D}^2$

$\mathrm{I} \propto \mathrm{D}^2$ This means I will be 4 times the original value of $\mathrm{I}$. As only $1$ option mentions this, the correct answer is D.

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# Question 133

**A gas at N.T.P. is suddenly compressed to $\left(\frac{1}{4}\right)^{\text{th}}$ of its original volume. The final pressure in (Given $\gamma =$ ratio of sp. heats $= \frac{3}{2}$ ) atmosphere is ( $\mathrm{P} =$ original pressure)**

**Options:**

A. 4P

B. $\frac{3}{2}$P

C. 8P

D. $\frac{1}{4}$P

**Answer: C**

## Solution:

In Adiabatic compression,

$\mathrm{PV}^{\gamma} =$ constant

Given $V_{\text{new}} = \frac{1}{4} V$ and $\gamma = \frac{3}{2}$

$\therefore \quad \frac{\mathrm{P}_{\text{new}}}{\mathrm{P}} = \left(\frac{\mathrm{V}}{\mathrm{V}_{\text{new}}}\right)^{\gamma} = \left(\frac{\mathrm{V}}{\frac{1}{4}\mathrm{V}}\right)^{3/2}$

$\therefore \quad \mathrm{P}_{\text{new}} = 8\mathrm{P}$

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# Question 134

## The excess of pressure in a first soap bubble is three times that of other soap bubble. Then the ratio of the volume of first bubble to other is

**Options:**

A. 1 : 3

B. 27 : 1

C. 1 : 9

D. 1 : 27

**Answer: D**

## Solution:

$$\Delta P \propto \frac{1}{r} \Rightarrow \frac{r_1}{r_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{1}{3}$$
$$\therefore \quad \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \frac{1}{27}$$

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# Question 135

## In a meter bridge experiment null point is obtained at $l$ cm from the left end. If the meter bridge wire is replaced by a wire of same material but twice the area of across-section, then the null point is obtained at a distance

**Options:**

A. $2l$ cm from left end.

B. $l$ cm from left end.

C. $l/2$ cm from left end.

D. $l/4$ cm from left end.

**Answer: B**

## Solution:

At null condition, $\frac{R_1}{R_2} = \frac{X}{100-X}$

This condition is independent of the cross-sectional area of the wire used.

$\therefore$ The null point will remain the same. (i.e., $l$ cm from left end)

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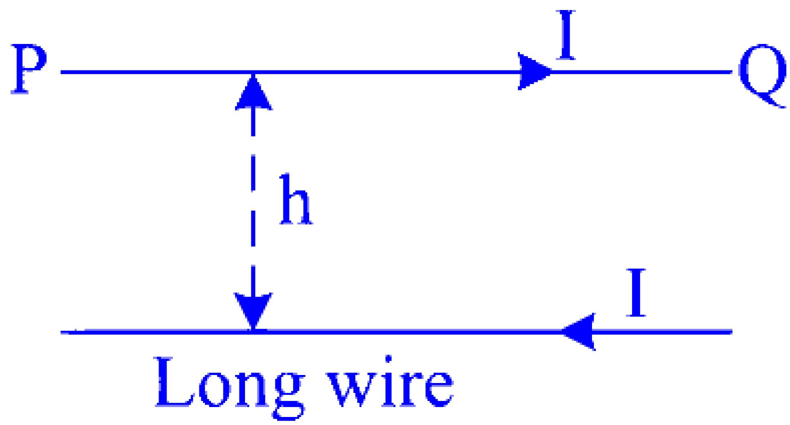
# Question 136

**A long straight wire carrying a current of $25\ \mathrm{A}$ rests on the table. Another wire PQ of length $1\ \mathrm{m}$ and mass $2.5\ \mathrm{g}$ carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will wire PQ rise? ($\mu_0 = 4\pi \times 10^{-7}$ SI unit)**




**Options:**

A. 3 mm

B. 4 mm

C. 5 mm

D. 8 mm

**Answer: C**

## Solution:

Given $\mathrm{I}_1 = \mathrm{I}_2 = 25\ \mathrm{A}, l = 1\ \mathrm{m}$,

$$\mathrm{B} = \frac{\mu_0 \mathrm{I}}{2\pi \mathrm{h}}$$
$$\mathrm{F} = \mathrm{BI}l \sin\theta = \mathrm{BI}l$$

Force applied on PQ $=$ Weight of the smaller current carrying wire

i.e, $\mathrm{mg} = \frac{\mu_0 \mathrm{I}^2 l}{2\pi \mathrm{h}}$

$\therefore \quad \mathrm{h} = \frac{4\pi \times 10^{-7} \times 250 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8} = 5\ \mathrm{mm}$

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# Question 137

## In a thermodynamic process, there is no exchange of heat between the system and surroundings. Then the thermodynamic process is

**Options:**

A. isothermal

B. isobaric

C. isochoric

D. adiabatic

**Answer: D**

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# Question 138

## According to kinetic theory of gases, which one of the following statements is wrong?

**Options:**

A. All the molecules of a gas are identical.

B. Collisions between the molecules of a gas and that of the molecules with the walls of the container are perfectly elastic.

C. The molecules do not exert appreciable force on one another except during collision.

D. The pressure exerted by a gas is due to the collision between the molecules of the gas.

**Answer: D**

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# Question 139

## The radii of two soap bubbles are $r_1$ and $r_2$. In isothermal condition they combine with each other to form a single bubble. The radius of resultant bubble is

**Options:**

A. $\mathrm{R} = \frac{\mathrm{r}_1 + \mathrm{r}_2}{2}$

B. $\mathrm{R} = \mathrm{r}_1 \left(\mathrm{r}_1 \mathrm{r}_2 + \mathrm{r}_2\right)$

C. $\mathrm{R} = \sqrt{\mathrm{r}_1^2 + \mathrm{r}_2^2}$

D. $\mathrm{R} = \mathrm{r}_1 + \mathrm{r}_2$

**Answer: C**

## Solution:

Under isothermal condition, T is constant.

This means the surface energy of the bubbles before combining will be equal to the surface energy after combining.

$$\text{i.e. } 4\pi r_1^2 T + 4\pi r_2^2 T = 4\pi R^2 T$$
$$\Rightarrow r_1^2 + r_2^2 = R^2$$
$$\therefore \quad R = \sqrt{r_1^2 + r_2^2}$$

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# Question 140

## The rays of different colours fail to converge at a point after passing through a thick converging lens. This defect is called

**Options:**

A. spherical aberration

B. distortion

C. coma

D. chromatic aberration

**Answer: D**

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# Question 141

## The ratio of potential difference that must be applied across parallel and series combination of two capacitors $C_1$ and $C_2$ with their capacitance in the ratio $1:2$ so that energy stored in these two cases becomes same is

**Options:**

A. $3:\sqrt{2}$

B. $\sqrt{2}:3$

C. $2:9$

D. $9:2$

**Answer: B**

## Solution:

Given: $\mathrm{C_1 : C_2 = 1 : 2}$

$$\therefore \quad \mathrm{C_2 = 2C_1}$$
$$\mathrm{C_P = C_1 + C_2 = C_1 + 2C_1 = 3C_1}$$
$$\mathrm{C_S = \frac{C_1C_2}{C_1 + C_2} = \frac{2C_1^2}{3C_1} = \frac{2}{3}C_1}$$

Let $V_P$ and $V_S$ be the potentials applied across the parallel and series combinations respectively, then using $\mathrm{E = \frac{1}{2}CV^2}$ we can write,

$$\frac{1}{2}C_PV_P^2 = \frac{1}{2}C_SV_S^2$$
$$\therefore \quad \frac{V_P^2}{V_S^2} = \frac{C_S}{C_P}$$
$$\frac{V_P}{V_S} = \sqrt{\frac{\frac{2}{3}C_1}{3C_1}} = \frac{\sqrt{2}}{3}$$

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# Question 142

## The potential energy of charged parallel plate capacitor is $v_0$. If a slab of dielectric constant $\mathrm{K}$ is inserted between the plates, then the new

## potential energy will be

**Options:**

A. $\frac{v_0}{\mathrm{K}}$

B. $v_0\ \mathrm{K}^2$

C. $\frac{v_0}{\mathrm{K}^2}$

D. $\mathrm{v}_0^2$

**Answer: A**

## Solution:

We know, $v_0=\frac{Q^2}{2C}$

On inserting the slab of dielectric constant k, the new capacitance $\mathrm{C}'=\mathrm{KC}$

$\therefore$ New potential energy $v_0'=\frac{Q^2}{2C'}$

$v_0^1=\frac{\mathrm{Q}^2}{2\mathrm{KC}}=\frac{v_0}{\mathrm{K}}$

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# Question 143

## A solid sphere rolls without slipping on an inclined plane at an angle $\theta$. The ratio of total kinetic energy to its rotational kinetic energy is

**Options:**

A. $\frac{7}{2}$

B. $\frac{5}{2}$

C. $\frac{7}{3}$

D. $\frac{5}{4}$

**Answer: A**

## Solution:

Moment of Inertia of a solid sphere, $I=\frac{2}{5}\mathrm{MR}^2$

Since there is no slipping,

$\mathrm{v}=\mathrm{R}\omega$

$\therefore$ Rotational kinetic energy

$$\mathrm{E}_{\mathrm{rot}}=\frac{1}{2}\mathrm{I}\omega^2$$
$$=\frac{1}{2}\times\frac{2}{5}\times\mathrm{M}\times\mathrm{R}^2\times\omega^2$$
$$=\frac{\mathrm{MR}^2\omega^2}{5}$$
$$=\frac{\mathrm{MV}^2}{5}\quad\text{.... (i)}$$

Total kinetic energy

$$E_{\mathrm{K}}=\frac{1}{2}I\omega^2+\frac{1}{2}\mathrm{MV}^2$$
$$=\frac{\mathrm{MV}^2}{5}+\frac{\mathrm{MV}^2}{2}$$
$$=\frac{7\mathrm{MV}^2}{10}\quad\text{..... (ii)}$$

Dividing (ii) by (i), we get,

$\frac{\mathrm{E}_{\mathrm{K}}}{\mathrm{E}_{\mathrm{rot}}}=\frac{\left(\frac{7\mathrm{MV}^2}{10}\right)}{\left(\frac{\mathrm{MV}^2}{5}\right)}=\frac{7}{2}$

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# Question 144

## Two discs of same mass and same thickness (t) are made from two different materials of densities '$d_1$' and '$d_2$' respectively. The ratio of the moment of inertia $I_1$ to $I_2$ of two discs about an axis passing through the centre and perpendicular to the plane of disc is

**Options:**

A. $\mathrm{d}_1:\mathrm{d}_2$

B. $\mathrm{d}_2:\mathrm{d}_1$

C. $1 : d_1 d_2$

D. $1 : \mathrm{d}_1^2\, \mathrm{d}_2$

**Answer: B**

## Solution:

The ratio of moments of inertia of two discs of the same mass and same thickness but of different densities is given by $\frac{I_1}{I_2} = \frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$

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# Question 145

## When a string of length '$l$' is divided into three segments of length $l_1, l_2$ and $l_3$. The fundamental frequencies of three segments are $\mathrm{n}_1, \mathrm{n}_2$ and $\mathrm{n}_3$ respectively. The original fundamental frequency '$n$' of the string is

**Options:**

A. $\mathrm{n} = \mathrm{n}_1 + \mathrm{n}_2 + \mathrm{n}_3$

B. $\sqrt{\mathrm{n}} = \sqrt{\mathrm{n}_1} + \sqrt{\mathrm{n}_2} + \sqrt{\mathrm{n}_3}$

C. $\frac{1}{\mathrm{n}} = \frac{1}{\mathrm{n}_1} + \frac{1}{\mathrm{n}_2} + \frac{1}{\mathrm{n}_3}$

D. $\frac{1}{\sqrt{\mathrm{n}}} = \frac{1}{\sqrt{\mathrm{n}_1}} + \frac{1}{\sqrt{\mathrm{n}_2}} + \frac{1}{\sqrt{\mathrm{n}_3}}$

**Answer: C**

## Solution:

The fundamental frequency of a string is given by

Given: $l = l_1 + l_2 + l_3$ ..... (i)

$\mathrm{n} = \frac{1}{2l}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}}$

$\Rightarrow \mathrm{n} \propto \frac{1}{l}$ or $\mathrm{n}l = \mathrm{k}$

$\therefore \quad l_1 = \frac{\mathrm{k}}{\mathrm{n}_1}, l_2 = \frac{\mathrm{k}}{\mathrm{n}_2}$ and $l_3 = \frac{\mathrm{k}}{\mathrm{n3}}$ .... (ii)

$\therefore$ Original length $l = \frac{k}{n}$ .... (iii)

Putting eq (ii) and (iii) into eq (i)

$$\frac{k}{n} = \frac{k}{n_1} + \frac{k}{n_2} + \frac{k}{n_3}$$

$$\therefore \quad \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$$

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# Question 146

**A seconds pendulum is placed in a space laboratory orbiting round the earth at a height '3R' from the earth's surface. The time period of the pendulum will be ( $R =$ radius of earth)**

**Options:**

A. zero

B. $\frac{2}{3}$ s

C. 4 s

D. infinite

**Answer: D**

## Solution:

In outer space, g = 0. Therefore, T = $\infty$.

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# Question 147

**A solid metallic sphere has a charge $+3Q$. Concentric with this sphere is a conducting spherical shell having charge $-Q$. The radius of the sphere is '$A$' and that of the spherical shell is '$B$'. $(B > A)$. The electric field at a distance 'R' $(A < R < B)$ from the centre is ( $\varepsilon_0 =$ permittivity of vacuum)**

**Options:**

A. $\frac{\mathrm{Q}}{2\pi\varepsilon_0\mathrm{R}}$

B. $\frac{3Q}{2\pi\varepsilon_0 R}$

C. $\frac{3\mathrm{Q}}{4\pi\varepsilon_0\mathrm{R}^2}$

D. $\frac{4Q}{2\pi\varepsilon_0 R^2}$

**Answer: C**

## Solution:

We know the electric field inside a shell is zero. So, we only consider the electric field due to the solid sphere.

$\therefore$ Using $E=\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, we get

$$E=\frac{1}{4\pi\varepsilon_0}\frac{(3Q)}{R^2}$$

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# Question 148

## A closed organ pipe of length '$L_1$' and an open organ pipe contain diatomic gases of densities '$\rho_1$' and '$\rho_2$' respectively. The compressibilities of the gases are same in both pipes, which are vibrating in their first overtone with same frequency. The length of the open organ pipe is (Neglect end correction)

**Options:**

A. $\frac{4\ \mathrm{L}_1}{3}$

B. $\frac{4L_1}{3}\sqrt{\frac{\rho_1}{\rho_2}}$

C. $\frac{4L_1}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

D. $\frac{3}{4L_1}\sqrt{\frac{\rho_1}{\rho_2}}$

**Answer: B**

## Solution:

Given both gases are vibrating in the first overtone with same frequency, we get

$$f_{closed} = f_{open}$$
$$\Rightarrow \frac{3v}{4\,L_1} = \frac{v}{L_2}$$

According to Laplace's correction

$$v = \sqrt{\frac{\gamma P}{\rho}}$$
$$\frac{3}{4\,L_1} \times \sqrt{\frac{\gamma P}{\rho_1}} = \frac{1}{L_2} \times \sqrt{\frac{\gamma P}{\rho_2}}$$
$$\therefore \quad L_2 = \frac{4\,L_1}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

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# Question 149

## From a metallic surface photoelectric emission is observed for frequencies $v_1$ and $v_2$ $(v_1 > v_2)$ of the incident light. The maximum values of the kinetic energy of the photoelectrons emitted in the two cases are in the ratio $1$ : x. Hence the threshold frequency of the metallic surface is

**Options:**

A. $\frac{v_1 - v_2}{x}$

B. $\frac{v_1 - v_2}{x-1}$

C. $\frac{x v_1 - v_2}{x-1}$

D. $\frac{x v_2 - v_1}{x-1}$

**Answer: C**

## Solution:

Using Einstein's photoelectric equation,

$$
\begin{aligned}
& E_k = hv - \phi_0 \\
& E_k = hv - hv_0 \\
\therefore \quad & E_{K_1} = h(v_1 - v_0) \text{ and } E_{K_2} = h(v_2 - v_0) \\
& \text{Given } \frac{E_{K_1}}{E_{K_2}} = \frac{1}{x} \\
& \Rightarrow \frac{v_1 - v_0}{v_2 - v_0} = \frac{1}{x} \\
& (v_1 - v_0)x = v_2 - v_0 \\
& v_1 x - v_0 x = v_2 - v_0 \\
\therefore \quad & v_1 x - v_2 = v_0 x - v_0 \\
& v_1 x - v_2 = v_0(x - 1) \\
\therefore \quad & v_0 = \frac{v_1 x - v_2}{x - 1}
\end{aligned}
$$

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# Question 150

## In an AC circuit, the current is $i = 5\sin\left(100t - \frac{\pi}{2}\right) A$ and voltage is $e = 200\sin(100t)$ volt. Power consumption in the circuit is $(\cos 90^\circ = 0)$

**Options:**

A. 200 W

B. 0 W

C. 40 W

D. 1000 W

**Answer: B**

## Solution:

$$
\begin{aligned}
& P = I_{rms} V_{rms} \cos\phi \\
& \phi = 90^\circ \\
\therefore \quad & P = 0 \quad (\because \cos 90^\circ = 0)
\end{aligned}
$$