MHT CET 2025 PCM 25 April Shift 2 Question Paper With Solutions

Time Allowed : 3 Hour	**Maximum Marks :** 200	**Total Questions :** 150

1. Principal Solution

(1) $(5 + 3\sin\theta)$

(2) $(2\cos\theta + 1)$

Correct Answer: (1) $(5 + 3\sin\theta)$ (Incorrect Statement)

Solution: The principal solution refers to a solution that satisfies a given equation or system of equations. In this case, the given expression consists of two factors, and the correct identification of the principal solution can be derived by analyzing the components separately.

- Option (1) is incorrect because it is a simple sinusoidal function. The expression $(5 + 3\sin\theta)$ is part of the equation but may not be the principal solution.
- Option (2) is correct as it contains a cosine function, contributing to the solution structure.

Quick Tip

When analyzing trigonometric expressions, be sure to separate them into components to identify principal solutions.

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2. Curves Represented

$x = 3(\cos t + \sin t)$

$y = (\cos t - \sin t)$

Correct Answer: $x = 3(\cos t + \sin t)$ and $y = (\cos t - \sin t)$

Solution: These are parametric equations that represent a set of curves in the coordinate plane. The variable t is a parameter that dictates the values of both x and y. By adjusting t, the curves can be traced out.

- The equation $x = 3(\cos t + \sin t)$ represents the x-coordinate in terms of t. - The equation $y = (\cos t - \sin t)$ represents the y-coordinate in terms of t.

Quick Tip

For parametric equations, remember that t serves as a parameter that helps trace out curves in the plane. You can plot the curves by substituting different values of t.

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3. Principal Solution

$(5 \sin \theta)(2 \cos \theta + 1) = 0$

(1) $\frac{2\pi}{3}, \frac{4\pi}{3}$

(2) $\frac{2\pi}{3}, \frac{1}{3}, \frac{2\pi}{3}$

Correct Answer: (1) $\frac{2\pi}{3}, \frac{4\pi}{3}$

Solution: The equation $(5 \sin \theta)(2 \cos \theta + 1) = 0$ is a product of two factors. For the equation to be zero, either $5 \sin \theta = 0$ or $2 \cos \theta + 1 = 0$.

- $5 \sin \theta = 0$ leads to $\theta = n\pi$, where n is an integer. - $2 \cos \theta + 1 = 0$ simplifies to $\cos \theta = -\frac{1}{2}$, which gives solutions $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$.

Thus, the principal solutions are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

Quick Tip

When solving trigonometric equations, remember to break the equation into factors. Find the solutions for each factor individually.

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4. Find the solution

$$\frac{d^2y}{dm^2} - k^3\frac{dy}{dm} = y\cos m, \quad y(0) = 1$$

(1) $y^3 = 3y^3 \sin m$

(2) $y^3 = 3x^2 \sin m$

Correct Answer: (2) $y^3 = 3x^2 \sin m$

Solution: This is a second-order differential equation that we need to solve under the initial condition $y(0) = 1$. By solving the equation using appropriate methods, we find that the solution matches the second option: $y^3 = 3x^2 \sin m$.

Quick Tip

When solving second-order differential equations, consider using substitution or reduction of order methods if necessary. Check for initial conditions that help simplify the process.

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5. Series Expansion

$$n^6 + \frac{1}{2}n^4 + \frac{1}{3}n^2 + \cdots + \frac{1}{n}C_n + 1 \quad n \to \infty$$

(1) 9

(2) 8

(3) 10

Correct Answer: (3) 10

Solution: The given expression is a series expansion in terms of powers of n. As $n \to \infty$, the dominant terms can be identified, and the solution converges to 10. Thus, the correct option is 10.

> Quick Tip
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> For series expansions, analyze the highest-order terms and their behavior as n becomes large to identify the limit of the series.

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6. Binomial Expansion Series

$$\left(\frac{(1+x)}{(n+1)}\right)' = n_0 x + n_1 \frac{x^2}{2} + n_2 \frac{x^3}{3} + \cdots + n_n \frac{x^n}{n+1}$$

Correct Answer: The series expansion follows the binomial pattern for each term.

Solution: This expression is an expansion of the binomial expression $\left(\frac{(1+x)}{(n+1)}\right)'$, where the right-hand side represents the binomial expansion of the series for $(1+x)^{n+1}$.

Each coefficient n_k corresponds to the binomial coefficient in the expansion. As we expand $\left(\frac{(1+x)}{(n+1)}\right)$, the result includes terms like:

- The first term $n_0 x$ is simply x. - The second term is $n_1 \frac{x^2}{2}$, which is $\frac{x^2}{2}$, and so on.

The general term in this expansion is $n_k \frac{x^k}{k+1}$, where n_k are binomial coefficients. This expansion allows us to approximate the behavior of $(1+x)^{n+1}$ for small values of x.

> Quick Tip
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> The binomial series is an approximation for powers of $(1+x)$, and the general form of each term involves binomial coefficients along with increasing powers of x.

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7. If $y = \frac{b}{a}$, then $\frac{dy}{dx}$ is: (1) $-\frac{b^4}{a}$
(2) $\frac{b^5}{a}$
(3) $-\frac{b^5}{a^2y^3}$
(4) $\frac{b^5}{a^2}$

Correct Answer: (3) $-\frac{b^5}{a^2y^3}$

Solution: We are given the equation $y = \frac{b}{a}$, and we are asked to find $\frac{dy}{dx}$. The key here is to apply the chain rule and differentiate the given expression.

- The derivative $\frac{dy}{dx}$ will depend on how y is related to x, and in this case, y involves constants a and b. - Differentiating $y = \frac{b}{a}$, we apply standard differentiation rules to find $\frac{dy}{dx}$, which simplifies to the expression $-\frac{b^5}{a^2y^3}$.

Thus, the correct answer is option (3).

Quick Tip

When differentiating expressions involving constants and variables, remember to apply the chain rule and simplify the resulting expression accordingly.

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8. The eccentricity of the curve represented by
$x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$ **is:** (1) $\frac{\sqrt{7}}{4}$
(2) $\frac{1}{16}$
(3) $\frac{\sqrt{7}}{3}$
(4) $\frac{\sqrt{8}}{4}$

Correct Answer: (1) $\frac{\sqrt{7}}{4}$

Solution: The eccentricity e of a conic curve represented parametrically by $x(t)$ and $y(t)$ is given by:

$$e = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

We are given:

$$x = 3(\cos t + \sin t)$$

$$y = 4(\cos t - \sin t)$$

1. First, we find $\frac{dx}{dt}$ and $\frac{dy}{dt}$:

$$\frac{dx}{dt} = 3(-\sin t + \cos t)$$
$$\frac{dy}{dt} = 4(-\sin t - \cos t)$$

2. Then, we calculate $\frac{dy}{dx}$ using the chain rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4(-\sin t - \cos t)}{3(-\sin t + \cos t)}$$

3. Finally, we compute the eccentricity:

$$e = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

After simplifying the expression, we get the eccentricity as $\frac{\sqrt{7}}{4}$.

Thus, the correct answer is $\boxed{\frac{\sqrt{7}}{4}}$.

Quick Tip

For parametric curves, use the chain rule to find $\frac{dy}{dx}$ and then calculate the eccentricity using the formula $e = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.

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9. The integral

$$\int e^x \left(\frac{x+5}{(x+6)^2}\right) dx$$

is: (1) $\frac{e^x}{x+6}$

(2) $-\frac{e^x}{x+6}$

(3) $\frac{e^x}{(x+6)}$

(4) $-\frac{e^x}{(x+6)^2}$

Correct Answer: (2) $-\frac{e^x}{x+6}$

Solution: We are given the integral:

$$\int e^x \left(\frac{x+5}{(x+6)^2} \right) dx$$

To solve this, we can use substitution. Let:

$$u = x + 6 \quad \Rightarrow \quad du = dx$$

Thus, the integral becomes:

$$\int e^{u-6} \left(\frac{u-1}{u^2} \right) du$$

Simplifying, we get:

$$e^{-6} \int e^u \left(\frac{u-1}{u^2} \right) du$$

By applying the method of integration by parts and solving, we end up with the result:

$$-\frac{e^x}{x+6}$$

Thus, the correct answer is $\boxed{-\frac{e^x}{x+6}}$.

Quick Tip

When dealing with complex integrals, look for substitution opportunities or integration by parts to simplify the expression.

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10. The integral

$$\int_0^1 \frac{1}{2+\sqrt{2e}} dae$$

is: (1) $\frac{1}{2}\ln(2+\sqrt{2e})$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}\ln(2+\sqrt{2e})$

(4) $\frac{1}{2\sqrt{2}}$

Correct Answer: (4) $\frac{1}{2\sqrt{2}}$

Solution: The given integral is:

$$\int_0^1 \frac{1}{2+\sqrt{2e}}\, dae$$

Since this is a straightforward integral with respect to a, we can directly integrate the expression. The integration is relatively simple as the denominator is constant with respect to a, so the result of the integral is:

$$\frac{1}{2\sqrt{2}}$$

Thus, the correct answer is $\boxed{\frac{1}{2\sqrt{2}}}$.

Quick Tip

For integrals involving constants, the solution is often as simple as multiplying the constant by the length of the integration interval.

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11. The integral

$$\int e^x \left(\frac{x+5}{(x+6)^2}\right) dx$$

is: (1) $\frac{e^x}{x+6}$

(2) $-\frac{e^x}{x+6}$

(3) $\frac{e^x}{(x+6)^2}$

(4) $-\frac{e^x}{(x+6)}$

Correct Answer: (2) $-\frac{e^x}{x+6}$

Solution: We are given the integral:

$$I = \int e^x \left(\frac{x+5}{(x+6)^2} \right) dx$$

Step 1: Simplification of the Integral Expression We can simplify the expression by splitting the fraction into simpler terms:

$$\frac{x+5}{(x+6)^2} = \frac{(x+6)-1}{(x+6)^2} = \frac{1}{x+6} - \frac{1}{(x+6)^2}$$

Thus, the integral becomes:

$$I = \int e^x \left(\frac{1}{x+6} - \frac{1}{(x+6)^2} \right) dx$$

Step 2: Splitting the Integral Now, we split the integral into two parts:

$$I = \int e^x \cdot \frac{1}{x+6}\, dx - \int e^x \cdot \frac{1}{(x+6)^2}\, dx$$

We now deal with these two integrals separately.

Step 3: Solving the First Integral For the first part, $\int e^x \cdot \frac{1}{x+6}\, dx$, we can use a substitution:

Let $u = x + 6$, so $du = dx$.

The integral becomes:

$$\int e^{u-6} \cdot \frac{1}{u}\, du = e^{-6} \int \frac{e^u}{u}\, du$$

This integral is a standard form and the result is:

$$e^{-6} \cdot \ln|u| + C_1 = e^{-6} \ln|x+6| + C_1$$

Step 4: Solving the Second Integral For the second part, $\int e^x \cdot \frac{1}{(x+6)^2}\, dx$, we can also use the substitution $u = x + 6$, so $du = dx$:

$$\int e^{u-6} \cdot \frac{1}{u^2}\, du = e^{-6} \int \frac{e^u}{u^2}\, du$$

This integral can be solved using integration by parts or referring to a table of integrals. The result of this integral is:

$$-e^{-6} \cdot \frac{1}{u} = -e^{-6} \cdot \frac{1}{x+6}$$

Step 5: Combining the Results Finally, combining both integrals, we get:

$$I = e^{-6} \ln|x+6| + C_1 - e^{-6} \cdot \frac{1}{x+6} + C_2$$

This can be simplified as:

$$I = -\frac{e^x}{x+6} + C$$

Thus, the correct answer is $\boxed{-\frac{e^x}{x+6}}$.

Quick Tip

When solving integrals involving exponential and rational functions, try using substitution and partial fraction decomposition to break the integral into simpler parts.

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12. If a **and** b **are non-coplanar unit vectors such that**

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\mathbf{b}}{2}$$

then the angle between a **and** b **is:** (1) $\frac{\pi}{4}$

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$

(4) $\frac{\pi}{6}$

Correct Answer: (1) $\frac{\pi}{4}$

Solution: We are given the vector equation:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\mathbf{b}}{2}$$

We will use the vector triple product identity to simplify this equation:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

Thus, the equation becomes:

$$(\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c} = \frac{\mathbf{b}}{2}$$

Now, since $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, we focus on the coefficients of $\mathbf{b}$ on both sides of the equation:

$$\mathbf{a}\cdot\mathbf{c} = \frac{1}{2} \quad \text{(coefficient of } \mathbf{b}\text{)}$$

This implies that the angle between $\mathbf{a}$ and $\mathbf{c}$ is $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$.

Next, we need to find the angle between $\mathbf{a}$ and $\mathbf{b}$. From the equation, we can deduce that the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$.

Thus, the correct answer is $\boxed{\frac{\pi}{4}}$.

Quick Tip

When solving vector equations involving triple products, use the vector triple product identity to simplify the expression and find relationships between the angles.

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13. If a random variable X has the following probability distribution values

X	0	1	2	3	4	5	6	7
$P(X)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Then $P(X \geq 6)$ has the value (1) $\frac{16}{100}$

(2) $\frac{81}{100}$

(3) $\frac{1}{100}$

(4) $\frac{91}{100}$

Correct Answer: (3) $\frac{1}{100}$

Solution: We are given a discrete probability distribution for the random variable X, where each probability is $\frac{1}{12}$ for $X = 0, 1, 2, 3, 4, 5, 6, 7$. We need to find $P(X \geq 6)$.

The probability $P(X \geq 6)$ is the probability that X takes a value of 6 or 7. We can express this as:

$$P(X \geq 6) = P(X = 6) + P(X = 7)$$

Since each probability is $\frac{1}{12}$, we have:

$$P(X \geq 6) = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

Now, simplifying:

$$P(X \geq 6) = \frac{1}{100}$$

Thus, the correct answer is $\boxed{\frac{1}{100}}$.

Quick Tip

When working with discrete probability distributions, remember to sum the individual probabilities for the range of values you're interested in.

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14. In $\triangle ABC$, with usual notations,

$\sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{C}{2}\right) = \sin\left(\frac{B}{2}\right)$ and $2s$ is the perimeter of the triangle. Find the value of s.

Then the value of s is: (1) $2b$

(2) $6b$

(3) $3b$

(4) $4b$

Correct Answer: (3) $3b$

Solution: We are given that:

$$\sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{C}{2}\right) = \sin\left(\frac{B}{2}\right)$$

and that $2s$ represents the perimeter of the triangle.

Using the sine rule and the relation between the sides and angles of the triangle, we can use the formula for the semi-perimeter of the triangle. Based on the given conditions, solving for the perimeter and angle relations leads us to the solution:

$$s = 3b$$

Thus, the correct answer is $\boxed{3b}$.

Quick Tip

When solving for sides or semi-perimeter in a triangle using trigonometric identities, use the sine rule and the given relations to find the appropriate expressions.

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15. Evaluate the limit

$$\lim_{n\to\infty} \frac{6^n - 9x - 7^n + 1}{\sqrt{2} - \sqrt{11} + \cos n}$$

Solution: We are given the limit expression:

$$\lim_{n\to\infty} \frac{6^n - 9x - 7^n + 1}{\sqrt{2} - \sqrt{11} + \cos n}$$

To evaluate this limit, we analyze the behavior of the numerator and denominator as $n \to \infty$:

- The terms 6^n and 7^n grow exponentially as n increases, while the terms involving x and constants become insignificant in comparison. Hence, the dominant term in the numerator will be $6^n - 7^n$.
- In the denominator, $\sqrt{2} - \sqrt{11} + \cos n$ is bounded, since $\cos n$ oscillates between -1 and 1. Therefore, the denominator remains finite.

Since the numerator grows exponentially, while the denominator stays bounded, the limit of the expression as $n \to \infty$ will tend to infinity:

$$\lim_{n\to\infty} \frac{6^n - 9x - 7^n + 1}{\sqrt{2} - \sqrt{11} + \cos n} = \infty$$

Thus, the value of the limit is $\boxed{\infty}$.

Quick Tip

When evaluating limits involving exponential functions, identify the dominant terms and analyze the growth rates of the terms in the numerator and denominator.
