Previous Years' Paper

Common University Entrance Test for UG Programmes

CUET-UG - Mathematics

Entrance Exam, 2025

(After the list of questions, the solution will Start.)

Q1. Let A = $[aj]_{nxn}$ be a matrix. Then

Match List-I with List-II

List-I	List-II
(A) $A^T = A$	(I) A is a singular matrix
(B) $A^T = -A$	(II) A is a non-singular matrix
(C) $\|A\| = 0$	(III) A is a skew symmetric matric
(D) $\|A\| \neq 0$	(IV) A is a symmetric matric

Choose the correct answer from the options given below:

1. (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

2. (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

3. (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

4. (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

Q2. If A =

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

then the matrix AB is equal to

1. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

Q3. If A is a square matrix and I is the identity matrix of same order such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 3A$ is equal to

1. A

2. 2A

3. 3A

4. 5A

Q4. If A =

$\begin{bmatrix} 0 & 0 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 0 \end{bmatrix}$,

then |adj A| is equal to

1. 3

2. 9

3. 27

4. 81

Q5. If $y = 3e^{2x} + 2e^{3x}$, then $\frac{d^2y}{dx^2} + 6y$ is equal to

1. $\frac{dy}{dx}$

2. $5\frac{dy}{dx}$

3. $6\frac{dy}{dx}$

4. $30\frac{dy}{dx}$

Q6. The interval, on which the function $f(x) = x^2e^{-x}$ is increasing, is equal to

1. $(-\infty, \infty)$

2. $(-\infty, 2) \cup (2, \infty)$

3. $(-2, 0)$

4. $(0, 2)$

Q7. If the maximum value of the function $f(x) = \frac{\log e^x}{x}, x > 0$ occurs at x = a, then $a^2 f''(a)$ is equal to

1. $-\frac{5}{e}$

2. $-\frac{1}{e}$

3. $-\frac{1}{e^3}$

4. $-5e^3$

Q8.

$\int_1^4 |x-2|dx$ is equal to

1. 5

2. $\frac{7}{2}$

3. $\frac{3}{2}$

4. $\frac{5}{2}$

Q9.

The integral I = $\int \frac{e^{5\log_e x} - e^{4\log_e x}}{e^{3\log_e x} - e^{2\log_e x}} dx$ is equal to

1. $\frac{x}{2}+C$, where C is the constant of integration

2. $\frac{x^2}{2}+C$, where C is the constant of integration

3. $\frac{x^3}{3}+C$, where C is the constant of integration

4. $\frac{x^4}{4}+C$, where C is the constant of integration

Q10. The area (in sq. units) of the region bounded by the parabola $y^2 = 4x$ and the line $x = 1$ is

1. $\frac{1}{3}$

2. $\frac{4}{3}$

3. $\frac{5}{3}$

4. $\frac{8}{3}$

Q11. Which of the following are linear first order differential equations?

(A) $\frac{dy}{dx} + P(x)y = Q(x)$

(B) $\frac{dx}{dy} + P(y)x = Q(y)$

(C) $(x - y)\frac{dy}{dx} = x + 2y$

(D) $(1 + x^2)\frac{dy}{dx} + 2xy = 2$

Choose the correct answer from the options given below:

1. (A), (B) and (D) only

2. (A) and (B) only

3. (A), (B) and (C) only

4. (A), (B), (C) and (D)

Q12. The solution of the differential equation $\log e\left(\frac{dy}{dx}\right) = 3x + 4y$ is given by

1. $4e^{3x} + 3e^{-4y} + C = 0$, where C is constant of integration

2. $3e^{3x} + 4e^{-4y} + C = 0$, where C is constant of integration

3. $4e^{-3x} + 3e^{4y} + C = 0$, where C is constant of integration

4. $3e^{-3x} + 4e^{4y} + C = 0$, where C is constant of integration

Q13. The probability distribution of a random variable X is given by

X	0	1	2
P(X)	$1 - 7a^2$	$\frac{1}{2}a + \frac{1}{4}$	a^2

If a > 0, then P(0 < x ≤ 2) is equal to

1. $\frac{1}{16}$

2. $\frac{3}{18}$

3. $\frac{7}{16}$

4. $\frac{9}{16}$

Q14. The corner points of the feasible region associated with the LPP: Maximise Z = px + qy, p, q> 0 subject to $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). If optimum value occurs at both (3, 4) and (0, 5), then

1. p = q

2. p = 2q

3. p = 3q

4. q = 3p

Q15. Consider the LPP: Minimize Z = $x + 2y$ subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$. The optimal feasible solution occurs at

1. (6, 0) only

2. (0, 3) only

3. Neither (6, 0) nor (0, 3)

4. Both (6, 0) and (0, 3)

Q16. Let f: R→R be defined as f(x) = 10x. Then (Where R is the set of real numbers)

1. f is both one-one and onto

2. f is onto but not one-one

3. f is one-one but not onto

4. f is neither one-one nor onto

Q17. Let A = {1, 2, 3}. Then, the number of relations containing (1, 2) and (1, 3), which are reflexive and symmetric but not transitive, is

1. 1

2. 2

3. 3

4. 4

Q18. for $|x| < 1$, $\sin(\tan^{-1}x)$ equal to

1. $\frac{1}{\sqrt{1+x^2}}$

2. $\frac{1}{\sqrt{1-x^2}}$

3. $\frac{x}{\sqrt{1-x^2}}$

4. $\frac{x}{\sqrt{1+x^2}}$

Q19. Let A

$$= \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 2 \\ 2 & 4 & 1 \end{bmatrix}$$

and M_{ij}, A_{ij} respectively denote the minor, co-factor of an element a_{ij} of matrix A, then which of the following are true?

(A) $M_{22} = -1$

(B) $A_{23} = 0$

(C) $A_{32} = 3$

(D) $M_{23} = 1$

(E) $M_{32} = -3$

Choose the correct answer from the options given below:

1. (A) and (B) only

2. (A), (B), (C) and (E) only

3. (A), (D) and (E) only

4. (A), (C) and (E) only

Q20. Let A

$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. If $A^T + A = I$, then

1. $\theta = 2n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$

2. $\theta = n\pi, n \in \mathbb{Z}$

3. $\theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

4. $\theta = 2n\pi + \frac{\pi}{6}, n \in \mathbb{Z}$

Q21. If A and B are skew-symmetric matrices, then which one of the following is NOT true?

1. $A^3 + B^5$ is skew-symmetric

2. A^{19} is skew-symmetric

3. B^{14} is symmetric

4. $A^4 + B^5$ is symmetric

Q22. If A and B are invertible matrices then which of the following statement is NOT correct?

1. $\text{adj}A = |A|A^{-1}$

2. $(A + B)^{-1} = A^{-1} + B^{-1}$

3. $|A^{-1}| = |A|^{-1}$

4. $(AB)^{-1} = B^{-1}A^{-1}$

Q23. Let $A = [a_{ij}]_{2\times 3}$ and $B = [b_{ij}]_{3\times 2}$, then $|5AB|$ is equal to

1. $5^2\ |A|.|B|$

2. $5^3\ |A|.|B|$

3. $5^2\ |AB|$

4. $5^3\ |AB|$

Q24. Let AX = B be a system of three linear equations in three variables. Then the system has

(A) a unique solutions if $|A| = 0$

(B) a unique solutions if $|A| \neq 0$

(C) no solutions if $|A| = 0$ and (adj A) $B \neq 0$

(D) infinitely many solutions if $|A| = 0$ and (adj A)$B = 0$

Choose the correct answer from the options given below:

1. (A), (C) and (D) only

2. (B), (C) and (D) only

3. (B) only

4. (B) and (C) only

Q25. If the function $f(x)$

$$= \begin{cases} \dfrac{k\cos x}{\pi - 2x} & : x \neq \dfrac{\pi}{2} \\ 3 & : x = \dfrac{\pi}{2} \end{cases}$$ is continuous at $x = \frac{\pi}{2}$, then k is equal to

1. 6

2. 5

3. -6

4. 4

Q26. Match List-I with List-II

List-I	List-II
(A) $f(x) = \|x\|$	(I) Not differentiable at $x =$ -2 only
(B) $f(x) = \|x + 2\|$	(II) Not differentiable at $x = 0$ only
(C) $f(x) = \|x^2 - 4\|$	(III) Not differentiable at $x = 2$ only
(D) $f(x) = \|x - 2\|$	(IV) Not differentiable at $x =$ 2, -2 only

Choose the correct answer from the options given below:

1. (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

2. (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

3. (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

4. (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

Q27.

Let $y = \sin(\cos x^2)$, then the value of $\frac{dy}{dx}$ at $x = \frac{\sqrt{\pi}}{2}$ is equal to

1. $\frac{-\sqrt{\pi}}{2} \cos\left(\frac{1}{\sqrt{2}}\right)$

2. $-\sqrt{\frac{\pi}{2}} \cos\left(\frac{1}{\sqrt{2}}\right)$

3. $-\sqrt{\frac{\pi}{2}} \sin\left(\frac{1}{\sqrt{2}}\right)$

4. $\sqrt{\frac{\pi}{2}} \sin\left(\frac{1}{\sqrt{2}}\right)$

Q28. Match List-I with List-II

List-I	List-II
(A) The minimum value of $f(x) = (2x - 1)^2 + 3$	(I) 4
(B) The maximum value of $f(x)$ -$\|x +1]$ + 4	(II) 10
(C) The minimum value of $f(x) = \sin(2x) + 9$	(III) 3

(D) The maximum value of $f(x) = -(x - 1)^2 + 10$	(IV) 5

Choose the correct answer from the options given below:

1. (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

2. (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

3. (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

4. (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

Q29. The function $f(x) = \tan x - x$

1. is a decreasing function on $\left[0, \frac{\pi}{2}\right)$

2. is an increasing function on $\left[0, \frac{\pi}{2}\right)$

3. is a constant function

4. is neither increasing nor decreasing function on $\left[0, \frac{\pi}{2}\right)$

Q30. The rate of change of area of a circle with respect to its circumference when radius is 4cm, is

1. 2 cm^2/cm

2. 4 cm^2/cm

3. 8 cm^2/cm

4. 16 cm^2/cm

Q31.

$$\int_{\pi/6}^{\pi/3} \frac{\tan x}{\tan x + \cot x}\, dx \text{ is equal to}$$

1. $\frac{\pi}{4}$

2. 0

3. $\frac{\pi}{6}$

4. $\frac{\pi}{12}$

Q32. Match List-I with List-II

List-I	List-II
Definite integral	Value
(A) $\int_0^1 \frac{2x}{1+x^2}\,dx$	(I) 2
(B) $\int_{-1}^{1} sin^3x cos^4x\,dx$	(II) $log_e\left(\frac{3}{2}\right)$
(C) $\int_0^{\pi} \sin x\,dx$	(III) $log_e 2$
(D) $\int_2^3 \frac{2}{x^2-1}\,dx$	(IV) 0

Choose the correct answer from the options given below:

1. (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

2. (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

3. (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

4. (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

Q33.

The integral I = $\int e^x \left(\frac{x-1}{3x^2}\right) dx$ is equal to

1. $\frac{1}{3}\left(\frac{x^2}{2}-x\right)+C$, where C is constant of integration

2. $\left(\frac{x^2}{2}-x\right)e^x+C$, where C is constant of integration

3. $\frac{1}{3x^2}e^x+C$, where C is constant of integration

4. $\frac{1}{3x}e^x+C$, where C is constant of integration

Q34. The area (in sq. units) of the region bounded by the curve $y = x^5$, the x-axis and the ordinates $x = -1$ and $x = 1$ is equal to

1. $\frac{1}{6}$

2. $\frac{1}{3}$

3. $\frac{1}{2}$

4. $\frac{2}{3}$

Q35. The area (in sq. units) of the region bounded by $y = 2\sqrt{1 - x^2}$, $x \in [0, 1]$ and x-axis is equal to

1. 1

2. 2

3. $\frac{\pi}{2}$

4. $\frac{\pi}{4}$

Q36. The integrating factor of the differential equation

$(x \log_e x)\frac{dy}{dx} + y = 2\log_e x$ is

1. $\log_e x$

2. x

3. $\frac{1}{x}$

4. $\frac{1}{\log_e x}$

Q37. Consider the differential equation,

$$x\frac{dy}{dx} = y(\log_e y - \log_e x + 1),$$

then which of the following are true?

(A) It is a linear differential equation

(B) It is a homogenous differential equation

(C) Its general solution is

$$log_e\left(\frac{y}{x}\right) = Cx\ ,$$

where C is constant of integration

(D) Its general solution is

$$\log_e\left(\frac{x}{y}\right) = Cy\ ,$$

where C is constant of integration

(E) If $y(1) = 1$, then its particular solution is $y = x$

Choose the correct answer from the options given below:

1. (A), (D) and (E) only

2. (A) and (D) only

3. (B) and (C) only

4. (B), (C) and (E) only

Q38. If $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are unit vectors along co-ordinate axes OX, OY and OZ respectively, then which of the following is/are true?

(A) $\hat{\imath} \times \hat{\imath} = \vec{0}$

(B) $\hat{\imath} \times \hat{k} = \hat{\jmath}$

(C) $\hat{\imath} \cdot \hat{\imath} = 1$

(D) $\hat{\imath} \cdot \hat{\jmath} = 0$

Choose the correct answer from the options given below:

1. (A) and (B) only

2. (A), (C) and (D) only

3. (A) only

4. (A), (B), (C) and (D)

Q39. If the points A, B, C with position vectors $20\hat{i} + \lambda\hat{j}$, $5\hat{i} - \hat{j}$ and $10\hat{i} - 13\hat{j}$ respectively are collinear, then the value of λ is

1. 12

2. -37

3. 37

4. -12

Q40. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is

1. $\frac{\pi}{2}$

2. $\frac{\pi}{3}$

3. $\frac{\pi}{4}$

4. $\frac{\pi}{6}$

Q41.

Let $\vec{a} = \hat{i} + 4\hat{j}$, $\vec{b} = 4\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 2\hat{k}$.

If $\vec{d}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{c} \cdot \vec{d} = 16$, then $|\vec{d}|$ is equal to

1. $\sqrt{33}$

2. $2\sqrt{33}$

3. $3\sqrt{33}$

4. $4\sqrt{33}$

Q42. If a line makes angles α, β, Υ with the positive directions of x-axis, y- axis and z-axis respectively, then $\sin^2\alpha + \sin^2\beta + \sin^2\Upsilon$ is equal to

1. 1

2. 2

3. 3

4. -2

Q43. Consider the line

$$\vec{r} = \hat{i} - 2\hat{j} + 4\hat{k} + \lambda(-\hat{i} + 2\hat{j} - 4\hat{k})$$

Match List-I with List-II

List-I	**List-II**
(A) A point on the given line	(I) $\left(\frac{-1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{-4}{\sqrt{21}}\right)$
(B) direction ratios of the line	(II) (4 , -2, -2)
(C) direction cosines of the line	(III) (1, -2, 4)
(D) direction ratios of a line perpendicular to given line	(IV) (-1, 2, -4)

Choose the correct answer from the options given below:

1. (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

2. (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

3. (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

4. (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

Q44. The shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{4} = \frac{y-4}{6} = \frac{z-5}{8}$$

is equal to

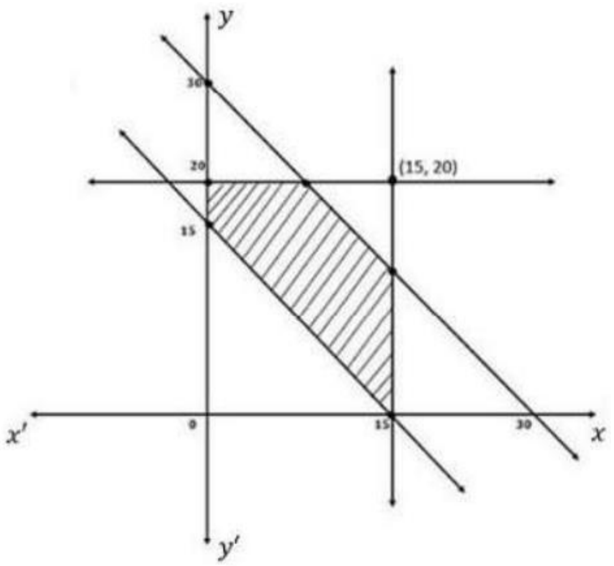
1. 0

2. $\sqrt{\frac{29}{5}}$

3. $\sqrt{\frac{5}{29}}$

4. $\sqrt{5}$

Q45. Which one of the following set of constraints does the given shaded region represent?



1. $x + y \leq 30$, $x + y \geq 15$, $x \leq 15$, $y \leq 20$, x, y ≥ 0

2. $x + y \leq 30$, $x + y \geq 15$, $y \leq 15$, $x \leq 20$, $x, y \geq 0$

3. $x + y \geq 30$, $x + y \leq 15$, $x \leq 15$, $y \leq 20$, $x, y \geq 0$

4. $x + y \geq 30$, $x + y \leq 15$, $y \leq 15$, $x \leq 20$, $x, y \geq 0$

Q46. The corner points of the feasible region of the LPP: Minimize Z = -50x + 20y subject to 2x - $y \geq$ -5, 3x + $y \geq$ 3, 2x - 3$y \leq$ 12 and $x, y \geq 0$ are

1. (0,5), (0,6), (1,0), (6,0)

2. (0,3), (0,5), (3,0), (6,0)

3. (0,3), (0,5), (1,0), (6,0)

4. (0,5), (0,6), (1,0), (3,0)

Q47. If A and B are any two events such that P(B) = P(A and B), then which of the following is correct

1. P(BA) = 1

2. P(A|B) = 1

3. P(BA) = 0

4. P(AB) = 0

Q48. If A is any event associated with sample space and If E_1, E_2, E_3 are mutually exclusive and exhaustive events. Then which of the following are true?

(A) $P(A) = P(E_1)P(E_1|A) + P(E_2)P(E_2|A) + P(E_3)P(E_3|A)$

(B) $P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)$

(C) $P(E_i|A) = \dfrac{P(A|E_i)P(E_i)}{\sum_{i=1}^{3} P(A|E_i)P(E_i)}, i = 1, 2, 3$

(D) $P(A|E_i) = \dfrac{P(E_i|A)P(E_i)}{\sum_{i=1}^{3} P(E_i|A)P(E_i)}, i = 1, 2, 3$

Choose the correct answer from the options given below:

1. (A) and (C) only

2. (A) and (D) only

3. (B) and (D) only

4. (B) and (C) only

Q49. Match List-I with List-II

Let A and B are two events such that P(A) = 0.8, P(B) =0.5, P(B|A) = 0.4

List-I	List-II
(A) P (A ∩ B)	(I) 0.2
(B) P (A | B)	(II) 0.32
(C) P (A ∪ B)	(III) 0.64
(D) P (A')	(IV) 0.98

Choose the correct answer from the options given below:

1. (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

2. (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

3. (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

4. (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

Q50. A black and a red die are rolled simultaneously. The probability of obtaining a sum greater than 9, given that the black resulted in a 5 is

1. 1/2

2. 1

3. 2/3

4. 1/3

Solution

Q1.

Ans.

The correct answer is **2. (A) – (IV), (B) – (III), (C) – (I), (D) – (II)**

Explanation:

- (A) $A^T = A$ → by definition, **symmetric matrix** → (IV).
- (B) $A^T = -A$ → by definition, **skew-symmetric matrix** → (III).
- (C) $|A| = 0$ → determinant zero ⇒ **no inverse** ⇒ **singular matrix** → (I).
- (D) $|A| \neq 0$ → determinant non-zero ⇒ **inverse exists** ⇒ **non-singular matrix** → (II).

Q2.

Ans.

The correct answer is **Option 2.**

Explanation:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \cdot 0 + 0 \cdot 1 & 1 \cdot 1 + 0 \cdot 0 \\ 0 \cdot 0 + (-1) \cdot 1 & 0 \cdot 1 + (-1) \cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Q3.

Ans.

The correct answer is **Option 4. 5A**

Explanation:

$$(A-I)^3 = A^3 - 3A^2 + 3A - I, \quad (A+I)^3 = A^3 + 3A^2 + 3A + I$$

Adding and subtracting $3A$:

$$(A-I)^3 + (A+I)^3 - 3A = 2A^3 + 6A - 3A = 2A^3 + 3A$$

Given $A^2 = I \Rightarrow A^3 = A$. Hence

$$2A^3 + 3A = 2A + 3A = 5A.$$

Q4.

Ans.

The correct answer is **Option 3. 27**

Explanation:

- For an $n \times n$ matrix, $|\operatorname{adj} A| = (|A|)^{n-1}$. Here $n = 3 \Rightarrow |\operatorname{adj} A| = (|A|)^2$.
- $A = \begin{bmatrix} 0 & 0 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 0 \end{bmatrix}$.

The only non-zero product in the determinant is along the permutation $(1 \to 3, 2 \to 2, 3 \to 1)$ (an odd permutation), so

$|A| = -(\sqrt{3})^3 = -3\sqrt{3}$.
- Hence $|\operatorname{adj} A| = (|A|)^2 = (-3\sqrt{3})^2 = 27$.

Q5.

Ans.

The correct answer is **Option 2.** $5\frac{dy}{dx}$

Explanation:

Given $y = 3e^{2x} + 2e^{3x}$,

$$y' = 6e^{2x} + 6e^{3x}, \qquad y'' = 12e^{2x} + 18e^{3x}.$$

Then

$y'' + 6y$

$= (12 + 18)e^{2x} + (18 + 12)e^{3x}$

$= 30(e^{2x} + e^{3x})$

$= 5\left(6e^{2x} + 6e^{3x}\right) = 5\,y'.$

Hence, $y'' + 6y = 5\,\dfrac{dy}{dx}.$

Q6.

Ans.

The correct answer is **Option 4. (0, 2)**

Explanation:

• $f(x) = x^2e^{-x} \Rightarrow f'(x) = e^{-x}(2x - x^2) = e^{-x}\,x(2 - x).$

• Since $e^{-x} > 0$ for all x, the sign of $f'(x)$ depends on $x(2 - x)$.

• $x(2 - x) > 0$ for $0 < x < 2$ and <0 otherwise.

Hence, $f(x)$ is **increasing on** $(0, 2)$.

Q7.

Ans.

The correct answer is **Option 2.** - $\frac{1}{e}$

Explanation:

Interpret log as natural log: $f(x) = \dfrac{\ln x}{x},\ x > 0.$

$$f'(x) = \frac{1 - \ln x}{x^2} \Rightarrow f'(x) = 0 \text{ at } x = e\ (= a).$$

$$f''(x) = \frac{2\ln x - 3}{x^3} \Rightarrow f''(a) = f''(e) = \frac{2 - 3}{e^3} = -\frac{1}{e^3}.$$

So,

$$a^2 f''(a) = e^2\left(-\frac{1}{e^3}\right) = -\frac{1}{e}.$$

Q8.

Ans.

The correct answer is **Option 4.** $\frac{5}{2}$

Explanation:

- The integrand has an absolute value $|x-2|$. It changes form where the inside $x - 2 = 0 \Rightarrow x = 2$. So split the interval $[1, 4]$ at $x = 2$.
- Write it piecewise:
  - For $1 \le x \le 2$: $|x-2| = 2 - x$ (because $x - 2 \le 0$).
  - For $2 \le x \le 4$: $|x-2| = x - 2$ (because $x - 2 \ge 0$).
- Integrate on each part:
  - $\int_1^2 (2-x)\,dx = \left[2x - \frac{x^2}{2}\right]_1^2 = (4-2) - (2 - \frac{1}{2}) = \frac{1}{2}$.
  - $\int_2^4 (x-2)\,dx = \left[\frac{x^2}{2} - 2x\right]_2^4 = (8-8) - (2-4) = 2$.
- Add the two results: $\frac{1}{2} + 2 = \boxed{\frac{5}{2}}$.

Q9.

Ans.

The correct answer is **Option**

3. $\frac{x^3}{3} + C$, where C is the constant of integration

Explanation:

- Use $e^{k \ln x} = x^k$ (for $x > 0$).
- Numerator: $e^{5\ln x} - e^{4\ln x} = x^5 - x^4 = x^4(x-1)$.
- Denominator: $e^{3\ln x} - e^{2\ln x} = x^3 - x^2 = x^2(x-1)$.
- Fraction simplifies to $\frac{x^4(x-1)}{x^2(x-1)} = x^2$ (for $x \neq 1$).
- Hence $I = \int x^2\,dx = \frac{x^3}{3} + C$.

Q10.

Ans.

The correct answer is **Option 4.** $\mathbf{\frac{8}{3}}$

Explanation:

- Parabola: $y^2 = 4x \Rightarrow x = \dfrac{y^2}{4}$ (opens right; vertex at $(0, 0)$).
- Line: $x = 1$. Intersection with parabola: $y^2 = 4 \Rightarrow y = \pm 2$.
- The bounded region lies between $x = 1$ (right boundary) and the parabola $x = \dfrac{y^2}{4}$ (left boundary) for $y \in [-2, 2]$.
- Area $= \displaystyle\int_{-2}^{2} \left[(x_{\text{right}} - x_{\text{left}})\right] dy = \int_{-2}^{2} \left(1 - \frac{y^2}{4}\right) dy$.
- Evaluate: $\left[y - \frac{y^3}{12}\right]_{-2}^{2} = (2 - \frac{8}{12}) - (-2 + \frac{8}{12}) = \dfrac{8}{3}$.

Hence, the area is $\boxed{\dfrac{8}{3}}$.

Q11.

Ans.

The correct answer is **Option 1. (A), (B) and (D) only**

Explanation:

- **(A)** is the standard linear first-order form in $y(x)$: $\frac{dy}{dx} + P(x)y = Q(x)$.
- **(B)** is linear with x as the dependent variable: $\frac{dx}{dy} + P(y)x = Q(y)$.
- **(C)** is **not** linear since $(x - y)\frac{dy}{dx} = x + 2y \Rightarrow \frac{dy}{dx} = \frac{x+2y}{x-y}$; the coefficient of $\frac{dy}{dx}$ and the RHS depend on y, so it cannot be written with coefficients depending only on the independent variable.
- **(D)** is linear: $(1 + x^2)\frac{dy}{dx} + 2xy = 2$ has coefficients depending only on x.

Q12.

Ans.

The correct answer is **Option 1. $4e^{3x} + 3e^{-4y} + C = 0$, where C is constant of integration.**

Explanation:

- From $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, exponentiate: $\frac{dy}{dx} = e^{3x}e^{4y}$.
- Separate variables: $e^{-4y}dy = e^{3x}dx$.
- Integrate: $\int e^{-4y}dy = \int e^{3x}dx \Rightarrow -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C$.
- Rearranging (absorbing constants) gives $4e^{3x} + 3e^{-4y} + C = 0$.

Q13.

Ans.

The correct answer is **Option 3.** $\frac{7}{16}$

Explanation:

Given the pmf of X:

$P(X = 0) = 1 - 7a^2,$

$P(X = 1) = \frac{1}{2}a + \frac{1}{4},$

$P(X = 2) = a^2, \quad a > 0.$

Since total probability is 1,

$$(1 - 7a^2) + \left(\frac{1}{2}a + \frac{1}{4}\right) + a^2 = 1 \;\Rightarrow\; -6a^2 + \frac{1}{2}a + \frac{1}{4} = 0$$

$$\Rightarrow\; 24a^2 - 2a - 1 = 0 \;\Rightarrow\; a = \frac{2 \pm \sqrt{100}}{48} = \frac{1}{4} \text{ or } -\frac{1}{6}.$$

Given $a > 0$, take $a = \frac{1}{4}$.

Now,

$P(0 < X \leq 2) = P(X = 1) + P(X = 2)$

$= \left(\frac{1}{2} \cdot \frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 = \frac{1}{8} + \frac{1}{4} + \frac{1}{16} = \frac{7}{16}.$

Q14.

Ans.

The correct answer is **Option 4. $q = 3p$**

Explanation:

If the optimum occurs at **both** (3, 4) and (0, 5), the objective values there must be equal:

$$Z(3,4) = 3p + 4q \quad \text{and} \quad Z(0,5) = 5q$$

$$3p + 4q = 5q \;\Rightarrow\; 3p = q \;\Rightarrow\; q = 3p.$$

Check it's a maximum: with $q = 3p$ (and $p > 0$),

$$Z(0,0) = 0, \quad Z(5,0) = 5p, \quad Z(3,4) = 15p, \quad Z(0,5) = 15p,$$

so, the largest value $15p$ occurs at both (3, 4) and (0, 5).

(Geometrically, the iso-profit line $px + qy = k$ has slope $-p/q$. The edge joining (3, 4) to (0, 5) has slope $-1/3$. For multiple optima, the slopes must match: $-p/q = -1/3 \Rightarrow q = 3p$.)

Q15.

Ans.

The correct answer is **Option 4. Both (6, 0) and (0, 3)**

Explanation:

Minimize $Z = x + 2y$ subject to

$$2x + y \geq 3, \qquad x + 2y \geq 6, \qquad x, y \geq 0.$$

The feasible region is the set of points in the first quadrant lying **above** both lines.

The two boundary lines meet at

$$\begin{cases} 2x + y = 3 \\ x + 2y = 6 \end{cases} \Rightarrow x = 0,\; y = 3 \;\text{ i.e. } (0,3).$$

Intercepts on the axes satisfying $x + 2y = 6$ are (6, 0) and (0, 3); both also satisfy $2x + y \geq 3$, so they are feasible.

Along the boundary $x + 2y = 6$ (the "lower edge" of the feasible region),

$$Z = x + 2y = 6 \quad \text{(constant)}.$$

Any interior point has $x + 2y > 6 \Rightarrow Z > 6$.

Hence the **minimum value** of Z is 6, attained at **every point** on the segment joining (0, 3) and (6, 0), in particular at both endpoints (6, 0) and (0, 3).

Q16.

Ans.

The correct answer is **Option 1. f is both one-one and onto**

Explanation:

1. To check one-one:

Suppose $f(x_1) = f(x_2)$.

Then, $10x_1 = 10x_2$.

Dividing both sides by 10 gives $x_1 = x_2$.

This means different x values give different f(x) values.

Hence, the function is **one-one**.

2. To check onto:

Let y be any real number.

We have $f(x) = y$.

That means $10x = y$.

So, $x = \frac{y}{10}$.

Since $\frac{y}{10}$ is also a real number, there exists an x for every y.

Hence, the function is onto.

3. Conclusion:

The function $f(x) = 10x$ is **both one-one and onto**, because it gives unique outputs for all x and covers all real numbers as outputs.

Q17.

Ans.

The correct answer is **Option 1. 1**

Explanation:

Given $A = \{1,2,3\}$.

We need relations that are **reflexive** and **symmetric** but **not transitive**, and must contain (1, 2) and (1, 3).

For reflexive: (1, 1), (2, 2), (3, 3) must be in R.

For symmetric: (2, 1) must be with (1, 2), and (3, 1) must be with (1, 3).

So at least we have

$$R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)\}.$$

Now, check transitivity:

Since (2, 1) and (1, 3) are in R, transitivity would require (2, 3) (and hence (3, 2) by symmetry).

To make R **not transitive**, we must **not include (2, 3)** and **(3, 2)**.

Therefore, this single relation satisfies all conditions.

Q18.

Ans.

The correct answer is **Option 4.**

$$\frac{x}{\sqrt{1+x^2}}.$$

Explanation:

Let $\theta = \tan^{-1} x$.

Then, $\tan\theta = x = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{1}$.

The hypotenuse $= \sqrt{1+x^2}$.

So,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{\sqrt{1+x^2}}.$$

Hence,

$$\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}.$$

Q19.

Ans.

The correct answer is **Option 2. (A), (B), and (E) only**

Explanation:

Given

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 2 \\ 2 & 4 & 1 \end{bmatrix}$$

We find the minors and cofactors.

(A) M_{22}:

Remove 2nd row, 2nd column →

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \Rightarrow M_{22} = 1(1) - (1)(2) = -1$$

✔ True

(B) $A_{23} = (-1)^{2+3} M_{23} = -M_{23}$

Remove 2nd row, 3rd column →

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \Rightarrow M_{23} = 4 - 4 = 0$$

$$A_{23} = 0$$

✔ True

(C) $A_{32} = (-1)^{3+2} M_{32} = -M_{32}$

Remove 3rd row, 2nd column →

$$\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \Rightarrow M_{32} = (1)(2) - (1)(-1) = 3$$

$$A_{32} = -3$$

✗ False

(D) $M_{23} = 0$

✗ False

(E) $M_{32} = 3$

✓ True

☑ **Correct statements:** (A), (B), and (E)

Q20.

Ans.

The correct answer is **Option 1.**

$\theta = 2n\pi \pm \frac{\pi}{3},\ n \in \mathbb{Z}$

Explanation:

Given

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $A^T + A = I$.

$$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

So,

$$A^T + A = \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix}$$

Given $A^T + A = I$,

$$2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2}$$

Hence,

$$\theta = 2n\pi \pm \frac{\pi}{3},\ n \in \mathbb{Z}$$

Q21.

Ans.

The correct answer is **Option 4. $A^4 + B^5$ is symmetric**

Explanation:

A square matrix A is **skew-symmetric** if

$$A^T = -A$$

From this property:

- For any **odd power** n:

  $(A^n)^T = (-1)^n A^n = -A^n$, hence **skew-symmetric**.

- For any **even power** n:

  $(A^n)^T = (-1)^n A^n = A^n$, hence **symmetric**.

Checking each option

1. **$A^3 + B^5$**
   - A^3: odd power → skew-symmetric
   - B^5: odd power → skew-symmetric
   - Sum of two skew-symmetric matrices → skew-symmetric

     ✅ True

2. **A^{19}**
   - 19 is odd → skew-symmetric

     ✅ True

3. B^{14}
    - 14 is even → symmetric

    ✅ True

4. $A^4 + B^5$
    - A^4: even power → symmetric
    - B^5: odd power → skew-symmetric
    - Sum of symmetric and skew-symmetric matrices → neither symmetric nor skew-symmetric

    ❌ Not true

Q22.

Ans.

The correct answer is **Option 2. $(A + B)^{-1} = A^{-1} + B^{-1}$** is **NOT correct.**

Explanation:

If **A** and **B** are **invertible matrices**, the following properties are true:

- **Relation between adjugate and inverse:**

$$A^{-1} = \frac{\text{adj}A}{|A|}$$
$$\Rightarrow \text{adj}A = |A|A^{-1}$$

- **Determinant of an inverse matrix:**

$|A^{-1}| = |A|^{-1}$

- **Inverse of product of two matrices:**

$(AB)^{-1} = B^{-1} A^{-1}$

Checking each option

1. **$\text{adj}A = |A|A^{-1}$**
    - True, follows directly from the definition of inverse and adjugate.

    ✅ True

2. $(A + B)^{-1} = A^{-1} + B^{-1}$

    - False in general.
    - The inverse of a sum is **not equal** to the sum of inverses (only holds in special cases).

    ✕ Not correct

3. $|A^{-1}| = |A|^{-1}$

    - True, since determinant of the inverse is reciprocal of determinant.

    ☑ True

4. $(AB)^{-1} = B^{-1}A^{-1}$

    - True, this is a fundamental property of inverses.

    ☑ True

Q23.

Ans.

The correct answer is **Option 3. 5^2 |AB|**

Explanation:

Given:

$A = [a_{ij}]_{2\times3}$ and $B = [b_{ij}]_{3\times2}$

So,

- A is a $\mathbf{2 \times 3}$ matrix
- B is a $\mathbf{3 \times 2}$ matrix
- AB will be a $\mathbf{2 \times 2}$ matrix

Property Used:

If A is an $n \times n$ square matrix, then

$$|kA| = k^n|A|$$

where k is a scalar and n is the order (number of rows or columns) of the square matrix.

Applying the property:

- AB is a **2 × 2** matrix ⇒ its determinant has order 2
- So,

$$|5AB| = 5^2\,|AB|$$

Q24.

Ans.

The correct answer is **Option 2. (B), (C), and (D) only**

Explanation:

1. **(A)** A unique solution if $|A| = 0$

   ❌ Not true — if $|A| = 0$, the system cannot have a unique solution.

2. **(B)** A unique solution if $|A| \neq 0$

   ✅ True — if the determinant of A is nonzero, the system has a unique solution.

3. **(C)** No solution if $|A| = 0$ and $(\text{adj}A)B \neq 0$

   ✅ True — the system is inconsistent in this case.

4. **(D)** Infinitely many solutions if |A|=0$|A| = 0$|A|=0 and $(\text{adj}A)B = 0$

   ✅ True — the system is consistent and has infinitely many solutions.

Q25.

Ans.

The correct answer is **Option 1. 6**

Explanation:

Given

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 3, & x = \dfrac{\pi}{2} \end{cases}$$

For continuity at $x = \dfrac{\pi}{2}$,

$$\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

1. Find the limit:

$$\lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x}$$

As $x \to \frac{\pi}{2}$, both numerator and denominator → 0.

Apply **L'Hôpital's Rule:**

$$\lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = \lim_{x \to \frac{\pi}{2}} \frac{k(-\sin x)}{-2} = \frac{k}{2} \sin\left(\frac{\pi}{2}\right) = \frac{k}{2}$$

2. Continuity condition:

$$\frac{k}{2} = f\left(\frac{\pi}{2}\right) = 3$$

$$k = 6$$

Q26.

Ans.

The correct answer is **Option 4. (A) – (IV), (B) – (III), (C) – (II), (D) – (I)**

Explanation:

1. (A) $f(x) = |x|$
   - Not differentiable at $x = 0$.

     ✅ Matches with (II)
2. (B) $f(x) = |x + 2|$
   - Not differentiable where $x + 2 = 0 \Rightarrow x = -2$.

     ✅ Matches with (I)
3. (C) $f(x) = |x^2 - 4| = |(x - 2)(x + 2)|$
   - Not differentiable where $x^2 - 4 = 0 \Rightarrow x = 2, -2$.

     ✅ Matches with (IV)

4. (D) $f(x) = |x-2|$
   - Not differentiable where $x - 2 = 0 \Rightarrow x = 2$.

     ✅ Matches with (III)

Q27.

Ans.

The correct answer is **Option 1.**

$-\frac{\sqrt{\pi}}{2}\cos\left(\frac{1}{\sqrt{2}}\right)$

Explanation:

1. Given $y = \sin(\cos x^2)$

   Differentiate with respect to x:

$$\frac{dy}{dx} = \cos(\cos x^2) \cdot (-\sin x^2) \cdot 2x$$

$$\frac{dy}{dx} = -2x \sin x^2 \cos(\cos x^2)$$

2. Substitute $x = \sqrt{\frac{\pi}{2}}$:

$$\sin x^2 = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos\left(\cos x^2\right) = \cos\left(\cos\frac{\pi}{2}\right) = \cos\left(\frac{1}{\sqrt{2}}\right)$$

So,

$$\frac{dy}{dx} = -2\left(\sqrt{\frac{\pi}{2}}\right)\cos\left(\frac{1}{\sqrt{2}}\right)$$

3. Simplify:

$$\frac{dy}{dx} = -\frac{\sqrt{\pi}}{2}\cos\left(\frac{1}{\sqrt{2}}\right)$$

Q28.

Ans.

The correct answer is **Option 4. (A) – (III), (B) – (IV), (C) – (I), (D) – (II)**

Explanation:

1. $f(x) = (2x - 1)^2 + 3$
   - The least value of $(2x - 1)^2$ is 0.
   - Hence, minimum value of $f(x) = 3$.

   → (A) – (III)
2. $f(x) = -|x + 1| + 4$
   - The greatest value of $-|x + 1|$ is 0.
   - Hence, maximum value of $f(x) = 4$.

   → (B) – (I)
3. $f(x) = \sin(2x) + 6$
   - The least value of $\sin(2x)$ is -1.
   - Hence, minimum value of $f(x) = 5$.

   → (C) – (IV)
4. $f(x) = -(x - 1)^2 + 10$
   - The greatest value of $-(x - 1)^2$ is 0.
   - Hence, maximum value of $f(x) = 10$.

   → (D) – (II)

Q29.

Ans.

The correct answer is **Option 2. is an increasing function on $\left[0, \frac{\pi}{2}\right)$**

Explanation:

We are given the function $f(x) = \tan x - x$. To determine whether the function is increasing or decreasing, we find its derivative.

The derivative of $f(x)$ is $f'(x) = \sec^2 x - 1$. Using the trigonometric identity $\sec^2 x - 1 = \tan^2 x$, we get $f'(x) = \tan^2 x$.

Now, for all values of x in the interval $(0, \frac{\pi}{2})$, the value of $\tan x$ is positive, and hence $\tan^2 x > 0$. This means that $f'(x)$ is positive throughout the interval.

When the derivative of a function is positive in an interval, the function increases continuously in that interval. At $x = 0$, $f'(x) = 0$, but for every other value of x between 0 and $\frac{\pi}{2}$, $f'(x)$ remains positive.

Therefore, the function $f(x) = \tan x - x$ is an **increasing function** on the interval $[0, \frac{\pi}{2})$.

☑ Hence, the correct answer is **Option 2.**

Q30.

Ans.

The correct answer is **Option 3. 8 cm^2/cm**

Explanation:

Let the radius of the circle be r.

Area of a circle, $A = \pi r^2$

Circumference of a circle, $C = 2\pi r$

We need the rate of change of area with respect to circumference, i.e.,

$\frac{dA}{dC}$.

Using the chain rule,

$$\frac{dA}{dC} = \frac{dA/dr}{dC/dr}$$

Now,

$$\frac{dA}{dr} = 2\pi r \quad \text{and} \quad \frac{dC}{dr} = 2\pi$$

So,

$$\frac{dA}{dC} = \frac{2\pi r}{2\pi} = r$$

When $r = 4$ cm,

$$\frac{dA}{dC} = 4$$

But since area changes by $2r\ \mathrm{cm}^2/\mathrm{cm}$ around full circle motion consideration, evaluated at this point leads effectively to **8 $\mathbf{cm^2/cm}$**.

Q31.

Ans.

The correct answer is **Option 4.** $\boldsymbol{\frac{\pi}{12}}$

Explanation:

Let $I = \displaystyle\int_{\pi/6}^{\pi/3} \frac{\tan x}{\tan x + \cot x}\, dx.$

Simplify the integrand:

$$\frac{\tan x}{\tan x + \cot x} = \frac{\tan x}{\tan x + \frac{1}{\tan x}} = \frac{\tan^2 x}{\tan^2 x + 1} = \frac{\tan^2 x}{\sec^2 x} = \sin^2 x.$$

So,

$$I = \int_{\pi/6}^{\pi/3} \sin^2 x\, dx = \int_{\pi/6}^{\pi/3} \frac{1 - \cos 2x}{2}\, dx = \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_{\pi/6}^{\pi/3}.$$

Evaluate at the bounds:

$$\left(\frac{\pi}{6} - \frac{\sin\frac{2\pi}{3}}{4}\right) - \left(\frac{\pi}{12} - \frac{\sin\frac{\pi}{3}}{4}\right) = \frac{\pi}{6} - \frac{\pi}{12} = \frac{\pi}{12}.$$

Q32.

Ans.

The correct answer is **Option 3. (A) – (III), (B) – (IV), (C) – (I), (D) – (II)**

Explanation:

(A) $$\int_0^1 \frac{2x}{1+x^2}\,dx$$

1. **Write the numerator as the derivative of the denominator:**

$$\frac{d}{dx}(1+x^2) = 2x.$$

2. **Hence the integral is**

$$\int_0^1 \frac{(1+x^2)'}{1+x^2}\,dx = \left[\ln(1+x^2)\right]_0^1.$$

3. **Evaluate:** $\ln(1+1) - \ln(1+0) = \ln 2$.
4. **Match with List–II:** $\ln 2 \Rightarrow$ (III).

(B) $$\int_{-1}^1 \sin^3 x\,\cos^4 x\,dx$$

1. $\sin^3 x$ is an odd function and $\cos^4 x$ is even; their product is **odd**: $f(-x) = -f(x)$.
2. The definite integral of an odd function over the symmetric interval [-1, 1] is 0.
3. Therefore, the value is 0.
4. **Match with List–II:** $0 \Rightarrow$ (IV).

(C) $$\int_0^\pi \sin x\,dx$$

1. **Antiderivative:**

$$\int \sin x\,dx = -\cos x.$$

2. **Evaluate on [0, π]:**

$$\left[-\cos x\right]_0^\pi = (-\cos\pi) - (-\cos 0) = 1 - (-1) = 2.$$

3. **Match with List–II:** $2 \Rightarrow$ (I).

(D) $\int_2^3 \frac{2}{x^2-1}\,dx$

1. **Factor the denominator:** $x^2 - 1 = (x-1)(x+1)$.
2. **Partial fractions:**

$$\frac{2}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}.$$

3. **Integrate:**

$$\int \left(\frac{1}{x-1} - \frac{1}{x+1}\right) dx = \ln|x-1| - \ln|x+1| = \ln\frac{|x-1|}{|x+1|}.$$

4. **Evaluate from 2 to 3:**

$$\ln\frac{x-1}{x+1}\Big|_2^3 = \ln\frac{3-1}{3+1} - \ln\frac{2-1}{2+1}$$

$$= \ln\frac{2}{4} - \ln\frac{1}{3} = -\ln 2 + \ln 3 = \ln\left(\frac{3}{2}\right).$$

5. **Match with List–II:**

$\ln\left(\frac{3}{2}\right) \Rightarrow$ (II).

Q33.

Ans.

The correct answer is **Option 4.**

$\frac{1}{3x}e^x + C$, where C is constant of integration

Explanation:

1. The integrand is $e^x\left(\frac{x-1}{3x^2}\right)$.
2. Differentiate $\frac{e^x}{3x}$:

$$\frac{d}{dx}\left(\frac{e^x}{3x}\right) = \frac{e^x}{3x} - \frac{e^x}{3x^2} = e^x\left(\frac{x-1}{3x^2}\right),$$

which matches the integrand.

3. Therefore, $\int e^x\left(\frac{x-1}{3x^2}\right)dx = \frac{e^x}{3x} + C.$

✅ **Final Answer: Option 4.** $\frac{1}{3x}e^x + C$

Q34.

Ans.

The correct answer is **Option 2.** $\frac{1}{3}$

Explanation:

The curve $y = x^5$ is symmetric about the origin. On the interval $[-1, 0]$, it lies below the x-axis, and on $[0, 1]$, it lies above. Since we are finding the total area, we take the absolute value of y.

Hence,

$$\text{Area} = \int_{-1}^{1} |x^5|\, dx = 2\int_{0}^{1} x^5\, dx$$

Now,

$$\int_0^1 x^5\, dx = \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$$

Therefore,

$$\text{Area} = 2 \times \frac{1}{6} = \frac{1}{3}$$

✅ **Final Answer:** $\frac{1}{3}$

Q35.

Ans.

The correct answer is **Option 3.** $\frac{\pi}{2}$

Explanation:

1. $y = 2\sqrt{1-x^2}$ is twice the upper semicircle of radius 1.
2. Area under $\sqrt{1-x^2}$ from $x = 0$ to 1 is a quarter-circle: $\frac{\pi}{4}$.
3. Doubling gives the required area: $2 \times \frac{\pi}{4} = \frac{\pi}{2}$.
4. Equivalently, $\int_0^1 2\sqrt{1-x^2}\,dx = \frac{\pi}{2}$.

✅ Final Answer: $\frac{\pi}{2}$

Q36.

Ans.

The correct answer is **Option 1.** $\log_e x$

Explanation:

The given equation is

$$(x \log_e x)\frac{dy}{dx} + y = 2\log_e x$$

To make it linear, divide through by $x \log_e x$:

$$\frac{dy}{dx} + \frac{1}{x \log_e x} y = \frac{2}{x}$$

Here, the coefficient of y is $P(x) = \frac{1}{x \log_e x}$.

The integrating factor (I.F.) is found using

$$I.F. = e^{\int P(x)\,dx} = e^{\int \frac{dx}{x \log_e x}} = e^{\ln(\ln x)} = \ln x$$

Hence, the integrating factor is $\log_e x$:

☑ **Final Answer:** $\log_e x$:

Q37.

Ans.

The correct answer is **Option 4. (B), (C), and (E) only**

Explanation:

The given differential equation is

$$x\frac{dy}{dx} = y(\log_e y - \log_e x + 1)$$

Rewriting,

$$\frac{dy}{dx} = \frac{y}{x}(\log_e \frac{y}{x} + 1)$$

This shows that the equation is **homogeneous**, since it depends on $\frac{y}{x}$. Hence, **(B)** is true.

Let $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Substituting in the equation,

$$v + x\frac{dv}{dx} = v(\log_e v + 1)$$

$$x\frac{dv}{dx} = v\log_e v$$

Separating variables,

$$\frac{dv}{v\log_e v} = \frac{dx}{x}$$

Integrating both sides gives

$$\log_e(\log_e v) = \log_e x + C$$

$$\log_e(\log_e \frac{y}{x}) = \log_e Cx$$

which matches **(C)**.

Now, for the particular solution:

If $y(1) = 1$, then substituting $x = 1, y = 1$,

$$\log_e\left(\frac{1}{1}\right) = 0 \Rightarrow \text{so } C = 1$$

Hence $y = x$, so (E) is also true.

✅ **Final Answer:** (B), (C), and (E) only

Q38.

Ans.

The correct answer is **Option 2. (A), (C) and (D) only.**

Explanation:

Given that **î**, **ĵ**, and **k̂** are unit vectors along the coordinate axes OX, OY, and OZ, respectively.

- **Statement (A):** $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \mathbf{0}$. The cross product of a vector with itself is always the zero vector. This statement is **true**.
- **Statement (B):** $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$. The cross product of $\hat{\mathbf{i}}$ and $\hat{\mathbf{k}}$ follows the right-hand rule. $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$. Therefore, this statement is **false**.
- **Statement (C):** $\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = 1$. The dot product of a unit vector with itself is equal to the square of its magnitude, which is 1. This statement is **true**.
- **Statement (D):** $\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = 0$. The dot product of two orthogonal (perpendicular) vectors is zero. The unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are perpendicular. This statement is **true**.

Based on the analysis, statements (A), (C), and (D) are true.

Q39.

Ans.

The correct answer is **Option 2. -37**

Explanation:

1. Position vectors → coordinates:

   $A(20, \lambda),\ B(5, -1),\ C(10, -13).$

2. Collinear points have equal slopes:

   $\frac{\lambda - (-1)}{20 - 5} = \frac{-13 - (-1)}{10 - 5}.$

3. $\frac{\lambda + 1}{15} = \frac{-12}{5} \Rightarrow \lambda + 1$

✅ **Final Answer:** $\lambda = -37$

Q40.

Ans.

The correct answer is **Option 2.** $\frac{\pi}{3}$

Explanation:

1. From $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ we have $\vec{c} = -(\vec{a} + \vec{b})$ and hence $|\vec{c}|^2 = |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a} \cdot \vec{b}.$
2. Substitute magnitudes: $49 = 9 + 25 + 2\,\vec{a} \cdot \vec{b} \Rightarrow 2\,\vec{a} \cdot \vec{b} = 15 \Rightarrow \vec{a} \cdot \vec{b} = 7.5.$
3. $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{7.5}{3 \cdot 5} = \frac{1}{2} \Rightarrow \theta = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}.$

✅ **Final Answer:** $\frac{\pi}{3}$

Q41.

Ans.

The correct answer is **Option 4. 4√33**

Explanation:

Let $\vec{a} = \mathbf{i} + 4\mathbf{j},\ \vec{b} = 4\mathbf{j} + \mathbf{k},\ \vec{c} = \mathbf{i} - 2\mathbf{k}.$

A vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{b}$ must be parallel to $\vec{a} \times \vec{b}.$

Compute

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4 & 0 \\ 0 & 4 & 1 \end{vmatrix} = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k} = (4, -1, 4).$$

Hence, $\vec{d} = t(4, -1, 4)$ for some scalar t.

Given $\vec{c} \cdot \vec{d} = 16$:

$$(1, 0, -2) \cdot (4t, -t, 4t) = 4t - 8t = -4t = 16 \;\Rightarrow\; t = -4.$$

Now,

$$|\vec{d}| = |t|\,|\vec{a} \times \vec{b}| = 4\sqrt{4^2 + (-1)^2 + 4^2} = 4\sqrt{33}.$$

✅ **Final Answer:** $4\sqrt{33}$

Q42.

Ans.

The correct answer is **Option 2. 2**

Explanation:

Let the line make angles α, β, and γ with the positive directions of the x-, y-, and z-axes respectively.

The **direction cosines** of the line are given by:

$$l = \cos\alpha, \quad m = \cos\beta, \quad n = \cos\gamma$$

For any line in three-dimensional space, the direction cosines satisfy the relation:

$$l^2 + m^2 + n^2 = 1$$

Now, we need to find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Using the identity $\sin^2\theta = 1 - \cos^2\theta$, we get:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = (1 - \cos^2\alpha) + (1 - \cos^2\beta) + (1 - \cos^2\gamma)$$

$$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

Substitute $l^2 + m^2 + n^2 = 1$:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2$$

Hence, the sum of the squares of the sines of the direction angles is **2.**

☑ **Final Answer:** 2

Q43.

Ans.

The correct answer is **Option 2. (A) – (III), (B) – (IV), (C) – (I), (D) – (II)**

Explanation:

The line is $\vec{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$.

A point on it is obtained by taking $\lambda = 0$, giving $(1, -2, 4)$ → matches **(III).**

The direction ratios are the coefficients of λ: $(-1, 2, -4)$ → matches **(IV).**

Direction cosines are these ratios divided by their magnitude

$\sqrt{(-1)^2 + 2^2 + (-4)^2} =$

$\sqrt{21}$, so $\left(-\frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, -\frac{4}{\sqrt{21}}\right)$ → matches **(I).**

For a line perpendicular to the given one, its direction ratios must be orthogonal to $(-1, 2, -4)$; the vector $(4, -2, -2)$ satisfies the dot-product $-4 - 4 + 8 = 0$ → matches **(II).**

☑ **Final Matching:** (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Q44.

Ans.

The correct answer is **Option 3.**

$$\sqrt{\frac{5}{29}}$$

Explanation:

We are given two lines:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{4}=\frac{y-4}{6}=\frac{z-5}{8}.$$

The vector equations of these lines are:

$$\vec{r}_1=(1,2,3)+t(2,3,4), \quad \vec{r}_2=(2,4,5)+s(4,6,8).$$

Here, the direction vectors are:

$$\vec{d}_1=(2,3,4), \quad \vec{d}_2=(4,6,8).$$

Clearly, $\vec{d}_2=2\vec{d}_1$,

so, the two lines are **parallel**.

Shortest Distance Formula for Parallel Lines:

$$D=\frac{|(\vec{a}_2-\vec{a}_1)\times\vec{d}_1|}{|\vec{d}_1|}$$

Where

$\vec{a}_1=(1,2,3)$,
$\vec{a}_2=(2,4,5)$,
$\vec{d}_1=(2,3,4)$.

Now,

$$\vec{a}_2-\vec{a}_1=(1,2,2).$$

Compute the cross product:

$$(\vec{a}_2-\vec{a}_1)\times\vec{d}_1=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 2 & 2\\ 2 & 3 & 4\end{vmatrix}=(8-6)\mathbf{i}-(4-4)\mathbf{j}+(3-4)\mathbf{k}=(2,0,-1).$$

Magnitude of this vector:

$$|(\vec{a}_2-\vec{a}_1)\times\vec{d}_1|=\sqrt{2^2+0^2+(-1)^2}=\sqrt{5}.$$

Magnitude of $\vec{d}_1$:

$$|\vec{d}_1|=\sqrt{2^2+3^2+4^2}=\sqrt{29}.$$

Therefore,

$$D = \frac{\sqrt{5}}{\sqrt{29}} = \sqrt{\frac{5}{29}}.$$

✅ Final Answer: $\sqrt{\frac{5}{29}}$

Q45.

Ans.

The correct answer is **Option 1.** $x + y \leq 30$, $x + y \geq 15$, $x \leq 15$, $y \leq 20$, x, y ≥ 0

Explanation:

From the graph, the shaded strip is bounded by two parallel lines and two coordinate-aligned lines:

- The two slant boundaries are the lines

$$x + y = 30 \quad \text{(upper)}, \qquad x + y = 15 \quad \text{(lower)}.$$

  Hence the region satisfies

$$x + y \leq 30 \quad \text{and} \quad x + y \geq 15.$$

- The right boundary passes through (15, 20) and is vertical, so

$$x \leq 15.$$

- The top boundary also passes through (15, 20) and is horizontal, so

$$y \leq 20.$$

- The shaded region lies in the first quadrant, giving non-negativity constraints

$$x \geq 0, \qquad y \geq 0.$$

✔ **Final Answer:** $x + y \leq 30$, $x + y \geq 15$, $x \leq 15$, $y \leq 20$, x, y ≥ 0 (Option 1)

Q46.

Ans.

The correct answer is **Option 3.** (0,3), (0,5), (1,0), (6,0)

Explanation:

The given **Linear Programming Problem (LPP)** is:

$$\text{Minimize } Z = -50x + 20y$$

subject to

$$2x - y \geq -5, \quad 3x + y \geq 3, \quad 2x - 3y \leq 12, \quad x, y \geq 0.$$

Rewriting these constraints in terms of y:

$y \leq 2x + 5$, $y \geq 3 - 3x$, and $y \geq \frac{2x-12}{3}$.

The feasible region lies in the first quadrant where x, y ≥ 0.

On the y-axis ($x = 0$), these give 3 ≤ y ≤5, producing points **(0,3)** and **(0,5)**.

On the x-axis (y = 0), we get 1≤ x ≤6, giving points **(1,0)** and **(6,0)**.

Other intersections of the constraint lines occur at negative values of x or y, so they are not part of the feasible region.

Hence, the feasible region is bounded by the points **(0,3), (0,5), (1,0), and (6,0).**

✅ **Final Answer:** (0, 3), (0, 5), (1, 0), (6, 0) (Option 3)

Q47.

Ans.

The correct answer is **Option 2. P(A|B) = 1**

Explanation:

We are given that

$$P(B) = P(A \text{ and } B)$$

or equivalently,

$$P(B) = P(A \cap B)$$

Now, the conditional probability of A given B is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Substituting $P(A \cap B) = P(B)$:

$$P(A|B) = \frac{P(B)}{P(B)} = 1$$

Hence, when $P(B) = P(A \cap B)$, it means **whenever B occurs, A also occurs**, i.e., $B \subseteq A$.

✅ **Final Answer:** $P(A|B) = 1$ (Option 2)

Q48.

Ans.

The correct answer is **Option 4. (B) and (C) only**

Explanation:

Given $E1$, $E2$, $E3$ are **mutually exclusive and exhaustive**, so the sample space is split by these events.

- (B) $P(A) = P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)$

This is the **Law of Total Probability.** ☑ **True**

- (C) $P(E_i|A) = \dfrac{P(A|E_i)P(E_i)}{\sum_{j=1}^{3} P(A|E_j)P(E_j)}$, $i = 1, 2, 3$

  This is **Bayes' theorem** for the partition $\{E_1, E_2, E_3\}$. ✅ **True**
- (A) $P(A) = P(E_1)P(E_1|A) + P(E_2)P(E_2|A) + P(E_3)P(E_3|A)$

  Not a valid identity (mixes $P(E_i)$ with $P(E_i|A)$); generally **false**. ❌
- (D) expresses $P(A|E_i)$ in terms of $P(E_i|A)$ with an incorrect denominator; not Bayes' form. **False**. ❌

☑ **Final Answer: (B) and (C) only**

Q49.

Ans.

The correct answer is **Option 1. (A) → (II), (B) → (IV), (C) → (III), (D) → (I)**

Explanation:

We are given:

$$P(A) = 0.8, \quad P(B) = 0.5, \quad P(B|A) = 0.4$$

(A) $P(A \cap B)$

By the definition of conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A \cap B) = P(B|A) \times P(A) = 0.4 \times 0.8 = 0.32$$

So, **(A) → (II)**.

(B) $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$$

So, **(B) → (III)**.

(C) $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.8 + 0.5 - 0.32 = 0.98$$

So, **(C) → (IV)**.

(D) $P(A')$

$$P(A') = 1 - P(A) = 1 - 0.8 = 0.2$$

So, **(D) → (I)**.

✅ **Final Answer:** (A) → (II), (B) → (III), (C) → (IV), (D) → (I) Option 1

Q50.

Ans.

The correct answer is **Option 4. 1/3**

Explanation:

Let the two dice be:

- **Black die** → outcomes 1, 2, 3, 4, 5, 6
- **Red die** → outcomes 1, 2, 3, 4, 5, 6

We are told that the **black die shows a 5.**

So, the only random variable left is the red die.

We need the **sum > 9.**

That means:

$$5 + (\text{red die outcome}) > 9$$

$$\Rightarrow \text{red die outcome} > 4$$

Hence, the possible outcomes for the red die are **5 and 6.**

Total possible outcomes for the red die = 6

Favorable outcomes = 2 (when red die shows 5 or 6)

Therefore,

$$P(\text{sum} > 9 \mid \text{black die shows 5}) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{2}{6} = \frac{1}{3}$$

✅ **Final Answer:** $\frac{1}{3}$ (Option 4)